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On the 2-factor index of a graph

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Abstract

The 2-factor index of a graph G, denoted by $f(G)$, is the smallest integer m such that the m-iterated line graph $L^m(G)$ of G contains a 2-factor. In this paper, we provide a formula for $f(G)$, and point out that there is a polynomial time algorithm to determine $f(G)$.

MSC: 05C45; 05C38

Keywords: 2-factor; 2-factor index; Branch-bond; Iterated line graph

1. Introduction

We use [1] for terminology and notation not defined here and consider only loopless finite graphs. Let G be a graph. For each integer $0 \leqslant i \leqslant \Delta(G)$, let $V_i(G)$ denote the set of vertices of G having degree i. A *branch* in G is a nontrivial path with end vertices that do not lie in $V_2(G)$ and with internal vertices of degree 2 (if existing). If a branch has length 1, then it has no internal vertices of degree 2. Let $B(G)$ denote the set of branches of G and $B_1(G)$ the subset of $B(G)$ in which every branch has exactly one end vertex in $V_1(G)$. A 2-*factor* in G is a spanning subgraph of G such that its vertices have degree 2. For any subgraph H of G, denote by $B_H(G)$ the set of branches of G whose edges are all in H. For any two subgraphs H_1 and H_2 of G, the *distance* $d_G(H_1, H_2)$ between H_1 and H_2 is defined to be $\min\{d_G(v_1, v_2) | v_1 \in V(H_1) \text{ and } v_2 \in V(H_2)\}$.

The *line graph* of $G = (V(G), E(G))$ has $E(G)$ as its vertex set, and two vertices are adjacent in $L(G)$ if and only if the corresponding edges are incident with a common vertex in G. The *m-iterated line graph* $L^m(G)$ is defined recursively by $L^0(G) = G$ and $L^m(G) = L(L^{m-1}(G))$. The *hamiltonian index* of a graph G, denoted by $h(G)$, is the smallest integer m such that $L^m(G)$ is hamiltonian, and the 2-*factor index* of a graph, denoted by $f(G)$, is the minimum integer m such that the m-iterated line graph contains a 2-factor.

Chartrand [2] showed that if a connected graph G is not a path, then the hamiltonian index of G exists. Lai [7] obtained a bound of $h(G)$. Because a hamiltonian cycle of G is a connected 2-factor of G, $f(G)$ exists for any connected graph

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G that is not a path. A *circuit* of a graph G is a connected nontrivial subgraph of G whose vertices have only even degrees. Harary and Nash-Williams characterized these graphs whose line graphs are hamiltonian.

Theorem 1 (*Harary and Nash-Williams [6]*). *Let G be a graph with at least three edges. Then $h(G)\leqslant 1$ if and only if $G \equiv K_{1,n}$, or G has a circuit H such that $d_G(e, H) = 0$ for any edge $e \in E(G)$.*

Gould and Hynds gave a characterization of graphs whose line graphs contain a 2-factor. A star is the bipartite graph $K_{1,m}$ $(m\geqslant 3)$, and the vertex of degree m in $K_{1,m}$ is called the center of the star. *A k-system that dominates* is a collection Γ of k edge-disjoint circuits and stars in G such that each edge e of G is either in one of the circuits or stars of Γ, e is adjacent to an edge of a circuit of Γ, or e is adjacent to the center of a star of Γ.

Theorem 2 (*Gould and Hynds [5]*). *Let G be a connected simple graph containing at least three edges. Then $f(G)\leqslant 1$ if and only if G has a k-system that dominates for some k.*

Xiong and Liu characterized the graphs for which the n-iterated line graph is hamiltonian, for any integer $n\geqslant 2$.

Theorem 3 (*Xiong and Liu [11]*). *Let G be a connected graph that is not a 2-cycle and let $n\geqslant 2$ be an integer. Then $h(G)\leqslant n$ if and only if $EU_n(G) \neq \emptyset$ where $EU_n(G)$ denotes the set of those subgraphs H of G which satisfy the following conditions*:

(i) *any vertex of H has even degree in H*;
(ii) $V_0(H) \subseteq \bigcup_{i=3}^{\Delta(G)} V_i(G) \subseteq V(H)$;
(iii) *$d_G(H_1, H - H_1)\leqslant n - 1$ for any subgraph H_1 of H*;
(iv) *$|E(b)|\leqslant n + 1$ for any branch b in $B(G)\backslash B_H(G)$*;
(v) *$|E(b)|\leqslant n$ for any branch in $B_1(G)$.*

Very recently, Ferrara and Gould proved the following result.

Theorem 4 (*Ferrara and Gould [3]*). *Let G be a connected graph with at least three edges. Then for any $n\geqslant 2$, $L^n(G)$ has a 2-factor if and only if $F_n(G) \neq \emptyset$ where $F_n(G)$ denotes the set of those subgraphs H of G that satisfy the following five conditions*:

(i′) *any vertex of H has even degree in H*;
(ii′) $V_0(H) \subseteq \bigcup_{i=3}^{\Delta(G)} V_i(G) \subseteq V(H)$;
(iii′) *$d_G(H_1, H - H_1)\leqslant n + 1$ for any subgraph H_1 of H*;
(iv′) *$|E(b)|\leqslant n + 1$ for any branch b in $B(G)\backslash B_H(G)$*;
(v′) *$|E(b)|\leqslant n$ for any branch in $B_1(G)$.*

We observe that Theorem 4 does not hold for $n = 0$ or 1. To see this, let $C = u_1u_2 \cdots u_{3s} \cdots u_t$ be a cycle of length t, $t\geqslant 3s\geqslant 6$, and x be a vertex outside C. Now let G_1 be the graph with $V(G_1) = V(C) \cup \{x\}$ and $E(G_1) = E(C) \cup \{xu_s, xu_{2s}, xu_{3s}\}$. It is easy to see that $C \cup \{x\} \in F_0(G_1)$ but G_1 has no 2-factor. To see that Theorem 4 does not hold for $n = 1$, let G_2 be the unique tree on $2n$ vertices with degree sequence $(x_1, x_2, \ldots, x_{n+1}, x_{n+2}, \ldots, x_{2n})$ where $x_i = 1$ for $i = 1, 2, \ldots, n + 1$ and $x_i = 3$ for $i = n + 2, \ldots, 2n$. It is easy to see that G_2 has no k-system that dominates for any k and the empty subgraph with the set of vertices of degree three in G_2 is in $F_1(G_2)$. This implies that $f(G_2)\geqslant 2$ and $F_1(G_2) \neq \emptyset$.

Note that the conditions on the subgraphs in $EU_k(G)$ of Theorem 3 and the subgraphs in $F_k(G)$ of Theorem 4 are the same except conditions (iii) and (iii′). The following natural result follows from the fact that all subgraphs F in $F_{f(G)+2}(G)$ are in $EU_{h(G)}(G)$ and all subgraphs H in $EU_{h(G)}(G)$ are in $F_{f(G)}(G)$.

Theorem 5. *Let G be a connected graph that is not a path. Then*

$$h(G) - 2\leqslant f(G)\leqslant h(G).$$

Proof. Since any hamiltonian cycle in a graph G is also a 2-factor in G, $f(G) \leqslant h(G)$. If $h(G)=0, 1, 2$, then obviously $f(G) \geqslant 0 \geqslant h(G)-2$. If $h(G) \geqslant 3$, then $h(G) \leqslant f(G)+2$ by Theorem 3 and since subgraphs F in $F_{f(G)+2}(G)$ are all in $EU_{h(G)}(G)$. □

Observing that conditions (ii′) and (iv′) in the definition of $F_k(G)$ imply condition (iii′) in the definition of $F_k(G)$, we obtain an equivalent version of Theorem 4 as follows.

Theorem 6. *Let G be a connected graph with at least three edges. Then for any* $n \geqslant 2$, $L^n(G)$ *has a 2-factor if and only if* $F_n(G) \neq \emptyset$ *where* $F_n(G)$ *denotes the set of those subgraphs H of G that satisfy the following four conditions*:

(I) *any vertex of H has even degree in H*;
(II) $V_0(H) \subseteq \bigcup_{i=3}^{\Delta(G)} V_i(G) \subseteq V(H)$;
(III) $|E(b)| \leqslant n+1$ *for any branch b in* $B(G) \backslash B_H(G)$;
(IV) $|E(b)| \leqslant n$ *for any branch in* $B_1(G)$.

Proof. Since the "only if" part is trivial, we only need to prove the "if" part of the theorem. It suffices to prove that the subgraph H satisfying the conditions (I)–(IV) also satisfies the conditions (i′)–(v′). We will prove this by contradiction. If possible, suppose that H is a subgraph satisfying (I)–(IV) but $d_G(H_1, H-H_1) \geqslant n+2$ for some subgraph H_1 of H, we claim that the shortest path P between H_1 and $H-H_1$ is a branch in $B(G) \backslash B_H(G)$, by (ii′). Hence by (iv′), $|E(P)| \leqslant n+1$, a contradiction. This implies that (iii′) holds for H. Thus we have completed the proof of Theorem 6. □

The main purpose of this paper is to establish a formula for $f(G)$.

2. Branch-bonds

In this section, we will introduce some notation and terminology about branch-bonds [10], which will be used in next section. For any subset S of $B(G)$, $G-S$ denotes the subgraph obtained from $G[E(G) \backslash E(S)]$ by deleting all internal vertices of degree 2 in any branch of S. A subset S of $B(G)$ is called a *branch cut* if $G-S$ has more components than G. A *branch-bond* is a minimal branch cut. If G is connected, then a branch cut S of G is a minimal subset of $B(G)$ such that $G-S$ is disconnected. It is easily shown that, for a connected graph G, a subset S of $B(G)$ is a branch-bond if and only if $G-S$ has exactly two components. We denote by $BB(G)$ the set of branch-bonds of G. Given $S, T \subseteq V(G)$, let $[S, T] = \{uv \in E(G): u \in S \text{ and } v \in T\}$. An *edge cut* is an edge set of the form $[S, \overline{S}]$, where S is a nonempty proper subset of $V(G)$ and $\overline{S} = V(G) \backslash S$. A minimal edge cut of G is called a *bond*. Note that a branch-bond of G is also a bond of G when every branch in the branch-bond is an edge.

McKee gave the following characterization of eulerian graphs.

Theorem 7 (*McKee [8]*). *A connected graph is eulerian if and only if each bond contains an even number of edges*.

The following characterization of eulerian graphs involves branch-bonds.

Theorem 8 (*Xiong et al. [10]*). *A connected graph is eulerian if and only if each branch-bond contains an even number of branches*.

3. A formula for $f(G)$

In this section we will establish a formula for $f(G)$, which relates to the concept of odd branch-bonds. A branch-bond is called *odd* if it consists of an odd number of branches. The *length of a branch-bond* $S \in BB(G)$, denoted by $l(S)$, is the length of a shortest branch in it. Let $BB_2(G) = \{S \in BB(G) \backslash BB_1(G) : S \text{ is odd}\}$ where $BB_1(G) = B_1(G)$, and, for $i=1, 2$,

$$h_i(G) = \begin{cases} \max\{l(S): S \in BB_i(G)\} & \text{if } BB_i(G) \neq \emptyset, \\ 0 & \text{if } BB_i(G) = \emptyset. \end{cases}$$

We will give a formula for $f(G)$ involving $h_i(G)$. First we present a lower bound for it.

Theorem 9. *Let G be a connected graph that is not a path. Then*

$$f(G) \geqslant \max\{h_1(G), h_2(G) - 1\}.$$

Proof. If $f(G) = 0$, then the definition of a 2-factor implies that $h_1(G) = 0$, i.e., $BB_1(G) = \emptyset$. Obviously $l(S) \leqslant 1$ for any branch-bond S with $|S| = 1$.

We further claim that $h_2(G) \leqslant 1$, which implies that Theorem 9 holds. We will prove this by contradiction. If possible, suppose that $h_2(G) \geqslant 2$, then there exists an odd branch-bond S_0 with $|S_0| \geqslant 3$ and $l(S_0) \geqslant 2$. Let F be a 2-factor of G. By the definition of a branch-bond, each cycle of F contains an even number of branches of S_0. Hence there exists a branch b_0 in the odd branch-bond S_0 such that b_0 is not in any cycle of F. However $|E(b_0)| \geqslant l(S_0) \geqslant 2$ implies that there exists a vertex u, of degree 2, such that u is in b_0 but u is not in any cycle of F, a contradiction. This settles the case that $f(G) = 0$.

If $f(G) = 1$, then, by Theorem 2, there exists a k-system Γ that dominates. Obviously $h_1(G) \leqslant 1$ and $l(S) \leqslant 2$ for any branch-bond $S \notin BB_1(G)$ with $|S| = 1$. We furthermore claim that $h_2(G) \leqslant 2$, which implies that Theorem 9 holds. We will prove this by contradiction. If possible, suppose that $h_2(G) \geqslant 3$, then there exists an odd branch-bond S_0 with $|S_0| \geqslant 3$ and $l(S_0) \geqslant 3$. By the definition of a branch-bond, any circuit of Γ contains an even number of branches of S_0. Hence there exists a branch b_0 in the odd branch-bond S_0 such that b_0 is not in any circuit of Γ. However, $|E(b_0)| \geqslant l(S_0) \geqslant 3$ implies that there is an edge uv, with $d(u) = d(v) = 2$, such that u and v in b_0 but uv is neither in one of stars of Γ nor has a vertex in one of the circuits of Γ, a contradiction. This settles the case that $f(G) = 1$.

It remains to consider the case that $f(G) \geqslant 2$. We can take an $S_i \in BB_i(G)$ such that $h_i(G) = l(S_i)$ for every $i \in \{1, 2\}$. For any subgraph $H \in F_{f(G)}(G)$, it is obvious that $E(b) \cap E(H) = \emptyset$ for any $b \in S_1$. The definitions of S_2 and H imply that there exists at least one branch $b \in S_2$ such that $E(b) \cap E(H) = \emptyset$. Hence by Theorem 6, we obtain $f(G) \geqslant h_1(G)$ by (IV) and $f(G) \geqslant h_2(G) - 1$ by (III). So $f(G) \geqslant \max\{h_1(G), h_2(G) - 1\}$, which completes the proof of Theorem 9. □

Now we state a formula for $f(G)$. Let

$$\beta(G) = \max\{h_1(G), h_2(G) - 1\}.$$

Theorem 10. *Let G be a connected graph that is not a path such that* $\beta(G) \geqslant 2$. *Then* $f(G) = \beta(G)$.

Proof. It suffices to prove that $f(G) \leqslant \beta(G)$ by Theorem 9. This theorem also implies $f(G) \geqslant \beta(G) \geqslant 2$. Hence by Theorem 6 we can assume that $H \in F_{f(G)}(G)$ is a subgraph with a maximal number of branches $b \in B_H(G)$ such that $|E(b)| \geqslant \beta(G) + 2$. Then we obtain the following fact.

Claim 1. *If S is a branch-bond in BB(G) which contains at least three branches, then* $|E(b)| \leqslant \beta(G) + 1$ *for any branch* $b \in S \backslash B_H(G)$.

Proof of Claim 1. We will prove this by contradiction. If possible, suppose that there is a branch-bond S with $|S| \geqslant 3$ and $b_0 \in S \backslash B_H(G)$ such that $|E(b_0)| \geqslant \beta(G) + 2$. Obviously b_0 is not a cycle. Let u and v be two end vertices of b_0. Let $S(u, b_0)$ be a branch-bond containing b_0 such that any branch of $S(u, b_0)$ has u as an end vertex. Obviously $|S(u, b_0)| \geqslant 2$.

By the following algorithm, we will first find a cycle of G that contains b_0 and then obtain a contradiction.

Algorithm b_0**.**

1. If $|S(u, b_0)|$ is even, then select a branch $b_1 \in S(u, b_0) \backslash (B_H(G) \cup \{b_0\})$ by Theorem 8. Otherwise, since $|E(b_0)| \geqslant \beta(G) + 2$, select a branch $b_1 \in S(u, b_0)$ with

   $$|E(b_1)| = l(S(u, b_0)) \leqslant h_2(G) \leqslant \beta(G) + 1$$

   (obviously $b_1 \neq b_0$) and let $u_1 (\neq u)$ be the other end vertex of b_1. If $u_1 = v$, then set $t := 1$ and stop. Otherwise $i := 1$.

2. Select a branch-bond $S(u, u_i, b_0)$ in G which contains b_0 but not $b_1, b_2, \ldots, b_i$ such that any branch in $S(u, u_i, b_0)$ has exactly one end vertex in $\{u, u_1, u_2, \ldots, u_i\}$. If $|S(u, u_i, b_0)|$ is even, then, by Theorem 8, select a branch

$$b_{i+1} \in S(u, u_i, b_0) \backslash (B_H(G) \cup \{b_0\}).$$

Otherwise, since $|E(b_0)| \geqslant \beta(G) + 2$, select a branch $b_{i+1} \in S(u, u_i, b_0)$ such that

$$|E(b_{i+1})| = l(S(u, u_i, b_0)) \leqslant h_2(G) \leqslant \beta(G) + 1$$

(obviously $b_{i+1} \neq b_0$), and let u_{i+1} be the end-vertex of b_{i+1} that is not in $\{u, u_1, u_2, \ldots, u_i\}$.
3. If $u_{i+1} = v$, then set $t := i + 1$ and stop. Otherwise replace i by $i + 1$ and return to step 2.

Note that $|B(G)|$ is finite, and $d_G(v) \geqslant 2$ implies that the Algorithm b_0 will stop after a finite number of steps. It is easy to see that $G[\bigcup_{i=0}^{t} E(b_i)]$ is connected. Furthermore, since $u_t = v$ and $|S(u, u_i, b_0)| \geqslant 2$, $G[\bigcup_{i=0}^{t} E(b_i)]$ has a cycle of G which contains b_0. Hence we have established the following fact.

Claim 1.1. *b_0 is in a cycle C_0 of $G[\bigcup_{i=0}^{t} E(b_i)]$.*

Let H' be the subgraph of G obtained from

$$G[(E(H) \cup (E(C_0) \backslash E(H))) \backslash (E(H) \cap E(C_0))]$$

by adding the remaining vertices of $\bigcup_{i=3}^{\Delta(G)} V_i(G)$ as isolated vertices in H'.

Obviously $|E(b)| \leqslant h_2(G) \leqslant \beta(G) + 1$ for $b \in B_H(G) \cap \{b_1, b_2, \ldots, b_t\}$. Hence, by Claim 1.1, H' satisfies (III). Obviously H' satisfies (I), (II) and (IV), and this implies that H' is also in $F_{f(G)}(G)$. But H' contains b_0 which contradicts the maximality of H. Thus Claim 1 is true.

Now we will complete the proof of Theorem 10. By the definition of $\beta(G)$, $|E(b)| \leqslant h_1(G) \leqslant \beta(G)$ for any branch $b \in B_1(G)$ and $|E(b)| \leqslant h_2(G) \leqslant \beta(G) + 1$ for the branch b in a branch-bond $S \notin BB_1(G)$ such that $|S| = 1$. The last fact and Claim 1 implies that $|E(b)| \leqslant \beta(G) + 1$ for any branch $b \in B(G) \backslash B_H(G)$. It follows that $H \in F_{\beta(G)}(G)$, and so $f(G) \leqslant \beta(G)$. Therefore we have completed the proof of Theorem 10. □

Remark 11. Note that Theorem 10 does not hold for a graph G with $\beta(G) \leqslant 1$. To see this, let G_0 be the graph depicted in Fig. 1. It is easy to see that $h_1(G_0) = 0$ and $h_2(G_0) = 2$, hence $\beta(G_0) = 1$. By Theorem 12, $f(G_0) \leqslant 2$. We claim that $f(G_0) = 2$. To see this, it suffices to show that G_0 has no k-system that dominates for any k. We will prove this by contradiction. If possible, suppose that G_0 has a k-system that dominates. It is easy to see that the unique cycle with all branches of length 4 of G_0 should be contained in Γ. Hence none of the vertices u_i is a center of some star since u_i

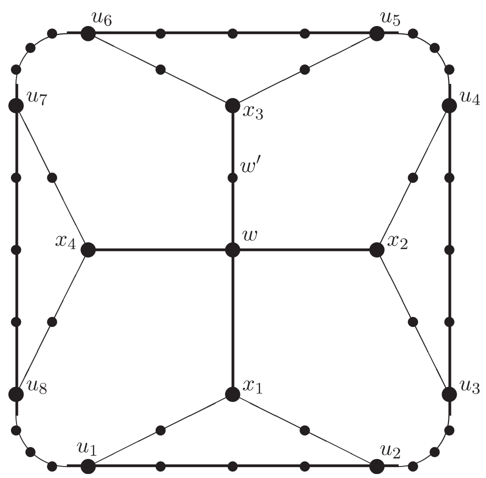

Fig. 1. A graph G_0 with $f(G_0) = 2$ and $\beta(G_0) = 1$.

has degree exactly three. So x_i should be a center of some star in S and hence w should not be a center of some star for wx_1, wx_2, wx_4 should be in the stars with centers x_1, x_2, x_4, respectively. The edge ww', however, is not contained in any star in Γ. This shows that Γ is not any k-system that dominates. This implies that $f(G_0) = 2$ by Theorem 2. If we replace some of these branches of length 4 by branches of length $l \geqslant 4$, then we can get infinite graph G with $f(G) = 2$ and $\beta(G) = 1$.

The following result deals with these graphs G with small $\beta(G)$.

Theorem 12. *Let G be a graph that is not a path such that $\beta(G) \leqslant 1$. Then $f(G) \leqslant 2$.*

Proof. By Theorem 6, we only need to prove that $F_2(G) \neq \emptyset$. Let H be a subgraph of G with (I) and (II) and with a maximal number of branches $b \in B_H(G)$ such that $|E(b)| \geqslant 3$. Then, in a way similar to the one in Claim 1 in the proof of Theorem 10, we obtain the following claim.

Claim 12.1. *If S is a branch-bond in $BB(G)$ which contains at least three branches, then $|E(b)| \leqslant 2$ for any branch $b \in S \backslash B_H(G)$.*

For any branch b of G, if $G[E(b)]$ is not a cycle of G then there exists a branch-bond $S \in BB(G)$ with $b \in S$. By $\beta(G) \leqslant 1$, we have $|E(b)| \leqslant 1$ for $b \in B_1(G)$, which implies that H satisfies (IV). By Claim 12.1, H satisfies (III). Hence $H \in F_2(G)$, and so $f(G) \leqslant 2$. Thus we have completed the proof of Theorem 12. □

A result in [4] implies the following.

Theorem 13 (*Fujisawa et al. [4]*). *Let G be a graph that is not a path such that $\beta(G) = 0$. Then $f(G) \leqslant 1$.* It would be interesting to consider the following question.

Question 14. Which graph G satisfies $f(G) = \beta(G) \leqslant 1$.

Remark 15. Note that the graph G_0 shown in Remark 11 is 2-connected and $F_1(G_0) \neq \emptyset$ since $C_0 \cup \{x_1, x_2, x_3, x_4, w\}$ is a subgraph in $F_1(G_0)$ where C_0 is the unique cycle with all branches of length 4. However $f(G_0) = 2$, this shows that Theorem 6 does not hold for $n = 1$ even for a 2-connected graph.

Remark 16. Woeginger [9] pointed out that there is a polynomial algorithm to determine $h_i(G)$ of G. Hence there is a polynomial algorithm to determine $\beta(G)$. So if $\beta(G) \geqslant 2$ then there is a polynomial algorithm to determine $f(G)$ by Theorem 10.

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