VISCOELASTICITY OF HUMAN BLOOD

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ABSTRACT Measurements made for oscillatory flow of blood in circular tubes show that blood possesses elastic properties which make consideration of its viscous properties alone inadequate. Results are for a frequency of 10 Hz while varying the amplitude of the velocity gradient for red blood cells in plasma at concentrations ranging from 0 to 100% apparent hematocrit. For velocity gradients less than 1–2 sec^{-1} both the viscous and elastic components of the shearing stress are linearly related to the gradient. For hematocrits above 20% the elastic component of the complex coefficient of viscosity increases with hematocrit approximately to the third power while the viscous component increases exponentially. Oscillatory flow measurements at very low hematocrits, when extrapolated to zero cell concentration, give the intrinsic viscosity of the average individual isolated red cell. The viscous part of this is found to be 1.7 which is compared with theoretical values from the rigid ellipsoid model for which the minimum possible value is 2.5. This difference is attributed to cell deformability. With increasing velocity gradient nonlinear properties develop. The viscous component of the complex viscosity becomes of the order of the steady flow viscosity at high gradients while the elastic component tends to decrease in inverse proportion to the gradient. Thus, the elastic component of the oscillatory stress tends to saturate, this tendency appearing at the approximate level of the yield stress.

INTRODUCTION

Pulsations and unsteady flow are natural in the in vivo circulation. In spite of this, the mechanical properties of blood have not been previously determined for unsteady flow. Instead, it has been sought to utilize the properties of blood derived from steady flow studies in unsteady flow situations (1–3). These properties include only the density and steady flow viscosity which is non-newtonian in character. In the present work, precise measurements have been made of the oscillatory flow of blood in circular tubes. It is found that the previous concept of blood as a purely viscous fluid is inadequate. Blood possesses an additional elastic property (4) and behaves as a viscoelastic material. The character of the dependence of these properties on both the amplitude of the oscillatory velocity gradient (shear rate) and the concentration of red blood cells in its plasma for human blood is determined. The elastic behavior increases rapidly with increasing hematocrit as direct cell-to-cell interactions become

more probable. While the viscous and elastic components of the shearing stress are linearly related to the velocity gradient at low gradients, they become nonlinearly related at high gradients. At sufficiently low hematocrits the measured properties are those due to diffuse noninteracting red cells suspended in plasma. Under these conditions, the elastic property is out of the range of present measurement techniques. The viscous property of the red cell, however, is clearly measurable and is thought to be strongly influenced by the cell's deformability. These new observations are discussed in the light of other contemporary work on the structure and flow properties of blood.

LIST OF SYMBOLS

a	Tube radius, centimeters.
C_v	Volume concentration.
D_r	Rotary diffusion constant for an ellipsoid, seconds^{-1}.
D_{sph}	Rotary diffusion constant for a sphere, seconds^{-1}.
G	Instantaneous value of velocity gradient, seconds^{-1}.
G_M	Maximum value of velocity gradient, seconds^{-1}.
$\bar{G}$	Mean value of velocity gradient, seconds^{-1}.
H	Hematocrit, per cent.
k	Boltzmann constant, erg-degree Kelvin^{-1}.
l	Tube length, centimeters.
P	Pressure, dynes per square centimeter.
R_1	Resistance per unit length, dyne-seconds per centimeter^6.
T	Temperature, degrees Kelvin.
U	Instantaneous volume velocity, cubic centimeters per second.
U_M	Maximum value of volume velocity, cubic centimeters per second.
v	Particle volume, cubic centimeters.
X_1	Reactance per unit length, dyne-seconds per centimeter^6.
Y	Dimensionless parameter.
Z_1	Impedance per unit length, dyne-seconds per centimeter^6.
ρ	Fluid density, grams per cubic centimeter.
ν_A, ν_B	Dimensionless parameters for ellipsoids.
η^*	Complex coefficient of viscosity, P.
η'	Viscous component of η^*, P.
η''	Elastic component of η^*, P.
η	Magnitude of η^*, P.
ϕ	Phase of η^*.
η^*_{sp}	Complex specific viscosity.
$[\eta^*]$	Complex intrinsic viscosity.
τ	Instantaneous shearing stress, dynes per square centimeter.
τ'	Viscous component of τ, dynes per square centimeter.
τ''	Elastic component of τ, dynes per square centimeter.
τ_M	Maximum value of τ, dynes per square centimeter.
τ_r	Relaxation time, second.
ω	Radian frequency, rads per second.

OSCILLATORY FLOW THEORY

The method used to determine the viscoelasticity of blood is based upon measurements of the pressure to volume flow relationship for oscillatory flow in rigid cylindrical tubes of circular cross-section. The theory for oscillatory tube flow of an ordinary newtonian fluid was verified (5, 6) experimentally and shown to depend upon the tube radius, fluid density, fluid viscosity, and the radian frequency as they appear in a dimensionless parameter. Subsequently this parametric form of the theory was applied by Womersley (3) to oscillatory blood flow. The theory for the oscillatory flow of a linear viscoelastic fluid was developed by Thurston (7) and was verified by experimental measurements (8). It is this viscoelastic theory which is necessary to understanding oscillatory blood flow. At high gradients where the properties of blood become nonlinear, the theory correspondingly becomes an approximation. The theory is used herein to obtain the complex coefficient of viscosity η^* of blood, while using tubes having diameters which are large compared with the dimensions of the red cell.

The coefficient η^* is determined by the relative magnitude and phase of the shearing stress τ in the fluid and the time rate of shear strain or velocity gradient G. The coefficient is defined as

$$\begin{aligned} \eta^* &= \tau/G, \\ &= \eta' - i\eta'', \\ &= \eta \exp(-i\phi), \end{aligned}$$

where η' and η'' are the real and imaginary parts of the viscosity, η is the magnitude of viscosity, and ϕ is the viscosity angle. The gradient G is written in complex form as

$$G = G_M \exp(i\omega t), \tag{2}$$

and the shearing stress is

$$\tau = \tau_M \exp[i(\omega t - \phi)]. \tag{3}$$

From these equations it is seen that the magnitude of the shearing stress τ_M and the real part τ_M' and the imaginary part τ_M'' of the complex stress are as follows:

$$\begin{aligned} \tau_M &= \eta G_M, \\ \tau_M' &= \eta' G_M, \\ \tau_M'' &= \eta'' G_M. \end{aligned} \tag{4}$$

In the event that $\eta'' = 0$ then $\phi = 0$ and the complex coefficient of viscosity reduces to an ordinary viscosity coefficient. Under this condition the stress component τ_M'' similarly vanishes. τ_M'' is considered an elastic component of the stress and is associ-

ated with an energy storage during the deformation of the blood while the τ'_M and η' are viscous components which are associated with energy dissipation. The instantaneous oscillatory flow is given by the volume velocity

$$U = U_M \exp(i\omega t). \tag{5}$$

The relation between the pressure drop p for a length of tube l and the volume velocity is specified by a complex impedance per unit length of tube Z_1,

$$\begin{aligned} Z_1 &= (p/l)/U, \\ &= R_1 + iX_1, \end{aligned} \tag{6}$$

where R_1 is a resistance per unit length and X_1 is the reactance per unit length of the tube. For N parallel tubes each of length l, the impedance of the assemblage is $Z_1(l/N)$. The theoretical expressions for R_1 and X_1 are dependent upon the dimensionless parameter Y

$$Y = a(\rho\omega/\eta)^{1/2}, \tag{7}$$

and the viscosity angle ϕ. Under the conditions of experimental measurements Y is less than 1 and these expressions take on the simple forms

$$\begin{aligned} R_1 &= 8\eta'/\pi a^4, \\ X_1 &= (4\rho\omega/3\pi a^2) - (8\eta''/\pi a^4). \end{aligned} \tag{8}$$

The velocity gradient in the tube is also a function of Y and ϕ. The mean value is simply related to the volume velocity by integration over the tube's cross-sectional area A under the conditions $Y < 1$

$$\begin{aligned} \bar{G} &= \int_A G\, dA/\pi a^2, \\ &= 8U/3\pi a^3. \end{aligned} \tag{9}$$

The gradient G ranges from 0 on the central axis of the tube to 3/2 the mean value at the tube wall. In the event that the fluid viscosity is nonlinear and is dependent upon the velocity gradient, then the theoretical flow analysis does not precisely apply. In this case only approximate mean values of the gradient and stress are determined.

From measurements of the magnitude and phase of the pressure-flow relationship the resistance R_1 and reactance X_1 are determined and the components of the viscosity may be calculated from equations 8 by using a known fluid density ρ, radian frequency ω, and tube radius. It should be noted that the earlier theory (5) gives an end point like equations 8 but differs in that the second term of X_1 involving η'' does

not appear. It is precisely this additional term which is essential to rationalize the reactance measurements for blood and to reveal its elastic component.

EXPERIMENTAL METHODS

The measurements were performed using a hydrodynamic test system (5, 6, 8) a schematic of which is shown in Fig. 1. Approximately 15 ml of blood fills the upper part of the test system including the tube sample. Measurements have been made using both glass and brass tubes. For measurements reported here the tube sample consisted of 50 brass tubes in parallel, each of length 0.654 cm and radius 0.0212 cm. The blood is separated from the water-filled lower chamber by a thin, flexible, plastic membrane. The pressure developed in the lower chamber is monitored by a transducer mounted in the chamber wall. The oscillatory flow is developed by an oscillating piston surface attached to the chamber through a flexible metal bellows. The drive shaft leads to the electrodynamic driver and transducer which measure both the displacement and velocity of motion of the drive. By knowing the velocity of motion and the effective area of the driver the volume velocity generated in the system may be determined. Electrical signals representative of the volume velocity and the pressure are analyzed as to magnitudes and relative phase angles in order to determine the impedance seen by the driver.

In order to achieve the accuracy necessary to resolve clearly the elastic component of blood, the small residual impedances associated with the hydrodynamic test system must be precisely known and compensated in the data analysis (8). At the frequency of interest here the influence of these residuals is minimized by lowering the over-all impedance of the tube sample. For this reason a large number of parallel tubes were used for the blood measurements. In this way the corrections due to the residuals is held generally to less than 5%. As a final check of the measurement procedure, data reduction, and correction for residuals, measurements are

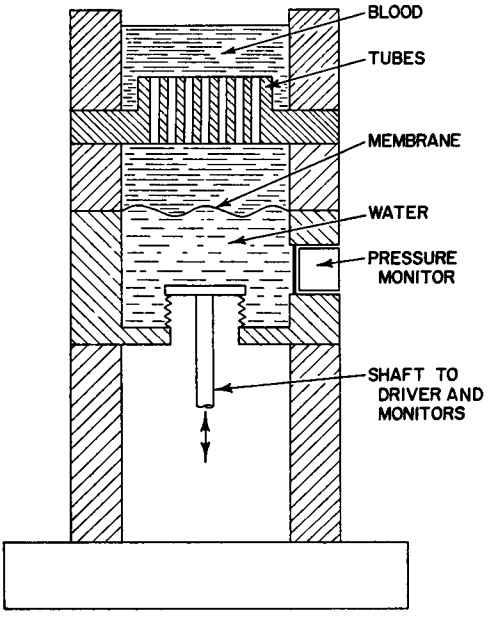

FIGURE 1 Hydrodynamic test system used for oscillatory tube flow measurements.

performed using fluids having known viscous properties, namely water and glycerol. The over-all precision is such than an elastic component of the viscosity as small as 1/100 of the viscous component can be resolved. Using a lock-in amplifier as the voltage-analyzing instrument, the signal amplitudes are determined to within ±2% and the phase angles to within ±0.2°.

A preparation of normal human blood was used in the measurements. The blood was as prepared for blood bank storage, each 100 ml containing 0.8 g citric acid (hydrous), 2.2 g sodium citrate (hydrous), and 2.45 g dextrose (hydrous). 3-day old blood was centrifuged at 4000 *g* for 20 min to separate the red blood cells from the plasma. The compacted red blood cells (designated as having an apparent hematocrit H of 100%) were recombined with the plasma to desired hematocrits. The density was determined for the plasma and all of the plasma-cell combinations. The fluid density ρ(grams per milliliter) was given by $\rho = 1.0255 + 0.000535 \times (H \text{ per cent})$.

The measurements reported herein were all carried out at a frequency of 10 Hz and at a temperature of 25°C. Supporting steady flow measurements were performed using a conventional Couette-type apparatus.

EXPERIMENTAL RESULTS

Measurements were carried out in order to determine how the stress components depend upon the velocity gradient using blood over hematocrits H from 1.3 to 100%. The measurements showed that at 10 Hz and for mean values of velocity gradient less than 1–2 sec^{-1}, the stress was directly proportional to the gradient. At higher gradients the blood became nonlinear in its response. For H less than 20% the elastic component becomes too small for precise measurement. For the higher hematocrits the character of the gradient dependence is typified by that shown in

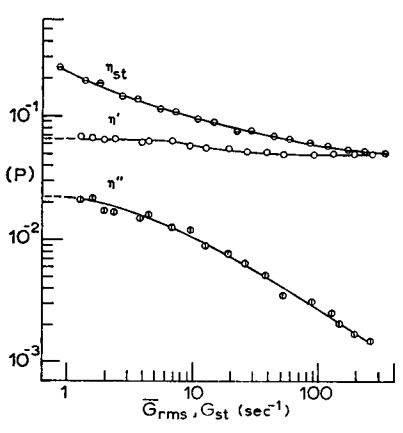

FIGURE 2

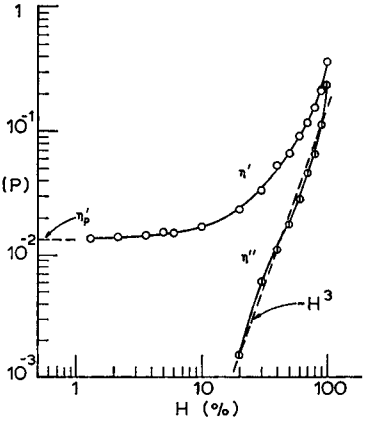

FIGURE 3

FIGURE 2 Gradient dependence of the steady flow viscosity and the viscous and elastic components of oscillatory flow viscosity at 10 Hz for 50% hematocrit blood.

FIGURE 3 Viscous and elastic components of the viscosity of blood at 10 Hz as a function of the hematocrit for velocity gradients less than 2 sec^{-1}.

Fig. 2 for the case of 50% hematocrit. Shown using logarithmic scales are the real and imaginary parts of the complex viscosity coefficient plotted vs. the root mean square (rms) value of the mean velocity gradient at a frequency of 10 Hz. Also shown is the steady flow viscosity as determined from the ratio of the shearing stress to the gradient from Couette measurement. It is seen that the viscous component η' and the elastic component η'' approach constant values for sufficiently low gradients. With increasing gradient the elastic component begins to decrease. This is accompanied by a slight decrease in η' which finally tends to arrive at a constant value at the highest gradients while η'' continues to decrease. The steady flow viscosity is continuously decreasing with increasing gradient but tending to approach a high gradient value of approximately 0.048 P, this value being very near the high gradient limit for the viscosity component η'.

The gradient dependence of η^* as shown in Fig. 2 is typical of other hematocrits above 20%. In order to see how the components of η^* changed with H in the low gradient range the components of η^* were measured as in Fig. 2, and the values obtained for $\bar{G}_{\text{rms}} < 2$ sec^{-1} were utilized to give limiting values. These results are shown in Fig. 3 using logarithmic scales on which limiting low gradient values of η' and η'' are plotted vs. the hematocrit. The measurements range upward from the plasma value for which $\eta_p' = 0.0133$ P, the hematocrits going from 1.3 to 100%. It is seen that η'' rapidly increases with increasing hematocrit and in the range of measurement from 20 to 90% is given approximately by $\eta'' \cong 1.6 \times 10^{-7} \times H^3$. For hematocrits below 20% the elastic component has become very much smaller than the viscous component and is out of the measurement range.

In the results of these measurements it is important to draw a distinction between the influence on the viscosity of blood arising from interactions between the red blood cells and that associated with the isolated, noninteracting red blood cells in the surrounding plasma. It is clear from the data of Fig. 3 that above hematocrits of 20% cell interactions dominate the viscoelastic behavior of the blood at low gradients. While for hematocrits less than 5% the measured viscous component is close to the truly dilute solution value. From these limits one may draw an approximate delineation between high and low concentration conditions.

In order to obtain the low concentration mechanical properties of the isolated red cell in its surrounding fluid, free from the effects of interactions with other cells, the intrinsic viscosity $[\eta^*]$ is sought. This is done by subtracting the plasma viscosity η_p from η^* and then extrapolating the ratio of this incremental viscosity to the volume concentration of red cells to the limiting condition of zero concentration. The intrinsic viscosity is thus defined as

$$[\eta^*] = [\eta'] - i[\eta''],$$

$$= \lim_{c_v \to 0} \eta^*_{sp}/c_v,$$

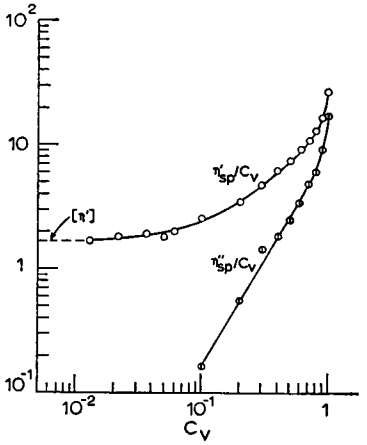

FIGURE 4 Components of the ratio of the specific viscosity to volume concentration of red blood cells vs. volume concentration.

$$\eta^*_{sp} = \eta'_{sp} - i\eta''_{sp},$$
$$= (\eta^* - \eta_p)/\eta_p, \qquad (10)$$

where η^*_{sp} is the complex specific viscosity, η_p is the viscosity of the plasma, and c_v is the volume concentration of the red cells which is the hematocrit per cent divided by 100. Using the numerical value of the plasma viscosity as previously noted, the real and imaginary parts of (η^*_{sp}/c_v) are shown in Fig. 4 plotted vs. the volume concentration. It is seen that at low concentration the real part of the ratio of the specific viscosity to volume concentration approaches 1.70 which is the intrinsic value $[\eta']$. The intrinsic value of the elastic component is not determined. The effect of cell-to-cell interaction at high concentrations is shown in Fig. 4 by the increase in the numerical values of the functions with volume concentration. The rate of increase of η'_{sp}/c_v is very nearly exponential with volume concentration. While it is most rapidly increasing at high volume concentrations, it is nevertheless approximately 40% above the intrinsic limit at $c_v = 0.1$. The rate of increase of the elastic component at low concentrations is undetermined. Above $c_v = 0.1$, however, it is at a more rapid rate than the viscous component.

Analysis of Results at Low Hematocrits

At sufficiently low hematocrits where the previously defined low concentration conditions prevail, the extrapolation to $c_v = 0$ gave the value of the intrinsic viscosity $[\eta']$. This condition may be modeled by the presence of isolated elastic and deformable formed elements (the red blood cells) suspended in a solvent (plasma) which may in itself possess a small amount of elasticity due to the concentration of plasma proteins. A simpler model is that of rigid ellipsoidal particles suspended in a purely

viscous solvent and the intrinsic properties of this model solution has been previously treated (9, 10). This dilute solution theory gives the real and imaginary parts of the complex intrinsic viscosity for oscillatory flow at low gradients as

$$[\eta^*] = \nu_A + \nu_B/(1 + i\omega\tau_r), \tag{11}$$

where the functions ν_A and ν_B are determined by the axial ratio for the suspended ellipsoidal particles and have been previously tabulated (11), τ_r is the relaxation time which is characteristic of the particle-solvent systems and is related to the rotary diffusion coefficient D_r for the rigid particle by

$$\tau_r = 1/6\, D_r\,. \tag{12}$$

From equation 11 it is seen that the intrinsic viscosity, under the condition that the radian frequency $\omega = 0$, becomes simply the sum $(\nu_A + \nu_B)$. This is also the value obtained in the theory for steady flow at zero gradient and concentration. The relaxation time may be conveniently calculated using

$$\tau_r = (D_{sph}/D_r)/(kT/\eta_p v), \tag{13}$$

where the ratio (D_{sph}/D_r) is a function of the axial ratio of the ellipsoid as tabulated elsewhere (11), k is the Boltzman constant, T is the absolute temperature, and v is the volume of the particle. If the red blood cell is approximated by an oblate ellipsoid having an axial ratio of 3, then the ratio D_{sph}/D_r is 1.464 and introducing a cell volume of 87×10^{-12} cm^3 and a temperature of 198°K gives a relaxation time of 43.0 sec. For measurements of 10 Hz the product $\omega\tau_r$ is 2700 and thus the intrinsic viscosity is predominantly determined by the function ν_A of equation 11. For an axial ratio of 3, ν_A equal 2.868, this value being considerably in excess of the measured value of $[\eta']$ of 1.70. The minimum value for the function ν_A is that for a spherical particle and has a value of 2.50. Thus one is led to the conclusion that the internal motions due to the flexibility of the red blood cell must have an appreciable influence upon the intrinsic viscosity of the solution. This is in qualitative agreement with steady flow studies which have been directed toward the determination of an internal viscosity for the red cell (12). With regard to the elastic component, using the above numerical values in equation 11 for rigid ellipsoids in calculation of $[\eta'']$ gives a value of 2.08×10^{-4}. This is clearly out of the range of present measurement capability. In view of the possible influence of cell flexibility and associated energy storage and dissipative mechanisms, however, the elastic part of the intrinsic viscosity might be substantially different from this computed value.

Analysis of Results at High Hematocrits

At high hematocrits the viscoelastic properties are strongly dependent upon both the hematocrit and the velocity gradient. The elastic component η'' of the complex

coefficient of viscosity increases sharply with H above 20%. It also changes from linear to nonlinear dependence on $\bar{G}_{rms}$ at approximately 1–2 sec^{-1}. It is of interest here to examine these properties in the light of previous studies of blood in steady flow.

It is well known that at low hematocrits the viscosity of blood is not dependent upon shear rate and hence is newtonian while at higher hematocrits it is non-newtonian (13, 14). The steady flow viscosity η_{st} for 50% hematocrit blood of Fig. 2 shows this non-newtonian property. The form of the gradient dependence of the shearing stress in steady flow has been studied extensively (15–20) and it has been found that the functional form of Casson's equation (21) is useful for analysis of the data at the lower shear rates. According to this equation the stress is related to the velocity gradient by

$$\tau_{st}^{1/2} = \tau_y^{1/2} + (\eta_N G)^{1/2}, \tag{14}$$

where η_N is a viscosity coefficient and τ_y is the yield stress, the value of which is usually determined by an extrapolation to $G = 0$.

In Fig. 5 the data of Fig. 2 are plotted in the form suggested by equation 14 where the square root of the stress components are shown vs. the square root of the velocity gradient. This is done for the steady flow stress τ_{st} as well as for the mean values of the oscillatory stress components $\bar{\tau}'_{rms}$ and $\bar{\tau}''_{rms}$ which are related through equation 4 to the mean velocity gradient $\bar{G}$ of equation 9. It is seen in Fig. 5 how a linear extrapolation of the steady flow data to zero gradient gives the yield stress $\tau_y = 0.0538$ dynes/cm.^{2} Also, the slope of the curve gives $\eta_N = 0.058$ P. The oscillatory stress components both tend to extrapolate to zero stress at zero gradient

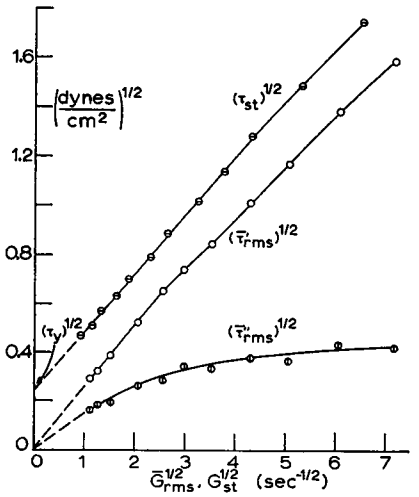

FIGURE 5 Square root of the shearing stress components vs. the square root of the gradient for the data of Fig. 2.

There the slope of the $\bar{\tau}'_{rms}$ curve gives $\eta' = 0.065\ P$ and the $\bar{\tau}''_{rms}$ curve gives $\eta'' = 0.0219\ P$. There is a pronounced tendency for the elastic component of the stress τ'' to saturate with increasing gradient. The beginning of this saturation is marked by the deviation from linearity of the $\bar{\tau}''_{rms}$ curve which occurs for $\bar{G}_{rms}$ between 1 and 2 sec^{-1}. The saturation occurs in a range of stress which is of the order of τ_y.

Measurements over a range of hematocrits from 30 to 80% show the mean value of the rms gradient at which the elastic component begins to deviate from linearity is in the range of 1 to 2 sec^{-1}. Thus, for these hematocrits the value of the stress component on becoming nonlinear has nearly the same dependence upon hematocrit as that of η''. From Fig. 3 this dependence was approximately on H to the third power. It has been found (16) that in steady flow the yield stress increases with hematocrit at approximately this same rate. Thus it would seem that the onset of nonlinear elastic stress and the yield stress may be interrelated.

DISCUSSION

The characterization of blood as a viscoelastic material when subjected to oscillatory flow is not dependent upon its classification as being a solid or a liquid. A determination of this classification can be based upon whether or not blood can sustain a static stress without continuing to flow. There exists substantial evidence that blood at higher hematocrits can sustain a static stress which is less than the yield stress and hence possesses the properties of an elastic solid. The fact that extrapolation to zero gradient according to the form of Casson's equation gives a value τ_y does not alone establish the case. The torque decay observations (17) show an elastic energy storage. Other supporting experiments include cell settling studies (22) and parallel plate stress system measurements (23). The combined visual and viscometric observations of Schmid-Schoenbein and coworkers (24–26) further support the idea of an elastic blood. They have observed the characteristics of naturally occurring aggregates of red cells and their ability to deform elastically under stress and their disruption and migration under flow. Chien (27, 28) has also critically examined the role of aggregation and cell deformation (29) in blood flow. With increasing shear rate it is found that the size of the aggregates decreases to the ultimate size of single cells (16, 26).

The viscoelasticity of blood may be considered in the light of the above. In this case, the linear behavior at very low amplitudes of velocity gradient is associated with blood in its unperturbed and fully aggregated state as its structure has not been degraded by flow. The precise value of η^* is then likely determined by the nature of the elastic coupling between the cells together with the internal elastic properties of the cell itself (12, 30). With increasing gradient the structure becomes disrupted at which level the viscoelasticity become nonlinear. For the measurements herein the elastic component of the oscillatory stress for disruption is in the neighborhood of the yield stress obtained by Casson extrapolation. This may be a fortuitous event

for it would be reasonably expected that these characteristics are frequency dependent.

The amplitude of the strain at which the blood becomes nonlinear may be determined from velocity gradient and the frequency of oscillation, since for oscillatory deformations the gradient is equal to the product of the strain times the radian frequency. For example, if the yield gradient is taken to the 1 sec^{-1} at a frequency of 10 Hz then the corresponding yield strain amplitude is 0.016. For small strains this corresponds to a rotation of elements in the fluid through an angle of 0.016 radians or approximately 1°. Over an 8 μ length which is characteristic of a red blood cell the end points would move through a distance of approximately 0.13 μ. This motion is thus sufficient to initiate structural changes.

From the viewpoint of viscoelasticity and oscillatory flow the question of whether or not blood is a solid or a liquid may be resolved by examining the behavior of η^* in the limit as ω approaches zero. If the product $(\omega\eta'')$ does not vanish in this limit, then the material has a springlike elastic quality which would classify it as a solid. The oscillatory measurements given here show similarities to steady flow measurements, particularly in comparing the high gradient values of η' and η_{st}, the low gradient value of η' and η_N, the yield stress τ_y and the saturation value of τ'', and the hematocrit dependence of τ_y and η''. It is not anticipated however, that the frequency of 10 Hz is low enough for close comparisons. A sufficiently low frequency would be one having a period longer than the longest relaxation time for processes in the blood. The calculated relaxation time of 43 sec for dilute solutions would not be expected to apply to concentrated solutions where free rotation of the cells is prohibited. Cell aggregates could possess both longer times due to gross configurational motions as well as shorter times due to internal deformations. To resolve these questions the characteristics of η^* must be obtained over a very wide frequency range.

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