# Package 'RCBR'

January 20, 2025

Title Random Coefficient Binary Response Estimation

Description Nonparametric maximum likelihood estimation methods for random coefficient binary response models and some related functionality for sequential processing of hyperplane arrangements. See J. Gu and R. Koenker (2020) [<DOI:10.1080/01621459.2020.1802284>](https://doi.org/10.1080/01621459.2020.1802284).

Version 0.6.2

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**Depends**  $R$  ( $>= 2.10$ ), Matrix

Suggests knitr, digest

Imports methods, Rmosek, REBayes, orthopolynom, Formula, mvtnorm

LazyData TRUE

SystemRequirements MOSEK (http://www.mosek.com) and MOSEK License for use of Rmosek,

License GPL  $(>= 2)$ 

URL <https://www.r-project.org>

NeedsCompilation no

Encoding UTF-8

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# **Contents**



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bounds.KW2 *Prediction of Bounds on Marginal Effects*

# Description

Given a fitted model by the exact NPMLE procedure prediction is made at a new design point with lower and upper bounds for the prediction due to ambiguity of the assignment of mass within the cell enumerated polygons.

# Usage

bounds.KW2(object, ...)

# Arguments



## Value

a list consisting of the following components:

phat Point prediction

lower lower bound prediction

upper upper bound prediction

xpoly indices of crossed polygons

#### <span id="page-2-0"></span> $GH$  3

# Author(s)

Jiaying Gu

# See Also

predict.KW2 for a simpler prediction function without bounds

# GH *Current Status Linear Regression*

# Description

Groeneboom and Hendrickx semiparametric binary response estimator (scalar case) score estimator based on NPMLE avoids any smoothing proposed by Groneboom and Hendrickx (2018).

# Usage

 $GH(b, X, y, eps = 0.001)$ 

# Arguments



# Value

A list with components:

- evaluation of a score function at parameter value
- estimated standard error
- sindex single index linear predictor

# References

Groeneboom, P. and K. Hendrickx (2018) Current Status Linear Regression, Annals of Statistics, 46, 1415-1444,

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Groeneboom and Hendrickx semiparametric binary response estimator (scalar case) score estimator based on NPMLE avoids any smoothing proposed by Groneboom and Hendrickx (2018).

#### Usage

 $GH. se(bstar, X, y, eps = 0.001, hc = 2)$ 

# Arguments



#### Value

A list with components:

- evaluation of a score function at parameter value
- estimated standard error
- sindex single index linear predictor

#### References

Groeneboom, P. and K. Hendrickx (2018) Current Status Linear Regression, Annals of Statistics, 46, 1415-1444,



# Description

These parameters can be passed via the ... argument of the rcbr function. defaults as suggested in Gautier and Kitamura matlab code

#### <span id="page-4-0"></span>Horowitz93 5

# Usage

GK.control(n,  $u = -20:20/10$ ,  $v = -20:20/10$ ,  $T = 3$ ,  $TX = 10$ ,  $Mn = 1/\log(n)^{2}$ )

#### Arguments



# Value

updated list

Horowitz93 *Horowitz (1993) Modal Choice Data*

#### Description

Modal choice data for journey to work in the Washington DC area from the late 1960's. The variables are: \* 'DCOST': difference in cost of car versus transit (transit - car) \* 'CARS': number of cars at home \* 'DOVTT': difference in out of vehicle time (transit - car) \* 'DIVTT': difference in in vehicle time (transit - car) \* 'DEPEND': coded 1 if by car, 0 if by mass transit

#### Usage

Horowitz93

#### Format

A data frame with 842 observations on 5 variables:

#### Source

[https://www.gams.com/latest/gamslib\\_ml/libhtml/gamslib\\_mws.html](https://www.gams.com/latest/gamslib_ml/libhtml/gamslib_mws.html)

# References

Horowitz, J L, (1993) Semiparametric estimation of a work-trip mode choice model. Journal of Econometrics, 58, 49-70.

<span id="page-5-0"></span>

These parameters can be passed via the ... argument of the rcbr function. The first three arguments are only relevant if full cell enumeration is employed for bivariate version of the NPMLE.

# Usage

```
KW.control(
  uv = NULL,u = NULL,v = NULL,initial = c(\emptyset, \emptyset),
  epsbound = 1,
  epstol = 1e-07,
  presolve = 1,
  verb = <math>\emptyset</math>)
```
# Arguments



# Value

updated list

<span id="page-6-0"></span>

Interface function for calls to optimizer from various REBayes functions There is currently only one option for the optimization that based on Mosek. It relies on the Rmosek interface to R see installation instructions in the Readme file in the inst directory of this package. This version of the function is intended to work with versions of Mosek after 7.0. A more experimental option employing the pogs package available from <https://github.com/foges/pogs> and employing an ADMM (Alternating Direction Method of Multipliers) approach has been deprecated, those interested could try installing version 1.4 of REBayes, and following the instructions provided there.

# Usage

KWDual(A, d, w, ...)

#### Arguments



# Value

Returns a list with components:



<span id="page-7-0"></span>. Mosek termination messages are treated as warnings from an R perspective since solutions producing, for example, MSK\_RES\_TRM\_STALL: The optimizer is terminated due to slow progress, may still provide a satisfactory solution, especially when the return status variable is "optimal".

#### Author(s)

R. Koenker

# References

Koenker, R and I. Mizera, (2013) "Convex Optimization, Shape Constraints, Compound Decisions, and Empirical Bayes Rules," *JASA*, 109, 674–685.

Mosek Aps (2015) Users Guide to the R-to-Mosek Optimization Interface, [https://docs.mosek.](https://docs.mosek.com/8.1/rmosek/index.html) [com/8.1/rmosek/index.html](https://docs.mosek.com/8.1/rmosek/index.html).

Koenker, R. and J. Gu, (2017) REBayes: An R Package for Empirical Bayes Mixture Methods, *Journal of Statistical Software*, 82, 1–26.

logLik.GK *log likelihood for Gautier Kitamura procedure*

#### Description

log likelihood for Gautier Kitamura procedure

# Usage

## S3 method for class 'GK' logLik(object, ...)

#### Arguments



#### Value

a scalar log likelihood

<span id="page-8-0"></span>

log likelihood for KW1 procedure

# Usage

```
## S3 method for class 'KW1'
logLik(object, ...)
```
# Arguments



#### Value

a scalar log likelihood

neighbours *Check Neighbouring Cell Counts*

# Description

Compare cell counts for each cell with its neighbours and return indices of the locally maximal cells.

#### Usage

```
neighbours(SignVector)
```
#### Arguments

SignVector n by m matrix of signs produced by NICER

# Value

Column indices of the cells that are locally maximal, i.e. those whose neighbours have strictly fewer cell counts. The corresponding interior points of these cells can be used as potential mass points for the NPMLE function rcbr.fit.KW.

<span id="page-9-0"></span>Find interior points and cell counts of the polygons (cells) formed by a line arrangement.

# Usage

```
NICER(A, b, initial = c(0, 0), verb = TRUE, epsbound = 1, epstol = 1e-07)
```
#### Arguments



# Details

Modified version of the algorithm of Rada and Cerny (2018). The main modifications include preprocessing as hyperplanes are added to determine which new cells are created, thereby reducing the number of calls to the witness function to solve LPs, and treatment of degenerate configurations as well as those in "general position." When the hyperplanes are in general position the number of polytopes (cells) is determined by the elegant formula of Zazlavsky (1975)

$$
m = \binom{n}{d} + n + 1
$$

. In degenerate cases, i.e. when hyperplanes are not in general position, the number of cells is more complicated as considered by Alexanderson and Wetzel (1981). The function polycount is provided to check agreement with their results in an effort to aid in the selection of tolerances for the witness function. Current version is intended for use with  $d = 2$ , but the algorithm is adaptable to  $d > 2$ , and there is an experimental version called NICERd in the package.

#### Value

A list with components:

- SignVector a n by m matrix of signs determining position of cell relative to each hyperplane.
- w a d by m matrix of interior points for the m cells

#### <span id="page-10-0"></span>NICERd 11

#### References

Alexanderson, G.L and J.E. Wetzel, (1981) Arrangements of planes in space, Discrete Math, 34, 219–240. Gu, J. and R. Koenker (2020) Nonparametric Maximum Likelihood Methods for Binary Response Models with Random Coefficients, *J. Am. Stat Assoc* Rada, M. and M. Cerny (2018) A new algorithm for the enumeration of cells of hyperplane arrangements and a comparison with Avis and Fukada's reverse search, SIAM J. of Discrete Math, 32, 455-473. Zaslavsky, T. (1975) Facing up to arrangements: Face-Count Formulas for Partitions of Space by Hyperplanes, Memoirs of the AMS, Number 154.

#### Examples

```
{
if(packageVersion("Rmosek") > "8.0.0"){
    A = \text{cbind}(c(1,-1,1,-2,2,1,3), c(1,1,1,1,1,-1,-2))B = matrix(c(3,1,7,-2,7,-1,1), ncol = 1)plot(NULL, xlim = c(-10, 10), ylim = c(-10, 10))for (i in 1:nrow(A))
 abline(a = B[i,1]/A[i,2], b = -A[i,1]/A[i,2], col = i)
    f = NICER(A, B)for (j in 1:ncol(f$SignVector))
       points(f$w[1,j], f$w[2,j], cex = 0.5)
    }
}
```
NICERd *New (Accelerated) Incremental Cell Enumeration (in) R*

#### Description

Find interior points and cell counts of the polygons (polytopes) formed by a hyperplane arrangement.

# Usage

```
NICERd(
  A,
  b,
  initial = rep(0, ncol(A)),verb = TRUE,
  accelerate = FALSE,
  epsbound = 1,
  epstol = 1e-07)
```
#### Arguments



# Details

Modified version of the algorithm of Rada and Cerny (2018). The main modifications include preprocessing as hyperplanes are added to determine which new cells are created, thereby reducing the number of calls to the witness function to solve LPs, and treatment of degenerate configurations as well as those in "general position." (for  $d = 2$  for now). When the hyperplanes are in general position the number of cells (polytopes) is determined by the elegant formula of Zaslavsky (1975)

$$
m = \binom{n}{d} + n + 1
$$

. In degenerate cases, i.e. when hyperplanes are not in general position, the number of cells is more complicated as considered by Alexanderson and Wetzel (1981). The function polycount is provided to check agreement with their results in an effort to aid in the selection of tolerances for the witness function for arrangement in  $d = 2$ . The current version is intended mainly for use with  $d = 2$ , but the algorithm is adapted to the general position setting with  $d > 2$ , although it requires hyperplanes in general position and may require some patience when both the sample size is large. if hyperplanes not general position (i.e. all cross at origin), turn off accelerate

#### Value

A list with components:

- SignVector a n by m matrix of signs determining position of cell relative to each hyperplane.
- w a d by m matrix of interior points for the m cells

#### References

Alexanderson, G.L and J.E. Wetzel, (1981) Arrangements of planes in space, Discrete Math, 34, 219–240. Rada, M. and M. Cerny (2018) A new algorithm for the enumeration of cells of hyperplane arrangements and a comparison with Avis and Fukada's reverse search, SIAM J. of Discrete Math, 32, 455-473. Zaslavsky, T. (1975) Facing up to arrangements: Face-Count Formulas for Partitions of Space by Hyperplanes, Memoirs of the AMS, Number 154.

<span id="page-12-0"></span>

Given a fitted model by the Guatier-Kitamura procedure plot the estimated density contours

# Usage

## S3 method for class 'GK'  $plot(x, \ldots)$ 

# Arguments



# Value

nothing (invisibly)



# Description

Given a fitted model by the rcbr NPMLE procedure plot the estimated mass points

# Usage

```
## S3 method for class 'KW2'
plot(x, smooth = 0, pal = NULL, inches = 1/6, N = 25, tol = 0.001, ...)
```
# Arguments



<span id="page-13-0"></span>14 polyzone

# Value

nothing (invisibly)

polycount *Check Cell Count for degenerate hyperplane arrangements*

# Description

When the hyperplane arrangement is degenerate, i.e. not in general position, the number of distinct cells can be checked against the formula of Alexanderson and Wetzel (1981).

# Usage

polycount(A, b, maxints = 10)

# Arguments



# Value

number of distinct cells

# References

Alexanderson, G.L and J.E. Wetzel, (1981) Arrangements of planes in space, Discrete Math, 34, 219–240.



# Description

Given an existing cell configuration represented by the Signvector and associated interior points w, identify the polygons crossed by the next new line.

#### Usage

```
polyzone(SignVector, w, A, b)
```
#### <span id="page-14-0"></span>prcbr that the contract of the

#### **Arguments**



# Value

vector of indices of crossed polygons

#### Author(s)

Jiaying Gu

prcbr *Profiling estimation methods for RCBR models*

# Description

Profile likelihood and (GEE) score methods for estimation of random coefficient binary response models. This function is a wrapper for rcbr that uses the offset argument to implement estimation of additional fixed parameters. It may be useful to restrict the domain of the optimization over the profiled parameters, this can be accomplished, at least for box constraints by setting omethod = "L-BFGS-B" and specifying the lo and up accordingly.

#### Usage

```
prcbr(
  formula,
  b0,
  data,
  logL = TRUE,
  omethod = "BFGS",
  lo = -Inf,up = Inf,...
\mathcal{L}
```
# Arguments

formula is of the extended form enabled by the Formula package. In the Cosslett, or current status, model the formula takes the form  $y \sim v \mid z$  where v is the covariate designated to have coefficient one, and z is another covariate or group of covariates that are assumed fixed coefficients that are to be estimated.

<span id="page-15-0"></span>

# Value

a list comprising the components:

bopt output of the optimizer for the profiled parameters beta

fopt output of the optimizer for the random coefficients eta

predict.GK *Prediction of Marginal Effects*

# Description

Given a fitted model by the Gautier Kitamura procedure predictions are made at new design points given by the newdata argument.

# Usage

## S3 method for class 'GK' predict(object, ...)

#### Arguments



# Value

a vector pf predicted probabilities

<span id="page-16-0"></span>

Given a fitted model by the rcbr NPMLE procedure predictions are made at new design points given by the newdata argument.

### Usage

## S3 method for class 'KW2' predict(object, ...)

#### Arguments



# Value

a vector pf predicted probabilities

#### See Also

bound.KW2 for a prediction function with bounds

rcbr *Estimation of Random Coefficient Binary Response Models*

# Description

Two methods are implemented for estimating binary response models with random coefficients: A nonparametric maximum likelihood method proposed by Cosslett (1986) and extended by Ichimura and Thompson (1998), and a (hemispherical) deconvolution method proposed by Gautier and and Kitamura (2013). The former is closely related to the NPMLE for mixture models of Kiefer and Wolfowitz (1956). The latter is an R translation of the matlab implementation of Gautier and Kitamura.

## Usage

```
rcbr(formula, data, subset, offset, mode = "GK", ...)
```
#### Arguments



## Details

The predict method produces estimates of the probability of a "success"  $(y = 1)$  for a particular vector, (z,v), when aggregated over the estimated distribution of random coefficients.

The logLik produces an evaluation of the log likelihood value associated with a fitted model.

# Value

of object of class GK, KW1, with components described in further detail in the respective fitting functions.

#### Author(s)

Jiaying Gu and Roger Koenker

#### References

Kiefer, J. and J. Wolfowitz (1956) Consistency of the Maximum Likelihood Estimator in the Presence of Infinitely Many Incidental Parameters, *Ann. Math. Statist*, 27, 887-906.

Cosslett, S. (1983) Distribution Free Maximum Likelihood Estimator of the Binary Choice Model, *Econometrica*, 51, 765-782.

Gautier, E. and Y. Kitamura (2013) Nonparametric estimation in random coefficients binary choice models, *Ecoonmetrica*, 81, 581-607.

Gu, J. and R. Koenker (2020) Nonparametric Maximum Likelihood Methods for Binary Response Models with Random Coefficients, *J. Am. Stat Assoc*

Groeneboom, P. and K. Hendrickx (2016) Current Status Linear Regression, preprint available from <https://arxiv.org/abs/1601.00202>.

#### <span id="page-18-0"></span>rcbr.fit that the control of the control o

Ichimura, H. and T. S. Thompson, (1998) Maximum likelihood estimation of a binary choice model with random coefficients of unknown distribution," *Journal of Econometrics*, 86, 269-295.

#### Examples

```
{
if(packageVersion("Rmosek") > "8.0.0"){
    # Simple Test Problem for rcbr
    n < - 60B0 = \text{rbind}(c(0.7, -0.7, 1), c(-0.7, 0.7, 1))z \le- rnorm(n)
    v \le - rnorm(n)s < - sample(0:1, n, replace = TRUE)
    XB0 <- cbind(1,z,v) %*% t(B0)
    u \leq -s * XBO[, 1] + (1-s) * XBO[, 2]y \le - (u > 0) - 0
    D \le - data.frame(z = z, v = v, y = y)
    f \leq rcbr(y \sim z + v, mode = "KW", data = D)
    plot(f)
    # Simple Test Problem for rcbr
    set.seed(15)
    n < -100B0 = \text{rbind}(c(0.7, -0.7, 1), c(-0.7, 0.7, 1))z \leq rnorm(n)
    v < - rnorm(n)
    s < - sample(0:1, n, replace = TRUE)
    XB0 <- cbind(1,z,v) %*% t(B0)
    u \leq -s \times XBO[, 1] + (1-s) \times XBO[, 2]y \leftarrow (u > 0) - 0D \le data.frame(z = z, v = v, y = y)
    f \le -rcbr(y \sim z + v, \text{ mode} = "GK", \text{ data} = D)contour(f$u, f$v, matrix(f$w, length(f$u)))
    points(x = 0.7, y = -0.7, col = 2)
    points(x = -0.7, y = 0.7, col = 2)
    f \le -rcbr(y - z + v, \text{ mode} = "GK", \text{ data} = D, T = 7)contour(f$u, f$v, matrix(f$w, length(f$u)))
    points(x = 0.7, y = -0.7, col = 2)points(x = -0.7, y = 0.7, col = 2)}
}
```
rcbr.fit *Fitting of Random Coefficient Binary Response Models*

#### Description

Two methods are implemented for estimating binary response models with random coefficients: A nonparametric maximum likelihood method proposed by Cosslett (1986) and extended by Ichimura and Thompson (1998), and a (hemispherical) deconvolution method proposed by Gautier and and Kitamura (2013). The former is closely related to the NPMLE for mixture models of Kiefer and

Wolfowitz (1956). The latter is an R translation of the matlab implementation of Gautier and Kitamura.

#### Usage

 $rcbr.fit(x, y, offset = NULL, mode = "KW", control)$ 

#### Arguments



# Details

The predict method produces estimates of the probability of a "success"  $(y = 1)$  for a particular vector,  $(z, v)$ , when aggregated over the estimated distribution of random coefficients.

#### Value

of object of class GK, KW1, with components described in further detail in the respective fitting functions.

#### Author(s)

Jiaying Gu and Roger Koenker

#### References

Kiefer, J. and J. Wolfowitz (1956) Consistency of the Maximum Likelihood Estimator in the Presence of Infinitely Many Incidental Parameters, *Ann. Math. Statist*, 27, 887-906.

Cosslett, S. (1983) Distribution Free Maximum Likelihood Estimator of the Binary Choice Model, *Econometrica*, 51, 765-782. Gautier, E. and Y. Kitamura (2013) Nonparametric estimation in random coefficients binary choice models, *Ecoonmetrica*, 81, 581-607.

Groeneboom, P. and K. Hendrickx (2016) Current Status Linear Regression, preprint available from <https://arxiv.org/abs/1601.00202>.

Ichimuma, H. and T. S. Thompson, (1998) Maximum likelihood estimation of a binary choice model with random coefficients of unknown distribution," *Journal of Econometrics*, 86, 269-295.

<span id="page-20-0"></span>rcbr.fit.GK *Gautier and Kitamura (2013) bivariate random coefficient binary response*

#### Description

This is an implementation based on the matlab version of Gautier and Kitamura's deconvolution method for the bivariate random coefficient binary response model. Methods based on the fitted object are provided for predict, logLik and plot.requires orthopolynom package for Gegenbauer polynomials

# Usage

rcbr.fit.GK(X, y, control)

# Arguments



#### Value

a list with components:

- u grid values
- v grid values
- w estimated function values on 2d u x v grid
- X design matrix
- y response vector

# Author(s)

Gautier and Kitamura for original matlab version, Jiaying Gu and Roger Koenker for the R translation.

# References

Gautier, E. and Y. Kitamura (2013) Nonparametric estimation in random coefficients binary choice models, *Ecoonmetrica*, 81, 581-607.

<span id="page-21-0"></span>rcbr.fit.KW1 *NPMLE fitting for the Cosslett random coefficient binary response model*

# Description

This is the original one dimensional version of the Cosslett model, also known as the current status model:

$$
P(y = 1|v) = \int I(\eta > v)dF(\eta).
$$

invoked with the formula  $y \sim v$ . By default the algorithm computes a vector of potential locations for the mass points of  $\hat{F}$  by finding interior points of the intervals between the ordered v, and then solving a convex optimization problem to determine these masses. Alternatively, a vector of predetermined locations can be passed via the control argument. Additional covariate effects can be accommodated by either specifying a fixed offset in the call to rcbr or by using the profile likelihood function prcbr.

# Usage

```
rcbr.fit.KW1(X, y, control)
```
# Arguments



#### Value

a list with components:

- x evaluation points for the fitted distribution
- y estimated mass associated with the v points
- logLik the loglikelihood value of the fit
- status mosek solution status

#### Author(s)

Jiaying Gu and Roger Koenker

# References

Gu, J. and R. Koenker (2018) Nonparametric maximum likelihood estimation of the random coefficients binary choice model, preprint.

<span id="page-22-0"></span>Exact NPMLE fitting requires that the uv argument contain a matrix whose rows represent points in the interior of the locally maximal polytopes determined by the hyperplane arrangement of the observations. If it is not provided it will be computed afresh here; since this can be somewhat time consuming, uv is included in the returned object so that it can be reused if desired. Approximate NPMLE fitting can be achieved by specifying an equally spaced grid of points at which the NPMLE can assign mass using the arguments u and v. If the design matrix X contains only 2 columns, so we have the Cosslett, aka current status, model then the polygons in the prior description collapse to intervals and the default method computes the locally maximal count intervals and passes their interior points to the optimizer of the log likelihood. Alternatively, as in the bivariate case one can specify a grid to obtain an approximate solution.

# Usage

rcbr.fit.KW2(x, y, control)

#### Arguments



#### Value

a list with components:

- uv evaluation points for the fitted distribution
- W estimated mass associated with the uv points
- logLik the loglikelihood value of the fit
- status mosek solution status

#### Author(s)

Jiaying Gu and Roger Koenker

#### References

Gu, J. and R. Koenker (2018) Nonparametric maximum likelihood estimation of the random coefficients binary choice model, preprint.

<span id="page-23-0"></span>

Find (if possible) an interior point of a polytope solving a linear program

# Usage

```
witness(A, b, s, epsbound = 1, epstol = 1e-07, presolve = 1, verb = 0)
```
# Arguments



# Details

Solves LP:  $maxoverw, epseps|SAw - eps >= Sb, 0 < eps <= epsbound S$  is diag(s), if at the solution  $eps > 0$ , then w is a valid interior point otherwise the LP fails to find an interior point, another s must be tried. Constructs a problem formulation that can be passed to Rmosek for solution.

# Value

List with components:

- w proposed interior point at solution
- fail indicator of whether w is a valid interior point

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