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The robustness of stability under link and node failures☆

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ABSTRACT

In the area of communication systems, *stability* refers to the property of keeping the amount of traffic in the system always bounded over time. Different communication system models have been proposed in order to capture the unpredictable behavior of some users and applications. Among those proposed models the *adversarial queueing theory* (AQT) model turned out to be the most adequate to analyze an unpredictable network. Until now, most of the research done in this field did not consider the possibility of the adversary producing failures on the network structure. The adversarial models proposed in this work incorporate the possibility of dealing with node and link failures provoked by the adversary. Such failures produce temporal disruptions of the connectivity of the system and increase the collisions of packets in the intermediate hosts of the network, and thus the average traffic load. Under such a scenario, the network is required to be equipped with some mechanism for dealing with those collisions.

In addition to proposing adversarial models for faulty systems we study the relation between the robustness of the stability of the system and the management of the queues affected by the failures. When the adversary produces link or node failures the queues associated to the corresponding links can be affected in many different ways depending on whether they can receive or serve packets, or rather that they cannot. In most of the cases, protocols and networks containing very simple topologies, which were known to be universally stable in the AQT model, turn out to be unstable under some of the newly proposed adversarial models. This shows that universal stability of networks is not a robust property in the presence of failures.

1. Introduction

As the world becomes more dependent on communications and computer networks, there is a growing concern about the effects of network failures. Failures can occur as the result of natural disasters, as the result of human action or by unintentional errors in software or control systems. However, failures do not only happen in relation to catastrophic or accidental situations. In a more natural way, disruptions also appear in wireless mobile networks, where some connections between nodes may fail or change quickly and unpredictably due to mobility, network components' configuration or topology variations. In fact, our work is inspired by the growing importance of this type of network and the necessity of theoretical frameworks for modelling their traffic flow and disruptions.

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We must be realistic and accept that our communication and computer networks nowadays are so widely extended, complex and heterogeneous, and they carry so much traffic, that suffering from overloading and failures is unavoidable. To address these issues, we need first to understand the types of failures that a system can suffer and the immediate consequences of those failures. Thus, appropriate models to study real network systems which suffer from failures are needed. Those models could help in detecting, understanding, and overcoming the conditions leading to these mentioned negative effects, as well as helping in their further prevention. All these goals are strongly concerned with network survivability, which is the ability of a network to maintain or restore an acceptable level of performance during network attacks or failures, and to mitigate or prevent service outages.

In this work we consider systems in which nodes and links may fail, and we study the impact of the organization and accessibility of the intermediate packet-storage devices at the network (i.e., the intermediate queues at the hosts) on the stability of the system. We use adversarial models in which the traffic and the failures are assumed to be produced in an unpredictable (and rather malicious) way. The theoretical study of stability via adversarial models has been a hot research topic in the last decade. *Stability* refers to the fact that the number of packets in the system remains bounded as the system dynamically evolves in time. This bound, which can be a function of the system parameters such as number of nodes or edges or the diameter, is not dependent on time. In this context, stability and universal stability (a stronger notion of this term) are key features to be preserved. Universal stability of networks has been shown to be a robust property as it is preserved under many variations of the AQT model [3,12,2,10]. And, except for the LIS protocol, the universally stable protocols remain stable under those variations of the AQT model [2,10]. We are interested in studying the influence of queue management during failures on the robustness of stability in faulty scenarios.

Stability in the adversarial queueing theory model

Stability is studied in relation to the three main components $(\mathcal{N}, \mathcal{A}, \mathcal{P})$ forming a communication system: the network $\mathcal{N}$, the traffic pattern defined by the adversary $\mathcal{A}$, and the scheduling protocol $\mathcal{P}$. *Networks* are modeled by directed graphs in which nodes represent the hosts and edges represent the links between those hosts. The *protocol* (or queueing policy) determines the order in which the packets requiring to cross a link are scheduled to be forwarded. The *adversary* controls the traffic pattern.

The Adversarial Queueing Theory (AQT) model proposed by Borodin et al. [11,4] has become in recent times an important model to study stability issues, since it can describe the behavior of both connectionless and short-term connection networks, as well as connection-oriented networks. The AQT model considers the time evolution of a packet-routing network as a game between the adversary and the queue policy of the system. The system is considered to be *synchronous*.

At each time step the adversary may inject a set of packets to some of the nodes. For each injected packet, the adversary specifies the route that it must traverse (*static routing*), after which the packet will disappear from the system. If more than one packet wishes to cross an edge e at the same time step, then the queueing policy chooses one of those packets to send across e. The remaining packets must wait in the queue. This game then advances to the next time step. The goal of the adversary is to try to prevent the protocol from guaranteeing load and delay bounds. On the contrary, the main goal of the model is to study conditions for stability of the network under different protocols.

In order not to trivially overload the system and in order to be able to guarantee delay bounds, it is necessary to restrict the traffic arriving to the network. The constraints on the traffic pattern must ensure that, over long periods of time, the maximum traffic injected in a link is roughly the amount of traffic that the link can forward. Two parameters (r, b) constrain an adversary in the AQT model, where $b \geq 0$ is *the burstiness* and $0 < r < 1$ is the *injection rate*. An adversary like this is henceforth referred to as an (r, b)-adversary. Let $N_e(I)$ be the number of packets injected by the adversary in a time interval I that have paths requiring a particular edge e. Then, an (r, b)-adversary in the AQT model must obey the following (leaky-bucket) constraint that restricts its injection power:

$$N_e(I) \leq \lceil r|I| \rceil + b. \tag{1}$$

An important term related to stability is that of *universal stability*, which is an extension of the stability property to systems with any configuration. It can be studied from the point of view of the network or from the point of view of the protocol. In broad terms, a network is said to be universally stable if the number of packets in the system is bounded whatever adversary is considered and whatever protocol is used for scheduling packets at the edges. Analogously, a protocol is said to be universally stable if the number of packets in the system is bounded whatever adversary is considered and whatever network topology is implemented underneath.

A considerable number of results concerning stability in the AQT model are available nowadays (see, e.g., [11,4,14,19,3,9,17,20–22], just to mention a few).

Greedy protocols

We consider a situation in which there is a queue associated to each link of the network. The role of such queues is to store those packets that require to cross the link but cannot be immediately forwarded because the link is busy or temporally unavailable. As is usual in the literature, we only consider *greedy protocols* which apply, in a work-conserving manner, their queueing policies according to some local or global criteria. In general, a protocol is greedy if whenever there is at least one

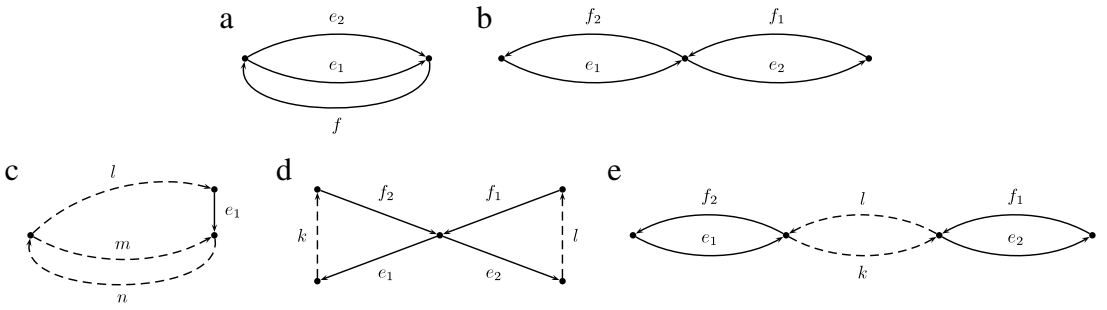


Fig. 1. Forbidden subdigraphs for detecting universally stability: (a) digraph U_1; (b) digraph U_2; (c) shape of a digraph in $\mathcal{E}(U_1)$; (d,e) shapes of two digraphs in $\mathcal{E}(U_2)$, where (e) corresponds to a two cycle extension combined with an arc extension. The dashed arcs represent paths with a certain length specified by their labels, where $k, l \geq 0$ and $m, n \geq 1$.

packet waiting in the queue, the queue serves a packet. The main queueing protocols considered in the study of stability are FIFO, LIFO, SIS, LIS, NTG, FTG, NFS and FFS. In this work, we will also consider the protocol LIQ.

The protocol LIFO (*last-in-first-out*) gives priority to the packet which joins the queue last; in FIFO (*first-in-first-out*), highest priority is given to the packet that arrives first in the queue. The protocol which gives priority to the packet last introduced into the system is SIS (*shortest-in-system*), while in LIS (*longest-in-system*) every queue gives priority to the packet that has been in the system the longest time. The protocol NTG (*nearest-to-go*) assigns highest priority to the packet that is closest to its destination and FTG (*farthest-to-go*) assigns it to the packet that is farthest away from its destination. NFS (*nearest-from-source*) and FFS (*farthest-from-source*) consider the same policies but taking the distance to the source node of the packets as reference point. LIQ (*longest-in-queue*) is a variation of LIS that gives priority to the packet that in total has been waiting in queues for the longest time. As usual we assume that any arising ties are broken by the adversary.

The universal stability of most of these protocols in the AQT model was already studied in the work of Andrews et al. [4], where FTG, NFS, SIS and LIS were shown to be universally stable, while FIFO, LIFO, NTG and FFS were shown not to be universally stable. Recently, LIQ has been shown to be universally stable in the NSCAQT model [13] (and thus also in the AQT model), which is a generalization of the AQT model in which the condition of system synchronism is relaxed.

Networks

Rings and directed acyclic graphs (which include lines, forks and crossings) are known to be universally stable in the AQT model [11,4]. In fact, the property of universal stability of networks in the AQT model was fully characterized in [3]. That characterization was described in terms of the existence of certain forbidden subtopologies, i.e., forbidden subdigraphs, in the network. More precisely (see Theorem 8 in [3]), a digraph is known to be universally stable in the AQT model if, and only if, it does not contain as subgraphs any of the digraphs in $\mathcal{E}(U_1) \cup \mathcal{E}(U_2)$, where U_1 and U_2 are the digraphs depicted in Fig. 1, and $\mathcal{E}(G)$ denotes the family of digraphs formed by G and all the digraphs obtained from G by successive arc or 2-cycle subdivisions. Also in [3], it was shown that the universal stability of a given digraph in the AQT model can be decided in polynomial time. The same property characterizes the set of networks stable under a protocol, for protocols, NTG [3] and FFS [1]. A different characterization for FIFO can be found in [22].

Related work

Initially, all the studies about the stability of networks and protocols focused on static systems. Later, different particularities related to dynamic networks started to be introduced. Although failures were not initially considered as such, some adversarial models were proposed that deal with variations of the capabilities of the edge for transmitting a packet.

In [12], links in packet routing networks can have capacities or speed/slowdowns (not both) that can change dynamically. However, since only finite slowdowns and positive capacities are considered in [12], the effect of failures is not really simulated, but roughly approximated. With such a model, the set of networks that are universally stable when considering dynamic capacities or slowdowns coincides with the networks that are universally stable in the AQT model. From the point of view of protocols, LIS is shown not to be universally stable, in contrast to its behavior under AQT. The capacities of the links in the network are also dynamic in [15,16,10], however all those models also limit significantly the change in the capacities and thus they do not study real faulty scenarios either. Moreover, the results obtained show that the set of universally stable networks remains the same as in the AQT model under these models. The configuration and evolution of the systems considered in all those works are also very different from those suffered by the systems we present here: in [12,15,16,10] the packets are always flowing, since the effect of a real failure (i.e., null link capacity or infinite slowdown) is never considered; thus packets are only buffered for congestion reasons, but never due to failures. Instead we consider that, when a failure

occurs, the packet using the failed link/node can not flow and must be retransmitted later.[1] Then, one needs to study how to store and manage those affected packets while the failure lasts. In our opinion, this is a more realistic way of tackling the problem.

In [8,7], other models were considered for studying distributed balancing algorithms for dynamically changing input streams and flow routing in dynamically changing networks. In those models, the injected packets are defined by specifying only source and destination (no path is pre-specified), and only the single receiver case is considered. The adversary is thus restricted to guarantee that a static multi-commodity flow problem has a solution. The proposed load balancing algorithms are shown to keep the system stable when the adversary injection rate is one. The main difference in the two models is that in [8] the adversary has to provide a solution to the associated multi-commodity flow problem, while in [7] the injection pattern must obey a condition that guarantees the existence of the solution. Our models and the models proposed in [8,7] have the common characteristic that, for every interval I, the adversary cannot inject to any edge e (or to any set S of nodes for the model in [7]) more packets than the number of packets that e can absorb (or the edges with only one extreme in S). However, the focus of [8,7] is on the balancing proposals and they do not deal with failures either.

Another interesting approach in the literature considers input/output blocking in the nodes [5,6]. Although this model does not consider failures on the network structure, the addition of input–output constraints that this model has on the arrival of packets seem to have some behavioral similarities with some of the models considered here. This might be the case for the models with failures at the links that block the reception of packets (i.e., namely the edge-nR and edge-nR-B models). From the point of view of networks, the results in [5,6] show that networks containing very simple topologies (such as the one we reach in this work) can become unstable. However, from a protocol point of view, the results obtained in both works are completely opposed, for example SIS is unstable while LIS is universally stable.

The first work where failures were considered as such was [2], where two AQT-based models allowing faults (namely the *failure model* and the *reliable model*) were proposed to consider dynamic networks in which links can stop transmission. Both models consider only the possibility of link failures. For any edge e and any interval I, the adversary must obey the constraint

$$N_e(I) + \alpha F_e(I) \leq r|I| + b,$$

where $F_e(I)$ is the number of steps during a time interval I in which the edge e is down. The failure model considers $\alpha = 1$, while the reliable model considers any α such that $r < \alpha \leq 1$. In both models, the adversary is not able to fail an edge permanently. Concerning the universal stability of networks, both models are equivalent to the AQT model. Concerning the universal stability of protocols, FTG, NFS, and SIS are universally stable but FIFO, LIFO, NTG and FFS are not as in the AQT model. However, the LIS protocol is universally stable in the AQT model, but it is not in either the failure or the reliable model. This first work settled the basis for the work we present here.

It is worth mentioning that none of these existing models for dealing with dynamic networks consider the possibility of changing the capabilities of the nodes of the topology, but only of the links. Also the failure and reliable models in [2] only considered link failures. Here we also consider node failures.

Our contributions

The main contribution of this paper is twofold: first, an important effort has been made on properly defining new concepts and techniques, and unifying existing ones, in a rigorous and formal way; second, an exhaustive study and analysis has been made of the effect of edge or/and node failures on the stability of networks and protocols in relation to the way those failures are administrated. Moreover, to the best of our knowledge, node failures were never studied before when studying stability in the context of adversarial systems.

Our focus on the rigorous definition and standardization of some concepts and techniques came after some years of study on stability of adversarial systems, when we observed that the generated literature had some clear gaps in unifying the definition of some basic concepts and describing properly the techniques used. An important part of this work aims at overcoming this problem and fixing that notation for the future. Concerning our contributions on this methodological side, it is worth mentioning the introduction of the notion of *system simulation*, and its usage in comparing the global behaviour of models. This technique, which was first used in [2], allows us to perform a systematic and incremental study of faulty adversarial models. For the first time in this topic of research, we provide in this work a formal definition for such a technique and for several other concepts which were often used in the literature but never formalized. Our definitions are general enough to adapt naturally when different models are considered. That required a clear definition of what a *model* is, an effort that was never made until now and that resulted in a slight mess in the existing literature.

On the other side, the exhaustive study and analysis performed of the effect of failures on the stability of the system provide us with interesting information on the most convenient way of dealing with failures depending on the topology of the network, and the protocols of the system.

[1] Observe that, when retransmissions are considered, the packets affected by a failure compete again with other packets when the failed link/node is restored.

In this work we deal with communication networks in which links, nodes, or both may fail. We propose adversarial models regulating the network behavior and restricting the traffic patterns occurring in this type of faulty system. We study the dependence between the conditions for the stability of the system, and how the packets affected by failures are managed, under non-trivial underloaded worst-case scenarios. The adversary is responsible not only for the traffic joining the network, but also for the failures of the links and nodes. Those failures have the effect of blocking temporally both those packets that arrive to a link/node while it is failed, and those waiting already to be transmitted through at the moment when the failure occurs. In addition to proposing adversarial models for such faulty networking systems, we study the dependence between the conditions for the stability of the system, and how the packets affected by failures are managed. As we will see, depending on how the system is organized and prepared to deal with failures, the dynamics of the system change and thus its conditions for stability.

First, we study systems in which the faulty elements are the links. We propose an adversarial model for them and study the conditions for stability under three different ways of managing the packets involved in the failures. This treatment gives rise to three versions of the proposed adversarial model for link-faulty networks, one for each management type, that we denote in the following as edge-R, edge-nR and edge-nR-B models. The difference relies on whether the queue of a failed link is able to receive (extension -R) packets or not (extension -nR) and, in case it is not, on whether the affected packets are kept in the normal queues or in a special buffer (extension -B) that the node keeps to that aim.

Then, we propose a adversarial model and study the stability of systems in which the faulty elements are the nodes. The model has four variations which we denote in the forthcoming as node-RnT, node-nRnT-B, node-nRnT and node-nRT models. The main difference among them relies on whether the queues of a failed node are able to receive (extension -R) packets or not (extension -nR) and on whether a failed node that cannot receive new packets while being failed is able to transmit (extension -T) those packets that are already queued, or not (extension -nT). In some of these cases, there might be also an additional buffer (extension -B) where the affected packets are kept. To the best of our knowledge, this is the first time that the stability of dynamic adversarial systems with failing nodes has been studied. As a natural extension to our study of systems with (exclusively) link or node failures, we consider also systems in which both the nodes and the links may fail. Obviously, the instability results observed in the non-combined models apply also in the combined ones, and thus the interest is in stability results. We show that when combining link models and node models with a common positive stability property, that property also holds in the model combining them.

The AQT model was the model of reference for the study of stability of systems under adversarial traffic conditions. Then, it is important to put in relation our results with those obtained in the basic AQT model, where no failures occur. We show that for the failure models edge-R, node-RnT, and their combination the set of universally stable networks coincides with the set of universally stable networks in the AQT model, while some universally stable protocol in the AQT becomes unstable. For the remaining models we show that, in contrast to what happened under AQT, already very simple direct-acyclic topologies are unstable. We show also that several protocols considered universally stable under AQT become now unstable.

Organization of the article

Section 2 describes the characteristics of the network scenario that this work assumes. The properties presented there are common to all the models that will be presented in this paper. Also in Section 2, most of the key concepts that are used in this work (especially those concerning the adversaries and stability) are formalized.

In Section 3, we concentrate on studying the effect of link failures and propose three adversarial models for describing systems that suffer from that type of failure. Complementarily, in Section 4 four adversarial models are presented whose focus is on studying the effect of node failures. All the adversarial models proposed in both sections are distinguished from each other in how the system is organized for managing those packets that suffer from failures. Under all those models, we study the stability of networks as well as the stability of some well-known protocols.

This separation between link and node failures allows us to study their direct effect on the dynamics of the systems and on the conditions for stability in a quite simple and readable way. Moreover, it allows us to extract clear conclusions about what happens in systems that can suffer only from link or node failures and highlight those combinations of link and node failure management that require further analysis. The reader will find those final results in Section 5. Finally, Section 6 gives some conclusions and reviews some open problems for future research.

2. Faulty model scenario

In this section, we detail the characteristics of the faulty network scenario that is assumed in this work. Hence, the following properties are common to all the models that will be introduced later.

Systems composed of network, adversary and protocol. The systems we deal with are composed of three main elements $(\mathcal{N}, \mathcal{A}, \mathcal{P})$, where $\mathcal{N}$ is the network topology, $\mathcal{A}$ is an adversary defining the traffic and failure pattern on $\mathcal{N}$, and $\mathcal{P}$ is a scheduling protocol. *Networks* are modeled by directed graphs in which nodes represent the hosts and edges represent the links between those hosts. We assume that the graphs might have multiple arcs but no loops. The *adversary* controls the traffic pattern and the failure pattern. The *protocol* determines the order in which the packets requiring to cross a link are scheduled to be forwarded.

As we will see, systems are always considered in the context of an adversarial model, which will fix how the adversary is restricted in its power for injecting and making failures, and how the infrastructure of the network is organized to deal with the packets affected by those failures.

Adversarial traffic flow and adversarial connection disruption. In this work we deal with faulty adversarial models in which the links and/or the nodes may fail temporarily. In such a dynamic scenario, the adversary is allowed to control not only the traffic injection, but also those link and node failures. The different models we present in this work apply different restrictions on the adversary. In broad terms, in any interval of time I, the adversary will be allowed to inject an amount of packets requiring a certain link e proportional to the steps in I in which the e is really available. The precise shape of the restrictions on the adversaries will be introduced later, when introducing each of the faulty models under consideration.

Queues at the head nodes of the links. We consider a situation in which there is a queue associated to each link of the network. For every link, such a queue is physically kept at the node which is the head of the link. The role of such queues is to store those packets that require to cross the link but cannot immediately, either because the link is busy due to the transmission of a different packet, or because some transmission disruption in the context of that link happened. We consider that every packet has unit size and has to be received integrally before it is forwarded to the next outgoing link, i.e., networks do not have *cut-through* capabilities.

In some cases, with the aim of not overloading the outgoing queues, we consider the existence of an additional buffer in the nodes. This consideration leads to the buffered models. In such models, that buffer is in charge of storing the blocked packets that suffered a failure. When the faulty element (link or node) is recovered from the failure, those packets affected by the failure are set to non-blocked. After that, the protocol chooses one packet among the non-blocked packets according to the scheduling policy of the system and puts the chosen packet, instantaneously, in the corresponding output queue (i.e., the queue corresponding to the next link in its path). Thus, only one packet can leave the buffer in one time step.

When a failure occurs (either in the edge associated to the queue, or in the node hosting the queue), the normal functional behavior of the queue may vary. We define different failure models depending on those variations. In some cases, the queue may be blocked for reception, while in some others it may be blocked for transmission. It may be even blocked for both reception and transmission. Fig. 2 depicts how those situations will be represented in our figures.

Synchronization. As is usual in studies of stability, we assume that the evolution of the packets in the network is synchronized. To maintain synchronization, each time step is divided into three basic phases: (1) *receive*, in which the packets (if any) arrive to the node and are placed in their corresponding outgoing queue; (2) *request connectivity* and *schedule*, in which the state of the node and every corresponding outgoing link is checked, and in which a packet to be forwarded is selected among all the available packets in all the outgoing queues (if any) according to the protocol of the system; and (3) *send*, in which, for each outgoing queue with packets, the packet chosen by the protocol leaves the queue.

This behavior varies slightly when some node failure occurs or when dealing with buffered nodes. How this behavior is changed in every case will be detailed later in the paper, in Sections 3 and 4, when presenting the different adversarial models under study.

Short-lived failures. We focus on short-lived failures, and consider that no link or node can be failed for an arbitrary large amount of time, i.e., there exists a constant w, independent of time, bounding the number of consecutive steps for which any edge or node can be down. That means that if any edge or node fails at any time step t then it will be recovered after at most $w + 1$ steps. This parameter will be one of the limits imposed on the sequence of failures that can be provoked by an adversary.

Evolution under failures. When a link failure is produced no packet kept in its queue can be served. But we also have to consider what to do with the incoming packets. Since they cannot be lost, either they are received by its queue, or they are not. In the latter case, either the packets wait in the previous link of their path or they are kept in an additional buffer at the link's head node which is common for all the output links of such a node.

When a node failure is produced we consider different managements depending on whether the queues of the output links of the failed nodes can transmit packets or not and in this latter situation we take into account whether there is an additional buffer where all the packets that need to go through the node can be kept.

When failures can occur, greedy protocols refer to those ones advancing a packet whenever the link is able to transmit and there is at least one available packet waiting in the queue. In all the situations we assume an ideal world in which no packet already in the system is lost.

Homogeneity. We assume that all the links (respectively, all the nodes) will manage the packets and the queues affected by failures in the same way. The different options considered for such management will be presented later in the paper. The case of non-homogeneous management, i.e., when different links (and possibly also different nodes) deal with failures in different ways, remains open.

Having in mind the common networking scenario presented in Section 2, we need to specify clearly what a faulty adversarial model is, what an adversary is, and the definition of the different concepts of stability we will use. Let us start with the definition of a faulty adversarial model.

Definition 1 (*Model $\mathcal{M}$*)**.** A faulty adversarial model $\mathcal{M}$ specifies the following three characteristics of a system:

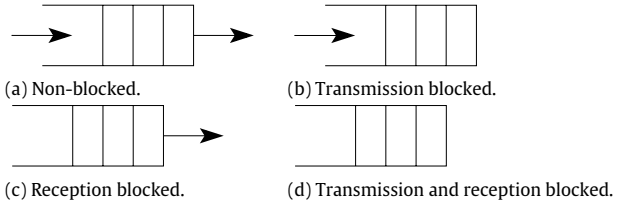


Fig. 2. Graphical representation of the functionality of the queues.

- whether the queues are or are not accessible during a failure,
- whether there are or there are not additional buffers, and
- what the restrictions are that apply to the adversary.

Since all the models we introduce in this work are aimed at adversarial systems in which failures may occur, we often skip the adjective *faulty* and refer just to *model* $\mathcal{M}$.

In our systems, the adversary represents the unpredictable source of traffic of the system and additionally, in the case of faulty scenarios, it is also the source of failures for the network elements. Although unpredictable, the behavior of an adversary is expected to be as harmful as possible, inside some limits that avoid trivial overflooding of the network. We need to provide a more formal definition for an adversary and its representation, which will be assumed for the rest of the paper.

Definition 2 (*Adversary*)**.** An adversary $\mathcal{A}$ for network $\mathcal{N}$ is an infinite sequence of triples. The t-th triple $\langle P_t, E_t, V_t\rangle$ corresponds to the changes in traffic and failures at time t; it is composed of the following elements:

- the set of packets P_t injected in the system at time t,
- the set of links E_t that fail at time t, and
- the set of nodes V_t that fail at time t.

In general we will omit the reference to the network in those cases where $\mathcal{N}$ is clear from the context.

In faulty models the adversary decides not only the packet injections but also the failures (in links, nodes or both), and thus this information must be part of its definition. Of course, since the AQT model is a non-faulty model, an adversary in AQT is described just by an infinite sequence of singletons; each singleton is formed by the set P_t of packets injected in the system at a time step t.

Before introducing the general restrictions that an adversary must obey we introduce some notation that will help in their description. Let $\mathcal{A}$ be an adversary for network $\mathcal{N}$ which at time step t has associated triple $\langle P_t, E_t, V_t\rangle$; then, for any edge $e = (u, v)$ of $\mathcal{N}$, we denote as

$$N_e(t), \text{ the number of packets } p \in P_t \text{ whose trajectory uses edge } e$$
$$F_e(t) = \begin{cases} 1 & e \in E_t \\ 0 & \text{otherwise} \end{cases}$$
$$H_e(t) = \begin{cases} 1 & u \in V_t \\ 0 & \text{otherwise} \end{cases}$$
$$D_e(t) = F_e(t) + H_e(t) - F_e(t)H_e(t).$$

Observe that $F_e(t)$ indicates whether the link e fails at time t, and $H_e(t)$ indicates whether the link e is unavailable because its head node fails at time t; thus, $D_e(t)$ gives evidence of whether the link e is available or not at time t, independently of what the reason is for it (link failure or head-node failure). We keep the notation $N_e(t)$ and $F_e(t)$ to be compatible with the usual notation in the AQT model and the *failure model* in [2]. The above functions can be extended as usual, by addition, to an interval of time $I = [t_0, t_1]$ composed of $|I| = t_1 - t_0 + 1$ time steps. For example $D_e(I) = \sum_{t\in I} D_e(t)$.

As we said, in order to avoid trivial overloading situations, the behavior of the adversaries must be restricted. Different restrictions on the power and shape of an adversary will lead to different adversarial models for faulty networks. In a faulty adversary, these restrictions are based on three elements:

- on the amount of traffic that the adversary has permission to inject,
- on the amount and type of failures that the adversary is allowed to produce, and
- on the maximum duration of any of those failures.

To capture these issues in the definition of a faulty adversary, we introduce the concept of an (r, b, ω)-adversary, which is defined as follows.

Definition 3 (*(r, b, ω)-Adversary in Model $\mathcal{M}$*)**.** An adversary $\mathcal{A}$ is an (r, b, ω)-adversary if the type of failures is the one allowed in model $\mathcal{M}$ and the sequence of injections and failures of $\mathcal{A}$ satisfies the following restrictions. Those restrictions are defined by the injection rate r, with $0 < r < 1$, the burstiness $0 \leq b$, and the maximum failure life-time $\omega \geq 0$. For any edge e and any time interval I, the adversary must respect the equations

$$N_e(I) \leq \lceil r(|I| - D_e(I)) \rceil + b \quad \text{and} \quad (|I| = D_e(I)) \Rightarrow |I| \leq \omega. \tag{2}$$

Observe that, with the previous definition, an (r, b)-adversary in the AQT model is always an $(r, b, 0)$-adversary, just by skipping the possibility of producing failures.

Confronting such (r, b, ω)-adversaries, we will study the stability of different systems with different characteristics. Stability refers to the fact that the number of packets in the system remains bounded as the system dynamically evolves in time. This bound, which can be a function of the system parameters, is not dependent on time. This desirable property is studied in relation to the three main components forming a system $(\mathcal{N}, \mathcal{A}, \mathcal{P})$: the network $\mathcal{N}$, the traffic pattern defined by the adversary $\mathcal{A}$ on $\mathcal{N}$, and the scheduling protocol $\mathcal{P}$.

Definition 4 (*System in Model $\mathcal{M}$*)**.** We say that $S = (\mathcal{N}, \mathcal{A}, \mathcal{P})$ is a *valid* system in model $\mathcal{M}$ if $\mathcal{A}$ is an (r, b, ω)-adversary in model $\mathcal{M}$, for some r, $0 < r < 1$, and some b, $\omega \geq 0$, that controls the type of failures allowed in model $\mathcal{M}$.

The evolution over time of a system under a model is fully determined by the model, i.e., by the queue management, accessibility of the queues, the existence of additional buffers, and the restrictions that the model puts on the allowed failure types, link, edges, or both. Thus, apart from the components of a system, the model in which the system is considered determines the contents of the queues (and the buffers, if any) at any time step.

Now, let us state the main definitions of stability. These stability concepts will be used in the remainder of the paper. We start by describing the most general notion of stability, i.e., the stability of a system in a certain model.

Definition 5 (*System Stability*)**.** Let $S = (\mathcal{N}, \mathcal{A}, \mathcal{P})$ be a valid system in model $\mathcal{M}$. S is stable in model $\mathcal{M}$ if the number of packets in the system (i.e., the number of packets in all the queues of the network) remains upper bounded by a time-independent constant $c(r, b, \omega, \mathcal{N})$ as the system dynamically evolves in time according to $\mathcal{M}$.

We formally define now the property of network stability in a certain model. In this notion of stability, the network and a rate r (with $0 < r < 1$), are fixed.

Definition 6 (*Network r-stability Under a Protocol $\mathcal{P}$*)**.** A network $\mathcal{N}$ is *r-stable* under protocol $\mathcal{P}$ in model $\mathcal{M}$, if for any $b, \omega \geq 0$ and any (r, b, ω)-adversary $\mathcal{A}$ for $\mathcal{N}$ in model $\mathcal{M}$, the system $(\mathcal{N}, \mathcal{A}, \mathcal{P})$ is stable in model $\mathcal{M}$.

A stronger concept is that of *universal stability*. In broad terms, a network $\mathcal{N}$ is said to be universally stable if the number of packets in the system is bounded whatever adversary $\mathcal{A}$ is considered and whatever protocol $\mathcal{P}$ is used for scheduling packets at the edges.

Definition 7 (*Network Universal Stability*)**.** A network $\mathcal{N}$ is universally stable in model $\mathcal{M}$ if it is r-stable in model $\mathcal{M}$ under any greedy protocol and for any $0 < r < 1$.

Like for networks, the concept of universal stability can be applied to protocols. In broad terms, a protocol $\mathcal{P}$ is said to be universally stable if the system $S = (\mathcal{N}, \mathcal{A}, \mathcal{P})$ is stable for every $\mathcal{N}$ and $\mathcal{A}$. The following is a more formal definition of this concept.

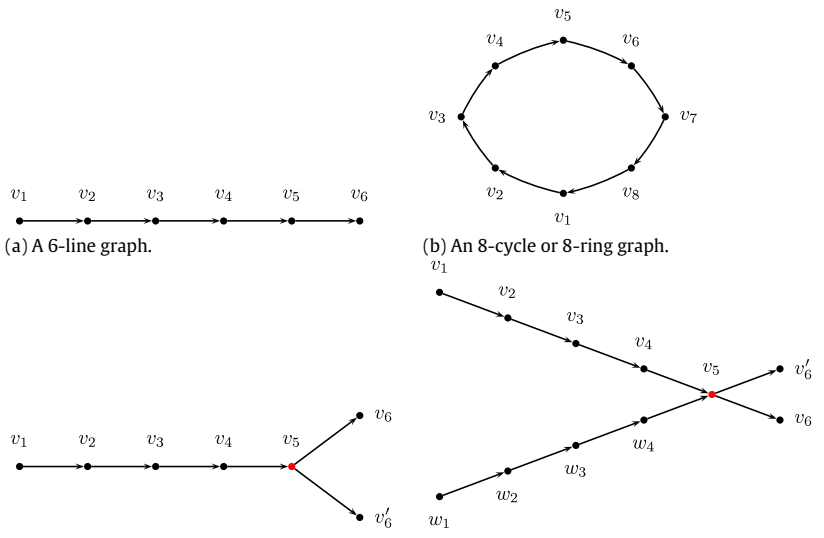
Definition 8 (*Protocol Universal Stability*)**.** A protocol $\mathcal{P}$ is universally stable in model $\mathcal{M}$ if, for any network $\mathcal{N}$ and any $0 < r < 1$, $\mathcal{N}$ is r-stable under protocol $\mathcal{P}$ in model $\mathcal{M}$.

As much as possible, it is always useful and interesting to infer new knowledge about the stability of some system, network or protocol from the already existing knowledge about some other systems, networks or protocols. In this section, we describe how systems can be simulated by other systems, which, as we will see, will be a useful tool for inferring some of the stability results that appear later in this work.

Definition 9 (*System Simulation*)**.** Let $S = (\mathcal{N}, \mathcal{A}, \mathcal{P})$ be a system in model $\mathcal{M}$, and let $S' = (\mathcal{N}, \mathcal{A}', \mathcal{P}')$ be a system in model $\mathcal{M}'$. The system S is *simulated* by the system S' if all the following conditions hold:

- when $\mathcal{A}$ injects at time t a packet p with route ρ in the system S, the adversary $\mathcal{A}'$ injects a corresponding packet p' at the same time t with the same route ρ in the system S',
- every time t that, according to $\mathcal{P}$, the packet p crosses the edge e of $\mathcal{N}$ in the system S, the packet p' crosses in S', according to $\mathcal{P}'$, the same edge e of $\mathcal{N}$ at the same time t.

It is possible to define a more generic definition of system simulation, however the previous one is enough for the system simulations used in the paper. Observe that when a system S can be simulated by another system S', the two systems have the same network and furthermore the number of packets present in the system S at time t is upperbounded by the number of packets present at time t in system S'. Therefore, we get



(a) A 6-line graph.
(b) An 8-cycle or 8-ring graph.
(c) A 5-fork graph; v_5 is its forking node.
(b) A 5-crossing graph; v_5 is its crossing vertex.

Fig. 3. Examples of line, fork, crossing, and ring graphs.

Lemma 1. *Let $S = (\mathcal{N}, \mathcal{A}, \mathcal{P})$ be a system in model $\mathcal{M}$ that can be simulated by a system $S' = (\mathcal{N}, \mathcal{A}', \mathcal{P}')$ in model $\mathcal{M}'$. If the system S' is stable in model $\mathcal{M}'$, then system S is stable in model $\mathcal{M}$.*

Since any (r, b)-adversary in the AQT model is an $(r, b, 0)$-adversary in a faulty adversarial model, the system behavior in a faulty adversarial model when no failure occurs will be the same as in the AQT model. Thus, for any of the faulty adversarial models $\mathcal{M}$ introduced in this paper, we can show trivially that

Fact 1. *Any system that is not stable in the* AQT *model, remains not stable in model $\mathcal{M}$.*

Finally we need some notation and definitions for directed graphs. Given a network $\mathcal{N}$ whose topology is described by the graph G,[2] we denote as $V(G)$ and $E(G)$ the sets of nodes and edges of G, respectively. As usual, an edge $e \in E(G)$ in a directed graph G is represented by the two endpoint vertices that define it, i.e., $e = (u, v)$, where $u \in V(G)$, is said to be the *head* of the edge, and $v \in V(G)$ is said to be its *tail*. This representation implicitly defines also the orientation of the edge; thus, the edge $e = (u, v)$ is oriented from u to v, i.e., from its head node to its tail node. The output degree of a vertex $v \in V(G)$ is the number of edges whose head node is v; on the contrary, its input degree is the number of edges whose tail node is v.

With this widely-used graph notation in mind, we introduce in the following some of the particular network topologies that will be mentioned during the rest of this work. Basically, the only topologies we need to distinguish are cycles and some specific types of directed acyclic graphs.

Definition 10 (*Simple Dags*)**.** Let G be a connected directed graph, and let $n > 1$.

(i) G is an n-cycle (or n-ring) graph if it has n vertices and n edges organized in the following way: $G = (\{v_1, \ldots, v_n\}, \{(v_i, v_{i+1}) \mid 1 \leq i < n\} \cup \{(v_n, v_1)\})$.
(ii) G is an n-line graph if it is obtained from an n-cycle graph, by removing one of its edges, i.e., $G = (\{v_1, \ldots, v_n\}, \{(v_1, v_2), \ldots, (v_{n-1}, v_n)\})$.
(iii) G is an n-fork graph if it is obtained from an n-line with two additional end vertices, in the following way: $G = (\{v_1, \ldots, v_n, v_{n+1}, v'_{n+1}\}, \{(v_1, v_2), \ldots, (v_{n-1}, v_n), (v_n, v_{n+1}), (v_n, v'_{n+1})\})$.
(iv) G is an n-crossing graph if it is obtained by joining an n-fork with an $(n-1)$-line in the following way: $G = (\{v_1, \ldots, v_n, v_{n+1}, v'_{n+1}, w_1, \ldots, w_{n-1}\}, \{(v_i, v_{i+1}) \mid 1 \leq i < n\} \cup \{(w_i, w_{i+1}) \mid 1 \leq i < n-2\} \cup \{(v_n, v_{n+1}), (v_n, v'_{n+1}), (w_{n-1}, v_n)\})$.

Fig. 3 depicts some examples. In the following, we sometimes refer to line graphs, fork graphs, crossing graphs and rings to denote the whole family of n-line graphs, n-fork graphs, n-crossing graphs and n-rings, respectively, for every $n > 1$. Lines and cycles have the particularity that the input and output degree of their vertices is at most one. As we will see, this fact will provoke the dynamics of systems with such topologies of behave alike. Fork graphs and crossing graphs have only one vertex with out degree two; we will refer to this vertex as the *forking*, respectively *crossing vertex*.

[2] When the context is clear enough and we want to focus on some property of the topology G of a network $\mathcal{N}$, we will commit an abuse of terminology and refer to G as the network.

3. Failures on links

In this section, we focus on the study of stability conditions in dynamic networks in which only the edges may fail. The constraints of an adversary in such faulty environments correspond to the restriction given in (2), in which case only links can be failed. We study the stability of networks and protocols when the management of those packets that suffer a failure is performed in different ways, depending on the accessibility of the queues. More precisely, we consider three situations.

(**type 1**) In spite of the failure of the link, the queue at the head of the link can still receive packets, in such a case packets are kept at the queue associated to the failed link.
(**type 2**) During a failure of a link the queue at its head is not accessible. In such a case, the packets have to wait in their previous location.
(**type 3**) During a failure of a link the queue at its head is not accessible, but there is a buffer at each node that can store those packets that want to traverse a failed outgoing link.

We call the different models arising under such assumptions the edge-R model (i), the edge-nR model (ii), and the edge-nR-B model (iii), respectively. Of course, in any of these models, we assume that a failed edge cannot transmit any packet, and thus no packet leaves the queue of a link during its failure.

3.1. The edge-R model

We consider the first model in which during a failure of a link e the packets wait to traverse e in its associated queue. Packets from incoming links and/or from the excess of the burstiness[3] might arrive to e since its queue is accessible. However, the arriving packets will wait in the queue until the edge recovers. When the link recovers, its queue always advances a packet if there is one. This behavior is depicted in Fig. 4(b).[4] Observe that this coincides with the failure management under the failure model introduced in [2] and, in the case of adversaries that do not create any failure, it coincides also with the behavior in the AQT model.We propose the edge-R model, in which the network has receiver links like the ones just described.

We will show that the edge-R model can be simulated by the *failure model* introduced in [2], in which the injection rate strongly constrains both the maximum number of failures and the maximum number of packet injections per edge. The management of a failure is the same in both models, i.e., they both manage failures with links that can receive packets although being failed. However, the number of injections and failures that an (r, b)-adversary in the failure model can produce is restricted to

$$N_e(I) + F_e(I) \leq r|I| + b.$$

Observe that this expression already restricts the duration of failures by itself, without the necessity of fixing an explicit upper bound for it. Observe also that, given a concrete injection rate r, our edge-R model allows more powerful adversaries than the failure model, both in terms of failure production and in terms of packet injection. We can establish the following relation.

Theorem 2. *Given a network $\mathcal{N}$, any (r, b)-adversary $\mathcal{A}$ for $\mathcal{N}$ in the failure model is an $(r, b, \lceil b/(1-r)\rceil)$-adversary in the edge-R model.*

Proof. The restriction for the edge-R model can be easily obtained from the failure model restriction, taking into account that $0 < r < 1$:

$$N_e(I) + rF_e(I) < N_e(I) + F_e(I) \leq r|I| + b.$$

The restriction $N_e(I) + F_e(I) \leq r|I| + b$ implies also that $F_e(I) \leq r|I| + b$. Therefore, in a time interval of length t in which an edge is failed, it must hold that $t \leq rt + b$, which implies that $t \leq b/(1-r)$. □

Moreover, any adversary in the edge-R model is also an adversary in the failure model with an increased injection rate. Such an increase depends on ω.

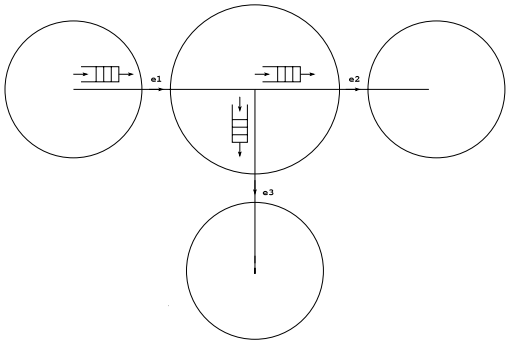
Theorem 3. *Given a network $\mathcal{N}$, any (r, b, ω)-adversary $\mathcal{A}$ in the edge-R model is an $(\frac{r+\omega}{\omega+1}, b)$-adversary in the failure model.*

Proof. Recall that ω bounds the number of consecutive steps in which any edge can be down. Since the duration of a failure in any edge e is bounded, the maximum number of failures that can occur to e in the edge-R model at any interval of time I is $\frac{\omega}{\omega+1}|I|$. Then, we can rewrite the constraint

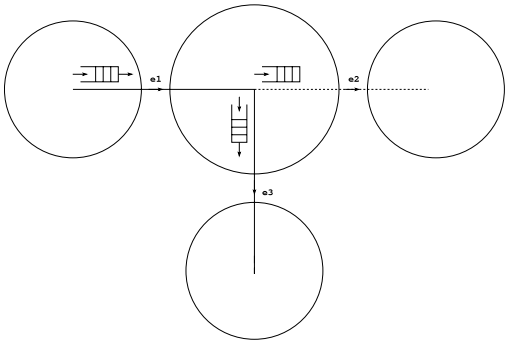
$$N_e(I) \leq r(|I| - F_e(I)) + b$$

[3] Note that, when an edge e is failed during an interval I, then $N_e(I) \leq b$ packets can still be injected.

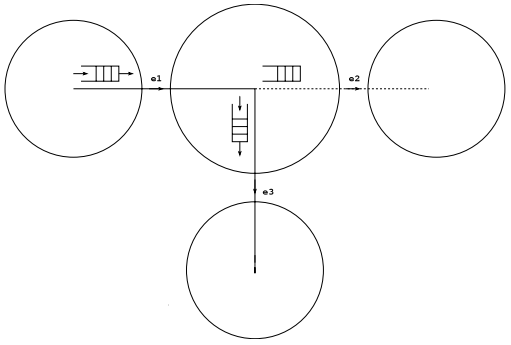
[4] With the aim of making the figure more clean and comprehensive, we depict the queue associated to each link right over the link (instead of at the head of the link, where it really is).



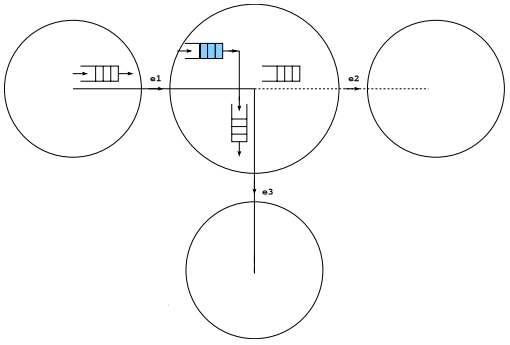
(a)No failure occurs: Packets incoming to the central node from $e1$ are scheduled to the queue at $e2$ or $e3$ depending on which is the next destination node in their path.



(b) Failure under the edge-R model: When edge $e2$ fails, its queue at the central node can still receive the packets addressed to $e2$, but it cannot transfer them forward. As soon as $e2$ recovers, its queue can forward packets normally again.



(c) Failure under the edge-nR model: When edge $e2$ fails, its queue at the central node can neither receive nor forward packets. In consequence, those packets addressed to $e2$ must be kept in the previous node. As soon as $e2$ recovers, its queue can again receive and forward packets normally.



(d) Failure under the edge-nR-B model: When edge $e2$ fails, its queue at the central node can neither receive nor forward packets. Those packets addressed to $e2$ are kept in the extra buffer of the node. As soon as $e2$ recovers, its queue can again receive packets (both from the incoming edge $e1$ and from the extra buffer) and forward packets normally.

Fig. 4. Accessibility of the queues under the different edge-failure models.

for an (r, b, ω)-adversary $\mathcal{A}$ in the edge-R model as

$$N_e(I) + F_e(I) \leq r|I| - rF_e(I) + F_e(I) + b;$$

therefore,

$$\begin{aligned} N_e(I) + F_e(I) &\leq r|I| - rF_e(I) + F_e(I) + b \\ &= r|I| + (1-r)F_e(I) + b \\ &\leq r|I| + ((1-r)\omega/(\omega+1))\,|I| + b \\ &= ((r+\omega)/(\omega+1))\,|I| + b. \end{aligned}$$

This expression corresponds to the restriction that applies to an (r', b)-adversary in the failure model, in which $r' = (r+\omega)/(\omega+1)$ and for which it holds that $0 < r' < 1$. □

Since the behavior of the queues and the systems in the failure model in [2] and in the edge-R model is the same, we have (from Theorems 2 and 3) that any valid system $S = (\mathcal{N}, \mathcal{A}, \mathcal{P})$ in the edge-R system is also a valid system in the failure model, and vice versa. Furthermore, at any time step, the number and position of any packet in S in the edge-R model is the same as in S in the failure model. Thus, the failure model and our edge-R model are equivalent from the point of view of universal stability of networks and protocols. Therefore, according to the equivalence between the failure and the AQT models, and to Theorem 2 (network universal stability equivalence among models) shown in [2], we can state the following corollary.

Corollary 4. *Directed acyclic graphs and rings are universally stable in the edge-R model.*

From the results in Theorems 4, 6, 8 (universal stability of SIS, FTG and NFS in different models) and 9 (universal instability of LIS) in [2], the following two corollaries can also be stated.

Corollary 5. *A network $\mathcal{N}$ is universally stable in the edge-R model if, and only if, $\mathcal{N}$ is universally stable in the* AQT *model.*

Corollary 6. SIS, FTG, *and* NFS *are universally stable in the edge-R model. However,* FIFO, LIFO, NTG, FFS, *and* LIS *are not universally stable in the edge-R model.*

In this last corollary, the protocol LIS is put in a different category with respect to the AQT adversarial model for non-faulty systems, where LIS was universally stable. This is due to the instability of LIS under the failure model, which was shown in [2]. The LIQ protocol remains unclassified, as this protocol is universally stable in the AQT model [13] and not known to be universally stable in the failure model. We can show that LIQ is not universally stable in the failure model.

Lemma 7. LIQ *is not universally stable in the failure model.*

The proof of this lemma is, as is the case for most of the proofs of instability in this work, based on induction. A set of rounds compose a step of the induction reasoning. The goal is to demonstrate that the number of packets in the system can increase from step to step (and, by applying the inductive hypothesis, they can increase infinitely). The configuration of the system at the end of every step must be the same as at the beginning (in terms of the type and the location of the packets). For the sake of simplicity, in those proofs we usually only reproduce the inductive step and sometimes we omit some additive constants in our analysis, which never change the final result.

In order to enhance the readability of the paper, we pushed most the proofs for instability based on induction to the end of the paper. This is the case for the proof of Lemma 7.

It is the case that, inductively using the same adversary given in the proof of Lemma 7, we can also show the instability under LIS. Therefore, taking into account Theorem 2, we can conclude the following result, whose proof is also to be found at the end of the paper.

Corollary 8. LIQ *and* LIS *are not universally stable in the edge-R model.*

3.2. The edge-nR model

Let us take now into consideration the case in which during a link failure the associated queue is not accessible. This fact forces that no packet can be injected requiring a failed edge. Moreover packets directed to an edge $e = (v, w)$ which come from an incoming link $e' = (u, v)$ will not be able to join the queue of e since we consider it is not accessible. They will stay in the queue of e'. In order to avoid losing packets, the packets in an active queue are classified as blocked or not. A packet is *blocked* if the link connecting to its next destination is down. The queuing policy applied to such a queue type will select one packet among those that are not blocked. Blocked packets remain in the queue. This behavior is depicted in Fig. 4(c).

We propose the edge-nR model, in which the network has non-receiver links like the ones just described. In this model, we show that there are very simple topologies, like line graphs, that turn out not to be universally stable. Observe that in this model when a packet cannot traverse a link it remains in the queue of the previous link and so it competes with a different set of packets in subsequent steps.

In the following we show that line graphs are not stable in the edge-nR model under SIS, LIFO, NTG, NFS, and FFS.

Theorem 9. *For every injection rate $0 < r < 1$, there is an $n_0 > 1$ such that any n-line graph with $n \geq n_0$ is not r-stable in the edge-nR model under protocols* SIS, LIFO, NTG, NFS, *and* FFS.

Furthermore, for the FTG protocol there are very simple directed acyclic graphs which are not stable. In the following we show that networks with topologies describing some forking can be made unstable.

Theorem 10. *For every injection rate $0 < r < 1$ there is an $n_0 > 1$ such that any n-fork graph with $n \geq n_0$ is not r-stable in the edge-nR model under* FTG.

From the previous Theorems 9 and 10, Corollary 11 follows.

Corollary 11. *Directed acyclic graphs and rings are* not *universally stable in the edge-nR model. More precisely, there are directed acyclic graphs which are not stable under protocols* SIS, LIFO, NTG, NFS, FFS *and* FTG.

All these protocols can bring the system to instability because they allow the accumulated packets to be rescheduled in such a way that old queued packets are kept on as blocked, while newer ones have priority over them. On the contrary, the intrinsic ordered nature of the protocols FIFO and LIS makes it difficult to accumulate packets. Although we suspect that acyclic topologies and even rings are stable under FIFO, LIS, and LIQ, these questions are left as open in this work. Nevertheless, we can show that LIS and LIQ are not universally stable protocols by using a simple cyclic topology, the graph given in Fig. 7.

Lemma 12. LIS *and* LIQ *are not universally stable in the edge-nR model.*

3.3. The edge-nR-B model

Finally, we consider a mixed situation in which the queue of a failed link is not accessible, but in which the packets that cannot join it are kept in an additional buffer placed at the head node of the link. This buffer is shared by all the links incident at the same head node (i.e., there is only one buffer per node), thus acting as an intermediate buffer for packets that have to continue to the queue of a link that was failed when they arrived. We assume that the buffer at every node is always accessible and always active. Moreover, the packets at the buffer are tagged with its state (*blocked* packet or *non-blocked* packet). Thus if a packet p traverses the link $e_1 = (u, v)$, and next p has to traverse $e_2 = (v, w)$ but e_2 goes down, then p is kept in the buffer at node v. It will be considered as a blocked packet while e_2 is failed, and it will change its state to non-blocked as soon as e_2 becomes alive again. Once the failed link is alive, its blocked packets become non-blocked. Then, according to the scheduling policy of the system, one packet among all those set to non-blocked is chosen and put instantaneously in its corresponding output queue. That behavior is repeated as long as there are non-blocked packets in the extra buffer. Thus, one non-blocked packet (but only one) will leave the extra buffer in one time step. This behavior is depicted in Fig. 4(d).

We propose the edge-nR-B model, in which the network has non-receiver links with extra buffers like the ones just described. In this model, we study a specific subset of networks: those whose vertices all have output degree at most one. This includes line graphs and ring graphs. In this case, the buffer in a node v of the graph will (only) store the packets that cannot be served towards the queue of the (unique) outgoing edge (v, w). It seems that, in this case, the edge-nR-B model has some similarities with the edge-R model. The main idea behind this is that each queue in the edge-R model can store the packets of the queue of the corresponding edges as well as the packets in the extra buffer of its source node.

Theorem 13. *Let $\mathcal{N}$ be a network in which all the nodes have output degree at most one. Any system $S = (\mathcal{N}, \mathcal{A}, \mathcal{P})$ in the edge-nR-B model can be simulated by a system $S' = (\mathcal{N}, \mathcal{A}, \mathcal{P}')$ in the edge-R model.*

Proof. When no failures occur, the protocol $\mathcal{P}' = \mathcal{P}$ makes the predicate hold trivially. When failures occur in the system S, the packets requiring to traverse any failed edge $e = (u, v) \in E(\mathcal{N})$ are kept in the extra buffer at node u. We simulate this system $S = (\mathcal{N}, \mathcal{A}, \mathcal{P})$ by $S' = (\mathcal{N}, \mathcal{A}, \mathcal{P}')$ in the edge-R model; the greedy protocol $\mathcal{P}'$ will be defined during the proof.

For system S, let us denote as $Q_u(t)$ and $Q_e(t)$ the contents at time t of the extra buffer at node $u \in V(\mathcal{N})$ and the queue corresponding to edge $e \in E(\mathcal{N})$, respectively. By $Q'_e(t)$ we refer to the queue at the same time t and the same edge e, but in the system S'. Protocol $\mathcal{P}'$ is defined in such a way that whenever link e is not failed and $Q'_e(t) = Q_u(t) \cup Q_e(t)$, the packet that traverses e is the same one as selected by $\mathcal{P}$. We describe for each time step t the behavior of S' simulating S, and we show by induction on t that each edge $e = (u, v)$ satisfies that:

- $Q'_e(t) = Q_e(t) \cup Q_u(t)$, and
- if p crosses in S the link e of $\mathcal{N}$ at time t, the corresponding packet crosses in S' the same link e at the same time t.

Let us consider the behavior of edge e at time step $t + 1$ in both systems in the following two cases.

1. e is alive. At time step t, the queue associated to e may receive three different sets of packets:
   - a set of packets L_e scheduled by the queues of the incoming edges,
   - a set of packets I_e injected by the adversary, and
   - a set of packets P_u coming from the buffer.

   Notice that P_u is either empty, when $Q_u(t) = \emptyset$, or contains only one packet, otherwise. After receiving these packets, it applies the protocol $\mathcal{P}$ and selects a packet p_e^t in order to be scheduled. Hence, $Q_e(t + 1) = (Q_e(t) \cup L_e \cup I_e \cup P_u) \setminus \{p_e^t\}$ and $Q_u(t + 1) = Q_u(t) \setminus P_u$.
   In the system S', the queue of edge e receives the same packets as in S: L_e by induction hypothesis and I_e because the adversaries are identical. Now, since by induction hypothesis $Q'_e(t) = Q_e(t) \cup Q_u(t)$ and e receives the same new packets, protocol $\mathcal{P}'$ can select the same packet p_e^t for being scheduled. Then we have that $Q'_e(t + 1) = (Q'_e(t) \cup L_e \cup I_e) \setminus \{p_e^t\} = (Q_e(t) \cup Q_u(t) \cup L_e \cup I_e) \setminus \{p_e^t\} = Q_e(t + 1) \cup Q_u(t + 1)$.

2. e is failed. In this case the queue of edge e in system S cannot receive any packet. If a packet $p_e^t \in L_e$ is scheduled to e at time step t, then it is stored in the extra buffer at node u at time step $t + 1$. If the adversary injects a set of packets I_e at time step $t + 1$ they are also stored in the extra buffer at u. Then we have that $Q_e(t + 1) = Q_e(t)$ and, as u has output degree 1, $Q_u(t + 1) = Q_u(t) \cup L_e \cup I_e$.
   In the system S', the queue of e cannot schedule any packet but it receives the same packets as the queue of u in S, packet p_i injected by the adversary and packet $p_e^t \in L_e$ scheduled at time step t. Then we have that $Q'_e(t + 1) = Q'_e(t) \cup L_e \cup I_e = Q_e(t) \cup Q_u(t) \cup L_e \cup I_e = Q_e(t + 1) \cup Q_u(t + 1)$.

Observe that the contents of the queue corresponding to edge $e = (u, v)$ in the system S' are the same as the sum of contents of the extra buffer at the node u plus the queue for the edge e in the system S'. The quantity of packets does not vary, but their location does. This applies for every edge e and time. □

As the following theorem proves, this simulation is in fact two sided.

Theorem 14. *Let $\mathcal{N}$ be a network in which all the nodes have output degree at most one. Any system $S = (\mathcal{N}, \mathcal{A}, \mathcal{P})$ in the edge-R model can be simulated by a system $S' = (\mathcal{N}, \mathcal{A}, \mathcal{P}')$ in the edge-nR-B model.*

Proof. When no failures occur, the protocol $\mathcal{P}' = \mathcal{P}$ makes the predicate hold trivially. When failures occur in the system S, the packets requiring to traverse any failed edge $e = (u, v) \in E(\mathcal{N})$ are kept in the queue of edge e. We simulate this system $S = (\mathcal{N}, \mathcal{A}, \mathcal{P})$ in the edge-R model by the system $S' = (\mathcal{N}, \mathcal{A}, \mathcal{P}')$ in the edge-nR-B model. Protocol $\mathcal{P}'$ is defined in such a way that the same packets requiring to traverse e would be kept either in the queue of e or in the extra buffer at u. As the output degree of u is one, all the packets in those queues must traverse e, we define $\mathcal{P}'$ in such a way that at any time step the packet selected to traverse e is the same selected by $\mathcal{P}$. Taking this into account, the rest of the proof is similar to that of Theorem 13. □

Combining these theorems with Corollary 4, we can state the following result.

Corollary 15. *Any graph in which all the nodes have output degree at most one (this includes line graphs and rings) is universally stable in the edge-nR-B model.*

However, a significant difference appears when considering nodes with output degree greater than one, e.g., when considering forks. In the following we show that for every injection rate, we can find a fork which is not stable under several of the usual greedy protocols, namely LIFO, SIS, NTG, FTG, NFS, and FFS.

Theorem 16. *For every injection rate $0 < r < 1$, there exists an $n_0 > 0$ such that the n-fork graph, for $n \geq n_0$, is not r-stable in the edge-nR-B model under any of the following protocols:* LIFO, SIS, NTG, FTG, NFS, *and* FFS.

In contrast to the edge-nR model, even if the queues of the failed links are not accessible, the packets requiring those links are kept in a different queue, i.e., the extra buffer. However, this fact does not imply significant changes in the networks which are stable. Already in networks whose nodes have output degree one, situations similar to those in the edge-nR model are obtained.

By considering the graph and adversary used for the proof of Lemma 7, we can also show that the protocols LIS and LIQ are not universally stable in the edge-nR-B model. Therefore, we have that

Lemma 17. LIS *and* LIQ *are not universally stable in the edge-nR-B model.*

4. Failures on nodes

In this section, we consider dynamic networks in which the nodes may fail. All the models presented in this section impose the restriction that only nodes can fail. We study the stability of networks when the management of those nodes that suffer a failure is performed in different ways. More precisely, we take into consideration the possibility that the nature of the failure affects the transmission and/or reception of packets. This feature leads us to consider three types of node failure management.

(**type 1**) During a node failure nodes are unable to transmit and to receive packets.
(**type 2**) During a node failure nodes are unable to transmit but able to receive packets.
(**type 3**) During a node failure nodes are able to transmit but unable to receive packets.

The type 1 of failure is the strongest one; under this type of failure packets do not arrive and leave, therefore it is natural to consider a queueing system in which all the queues of the outgoing links of a node are not accessible during a failure. Observe that in such a case packets whose next link to traverse has a failed head node must wait

(i) in the queue of the previous link.

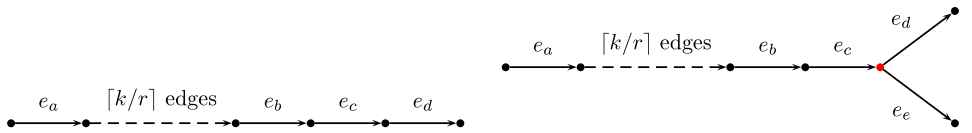
In the type 2 of failure packets are not leaving from the node, but can arrive. Thus, in the associated queuing systems the packets arriving to the node can be kept

(ii) in the queue associated to the edge, or
(iii) in an extra buffer at the node. Here we assume that even though the node can receive its output link queues are not accessible.

The type 3 of failures leave us with a failed node which is unable to receive but it can still transmit. Then those packets that have to arrive to the failed node cannot follow their next link and are kept

(iv) in the queue of the previous link, while packets at a failed node can be transmitted provided that the next node is not failed.

Each of these options will be considered in the adversarial models we present in this section. Option (ii) will characterize the node-RnT model, while option (iii) will define the node-nRnT-B model. Option (i) is used in the node-nRnT model and, finally, option (iv) will be considered in the node-nRT model.



(a) $(5 + \lceil k/r \rceil)$-line graph. (b) $(4 + \lceil k/r \rceil)$-fork graph.

Fig. 5. The line graphs and fork graphs used in Theorems 9 and 10.

4.1. The node-RnT model

We consider the first type of node failure management in which a failure of a node v means that the packets wait to traverse v in their corresponding output queue. Packets from incoming links and/or from the excess of the burstiness might arrive to v since its queue is accessible. However, the arriving packets will wait in the corresponding output queue until the node recovers. When the node recovers, every outgoing queue advances a packet if there is one. This behavior is depicted in Fig. 6(b).[5] We consider now the node-RnT model, in which the network has receiver non-transmitter nodes.

Using the fact that a node failure represents somehow the failure of all its outgoing edges, we will show that the node-RnT model can be simulated by the edge-R model. For doing so we have to take into account that the management of a failure is the same in both models, i.e., they both manage failures with receiver links. However, the adversaries in the edge-R model are more powerful as they can make one outgoing edge fail independently of the other outgoing edges.

Theorem 18. *Any system $S = (\mathcal{N}, \mathcal{A}, \mathcal{P})$ in the node-RnT model can be simulated by a system $S' = (\mathcal{N}, \mathcal{A}', \mathcal{P})$ in the edge-R model.*

Proof. The new adversary $\mathcal{A}'$ is obtained from $\mathcal{A}$ by keeping the same sequence of packet injections and replacing any failure of a node $u \in V(\mathcal{N})$ by the failure of all the outgoing edges of u. From the definition of the adversaries it follows that, for every link e and any time interval I, the amount $H_e(I)$ incurred by adversary $\mathcal{A}$ in system S coincides with the value $F_e(I)$ incurred by adversary $\mathcal{A}'$ in S'. Therefore, from the definition of the models it is straightforward to show that when $\mathcal{A}$ is an (r, b, ω)-adversary in model node-RnT then $\mathcal{A}'$ is also an (r, b, ω)-adversary in model edge-R, and that the two systems act alike. □

With this result, any (positive) network or protocol stability result from the edge-R model can be transferred to the node-RnT model. Thus, from Corollary 4 we obtain the following result.

Corollary 19. *Directed acyclic graphs and rings are universally stable in the node-RnT model.*

Corollary 5 establishes that the set of networks that are universally stable in the edge-R model coincide with the set of networks that are universally stable in the AQT model. Therefore, taking into account Fact 1, we obtain

Corollary 20. *A network $\mathcal{N}$ is universally stable in the node-RnT model if, and only if, $\mathcal{N}$ is universally stable in the* AQT *model.*

From the results in [3], we also know that testing the property of universal stability in networks can be done in polynomial time. Thus we can state that

Corollary 21. *The universal stability of a given network in the node-RnT model can be decided in polynomial time.*

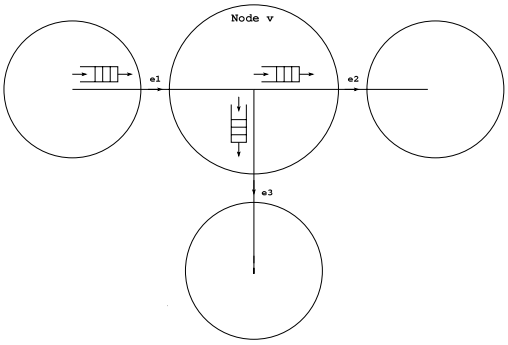
For protocols, we have a similar scenario. Taking into account Fact 1, any instability result for protocols in the AQT model extends to the node-RnT model and, taking into account Theorem 18, any universal stability result in the edge-R model extends to the node-RnT model. We then get

Corollary 22. SIS, FTG, *and* NFS *are universally stable in the node-RnT model, while* FIFO, LIFO, NTG, *and* FFS *are not universally stable in the node-RnT model.*

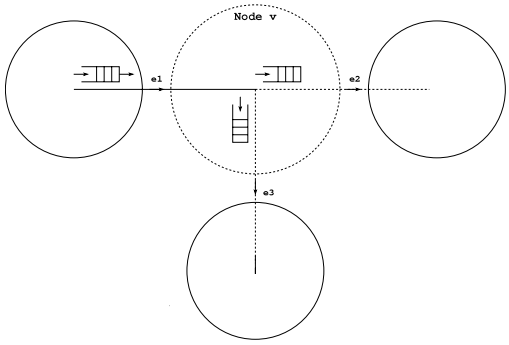
Only LIS and LIQ remain unclassified as those protocols are universally stable in the AQT model but not universally stable in the edge-R model. The following lemma shows that they are not universally stable in the node-RnT model.

Lemma 23. LIS *and* LIQ *are not universally stable in the node-RnT model.*

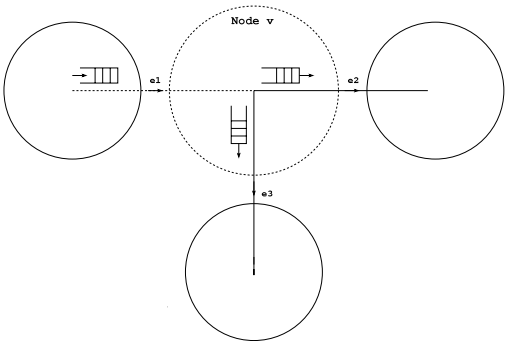
[5] With the aim of making the figure more clean and comprehensive, nodes are depicted now as bigger blocks, with a dispatcher switch that puts every incoming packet in the corresponding output queue. The queue for each outgoing link is depicted now inside the node, as it really is.



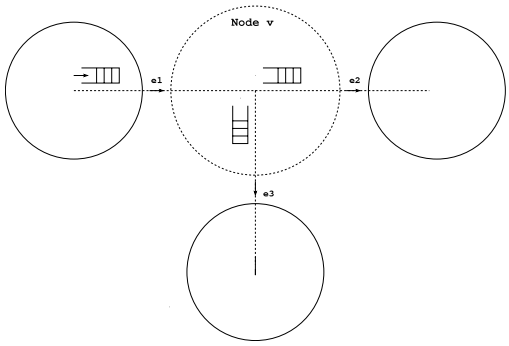
(a) No failure occurs: Packets incoming to the central node v from $e1$ are scheduled to the queue at $e2$ or $e3$ depending on which is the next destination node in their paths.



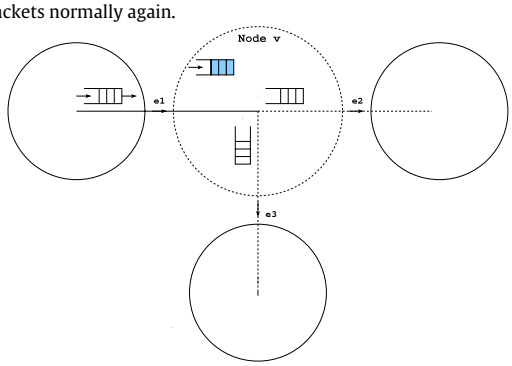
(b) Failure under the node-RnT model: When node v fails, its queues can still receive the packets addressed to its outgoing edges, however none of the queues can transfer packets forward. As soon as node v recovers, all the queues in it can again forward packets normally.



(c) Failure under the node-nRT model: When node v fails, its queues can still forward the packets queued in them through their corresponding outgoing edges, however none of the queues can receive new packets. In consequence, those packets that need to traverse node v must be kept in the previous node. As soon as node v recovers, its queues can receive packets normally again.



(d) Failure under the node-nRnT model: When node v fails, its queues can neither receive nor forward packets. In consequence, those packets that need to traverse node v must be kept in the previous node. As soon as node v recovers, its queues can receive packets normally again.



(e) Failure under the node-nRnT-B model: When node v fails, its queues can neither receive nor forward packets. Those packets that need to traverse node v are kept in the extra buffer of the node. As soon as node v recovers, its queues can again receive packets (both from the incoming edge $e1$ and from the extra buffer) and forward packets normally.

Fig. 6. Accessibility of the queues under the different node-failure models.

4.2. The models with non-receiving nodes

The common property of all the models we consider in this section is that, during the failure of a node, the queues can not receive packets. In other words, they are not accessible during the failure. Under this assumption, two scenarios need to be studied: (a) the case where, in spite of being unaccessible for new packets, the queues can forward the packets stored in

them (see Fig. 6(c)), and (b) the case where they are also blocked for transmission (see Fig. 6(d)). In this latter case, if a node v is failed the packets wanting to cross an incoming link (u, v) must wait at the output queue at u until v recovers.

According to these cases, we propose two models, the node-nRT model, in which the network has non-receiver but transmitter nodes, and the node-nRnT model, in which the network has non-receiver non-transmitter nodes. We will consider also the node-nRnT-B model, a variation of the latter in which u keeps in an extra buffer the packets that can not be transferred to v, rather than keeping them in its output queue. In this model, there is only one additional buffer per node, which is shared by all the packets that at some time step could not be forwarded from v. As before, we assume that the extra buffers at the nodes are always accessible and always active.

In relation to the buffered models presented earlier in this work, there is a significant difference concerning the state of the packets. Since the element failing is now the node, and since a failed node does not transmit any packet, all the packets stored in the buffer will always be tagged as *blocked* (while the node is failed) or as *non-blocked* (while the node is alive). Somehow, the failure of a node implies the failure of all the outgoing links. In those periods in which the node is not failed, the node queue policy will select one packet among the packets in the buffer and put it instantaneously in its corresponding output queue. Once there, it will compete with the other packets in the output queue for being forwarded. This behavior is depicted in Fig. 6(e).

As a first result we show that, when non-buffered failed nodes cannot receive packets, every network can be made unstable. This bad behavior is due to the fact that the restrictions imposed by the model on the adversaries do not take into account the failures at the tail of the link.

Theorem 24. *Any network with at least one link can be made unstable in the node-nRnT and the node-nRT models.*

Proof. We show that for any r, $0 < r \leq 1$, the 2-line graph is not r-stable under any greedy protocol. Consider a 2-line graph, and let us denote as (v_1, v_2) its unique edge. Let us assume that initially there are q packets stored in the queue at v_1 whose destination is the next adjacent vertex v_2. An adversary playing the following round indefinitely can make the network unstable for any of the usual greedy protocols.

Round 1: For q time steps, the adversary injects rq packets at v_1, whose destination is v_2. During that time, the vertex v_2 is failed. This failure makes the packets stored at v_1 remain there.

Round 2: The vertex v_2 recovers for one time step.

At the end of the second round, there will be $rq + q - 1$ packets stored in the queue at v_1 with destination v_2. By infinite repetitions of this pattern of injections and failures, this adversary would make the system unstable for any r such that $rq > 1$, as $r > 0$, we have that this happens for any $q > 1/r$. Thus for any required r, a sufficiently large initial load q can be found such that this inequality holds. Observe that, implicitly, $\omega = q$ and that this result is independent of the protocol used in the system and also independent of the length of the line. □

As a consequence of the previous theorem we have also that

Corollary 25. *There are no universally stable greedy protocols in either the node-nRnT model the node-nRT model.*

When extra reception buffers are considered, namely under the node-nRnT-B model, the situation changes. Let us first focus on a specific subset of networks: those whose vertices all have output degree at most one. This includes line graphs and ring graphs. In this case, it is easy to see that the failure of a node (in terms of both injections and buffer occupancy) has the same impact as a failure of the outgoing edge in the edge-nR-B model. As a consequence of this fact, we have the following result.

Theorem 26. *Let $\mathcal{P}$ be a greedy protocol and $\mathcal{N}$ a network in which all the nodes have output degree at most one. Any system $\mathcal{S} = (\mathcal{N}, \mathcal{A}, \mathcal{P})$ in the node-nRnT-B model can be simulated by a system $\mathcal{S}' = (\mathcal{N}, \mathcal{A}', \mathcal{P}')$ in the edge-R model.*

Proof. The new adversary $\mathcal{A}'$ is obtained from $\mathcal{A}$ by keeping the same sequence of packet injections and replacing any failure of a node u by the failure of the unique outgoing edge of u. From the definition of the adversaries, it follows that for every link e and any time interval I, the amount $H_e(I)$ incurred by adversary $\mathcal{A}$ in system S coincides with the value $F_e(I)$ incurred by adversary $\mathcal{A}'$ in S'. Therefore, from the definition of the models it is straightforward to show that when $\mathcal{A}$ is an (r, b, ω)-adversary in model node-nRnT-B then $\mathcal{A}'$ is also an (r, b, ω)-adversary in model edge-R.

We consider the protocol $\mathcal{P}'$ defined in such a way that, from any node u, the packet that is transmitted through the unique outgoing edge $e = (u, v) \in E(\mathcal{N})$ is selected by $\mathcal{P}$ when considering the packets present in the queue Q_e associated to e and the extra buffer Q_u. The sequence of injections and the definition of $\mathcal{P}'$ guarantees that at any time step the set of packets stored in the queue Q'_e associated to $e' \in E(\mathcal{N})$ coincides with $Q_e \cup Q_u$, and thus the simulation follows. □

As a consequence of the previous theorem we have that those networks which are universally stable in the edge-R model and whose vertices have output degree at most one remain universally stable in the node-nRnT-B model.

Corollary 27. *Line graphs and rings are universally stable in the node-nRnT-B model.*

However, we can show that in the node-nRnT-B model there are directed acyclic graphs other than line graphs that are not stable under any greedy protocol.

Theorem 28. *For every injection rate $\frac{1}{2} < r < 1$, there is an $n_0 > 1$ such that any n-crossing graph with $n \geq n_0$ is not r-stable in the node-nRnT-B model under any greedy protocol.*

Corollary 29. *There are directed acyclic graphs that are not universally stable in the node-nRnT-B model. In fact, there are directed acyclic graphs that are not stable in the node-nRnT-B model under any greedy protocol.*

5. Failures on both links and nodes

Until now, we have considered separately the case in which the communication network can only suffer link failures from the case in which the network can only suffer node failures. In this section we tackle the general case in which both link and node failures might occur in the system. The restrictions on an (r, b, w)-adversary in an adversarial model $\mathcal{M}$ are described by Eq. (2).

We consider combined faulty adversarial models in which the queue management for failed links implements one of the models presented in Section 3 (namely the edge-R, edge-nR or edge-nR-B model), and the queue management for failed nodes follows one of the models presented in Section 4 (namely the node-RnT, node-nRnT-B, node-nRnT or node-nRT model). Since both the model implemented at the links and the model implemented at the nodes apply over the same set of queues, we must carefully specify how their access to them is organized. When a node failure occurs (independently of whether some of the outgoing edges also fail), the system uses the adversarial model implemented by the node. If, on the contrary, a node does not fail but some of its outgoing edges do, then the system uses the adversarial model implemented by the links to deal with that failure.

When, in a combination of models, only the model for links (respectively, for nodes) is buffered, then the extra buffer at the nodes is used only to store the packets suffering the failures of the links (respectively, of the nodes). In the combined (edge-nR-B,node-nRnT-B) model, in which both the model for links and the model for nodes are buffered, we consider that the extra buffer at each node is also unique, and thus the buffer is shared by both models.

Table 1 summarizes the results obtained concerning the universal stability of rings and directed acyclic graphs, and the universal stability of the protocols SIS, NFS, FTG, LIS and LIQ for the models presented in Section 3. Table 2 summarizes the results obtained concerning the universal stability of the same networks and protocols for the models presented in Section 4.

Table 1
Universal stability property in the combination of adversarial models in which the nodes do not fail, i.e., they implement the AQT model, but the links do. The symbol $\surd$ means that the network (respectively, the protocol) is universally stable, while the symbol $\times$ means it is not. The symbol other-USP represents the other AQT universal stable protocols mentioned in this paper, i.e. SIS, NFS, and FTG.

Models	edge-R	edge-nR-B	edge-nR
non-faulty node	ring $\surd$	ring $\surd$	ring $\times$
	dags $\surd$	dags $\times$	dags $\times$
	LIS, LIQ $\times$	LIS, LIQ $\times$	LIS, LIQ $\times$
	other-USP $\surd$	other-USP $\times$	other-USP $\times$

Table 2
Maintenance of the universal stability property in faulty adversarial models. The symbols have the same meanings as in Table 1. The results for the combination of non-faulty nodes and non-faulty edges are omitted, since they are the same as in Table 1.

Models	node-RnT	node-nRnT-B	node-nRnT node-nRT
non-faulty edge	ring $\surd$	ring $\surd$	ring $\times$
	dags $\surd$	dags $\times$	dags $\times$
	LIS, LIQ $\times$	LIS, LIQ $\times$	LIS, LIQ $\times$
	other-USP $\surd$	other-USP $\times$	other-USP $\times$

It is clear that a system with a combination of models, in which at least one of them can produce some type of instability, can automatically suffer from the same instability. Thus, those results on instability that we obtained in Sections 3 and 4 (see Tables 1 and 2) apply directly to the combined adversarial models incorporating any of the models in which the negative result arises. In the following we study the universal stability of networks and protocols in combined adversarial models, considering only those combinations of models that guarantee universal stability when considered separately.

When considering the universal stability of rings, the combined models still to be studied are those combining either edge-R or edge-nR-B in their links, and node-RnT or node-nRnT-B in their nodes. When considering the universal stability of directed acyclic graphs, only the combined model (edge-R,node-RnT) needs still to be studied. We study all these combined models in the following. First, let us point out the fact that systems in the (edge-R,node-RnT) model can be simulated by systems in the edge-R model. As we will see, this will have further implications.

Theorem 30. *Any system $S = (\mathcal{N}, \mathcal{A}, \mathcal{P})$ in the (edge-R,node-RnT) model can be simulated by a system $S' = (\mathcal{N}, \mathcal{A}', \mathcal{P})$ in the edge-R model.*

Proof. Let $\mathcal{A}$ be an (r, b, w)-adversary for $\mathcal{N}$ in the (edge-R,node-RnT) model. Assume that $\mathcal{A}$ at time t has an associated triple $\langle P_t, E_t, V_t\rangle$ following the same idea used in Theorem 18, we replace node failures by outgoing link failures. Thus, $\mathcal{A}'$ at time t has associated tuple $\langle P_t, E'_t\rangle$ where

$$E'_t = E_t \cup \left(\bigcup_{u \in V_t} \{e \in E(\mathcal{N}) \mid e = (u, v)\} \right).$$

As the behaviour of the queues in the (edge-R,node-RnT) model is the same as in the edge-R model, the result follows. □

With this result, any (positive) network or protocol stability result from the edge-R model can be transferred to the (edge-R,node-RnT) model. Thus, from Corollary 4 we obtain the following result.

Corollary 31. *Directed acyclic graphs and rings are universally stable in the (edge-R,node-RnT) model.*

Moreover, recall that Corollary 5 stated that universal stability of networks in the edge-R model is equivalent to universal stability of networks in the AQT model. Taking also into account that the set of networks that are not universally stable in the AQT model is also not stable in the edge-R, we can conclude the equivalence also between the (edge-R,node-RnT) model and the AQT model.

Corollary 32. *A network $\mathcal{N}$ is universally stable in the (edge-R,node-RnT) model if, and only if, $\mathcal{N}$ is universally stable in the* AQT *model.*

Since universal stability in the AQT model can be decided in polynomial time [3], we can then state that

Corollary 33. *The universal stability of a given network in the (edge-R,node-RnT) model can be decided in polynomial time.*

From Theorem 26 we know that any system $\mathcal{S} = (\mathcal{N}, \mathcal{A}, \mathcal{P})$ in the node-RnT or node-nRnT-B model, in which all the nodes of $\mathcal{N}$ have output degree at most one, can be simulated by a system $\mathcal{S}' = (\mathcal{N}, \mathcal{A}', \mathcal{P}')$ in the edge-R model. Therefore, for the particular case of lines and rings, we have the following result when taking into account Corollary 4.

Theorem 34. *Lines and rings are universally stable in the* (*edge-R*, *node-RnT*) *model and in the* (*edge-R*, *node-nRnT-B*) *model.*

In Theorem 18 we showed that any system $S = (\mathcal{N}, \mathcal{A}, \mathcal{P})$ in the node-RnT model can be simulated by a system $S' = (\mathcal{N}, \mathcal{A}', \mathcal{P})$ in the edge-R model. Also from Theorem 13, we know that any system $\mathcal{S} = (\mathcal{N}, \mathcal{A}, \mathcal{P})$ in the edge-nR-B model, in which all the nodes of $\mathcal{N}$ have output degree at most one, can be simulated by a system $\mathcal{S}' = (\mathcal{N}, \mathcal{A}', \mathcal{P}')$ in the edge-R model. Those simulations can be extended to combined models in which the adversarial model of the nodes does not change. Therefore, for the particular case of lines and rings, we have the following result.

Theorem 35. *Lines and rings are universally stable in the* (*edge-nR-B*, *node-RnT*) *model and in the* (*edge-nR-B*, *node-nRnT-B*) *model.*

As a consequence of the instability results obtained in Sections 3 and 4, the instability of some of the protocols considered in this work can be directly stated in some combined models. Thus, only the universal stability of SIS, NFS and FTG in the (edge-R,node-RnT) combined adversarial model remains to be studied. As we have just shown in Theorem 30, systems in the (edge-R,node-RnT) model can be simulated by systems in the edge-R model, and this allows us to transfer any positive network or protocol stability result from the edge-R model to the combined (edge-R,node-RnT) model. Thus, from Corollary 6 we obtain the following result.

Corollary 36. SIS, FTG, *and* NFS *are universally stable in the (edge-R,node-RnT) model, while* FIFO, LIFO, LIS, LIQ, NTG, *and* FFS *are not universally stable in the (edge-R,node-RnT) model.*

6. Conclusions

The main contribution of this paper is twofold: first, an important effort has been made in properly defining and unifying concepts and techniques in a rigorous and formal way; second, exhaustive study and analysis have been made of the effect of edge or/and node failures on the stability of networks and protocols in relation to the way those failures are administrated. Moreover, to the best of our knowledge, node failures were never studied before when studying stability in the context of adversarial systems. After some years of study on the stability of adversarial systems, we observed that the generated literature had some clear gaps in unifying the definition of some basic concepts and describing properly the techniques used. An important part of this work aims at overcoming this problem and fixing that notation for the future.

Using as base model the AQT for static networks, we have proposed extensions for dynamic packet-switched networks in which short-lived link and/or edge failures might occur. Different models can be considered, depending on how the management of link failures or node failures affects the accessibility to their queues and depending also on where the affected packets are stored during the failure. Table 3 summarizes the most relevant results of our work concerning the universal stability of networks and protocols. The rows in Table 3 consider the node-faulty models, while the columns consider the edge-faulty models, both from less to more restrictive. Thus, the results at the leftmost upper cell represent the results for AQT, and the rest of the results in the first row and first column of the table state the results for the homogeneous

Table 3
Summary of results: Maintenance of the universal stability property in faulty adversarial models. Rows depict different node-failure models, while columns depict different edge-failure models, and thus each cell summarizes the results for systems combining the corresponding node and edge failure models. Then, the results at cell (1, 1) (non-faulty nodes and non-faulty edges) correspond to the basic AQT model. The symbol √ means that the network (respectively, the protocol) is universally stable, while the symbol × means that it is not. The symbol other-USP represents the other AQT universal stable protocols mentioned in this paper, i.e. SIS, NFS, and FTG.

MODELS	non-faulty edge	edge-R	edge-nR-B	edge-nR
non-faulty node	ring √ dags √ LIS, LIQ √ other-USP √	ring √ dags √ LIS, LIQ × other-USP √	ring √ dags × LIS, LIQ × other-USP ×	ring × dags × LIS, LIQ × other-USP ×
node-RnT	ring √ dags √ LIS, LIQ × other-USP √	ring √ dags √ LIS, LIQ × other-USP √	ring √ dags × LIS, LIQ × other-USP ×	ring × dags × LIS, LIQ × other-USP ×
node-nRnT-B	ring √ dags × LIS, LIQ × other-USP ×	ring √ dags × LIS, LIQ × other-USP ×	ring √ dags × LIS, LIQ × other-USP ×	ring × dags × LIS, LIQ × other-USP ×
node-nRnT node-nRT	ring × dags × LIS, LIQ × other-USP ×	ring × dags × LIS, LIQ × other-USP ×	ring × dags × LIS, LIQ × other-USP ×	ring × dags × LIS, LIQ × other-USP ×

cases, i.e., for the (exclusively) edge-faulty or node-faulty models. The rest of the cells show the results for the combined models, where both link and nodes might fail.

Concerning networks, it is worth mentioning the fact that only in one of the model combinations (edge-R with node-RnT) is the property of universal stability robust with relation to AQT. In the rest of the combinations, even very simple directed acyclic network topologies (and thus of course any network containing those topologies) can be made unstable when failures occur. Ring topologies remain universally stable under more models (and combinations of them) than simple directed acyclic topologies do. Interestingly enough, rings become not universally stable as soon as the failures at the edges and/or at the nodes block their respective receiving capabilities and no extra buffers are available.

It is clear from our results that the combination of the edge-R model with the node-RnT model, in which the queues are always accessible, would be preferable since it is robust (in terms of network stability) when related to the AQT model. This management form assures the same stability conditions for faulty communication networks as does the AQT model for non-faulty ones. This is an especially interesting and desirable property, since it is transferring the conditions for stability from a non-faulty environment to a faulty one; however, it is quite unrealistic as an assumption. When the access to the queues corresponding to failed edges cannot be assured, then a management in the way considered in the buffered models is preferable since it assures at least universal stability of simple topologies such as lines and rings.

Concerning protocols, it is interesting to observe how LIS and LIQ, which are known to be universally stable under AQT, easily lose this property as soon as failures are considered, even when they occur only at nodes or only at links and even when those failures are managed in the simplest way. The protocols SIS, NFS, and FTG (which are also universally stable under AQT) remain so under the receiving models edge-R and node-RnT and their combination, but they also lose the property as soon as failures impede the reception of packets.

Open problems

Focusing on the proposed models, it would be especially interesting to provide a characterization for universal stability of networks in the models in which some of the AQT universally stable networks are not universally stable. This characterization will allow us to asses the equivalence of models from the point of view of network universal stability, as we have done between the AQT model and the edge-R and node-RnT models and the model that combines both. Although we believe that the edge-nR-B model is equivalent, in this sense, to the node-nRnT-B model, to prove this fact an exact characterization of the universal stable networks in both models is needed. In relation to this, another related interesting question is whether the universally stability of networks under faulty models can be decided in polynomial time.

The models presented in this work consider that all the links, respectively nodes, of the network share the same failure management strategy. It would be of interest to study the stability of heterogeneous systems, i.e., systems whose nodes and links do not share the same failure management forms. For a heterogeneous faulty environment, it seems natural to consider that the adversary could also be restricted by the constraint proposed in Eq. (2) (but, of course, other constraints might also be appropriate). An important matter in the study of faulty heterogeneous systems would be the study of the influence of certain types of failure management and the importance of their incidence and location in the system. This would give us some knowledge about system configurations with certain stability guarantees or, from the opposite point of view, knowledge about which system configurations are not convenient because of their potential to provoke instability.

Observe that we have obtained very strong instability results as soon as the queues kept at a node are not accessible during a failure. This is due to the fact that we can make the tail of a node fail and use this fact to overflow the network

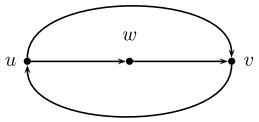


Fig. 7. Network $\mathcal{N}$, with $V(\mathcal{N}) = \{u, v, w\}$ and $E(\mathcal{N}) = \{(u, w), (w, v), (u, v), (v, u)\}$.

while preserving the adversary restrictions. A better understanding of the behavior of such a system under other adversary restrictions is an interesting open problem.

The kind of failures considered in this work are due to misbehavior of the communicating links or in the accessibility to the queues. We have not considered other sources of failure inspired by the classical Byzantine faults (see [18]), which are due to misbehavior in the computation. In the AQT setting it would also be of interest to analyze other faults coming from potential misbehavior in the outcome of the queuing policy (violating the pre-established rule) or deviations in the time of serving packets (violating the greedy assumption).

Other models for dealing with adversarial traffic and failure under other forms of failure management are also of interest, for example, adversarial models that drop those packets that suffer long delays due to failures along their traversal, and require a posterior packet retransmission. One might also require additional properties on the trajectories of the packets, in order to minimize (or to overcome) the effect of a failure, for example to require that, during a given period of time, the packets in the system are guaranteed to describe edge/vertex disjoint trajectories. Also related to this latter option of requiring extra properties on the paths, one could consider an alternative approach, which is to compute backup or repair routes that allow the failure to be repaired locally by re-routing the affected packets to their backup routes.

The methodology used in this work to obtain most of our proofs of stability is based on system simulations. We believe that such a technique will be helpful in comparing the global behaviour of different adversarial models that incorporate other features of interest.

Appendix. More proofs for instability based on induction

Lemma 7. LIQ *is not universally stable in the failure model.*

Proof. Consider the network $\mathcal{N} = (V, E)$, with vertex set $V(\mathcal{N}) = \{u, v, w\}$ and edge set $E(\mathcal{N}) = \{(u, w), (w, v), (u, v), (v, u)\}$, depicted in Fig. 7. Assume an initial configuration consisting of s packets, $s/2$ of them located at the node w that want to cross the path $\{wvu\}$, and the other $s/2$ packets located at the node u that want to cross the path $\{uvu\}$. The following adversary makes the network described above unstable under LIQ.

Round 1: For the first s steps, we inject a set α of rs packets that want to traverse the path $\{vuv\}$. During all that time, a constant flow of packets from the initial configuration will also arrive to v. Since the protocol is LIQ, and since those packets from the initial configuration have been queued up in the system for a longer time than the newly-injected packets from α, they have priority to pass node v. Thus, the packets from α will remain stacked at node v at the end of the round.

Round 2: For the next rs steps we inject a set β of r^2s packets that follow the path $\{vuwv\}$ together with a set γ of r^2s packets that follow edge (u, v). Those packets compete with the set α. At the end of the round there will be r^2s packets waiting to traverse the path $\{vuwv\}$ and r^2s packets waiting to traverse $\{uv\}$.

Round 3: For the next r^2s steps we inject a set δ of r^3s packets that follow the path $\{uv\}$ together with a set η of r^3s packets that follow the path $\{uwvu\}$. Those packets compete with the surviving packets and at the end of the round there will be r^3s packets waiting to traverse $\{uv\}$ and r^3s packets waiting to traverse the path $\{uwvu\}$.

Round 4: For the next r^3s steps we make link (w, v) fail for r^4s steps and inject a set of r^4s packets that follow the path $\{uvu\}$. Those packets compete with the surviving packets of the previous round and the failures keep some of the packets waiting to traverse link (w, v). So, at the end of the round there will be r^4s packets waiting to traverse the path $\{wvu\}$ and r^4s packets waiting to traverse the path $\{uvu\}$.

At the end of the fourth round, we are in the initial situation but with a set of $2r^4s$ packets in the system instead of s. For $r > 0.841$, we have that $2r^4s > s$ and thus by repeating the set of rounds the number of packets in the system grows unboundedly, thus making the system not stable under LIQ.

Observe that the proposed adversary obeys the restrictions of the failure model. □

Theorem 9. *For every injection rate* $0 < r < 1$*, there is an* $n_0 > 1$ *such that any n-line graph with* $n \geq n_0$ *is not r-stable in the edge-nR model under protocols* SIS, LIFO, NTG, NFS, *and* FFS.

Proof. Let $k = \lceil 2/r \rceil$, $\ell = \lceil k/r \rceil$ and $n_0 = 5 + \ell$. Observe that $rk > 1$. Consider an n_0-line graph with edges labelled as in Fig. 5(a). We assume that initially the network is empty and that the protocol is SIS. The adversary operates in the following rounds.

Round 1: For the first ℓ steps, we inject a set α of k packets that want to traverse the line starting at e_a and ending at e_d. Note that due to the length of the line all the injected packets will stay in the system at the end of the round, distributed along the line.

Round 2: Edges e_d fails for n_0 steps, this guarantees that all the α packets will be waiting at e_c at the end of the round. So, at the end of this round there are k packets queued at e_c.

Round 3: For the next ℓ steps, e_d fails and we inject a set β of k packets that want to traverse the line starting at e_a and ending at e_c.

Round 4: Edge e_d and e_c fail for n_0 steps. Again this provides enough time to guarantee that all the β packets are waiting at e_b. At the end of this round there are k packets at the queue of e_c and k packets at the queue of e_b, all of them requiring e_c.

Round 5: For the next k steps, we inject a set γ of rk packets that want to traverse the line graph starting at e_a and ending at e_d. Since the scheduling policy is SIS the β packets queued at e_b leave and, at the end of this round, there will be the k packets of the α set still at e_c. Furthermore, there are rk packets traversing the line with final destination e_d.

Round 6: Edge e_d fails for n_0 steps. Thus all the set γ will be waiting at e_c after those steps. At the end of this round there are $k + rk$ packets queued at e_c, all with destination e_d.

The adversary continues repeating rounds 3 to 6, thus creating at every completed phase an additional increase at edge e_c of $rk > 1$ packets with respect to the quantity of packets queued at this edge at the beginning of every phase. The above construction shows instability of a line graph for SIS, but can be also applied for LIFO and NTG, NFS and FFS (with an adequate tie breaking). Furthermore, observe that in the above description we have that $\omega = n_0$.

The above adversary will make unstable any network containing a path of length n_0, in particular any n-line graph with $n \geq n_0$. □

Theorem 10. *For every injection rate* $0 < r < 1$ *there is an* $n_0 > 1$ *such that any n-fork graph with* $n \geq n_0$ *is not r-stable in the edge-nR model under* FTG.

Proof. Let $k = \lceil 2/r \rceil$, $\ell = \lceil k/r \rceil$ and $n_0 = 4 + \ell$. Observe that $rk > 1$. Consider an ℓ-fork graph with edges labelled as in Fig. 5. The adversary operates in phases. During each phase $\phi \geq 0$ (rounds 1–4) we will accumulate rk packets at the queue of e_c. Let us suppose that at the beginning of phase ϕ there are $k + \phi rk$ packets queued at edge e_c with destination e_d.[6] The four rounds composing every phase $\phi \geq 0$ are described in the following. It is easy to see by induction on the number of phases ϕ that the number of packets in the system grows up unboundedly.

Round 1: For ℓ steps, k packets with destination e_e are injected in edge e_a. During all these steps edge e_d fails, thus accumulating all the packets still queued at edge e_c.

Round 2: Edges e_d and e_e fail for n_0 steps. At the end of the round, packets with destination e_d or e_e are waiting at the queue of e_c. At the queue there are $k + \phi rk$ packets with destination e_d and k more packets with destination e_e.

Round 3: For the next k steps, the adversary injects a set β of rk packets at edge e_a with destination e_d. Since neither edge e_d nor edge e_e fails a tie arises. FTG solves the tie in such a way that the k packets at the queue of e_c with destination e_e will flow to e_e.

Round 4: Edge e_d fails for the next n_0 steps, time enough to guarantee that the β packets get queued at edge e_c. At the end of this round there are $k + (\phi + 1)rk$ packets at the queue of e_c with destination e_d.

The adversary continues repeating the same sequence of rounds, thus creating at every completed phase an additional increase of rk packets. Furthermore, observe that in the above description we have $\omega = n_0$. Again the adversary will make unstable any network containing an n_0-fork graph. □

Lemma 12. LIS *and* LIQ *are not universally stable in the edge-nR model.*

Proof. Consider the network $\mathcal{N} = (V, E)$, with vertex set $V(\mathcal{N}) = \{u, v, w\}$ and edge set $E(\mathcal{N}) = \{(u, w), (w, v), (u, v), (v, u)\}$, depicted in Fig. 7. Assume an initial configuration consisting of $s/2$ packets located at the node u that want to cross the path $\{uwvu\}$ and a set of $s/2$ packets located at the node u that wants to cross the path $\{uvu\}$.

The adversary follows the same four rounds as the adversary used for the proof of Lemma 7. Since no failures occur during the first three rounds, at the end of the third round, both under LIQ and LIS, the packet distribution on the network is as follows:

- there are $r^3 s$ packets waiting to traverse only the edge (u, v), and
- $r^3 s$ packets waiting at node u to traverse the path $\{uwvu\}$.

In the fourth round, the adversary makes the link (w, v) fail and, since we are dealing with the edge-nR model, the packets remain blocked at their initial position at node u (i.e., with path $\{uwvu\}$ still to traverse).

Thus, at the end of the fourth round, the system has a configuration analogous to the initial one but with $2r^4 s$ packets in the system instead of s. For any injection ratio that makes $2r^4 s > s$ (i.e., for $r > 0.841$), the infinite repetition of the described rounds makes the system not stable, under both LIQ and LIS. □

[6] To start from an empty initial configuration just consider two initial preceding rounds: (first round) for k/r steps, inject k packets at e_a with destination e_d, and (second round) edge e_d fails enough to guarantee that all the packets get blocked at the queue of e_c.

Theorem 16. *For every injection rate $0 < r < 1$, there exists an $n_0 > 0$ such that the n-fork graph, for $n \geq n_0$, is not r-stable in the edge-nR-B model under any of the following protocols:* LIFO, SIS, NTG, FTG, NFS, *and* FFS.

Proof. Given a valid injection rate r, let $k = \lceil 2/r \rceil$, $\ell = \lceil k/r \rceil$ and $n_0 = 4 + \ell$. Observe that $rk > 1$. Consider a $4 + \ell$-fork like the one depicted in Fig. 5, whose length depends on r. In that network, we denote as v_f the source node of the forking, i.e., let v_f be the target node of link e_c and the source node for links e_d and e_e. Again we suppose that the initial configuration is empty. The following adversary can make the network not stable for any of the above-mentioned greedy protocols.

Round 1: For the first ℓ steps, the adversary injects a set α of k packets of the form $(e_a, \ldots, e_d)$.

Round 2: The edge e_d fails for the next n_0 steps, time enough to make all the set α accumulate in the queue of node v_f.

Round 3: For ℓ steps, the edge e_d continues failing and a set β of k packets is injected, which wish to traverse the path $(e_a, \ldots, e_e)$.

Round 4: The edges e_d and e_e fail for the next n_0 steps. This is enough to accumulate the packets from the α and β sets in the queue of node v_f. At the end of this round there are $2k$ packets in the queue of this node.

Round 5: We consider now an interval of time of k steps. This represents that, except in the case that both edges e_d and e_e fail, k packets will leave the queue at edge v_f. The adversary injects a set γ of rk packets of the form $(e_a, \ldots, e_d)$. The packets which abandon the queue at v_f are the packets from the β set. An additional round has to be considered.

Round 6: The edge e_d fails for n_0 steps, thus blocking the packets from α and the packets from γ at the queue of the vertex v_f.

At the end of the sixth round, there are $k + rk$ packets in the queue of v_f with destination e_d. By indefinitely repeating rounds 3 to 6, after rounds 1 and 2, the system accumulates rk additional packets at every iteration. The number of packets in the system grows unboundedly, thus making the system unstable under the mentioned protocols. For the protocols NTG, FTG, NFS and FFS, the forms of the packets are valid to bring the system to instability, solving ties appropriately. Furthermore, observe that in the above description we have $\omega = n_0$. □

Lemma 17. LIS *and* LIQ *are not universally stable in the edge-nR-B model.*

Proof. Consider the graph used in the proof of Lemma 7 (see Fig. 7). Consider a slightly different initial configuration consisting of s packets, where $s/2$ of them are located at the node u and want to cross the path $\{uwvu\}$, and $s/2$ packets are located at the node u and want to cross the path $\{uvu\}$. Consider also the adversary given for the proof of Lemma 7. Since no failures occur in the first three rounds, the configuration obtained after the third round is the same when applying that adversarial pattern under LIS and LIQ in the edge-nR-B model, i.e., at the end of the third round there will be $r^3 s$ packets waiting to traverse $\{uv\}$ and $r^3 s$ packets waiting to traverse the path $\{uwvu\}$.

Let us consider now also the same injection and failure pattern for the fourth round, i.e., for $r^3 s$ steps, we make link (w, v) fail for $r^4 s$ steps and inject a set of $r^4 s$ packets that follow the path $\{uvu\}$. Those packets compete with the surviving packets of the third round and the failures keep some of the packets waiting to traverse link (w, v). In the edge-nR-B model, those packets wait in the queue of the previous link (u, w) (in contrast to what happens in Lemma 7, where they wait in the queue of (w, v) itself). At the end of the fourth round there will be $r^4 s$ packets waiting to traverse the path $\{uwvu\}$ and $r^4 s$ packets waiting to traverse the path $\{uvu\}$.

At the end of the fourth round, we are in the initial situation but with a set of $2r^4 s$ packets in the system instead of s. The number of packets grows with respect to the initial configuration for $r > 0.841$, i.e., when $2r^4 s > s$. □

Lemma 23. LIS *and* LIQ *are not universally stable in the node-RnT model.*

Proof. This result can be easily shown by applying the adversary given in the proof of Lemma 7 over the graph given in Fig. 7, where every failure of edge (w, u) is replaced by failure of the node w. □

Theorem 28. *For every injection rate $\frac{1}{2} < r < 1$, there is an $n_0 > 1$ such that any n-crossing graph with $n \geq n_0$ is not r-stable in the node-nRnT-B model under any greedy protocol.*

Proof. Let $n > 2$ and consider the n-crossing graph depicted in Fig. 8, where we denote as λ_1 and λ_2 the two n-line graphs with endpoints in the crossing vertex v_c, and as v_1 and v_2 the tail vertices of the two edges outgoing from the crossing vertex.

Let us assume that initially there are $2n$ packets stored in the extra buffer of the crossing vertex v_c. An adversary playing the following rounds indefinitely will make the network unstable, independently of the greedy protocol used.

Round 1: For the first n steps, the adversary injects rn packets at the first node of λ_1 with destination v_1, and also rn packets at the first node of λ_2 with destination v_2. Those injections are done simultaneously at regular intervals of length $1/r$ starting at time 1. Since $|\lambda_1| = |\lambda_2| = n$, none of the injected packets have arrived to the crossing vertex v_c at the end of this round.

Round 2: The crossing vertex v_c fails for n steps, thus at the end of the round all the packets injected in the previous round will be stored in its extra buffer.

Round 3: For the next $1/r$ steps no packet is injected and no node is failed.

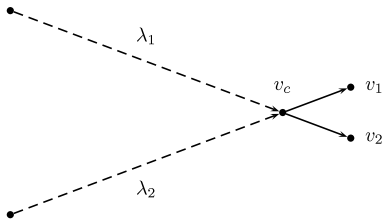


Fig. 8. The n-crossing graph used in Theorem 28.

At the end of the third round, there will be $2rn + n - 1/r$ packets stored in the extra buffer of the crossing vertex v_c. Observe that the last round provides time to guarantee that in the interval between the last failure and the first injection the adversary restriction is fulfilled.

By infinite repetitions of the above pattern of injections and failures, this adversary would make the system unstable for any r such that $2rn + n - 1/r \geq 2n + 1$, in particular this holds for any r such that

$$r \geq \frac{1}{4} \cdot \frac{n + 1 + \sqrt{n^2 + 10n + 1}}{n}.$$

Observe that this result is independent of the protocol used in the system. Observe also that 0.5 is the lower bound for the instability of the n-crossing graphs, since

$$\lim_{n \to \infty} \left(\frac{1}{4} \cdot \frac{n + 1 + \sqrt{n^2 + 10n + 1}}{n} \right) = \frac{1}{2}. \quad \square$$

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