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REVIEW ARTICLE

Eclipse intervals for satellites in circular orbit under the effects of Earth's oblateness and solar radiation pressure

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Abstract In this work, the circumstances of eclipse for a circular satellites' orbit are studied. The time of passage of the ingress and egress points is calculated. Finally, the eclipse intervals of satellites' orbit are calculated. An application was done taken into account the effects of solar radiation pressure and Earth's oblateness on the orbital elements of circular orbit satellite.

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1. Introduction

The problem of computing Earth satellite entry and exit positions through the Earth's umbra and penumbra is a problem dating from the earliest days of the space age, but it is still of the most importance to many space projects for thermal and power considerations. It is also important for optical tracking of a satellite. To a lesser extent, the satellite external torque history and the sensor systems are influenced by the time the satellite spends in the Earth's shadow. The specification of the shadow intervals through the satellites' orbit is a very important study, owing to the following reasons:

1. The satellite heat balance.
2. Passive optical tracking.
3. Solar power source.
4. Orbital stability of satellites suffering from solar radiation pressure.

The umbra is the conical total shadow projected from the Earth on the side opposite to the Sun. In this region, the intensity of the solar radiation is zero. The penumbra is the partial shadow between the umbra and the full-light region. In the penumbra, the light of the Sun is only partially cut off by the Earth, and the intensity is between 0 and 1. All textbooks discussing the problem (e.g. Geyling and Westerman, 1971 and Escobal, 1985) even the recent work by Mullins (1991), suggest the use of a quartic equation analytic solution. Because the quartic is a result of squaring the equation of interest, one must check all four solutions and discard the spurious ones.

From the literature survey, it can be found that the shadow state prediction model falls into two categories namely; spherical Earth conical shadow model and cylindrical shadow model, respectively. Vallado (2007) developed both shadow models numerically using Newton–Raphson technique involving the solution of quartic polynomial of the true anomaly.

2. The Earth's shadow

The Earth and Sun are nearly spherical bodies and the Sun is about 100 times the Earth's size. Therefore, the shadow of the Earth is a cone with the centers of the Sun and Earth. As the vertex is going away from the Sun, the shadow is divided into two main parts as follows Fig. 1:

1. The umbra of the shadow which extends about 138,900 km.
2. The penumbra of the shadow in which the sunlight is partially excluded.

These two parts are shown clearly when one studies the eclipse of the Moon. To study the satellite's eclipse it is more convenient to assume that:

1. The Earth's shadow is circular cylinder with a diameter equal to the mean diameter of the Earth (Fig. 2).
2. The shadow has no penumbra.
3. The atmospheric refractions are neglected.
4. The synodic and sidereal periods are identical, owing to the small period of satellite.

2.1. Conditions of eclipse

To study the conditions of artificial satellite eclipse, Fig. 2 illustrates the projection of the Earth's shadow on the orbital plane of an satellite which is an ellipse with semi-minor axis

$$b = r_{\oplus} \quad (1)$$

and semi-major axis

$$a = r_{\oplus} \operatorname{cosec} i_{\odot}, \quad (2)$$

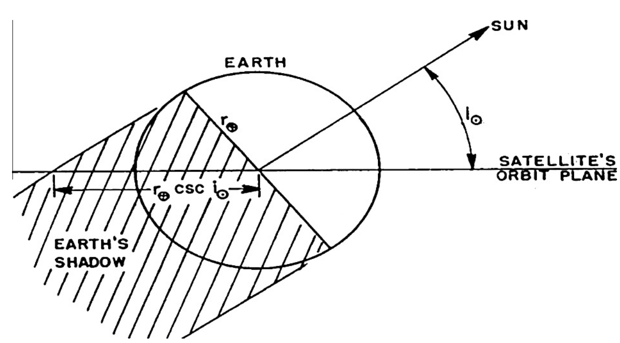

Figure 2 The Earth's shadow with no penumbra (case of artificial satellites).

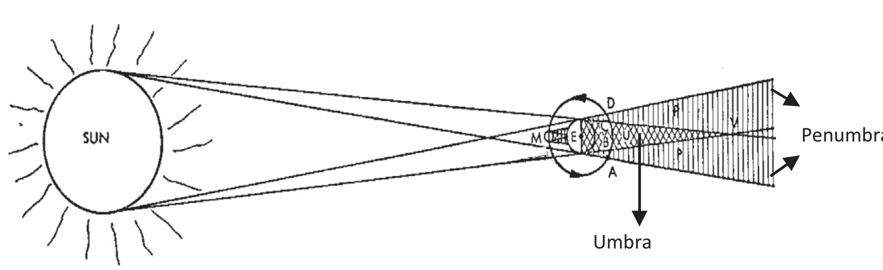

Figure 1 The shadow of the Earth and its two main parts.

where $i_{\odot}$ defines the inclination of the Sun onto the orbital plane of the satellite. It is known that the equation of an ellipse is given by

$$r^2 = \frac{b^2}{1 - e^2(\cos\theta)^2}, \tag{3}$$

where

$$e = \frac{\sqrt{a^2 - b^2}}{a}. \tag{4}$$

and θ Geocentric angle measured in the orbit plane between the satellite and the conjunction point.

Eq. (4) illustrates the size of the ellipse. Substituting Eqs. (1) and (2) into Eq. (4), with $e = e_{SH}$, and after some little reduction, yields

$$e_{SH} = \cos i_{\odot}. \tag{5}$$

where (SH = shadow).

Then Eq. (3) can be written as

$$r^2 = \frac{r_{\oplus}^2}{1 - (\cos i_{\odot})^2(\cos\theta)^2}. \tag{6}$$

The satellite enters or leaves the shadow when $r = r_{SH}$, then from Eq. (6) the term $\cos\theta$ can be obtained, so

$$\cos\theta = \pm\sqrt{\frac{1 - \left(\frac{r_{\oplus}}{r_{SH}}\right)^2}{\cos i_{\odot}}}. \tag{7}$$

For the value of $90^{\circ} < \theta < 270^{\circ}$, yields $\cos\theta$ to be negative, then the limits of the eclipse are obtained from the above Eq. (7), then when $\theta = 180^{\circ}$, this yields

$$r = r_{\oplus}\mathrm{cosec}\, i_{\odot}. \tag{8}$$

From Eq. (8), if $r < r_{\oplus}\mathrm{cosec}\, i_{\odot}$, then an eclipse will occur, if $r > r_{\oplus}\mathrm{cosec}\, i_{\odot}$, an eclipse will not occur.

Fig. 3 illustrates the relation between r and $i_{\odot}$, and gives the limits of eclipse (height in the (Fig. 3) instead of r).

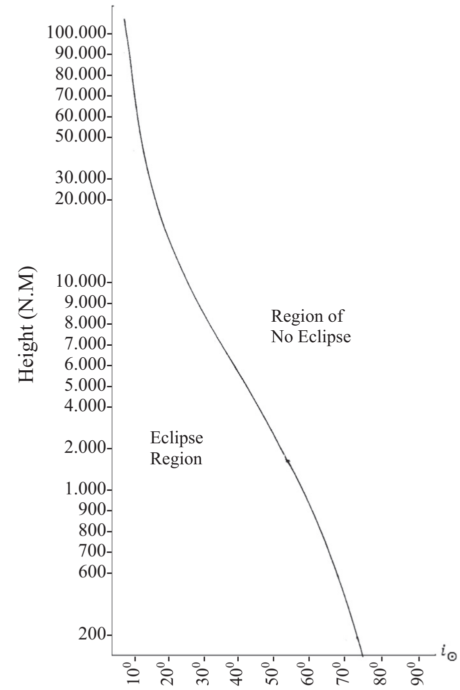

Figure 3 Geocentric angle between Sun and Satellite.

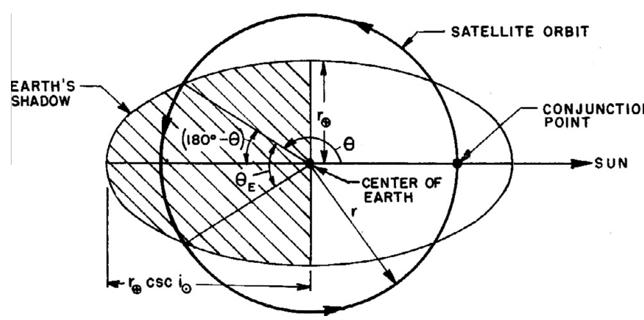

Figure 4 The shadow area, the angle of ingress, and the angle of egress.

3. Eclipse for circular orbit satellite

The duration angle of eclipse for a circular orbit satellite θ_E can be obtained from

$$\theta_E = 2(180^{\circ} - \theta), \tag{9}$$

where θ is obtained from Eq. (7).

From Fig. 4, within the eclipse duration, a satellite enters the shadow at $(180^{\circ} - \theta)$, and leaves the shadow at $(180^{\circ} + \theta)$. Now the shadow interval can be obtained from

$$t_{SH} = \frac{\theta_E}{360^{\circ}} P, \tag{10}$$

where P is the period of the satellite, Stoddard (1961).

Fig. 5 shows the duration of eclipse for various altitudes and various inclination of Sun ($i_{\odot}$) to the plane of satellite orbit. Notice that the range of duration of eclipse is between 32 min and 47 min.

3.1. Inclination of the Sun to the orbital plane

It is very important to obtain the inclination of the Sun ($i_{\odot}$) to the orbital plane. The most convenient in this application is the right ascension and declination system of coordinates. The inclination i of the orbit plane to the plane of equator where $0^{\circ} < i < 90^{\circ}$, and the right ascension of the ascending node of the orbital plane α_{Ω} are only the required orbital elements. From the spherical triangle ABS using the cosines law (Fig. 6), then

$$\sin i_{\odot} = \cos i \sin\delta_{\odot} + \sin i \cos\delta_{\odot}\sin(\alpha_{\Omega} - \alpha_{\odot}) \tag{11}$$

If $\sin i_{\odot}$ is negative, then $i_{\odot}$ must be the difference between 180° and $i_{\odot}$ obtained by Eq. (11).

4. Times of eclipse and geocentric elongation at any time for a circular orbit

Since there is no certain point on the circular orbit to specify the time of passage (in elliptical orbits this point is the perigee), so, it is more suitable in this case that the time of passage of the ascending node is considered as a reference. In Fig. 7 A is the

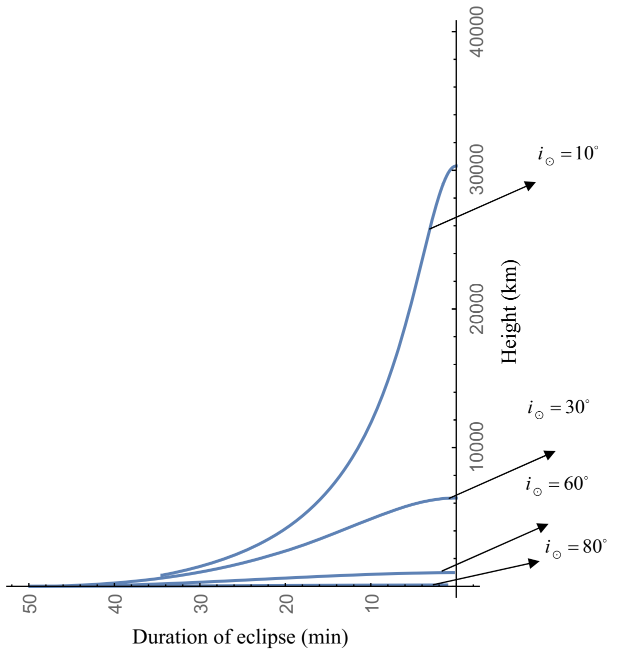

Figure 5 Duration of eclipse for various altitudes and Sun inclination ($i_{\odot}$).

north pole of the equator, B is the north pole of the orbital plane, and S is the Sun. It is needed to find the distance θ_{Ω} between the conjunction point C and the node Ω measured in the satellite's motion. From the spherical triangle ABS using sines law

$$\cos\theta_{\Omega} = \frac{\cos\delta_{\odot}\cos(\alpha_{\Omega} - \alpha_{\odot})}{\cos i_{\odot}}. \tag{12}$$

and from the cosines law

$$\sin\theta_{\Omega} = \frac{\cos i \sin i_{\odot} - \sin\delta_{\odot}}{\sin i \cos i_{\odot}}. \tag{13}$$

From Eqs. (12) and (13) θ_{Ω} can be obtained.

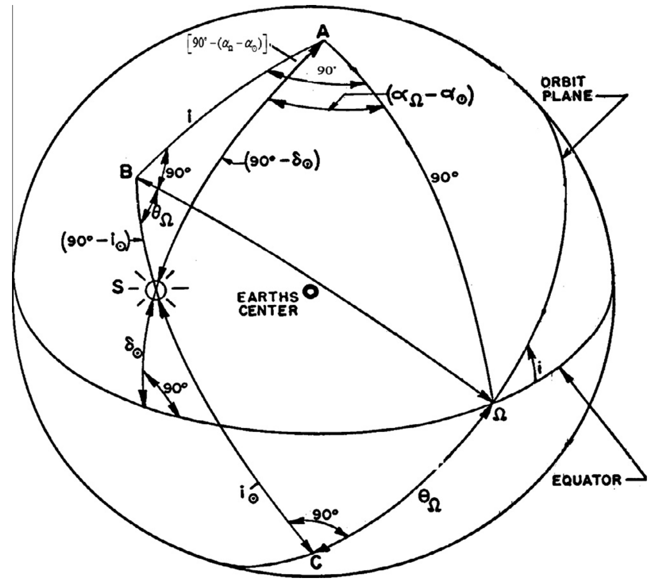

Figure 7 The distance θ_{Ω} between the conjunction point C and the node Ω.

In the circular orbits when the satellite passes through the ascending node the time of passage is T_{Ω}, then the time of the conjunction passage T_C is given by

$$T_C = T_{\Omega} - \left(\frac{\theta_{\Omega}}{360^{\circ}}\right)P \tag{14}$$

or

$$T_C = T_{\Omega} + \left(\frac{\theta - \theta_{\Omega}}{360^{\circ}}\right)P \tag{14.1}$$

From Fig. 4, it is clear that the angle of ingress, the shadow θ_i and the angle of exit the shadow θ_e can be defined as

$$\theta_i = \theta, \tag{15}$$

$$\theta_e = 360^{\circ} - \theta_E. \tag{16}$$

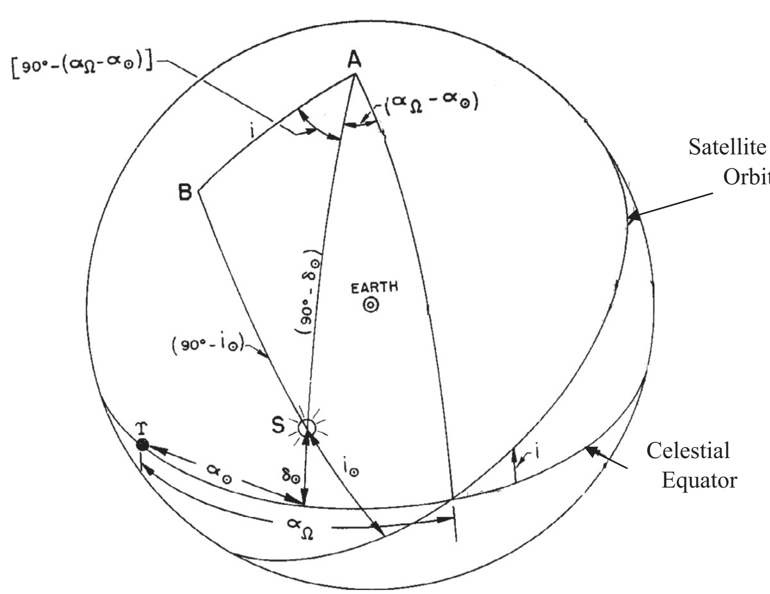

Figure 6 The angle between Sun and orbital plane.

Using Eqs. (15) and (16) the time of ingress of the shadow and the time of egress of the shadow are calculated as follows:

$$T_i = T_\Omega + \frac{(\theta_i - \theta_\Omega)}{360^\circ} P \quad (17)$$

$$T_e = T_\Omega + \frac{(\theta_e - \theta_\Omega)}{360^\circ} P \quad (18)$$

This eclipse will repeat at integral multiples of the period.

5. Effects of solar radiation pressure and earth's oblateness

5.1. Solar radiation pressure

It is known that the momentum can be exchanged during interaction with a solid surface. So, the light behaves like a medium of material particles continuously emitted by the sun.

A satellite whose surface has a reflection coefficient α, placed at a distance d from the sun and receiving the solar radiation at an angle of incidence χ will experience an acceleration under the influence of solar radiation pressure, determined by

$$\bar{p} = -\frac{\beta_1}{d^2}\bar{R}_s \quad (19)$$

$$\beta_1 = \frac{A}{m}\frac{\Phi_0}{C}(1+\alpha)a_s^2\cos^2\chi \quad (20)$$

where Φ_0 is the solar constant, C is the speed of light, a_s is the mean distance Earth–Sun, and $\bar{R}_s$ is a unit vector in the direction Earth–Sun given, in a geocentric equatorial frame by

$$\bar{R}_s = \cos\Lambda_\odot \bar{i} + \cos\varepsilon\sin\Lambda_\odot \bar{j} + \sin\varepsilon\sin\Lambda_\odot \bar{k} \quad (21)$$

where $\Lambda_\odot$ is the true celestial longitude of the Sun and ε is the obliquity of the ecliptic. $\Lambda_\odot$ is expressed in terms of the orbital elements as $\Lambda_\odot = f_\odot + \Omega_\odot$. Due to the Earth's shadow, $\bar{R}$ is a distance continuous function of time, where the data of the Sun are shown in Table 1.

5.2. The Earth's oblateness

It is well known that the perturbing potential V depending on time and the position of the satellite, including the asphericity of the earth is given by

$$V = -\frac{\mu}{r} + \frac{\mu}{r}\sum_{n=2}^{\infty} J_n \left(\frac{R}{r}\right)^n P_n(\cos\theta) \quad (22)$$

where $P_n(\cos\theta)$ are the associated Legendre polynomial taken up to J_6 and $\cos\theta = \frac{x_3}{r}$, and r is the altitude of the satellite.

The first term in Eq. (22) represents the potential when the Earth is a uniform sphere while remaining terms represent the perturbations due to oblateness of the Earth up to J_n, which are taken into account during the calculations.

6. Result and discussion

Now, a code of Mathematica was constructed to obtain the effects of radiation pressure and oblateness of Earth, taken into account the shadow intervals, on a satellite of circular orbit, which is launched at 21 May 1958 (Stoddard, 1961). The data of this satellite are shown in Table 2.

Table 1 The data of Sun.

The speed of light	2.997929×10^8 m s^{-1}
The solar constant	1367 w m^{-2}
Mean distance Earth–Sun	149.759×10^9 m
A reflection coefficient of surface	0
Obliquity of sun	23.5°
Mean longitude of sun	179°
The angle of incident radiation	60°

Table 2 The data of the satellite.

Area of satellite surface	0.319019 m^2
Mass of satellite	35.443 kg
Height	735.8 km
Right ascension of sun	57°.525
Declination of sun	20°.033
Right ascension of satellite	260°.72
Inclination	140°.0
Argument of perigee	62°.32
Radius	7104.1 km
Radius of earth	6368.3 km
Eccentricity	0.0
Period	99.54 min

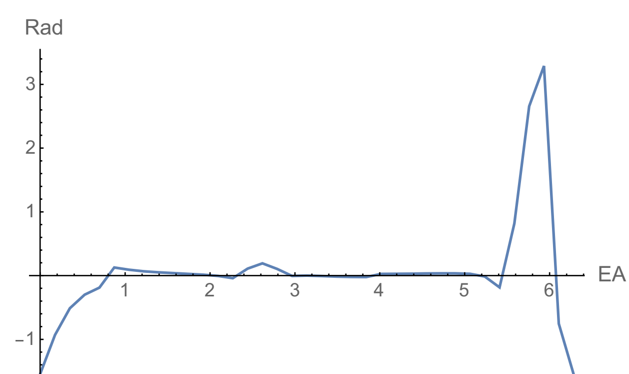

Figure 8 Effects of solar radiation pressure and oblateness on the semi-major axis (Semi-major axis = radius in circular orbit).

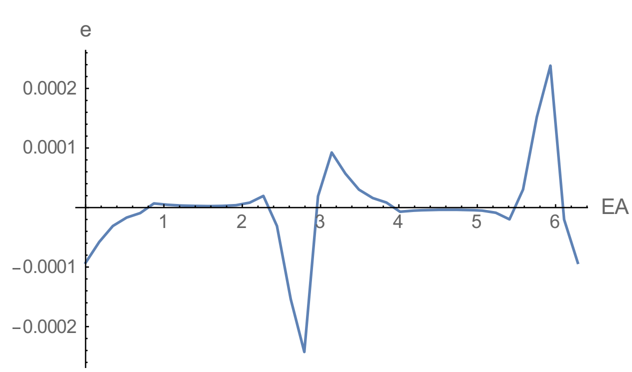

Figure 9 Effects of solar radiation pressure and oblateness on the eccentricity.

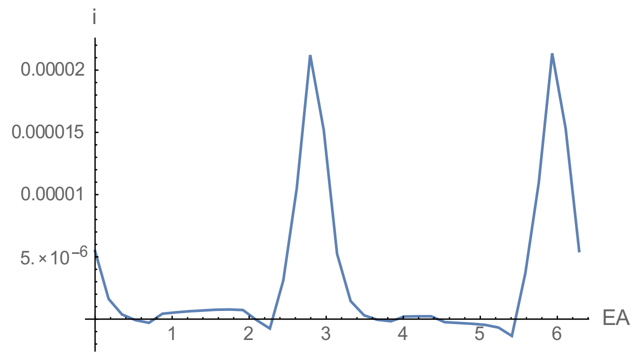

Figure 10 Effects of solar radiation pressure and oblateness on the inclination.

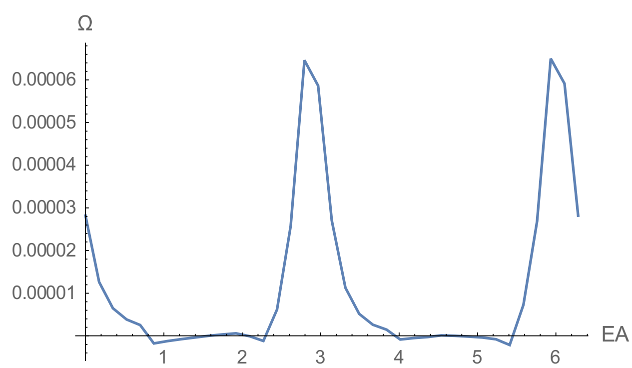

Figure 11 Effects of solar radiation pressure and oblateness on the right ascension of ascending node.

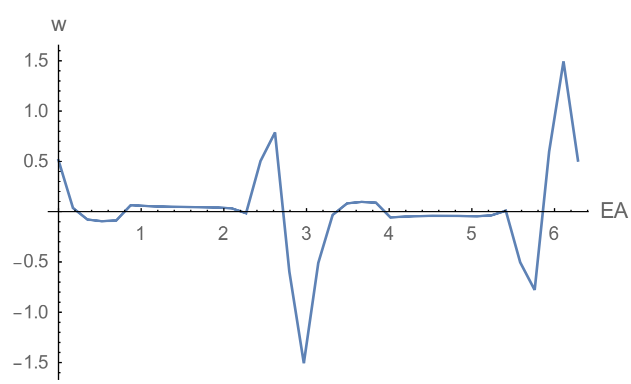

Figure 12 Effects of solar radiation pressure and oblateness on the argument perigee.

The code consists of two parts: the first to obtain the intervals of the shadow effect, i.e., when the satellite is in the shadow of the Earth, and also to obtain the times of ingress and egress of the shadow respectively.

The second part to obtain the effect of solar radiation pressure by solving Lagrange's planetary Equations in the

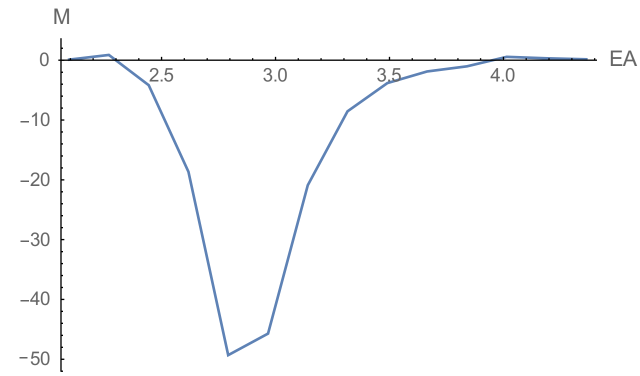

Figure 13 Effects of solar radiation pressure and oblateness on the mean anomaly.

Gaussian form, and to obtain the effect of the Earth's oblateness by solving Lagrange's planetary Equations numerically respectively using the Runge–Kutta fourth order method. The results for one revolution are obtained as follows:

$TPH0 = 1$ h $\quad TPM0 = 20.5$ min

where (TPH0) is the zero time by hour and (TPM0) is the zero time by minute.

	h	m
Time of ingress =	1	5.31781
Time of egress =	1	38.0014

It is clear that the duration of eclipse is about 32.7 min.

Figs. 8–13 illustrate the effects of radiation pressure and oblateness of the earth on the orbital elements of a circular satellite orbit taken into account the intervals of eclipse.

7. Conclusion

Since the shadow effects are very important during the motion of the satellite around the Earth, it must be taken into account when the perturbation problems are treated.

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