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The effects of variable viscosity and thermal conductivity on a thin film flow over a shrinking/stretching sheet

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ABSTRACT

The effects of variable viscosity and thermal conductivity on the flow and heat transfer in a laminar liquid film on a horizontal shrinking/stretching sheet are analyzed. The similarity transformation reduces the time independent boundary layer equations for momentum and thermal energy into a set of coupled ordinary differential equations. The resulting five-parameter problem is solved by the homotopy perturbation method. The results are presented graphically to interpret various physical parameters appearing in the problem.

1. Introduction

The flow and heat transfer phenomena of a viscous liquid over a stretching surface have promising applications in a number of technological processes such as metal and polymer extrusion, continuous casting, drawing of plastic sheets, etc. Crane [1] first studied the steady two-dimensional boundary layer flow due to the stretching of a flat elastic sheet. Since then the problem has been extensively studied by taking into account many different physical features either separately or in various combinations. In most of the cases, the solution turns out to be again in a closed form closely mimicking the solution of the original problem. Later on several researchers, see e.g. the references cited in [2], have explored various aspects of the accompanying heat transfer occurring in the infinite fluid medium surrounding the stretching sheet. Thin film flow due to stretching phenomena has been discussed by a number of workers such as Wang and Pop [3], Liu and Anderson [4], Mahmood et al. [5], Wang [6], Dandapat et al. [7], Anderson et al. [8] and Siddiqui et al. [9]. The stretching problems [10–14] for steady and unsteady flows have been studied extensively in various aspects, such as for non-Newtonian fluids, MHD flows, porous plates, porous medium, with and without heat transfer analysis. Although the stretching problems [10–14] have been studied extensively, much less emphasis has been given to a shrinking sheet problem. Shrinking sheet is a surface which decreases in size to a certain area due to an imposed suction or external heat. Shrinking film is one of the common applications of shrinking problems in industries. The shrinking film is very useful in packaging of bulk products since it can be unwrapped easily with adequate heat. Shrinking problem can also be applied to study the capillary effects in smaller pores, the shrinking-swell behavior and the hydraulic properties of agricultural clay soils since associated changes in hydraulic

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and mechanical properties of such soils will seriously hamper predictions of the flow and transport processes which are essential for agricultural development and environmental management strategies.

The homotopy perturbation method introduced by He [15] has been accepted as an elegant tool in the hands of researchers looking for simple yet highly effective solutions to complicated problems in many diverse arenas of science and technology. In a series of papers, He [16–18] has outlined and refined the HPM, showing its usefulness by solving algebraic, nonlinear ordinary differential equations, bifurcation, partial differential equations and problems involving discontinuities. This method, which does not require a small parameter in an equation, has a significant advantage in that it provides an analytical approximate solution to a wide range of nonlinear problems in applied sciences [19–28]. In general, the solutions produced by the HPM are as accurate as the solutions given by the other methods like decomposition method [29–33], the iteration method [34–36], the tanh method [37] and the transformed rational function method [38].

The tanh method [37] or more generally the transformed rational function method [38] can provide closed-form solutions, which are very efficient if computer algebra systems are adopted. Ma and Fuchssteiner [37] presented some explicit traveling wave solutions to a Kolmogorov–Petrovskii–Piskunov equation through two ansatz. Using a Cole–Hopf transformation, this Kolmogorov–Petrovskii–Piskunov equation was written as a bilinear equation and two solutions to describe nonlinear interaction of traveling waves are generated. Backlund transformations of the linear form and some special cases were considered. Finally, they remarked that any Riccati equation possesses an important property: given a particular solution, its general solution may be found by quadratures.

Ma and Lee [38] used the transformed rational function method to obtain exact solutions to the $3 + 1$ dimensional Jimbo–Miwa equation. This new method provides a more systematical and convenient handling of the solution process of nonlinear equations, unifying the tanh function type methods, the homogeneous balance method, the exp-function method, the mapping method, and the F-expansion type methods. Its key point is to search for rational solutions to variable-coefficient ordinary differential equations transformed from given partial differential equations.

It is really worth considering or commenting upon if such methods work for solving boundary-valued problems. Even more specifically, it is very interesting to check if there exist linear subspaces of the solution space of the considered problem [39]. Ma and Fan [39] explore (for example, for exponential waves) that the linear superposition principle does not apply to nonlinear differential equations. But, it can hold for some special kind of wave solutions to Hirota bilinear equations.

Keeping in view the above analysis, the aim of the present investigation is to study the thin film flow of viscous fluid over a shrinking/stretching sheet with variable temperature dependent viscosity and thermal conductivity by means of HPM. In addition, the investigations regarding the fluid problems due to shrinking sheet are not available in the literature by means of the homotopy perturbation method.

2. Formulation for shrinking sheet problem

Consider a thin elastic sheet that emerges from a narrow slit at the origin of a Cartesian coordinate system. The continuous sheet at $y = 0$ moves in its own plane with velocity

$$U = bx \tag{2.1}$$

where $b > 0$ corresponding to stretching and $b < 0$ corresponding to the shrinking and the surface temperature T_s is defined as

$$T_s = T_0 + T_{ref}\left[\frac{bx^2}{2v_0}\right]. \tag{2.2}$$

Here T_0 is the temperature at the slit and T_{ref} can be taken as a constant reference temperature such that $0 \leq T_{ref} \leq T_0$. Variation of the viscosity and thermal conductivity with temperature are assumed to be in the form:

$$\mu = \mu_0 e^{-\xi(T-T_0)}, \tag{2.3}$$

$$k = k_0[1 + c(T - T_0)], \tag{2.4}$$

where μ_0 and k_0 are the viscosity and conductivity of the fluid respectively at slit temperature T_0. The velocity and temperature field in the thin liquid layer are governed by the two-dimensional boundary layer equations for mass, momentum and thermal energy:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{2.5}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = \frac{1}{\rho_0}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right), \tag{2.6}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{1}{\rho_0 c_p}\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right), \tag{2.7}$$

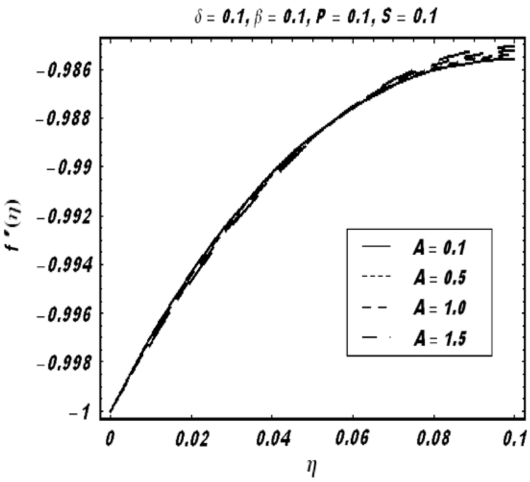

Fig. 1. Variation of f' for different values of A (shrinking case).

with the boundary conditions

$$\begin{aligned} &u = U; \qquad v = -V_0; \qquad T = T_s \quad \text{at } y = 0, \\ &\mu \partial u/\partial y = \partial T/\partial y = 0; \qquad v = u\mathrm{d}h/\mathrm{d}x \quad \text{at } y = h \end{aligned} \tag{2.8}$$

where ρ_0 is the density, μ is the viscosity, T is the temperature, c_p is the specific heat, V_0 is the suction velocity, and h is the thickness of the film. Let us introduce the dimensionless variables f and θ and the similarity variable η as

$$\psi = (b\nu)^{1/2} x f(\eta), \tag{2.9}$$

$$\eta = \left(\frac{b}{\nu_0}\right)^{1/2} y, \tag{2.10}$$

$$T = T_0 + T_{ref}\left[\frac{bx^2}{2\nu_0}\right]\theta(\eta). \tag{2.11}$$

In which $\psi(x, y)$ is the physical stream function which automatically ensures mass conservation (2.5). The velocity components are readily obtained as

$$u = \partial\psi/\partial y = bxf'(\eta), \tag{2.12}$$

$$v = -\partial\psi/\partial x = -(b\nu_0)^{1/2} f(\eta). \tag{2.13}$$

The mathematical problem defined in Eq. (2.5) to (2.8) are then transformed into a set of ordinary differential equations and their associated boundary conditions:

$$\begin{aligned} &ff'' - (f')^2 = (1 - A\theta)\left[A\theta' f'' - f'''\right], \\ &P\left[2\theta f' - f\theta'\right] = (1 - \delta\theta)\theta'' - \delta(\theta')^2, \end{aligned} \tag{2.14}$$

$$\begin{aligned} &f(0) = S, \qquad f'(0) = -1, \qquad f(\beta) = 0, \qquad f''(\beta) = 0, \\ &\theta(0) = 1, \qquad \theta'(\beta) = 0 \end{aligned} \tag{2.15}$$

where

$$\begin{aligned} &\delta = -C[T_s - T_0], \qquad A = -\varsigma(T_s - T_0), \qquad \beta = (b/\nu_0)^{1/2} h, \\ &S = \left(\frac{b\nu_0}{V_0^2}\right)^{-1/2} \quad \text{and} \quad \theta = \frac{T - T_0}{T_s - T_0}. \end{aligned} \tag{2.16}$$

The five dimensionless parameters, appear explicitly in the transformed problem, such as the Prandtl number $P = \nu_0\rho_0 c_p/k_0$, the variable viscosity parameter A, thermal conductivity parameter δ, steadiness parameter S, height of fluid film β.

According to HPM [15–18], Eq. (2.14) takes the form

$$\begin{aligned} &(1 - p)L_1(f - f_0) + p\left(f'' - (f')^2 - (1 - A\theta)\left[A\theta' f'' - f'''\right]\right) = 0 \\ &(1 - p)L_2(\theta - \theta_0) + p\left(P\left[2\theta f' - f\theta'\right] - (1 - \delta\theta)\theta'' + \delta(\theta')^2\right) = 0 \end{aligned} \tag{2.17}$$

$$f = f_0 + pf_1 + p^2 f_2 + \cdots \qquad \theta = \theta_0 + p\theta_1 + p^2\theta_2 + \cdots. \tag{2.18}$$

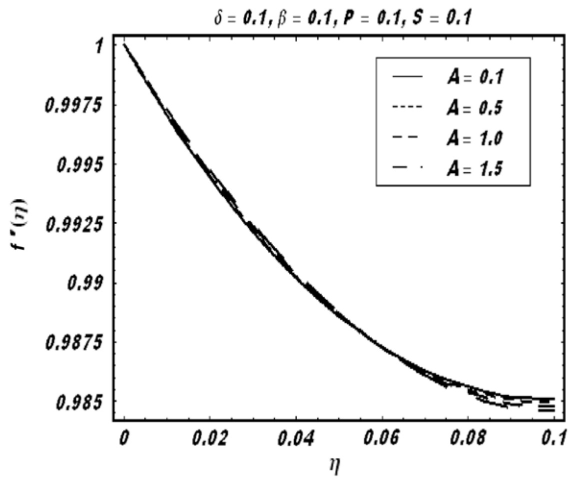

Fig. 2. Variation of f' for different values of A (stretching case).

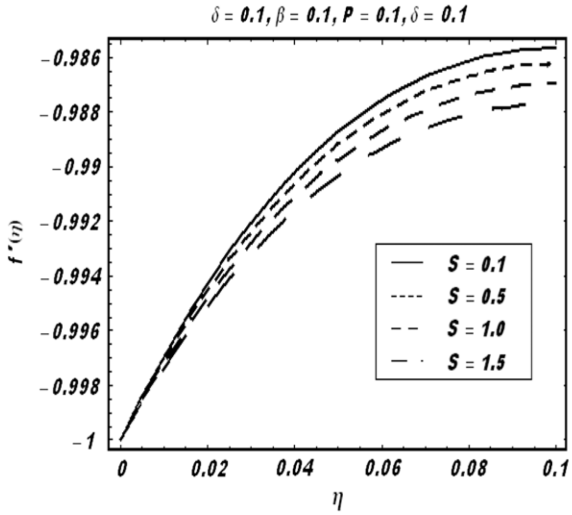

Fig. 3. Variation of f' for different values of S (shrinking case).

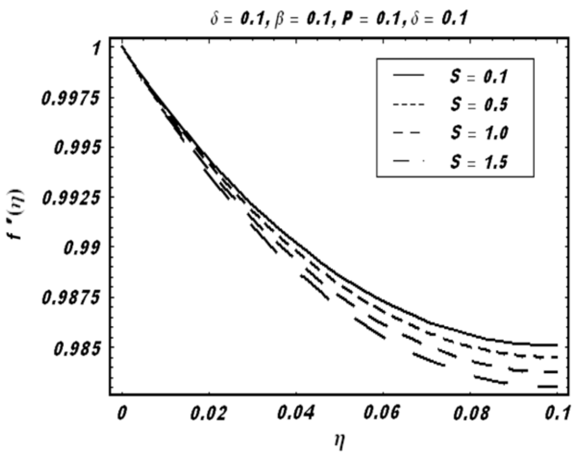

Fig. 4. Variation of f' for different values of S (stretching case).

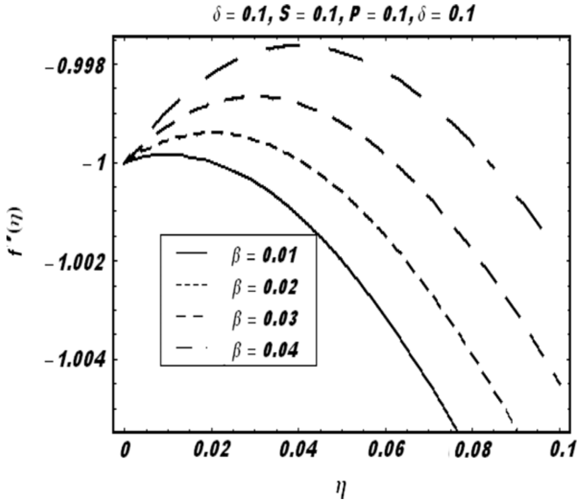

Fig. 5. Variation of f' for different values of β (shrinking case).

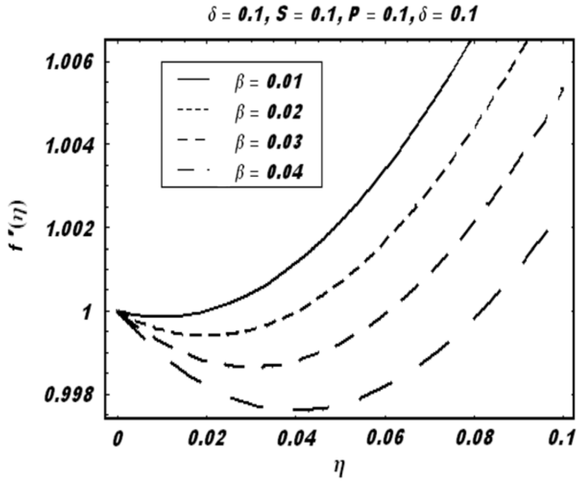

Fig. 6. Variation of f' for different values of β (stretching case).

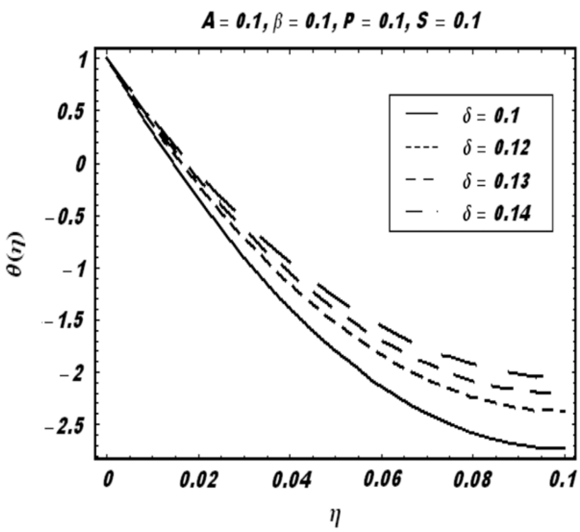

Fig. 7. Variation of θ for different values of δ (shrinking case).

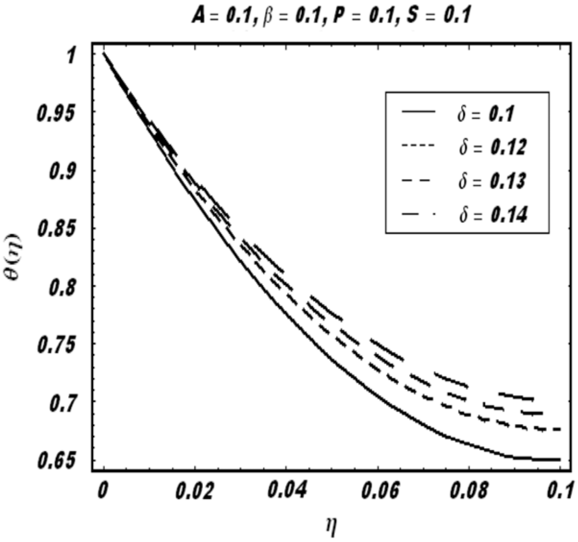

Fig. 8. Variation of θ for different values of δ (stretching case).

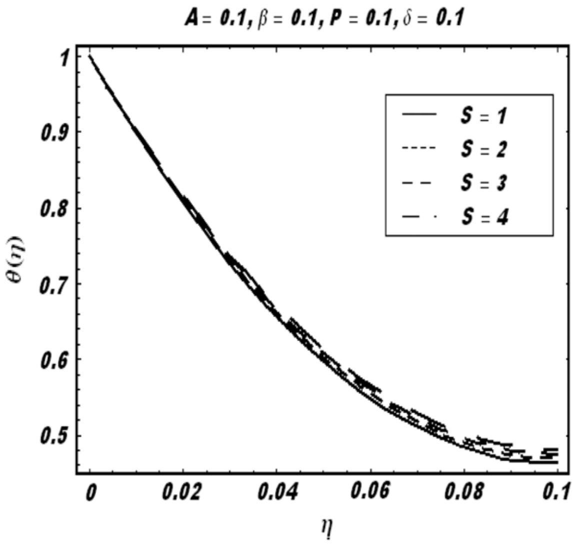

Fig. 9. Variation of θ for different values of S (shrinking case).

Assuming $L_1 f = 0$ and $L_2\theta = 0$, substituting f and θ from Eq. (2.18) into Eqs. (2.17) and (2.15) and some simplification and rearrangement based on powers of p-terms, we have

$$\begin{aligned} & p^{(0)} : L_1 f_0 = 0 \quad \text{and} \quad L_2\theta_0 = 0 \\ & f_0(0) = S, \qquad f_0'(0) = -1, \qquad f_0(\beta) = 0, \qquad f_0''(\beta) = 0 \\ & \theta_0(0) = 1, \qquad \theta_0'(\beta) = 0 \end{aligned} \tag{2.19}$$

where L_1 and L_2 are defined as

$$L_1 = \frac{\partial^4}{\partial \eta^4}, \qquad L_2 = \frac{\partial^2}{\partial \eta^2}. \tag{2.20}$$

Keeping in view Eq. (2.19), we obtain initial guesses are as follows

$$f_0(\eta) = S - \eta + \left(\frac{\eta^3}{\beta} - 3\eta^2\right)\left(\frac{S}{2\beta^2} - \frac{1}{2\beta}\right), \qquad \theta_0(\eta) = \frac{\eta^2}{2} - \beta\eta + 1. \tag{2.21}$$

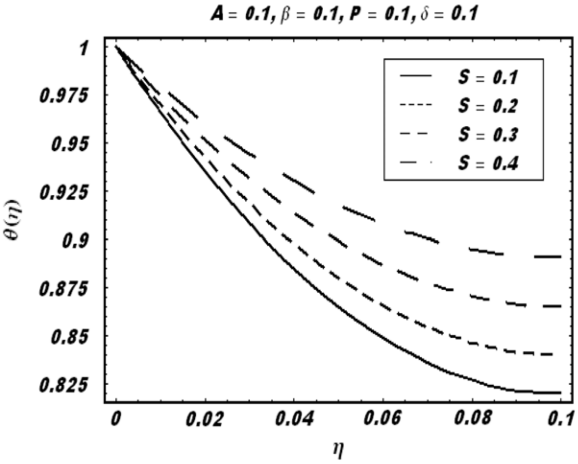

Fig. 10. Variation of θ for different values of S (stretching case).

$$
\begin{aligned}
&p^{(1)}: L_1 f_1 + f_0'' - (f_0')^2 - (1 - A\theta_0)\left[A\theta_0' f_0'' - f_0'''\right] = 0,\\
&p^{(1)}: L_2\theta_1 - L_2\theta_0 + P\left[2\theta_0' f_0' - f_0\theta_0'\right] - (1-\delta\theta_0)\theta_0'' + \delta(\theta_0')^2 = 0,\\
&f_1(0) = 0, \quad f_1'(0) = 0, \quad f_1(\beta) = 0, \quad f_1''(\beta) = 0 \quad \theta_1(0) = 0, \quad \theta_1'(\beta) = 0\\
&\vdots
\end{aligned} \tag{2.22}
$$

$$
\begin{aligned}
&p^{(j)}: L_1 f_j - L_1 f_{j-1} + f_{j-1}'' - \sum_{k=0}^{j-1} f_k' f_{j-1-k}' - A\sum_{k=0}^{j-1}\theta_0' f_{j-1-k}'' + A^2 \sum_{k=0}^{m-1}\theta_{m-1-k}\sum_{l=0}^{k}\theta_{k-l}' f_l''\\
&\quad - A\sum_{k=0}^{j-1}\theta_k f_{j-1-k}''' + f_{j-1}''' = 0,\\
&p^{(j)}: L_2\theta_j - L_2\theta_{j-1} + 2P\sum_{k=0}^{j-1}\theta_k' f_{j-1-k}' - P\sum_{k=0}^{j-1}\theta_k' f_{j-1-k} + \delta\sum_{k=0}^{j-1}\theta_k''\theta_k\\
&\quad + \delta\sum_{k=0}^{j-1}\theta_k'\theta_{j-1-k}' - \theta_k'' = 0\\
&f_j(0) = 0, \quad f_j'(0) = 0, \quad f_j(\beta) = 0, \quad f_j''(\beta) = 0, \quad \theta_j(0) = 0, \quad \theta_j'(\beta) = 0\\
&\vdots
\end{aligned} \tag{2.23}
$$

On solving Eqs. (2.22) and (2.23) in any software like MATHEMATICA, MAPLE or MATLAB we can obtain any order of approximation.

2.1. Stretching sheet problem

For stretching phenomena, the governing equation is (2.14), however, the boundary conditions for stretching case are as follows:

$$
\begin{aligned}
&f(0) = S, \quad f'(0) = 1, \quad f(\beta) = 0, \quad f''(\beta) = 0,\\
&\theta(0) = 1, \quad \theta'(\beta) = 0.
\end{aligned} \tag{2.24}
$$

The solution of the above boundary value problem has been calculated using the procedure given in [15–28]. To avoid the repetition, the complete solution is not defined, however, the initial guesses are as follows

$$
f_0(\eta) = S + \eta + \left(\frac{\eta^3}{\beta} - 3\eta^2\right)\left(\frac{S}{2\beta^2} + \frac{1}{2\beta}\right), \qquad \theta_0(\eta) = \frac{\eta^2}{2} - \beta\eta + 1. \tag{2.25}
$$

Graphical behavior of the physical parameters has been calculated for 10th order approximation.

3. Results and discussion

In this study our main focus is to investigate the effects of viscosity, thermal conductivity, Prandtl number and height of fluid film.

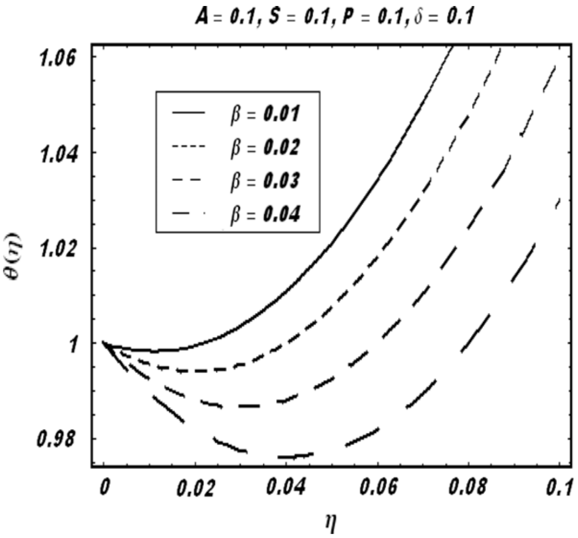

Fig. 11. Variation of θ for different values of β (shrinking case).

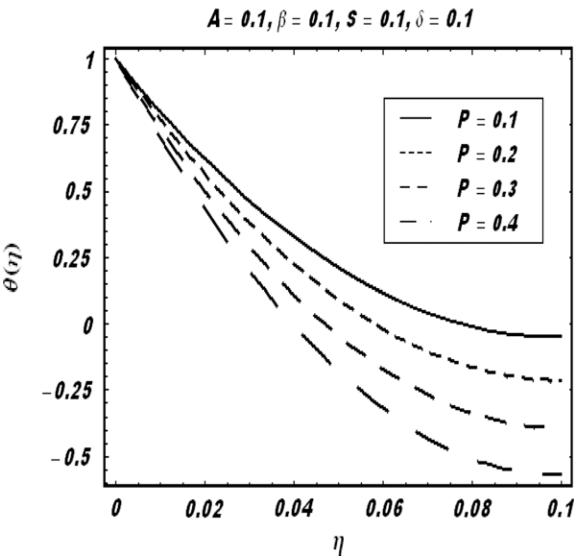

Fig. 12. Variation of θ for different values of P (shrinking case).

The graphs for the function $f'(\eta)$ and $\theta(\eta)$ are plotted against η for different values of the parameters appearing in Eqs. (2.14) and (2.15). Figs. 1 and 2 show the effects of A on the velocity component f' for shrinking and stretching respectively. Figs. 1 and 2 depict that there is a very slight change in the velocity f' with the increasing values of A for shrinking and stretching respectively. Figs. 3 and 4 explains the effects of S on the velocity component f' for shrinking and stretching respectively. It is observed that with the increasing values of S, f' decreases for both cases (shrinking/stretching). The height of fluid film β has opposite effects on f' for shrinking and stretching. It is seen that f' increases for shrinking and decreases for stretching (see Figs. 5 and 6). The variation of δ on θ is plotted in Figs. 7 and 8. It is observed that the behavior of δ on θ is similar for shrinking and stretching, increases with the increasing values of δ. Figs. 9 and 10 explain the variation of S on θ. It is observed that the behavior of S on θ is similar to that of δ. Figs. 11 and 12 show the effects of β and Prandtl number on θ respectively. It is seen from the figures that θ decreases for different values of β and P.

4. Conclusion

Steady flow of a thin liquid film over a shrinking/stretching sheet is studied in the light of variation of fluid properties due to temperature differences. In this study, emphasis is given on how velocity field, temperature distribution changes due to the variation of viscosity, thermal conductivity. We have used similarity transformation that reduces the governing equations to a system of nonlinear ODEs which are then solved analytically by HPM. The analytical results for the effects of variable viscosity and thermal conductivity on a thin film flow over a shrinking/stretching sheet by means of HPM yet not available in the literature. Such kinds of analytical results have never been reported by using the homotopy perturbation method.

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