Digital Imaging and Communications in Medicine (DICOM)

Part 14: Grayscale Standard Display Function

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FOREWORD

The American College of Radiology (ACR) and the National Electrical Manufacturers Association (NEMA) formed a joint committee to develop a standard for Digital Imaging and Communications in Medicine (DICOM). While other parts of the DICOM Standard specify how digital image data can be moved from system to system, it does not specify how the pixel values should be interpreted or displayed. PS 3.14 specifies a function that relates pixel values to displayed Luminance levels.

A digital signal from an image can be measured, characterized, transmitted, and reproduced objectively and accurately. However, the visual interpretation of that signal is dependent on the varied characteristics of the systems displaying that image. Currently, images produced by the same signal may have completely different visual appearance, information, and characteristics on different display devices.

In medical imaging, it is important that there be a visual consistency in how a given digital image appears, whether viewed, for example, on the display monitor of a workstation or as a film on a light-box. In the absence of any standard which regulates how these images are to be visually presented on any device, a digital image which has good diagnostic value when viewed on one device could look very different and have greatly reduced diagnostic value when viewed on another device. Accordingly, PS 3.14 was developed to provide an objective, quantitative mechanism for mapping digital image values into a given range of Luminance. An application which knows this relationship between digital values and display Luminance can produce better visual consistency in how that image appears on diverse display devices. The relationship that PS 3.14 defines between digital image values and displayed Luminance is based upon measurements and models of human perception over a wide range of Luminance, not upon the characteristics of any one image presentation device or of any one imaging modality. It is also not dependent upon user preferences, which can be more properly handled by other constructs such as the DICOM Presentation Lookup Table.

PS 3.14 was reviewed by other standardization organizations including CEN TC251 in Europe and JIRA in Japan, and by other organizations including IEEE, HL7, and ANSI in the USA.

PS 3.14 was developed according to NEMA procedures.

PS 3.14 is structured using the guidelines established in the following document:

ISO/IEC Directives, 1989 Part 3: Drafting and Presentation of International Standards.

1. Scope and Field of Application

PS 3.14 specifies a standardized Display Function for display of grayscale images. It provides examples of methods for measuring the Characteristic Curve of a particular Display System for the purpose of either altering the Display System to match the Grayscale Standard Display Function, or for measuring the conformance of a Display System to the Grayscale Standard Display Function. Display Systems include such things as monitors with their associated driving electronics and printers producing films that are placed on light-boxes or alternators.

PS 3.14 is neither a performance nor an image display standard. PS 3.14 does not define which Luminance and/or Luminance Range or optical density range an image presentation device must provide. PS 3.14 does not define how the particular picture element values in a specific imaging modality are to be presented.

PS 3.14 does not specify functions for display of color images, as the specified function is limited to the display of grayscale images. Color Display Systems may be calibrated to the Grayscale Standard Display Function for the purpose of displaying grayscale images. Color images, whether associated with an ICC Profile or not, may be displayed on standardized grayscale displays, but there are no normative requirements for the display of the luminance information in a color image using the GSDF.

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2. Normative References

The following standards contain provisions, which, through reference in this text, constitute provisions of this Standard. At the time of publication, the editions indicated were valid. All standards are subject to revision, and parties to agreements based on this Standard are encouraged to investigate the possibilities of applying the most recent editions of the standards indicated below.

ISO/IEC Directives, 1989 Part 3 - Drafting and presentation of International Standards.

3. Definitions

For the purposes of PS 3.14 the following definitions apply.

<u>Characteristic Curve</u>: The inherent Display Function of a Display System including the effects of ambient light. The Characteristic Curve describes Luminance versus DDL of an emissive display device, such as a CRT/display controller system, or Luminance of light reflected from a print medium, or Luminance derived from the measured optical density versus DDL of a hard-copy medium and the given Luminance of a light-box. The Characteristic Curve depends on operating parameters of the Display System.

Note: The Luminance generated by an emissive display may be measured with a photometer. Diffuse optical density of a hard-copy may be measured with a densitometer.

<u>Contrast Sensitivity</u> characterizes the sensitivity of the average human observer to Luminance changes of the Standard Target. Contrast Sensitivity is inversely proportional to Threshold Modulation.

<u>Contrast Threshold</u>: A function that plots the Just-Noticeable Difference divided by the Luminance across the Luminance Range.

<u>Digital Driving Level</u> (DDL): A digital value which given as input to a Display System produces a Luminance. The set of DDLs of a Display System is all the possible discrete values that can produce Luminance values on that Display System. The mapping of DDLs to Luminance values for a Display System produces the Characteristic Curve of that Display System. The actual output for a given DDL is specific to the Display System and is not corrected for the Grayscale Standard Display Function.

<u>Display Function</u>: A function that describes a defined grayscale rendition of a Display System, the mapping of the DDLs in a defined space to Luminance, including the effects of ambient light at a given state of adjustment of the Display System. Distinguished from Characteristic Curve, which is the inherent Display Function of a Display System.

<u>Display System</u>: A device or devices that accept DDLs to produce corresponding Luminance values. This includes emissive displays, transmissive hardcopy viewed on light boxes, and reflective hardcopy.

<u>Illuminance</u>: Light from the environment surrounding the Display System which illuminates the display medium. It contributes to the Luminance that is received by an observer from the image display. Ambient Light reduces the contrast in the image.

<u>Just-Noticeable Difference</u> (JND): The Luminance difference of a given target under given viewing conditions that the average human observer can just perceive.

<u>JND Index</u>: The input value to the Grayscale Standard Display Function, such that one step in JND Index results in a Luminance difference that is a Just-Noticeable Difference.

<u>Luminance</u> is the luminous intensity per unit area projected in a given direction. The Système Internationale unit (used in PS 3.14) is candela per square meter (cd/m^2), which is sometimes called nit. Another unit often used is footlambert (fL). 1 fL = 3.426 cd/m².

<u>Luminance Range</u>: The span of Luminance values of a Display System from a minimum Luminance to a maximum Luminance.

<u>P-Value</u>: A device independent value defined in a perceptually linear grayscale space. The output of the DICOM Presentation LUT is P-Values, ie. the pixel value after all DICOM defined grayscale transformations have been applied. P-Values are the input to a Standardized Display System.

<u>Grayscale Standard Display Function</u>: The mathematically defined mapping of an input JND index to Luminance values defined in PS 3.14.

<u>Standardized Display System</u>: A device or devices that produce Luminance values which are related to input P-Values by the Grayscale Standard Display Function. How this is performed is not defined, though it may be achieved by transformation of P-Values into DDLs accepted by a Display System.

Standard Luminance Level: Any one of the Standard Luminance levels in Table B-1.

<u>Standard Target</u>: A 2-deg x 2-deg square filled with a horizontal or vertical grating with sinusoidal modulation of 4 cycles per degree. The square is placed in a uniform background of a Luminance equal to the mean Luminance of the Target.

Note: The Standard Target is defined in terms of the subtended viewing angle, not in terms of the distance from the viewer to the target.

<u>Threshold Modulation</u>: The minimum Luminance modulation required by the average human observer to detect the Standard Target at a given mean Luminance level. The Threshold Modulation corresponds to the Just-Noticeable Difference in Luminance of the Standard Target.

4. Symbols and Abbreviations

The following symbols and abbreviations are used in PS 3.14.

ACR American College of Radiology

ANSI American National Standards Institute

CEN TC251 Comite' Europeen de Normalisation - Technical Committee 251 - Medical Informatics

DICOM Digital Imaging and Communications in Medicine

HL7 Health Level 7

IEEE Institute of Electrical and Electronics Engineers

ISO International Standards Organization

JIRA Japan Industries Association of Radiation Apparatus

NEMA National Electrical Manufacturers Association

5. Conventions

The following conventions are used in PS 3.14:

The terminology defined in Section 3 above is capitalized throughout PS 3.14.

6. Overview

PS 3.14 defines, mathematically, the Grayscale Standard Display Function of Standardized Display Systems. These systems may be printers producing hard-copies viewed on light-boxes or electronic Display Systems for soft-copies.

Hard-copies may consist of transmissive films as well as reflective prints. The image in these prints is represented by optical density variations in transmission or diffuse reflection. To an observer, every element of the image appears with a certain Luminance depending on the Illuminance and the optical density of the image element.

Soft-copies may be produced by emissive Display Systems (such as CRT monitors) or electronic light valves (such as light sources and liquid crystal displays).

For the purpose of PS 3.14, Display Systems take a Digital Driving Level and produce Luminance or optical density variations that represent the image. Predictable application of image transformations, such as the modality, value-of-interest, and presentation look-up tables specified in the DICOM standard, requires knowledge of the Characteristic Curve of the Display System. Standardizing the response function expected of the Display System simplifies the application of such image transformations across several different Display Systems such as encountered in a network environment.

PS 3.14 does not define when conformance with the Grayscale Standard Display Function is achieved or how to characterize the degree of conformance reached.

Note: A definition of conformance would require thorough evaluations of human visual system sensitivity to deviations of Display Functions from the Grayscale Standard Display Function for medical images.

Figures 6-1 and 6-2 show the context for the Grayscale Standard Display Function. The Grayscale Standard Display Function is part of the image presentation. There will be a number of other modifications to the image before the Grayscale Standard Display Function is applied. The image acquisition device will adjust the image as it is formed. Other elements may perform a "window and level" to select a part of the dynamic range of the image to be presented. Yet other elements can adjust the selected dynamic range in preparation for display. The Presentation LUT outputs P-Values (presentation values). These P-Values become the Digital Driving Levels for Standardized Display Systems. The Grayscale Standard Display Function maps P-Values to the log-luminance output of the Standardized Display System. How a Standardized Display System performs this mapping is implementation dependent.

The boundary between the DICOM model of the image acquisition and presentation chain, and the Standardized Display System, expressed in P-Values, is intended to be both device independent and conceptually (if not actually) perceptually linear. In other words, regardless of the capabilities of the Standardized Display System, the same range of P-Values will be presented isimilarlyî.

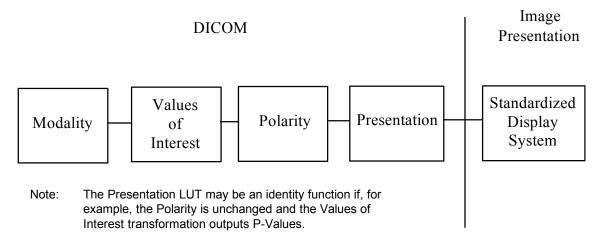


Figure 6-1. The Grayscale Standard Display Function is an element of the image presentation after several modifications to the image have been completed by other elements of the image acquisition and presentation chain.

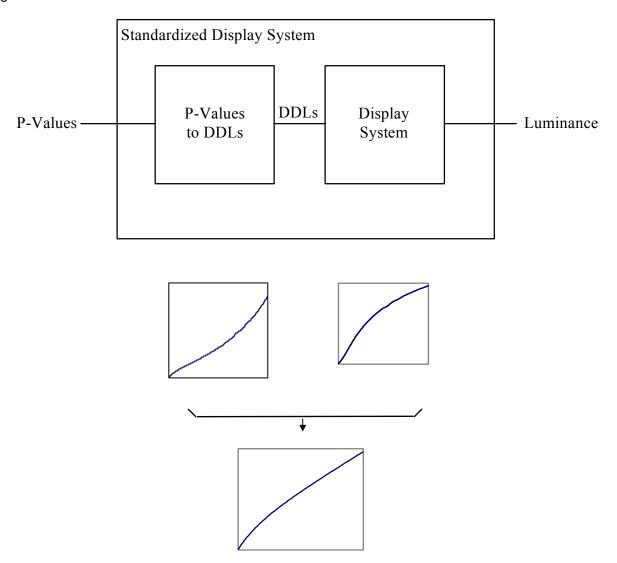


Figure 6-2. The conceptual model of a Standardized Display System maps P-Values to Luminance via an intermediate transformation to Digital Driving Levels of an unstandardized Display System.

The main objective of PS 3.14 is to define mathematically an appropriate Grayscale Standard Display Function for all image presentation systems. The purpose of defining this Grayscale Standard Display Function is to allow applications to know *a priori* how P-Values are transformed to viewed Luminance values by a Standardized Display System. In essence, defining the Grayscale Standard Display Function fixes the "units" for the P-Values output from the Presentation LUT and used as Digital Driving Levels to Standardized Display Systems.

A second objective of PS 3.14 is to select a Display Function which provides some level of similarity in grayscale perception or basic appearance for a given image between Display Systems of different Luminance and which facilitates good use of the available Digital Driving Levels of a Display System. While many different functions could serve the primary objective, this Grayscale Standard Display Function was chosen to meet the second objective. With such a function, P-Values are approximately linearly related to human perceptual response. Similarity does not guarantee equal information content. Display Systems with a wider Luminance Range and/or higher Luminance will be capable of presenting

more just-noticeable Luminance differences to an observer. Similarity also does not imply strict perceptual linearity, since perception is dependent on image content and on the viewer. In order to achieve strict perceptual linearity, applications would need to adjust the presentation of images to match user expectations through the other constructs defined in the DICOM Standard (eg. VOI and Presentation LUT). Without a defined Display Function, such adjustments on the wide variety of Display Systems encountered on a network would be difficult.

The choice of the function is based on several ideas that are discussed further in Annex A.

Annex B contains the Grayscale Standard Display Function in tabular form.

Informative Annex C provides an example procedure for comparing mathematically the shape of the actual Display Function with the Grayscale Standard Display Function and for quantifying how well the actual discrete Luminance intervals match those of the Grayscale Standard Display Function.

Display Systems often will have Characteristic Curves different from the Grayscale Standard Display Function. These devices may contain means for incorporating externally defined transformations that make the devices conform with the Grayscale Standard Display Function. PS 3.14 provides examples of test patterns for Display Systems with which their behavior can be measured and the approximation to the Grayscale Standard Display Function evaluated (see Informative Annex D.1, D.2, D.3).

7 The Grayscale Standard Display Function

As explained in greater detail in Annex A, the Grayscale Standard Display Function is based on human Contrast Sensitivity. Human Contrast Sensitivity is distinctly non-linear within the Luminance Range of the Grayscale Standard Display Function . The human eye is relatively less sensitive in the dark areas of an image than it is in the bright areas of an image. This variation in sensitivity makes it much easier to see small relative changes in Luminance in the bright areas of the image than in the dark areas of the image. A Display Function that adjusts the brightness such that equal changes in P-Values will result in the same level of perceptibility at all driving levels is "perceptually linearized". The Grayscale Standard Display Function incorporates the notion of perceptual linearization without making it an explicit objective of PS 3.14.

The employed data for Contrast Sensitivity are derived from Barten's model of the human visual system (Ref. 1, 2 and Annex B). Specifically, the Grayscale Standard Display Function refers to Contrast Sensitivity for the Standard Target consisting of a 2-deg x 2-deg square filled with a horizontal or vertical grating with sinusoidal modulation of 4 cycles per degree. The square is placed in a uniform background of Luminance equal to the mean Luminance L of the Target. The Contrast Sensitivity is defined by the Threshold Modulation at which the grating becomes just visible to the average human observer. The Luminance modulation represents the Just-Noticeable Difference (JND) for the Target at the Luminance L.

Note: The academic nature of the Standard Target is recognized. With the simple target, the essential objectives of PS 3.14 appear to be realizable. Only spurious results with more realistic targets in complex surroundings were known at the time of writing PS 3.14 and these were not assessed.

The Grayscale Standard Display Function is defined for the Luminance Range from 0.05 to 4000 cd/m². The minimum Luminance corresponds to the lowest practically useful Luminance of cathode-ray-tube (CRT) monitors and the maximum exceeds the unattenuated Luminance of very bright light-boxes used for interpreting X-Ray mammography. The Grayscale Standard Display Function explicitly includes the effects of the diffused ambient Illuminance.

Within the Luminance Range happen to fall 1023 JNDs (see Annex A).

7.1 GENERAL FORMULAS

The Grayscale Standard Display Function is defined by a mathematical interpolation of the 1023 Luminance levels derived from Barten's model. The Grayscale Standard Display Function allows us to calculate luminance, L, in candelas per square meter, as a function of the Just-Noticeable Difference (JND) Index, j:

$$\log_{10} L(j) = \frac{a + c \cdot Ln(j) + e \cdot (Ln(j))^2 + g \cdot (Ln(j))^3 + m \cdot (Ln(j))^4}{1 + b \cdot Ln(j) + d \cdot (Ln(j))^2 + f \cdot (Ln(j))^3 + h \cdot (Ln(j))^4 + k \cdot (Ln(j))^5}$$

with Ln referring to the natural logarithm, j the index (1 to 1023) of the Luminance levels L_j of the JNDs, and a = -1.3011877, b = -2.5840191E-2, c = 8.0242636E-2, d = -1.0320229E-1, e = 1.3646699E-1, f = 2.8745620E-2, g = -2.5468404E-2, h = -3.1978977E-3, k = 1.2992634E-4, m = 1.3635334E-3.

The logarithms to the base 10 of the Luminance L_j are very well interpolated by this function over the entire Luminance Range. The relative deviation of any log(Luminance)-value from the function is at most 0.3%, and the root-mean-square-error is 0.0003. The continuous representation of the Grayscale Standard Display Function permits a user to compute discrete JNDs for arbitrary start levels and over any desired Luminance Range.

Notes: 1. To apply the above formula to a device with a specific range of L values, it is convenient to also have the inverse of this relationship, which is given by:

$$\begin{split} j(L) &= A + B \cdot Log_{10}(L) + C \cdot (Log_{10}(L))^2 + D \cdot (Log_{10}(L))^3 + E \cdot (Log_{10}(L))^4 + \\ F \cdot (Log_{10}(L))^5 + G \cdot (Log_{10}(L))^6 + H \cdot (Log_{10}(L))^7 + I \cdot (Log_{10}(L))^8 \\ \text{where Log}_{10} \text{ represents logarithm to the base 10, and A = 71.498068, B = 94.593053, C = 41.912053, D = 9.8247004, E = 0.28175407, F = -1.1878455, G = -0.18014349, H = 0.14710899, I = -0.017046845. \end{split}$$

- 2. When incorporating the formulas for L(j) and j(L) into a computer program, the use of double precision is recommended.
- 3. Alternative methods may be used to calculate the JND Index values. One method is use a numerical algorithm such as the Van Vijngaarden-Dekker-Brent method described in *Numerical Recipes in C* (Cambridge University press, 1991). The value j may be calculated from L iteratively given the Grayscale Standard Display Function's formula for L(j). Another method would be to use the Grayscale Standard Display Function's tabulated values of j and L to calculate the j corresponding to an arbitrary L by linearly interpolating between the two nearest tabulated L,j pairs.
- 4. No specification is intended as to how these formulas are implemented. These could be implemented dynamically, by executing the equation directly, or through discrete values, such as a LUT, etc.

Annex B lists the Luminance levels computed with this equation for the 1023 integer JND Indices and Fig. 7-1 shows a plot of the Grayscale Standard Display Function. The exact value of the Luminance levels, of course, depends on the start level of 0.05 cd/m^2 .

The Characteristic Curve of a Display System represents the Luminance produced by a Display System as a function of DDL and the effect of ambient Illuminance. The Characteristic Curve is measured with Standard Test Patterns (see Annex D). In general, the Display Function describes, for example,

a) the Luminance (including ambient Illuminance) measured as a function of DDL for emissive displays such as a CRT-monitor/digital display controller system,

- b) the Luminance (including ambient Illuminance) as a function of DDL measured for a transmissive medium hung in front of a light-box after a printer produced an optical density, depending on DDL, on the medium,
- the Luminance (including ambient light) as a function of DDL measured for a diffusely reflective medium illuminated by a office lights after a printer produced a reflective density, depending on DDL, on the medium.

By internal or external means, the system may have been configured (or calibrated) such that the Characteristic Curve is consistent with the Grayscale Standard Display Function.

Some Display Systems adapt themselves to ambient light conditions. Such a system may conform to the Grayscale Standard Display Function for one level of ambient Illuminance only, unless it had the capability of adjusting its Display Function without user-intervention so that it remains in conformance with the Grayscale Standard Display Function.

7.2 TRANSMISSIVE HARDCOPY PRINTERS

For transmissive hardcopy printing, the relationship between luminance, L, and the printed optical density, D, is:

$$L = L_a + L_0 \cdot 10^{-D}$$

where:

L₀ is the luminance of the light box with no film present, and

La is the luminance contribution due to ambient illuminance reflected off the film.

If film is to be printed with a density ranging from D_{min} to D_{max} , the final luminance will range between:

$$L_{\rm min} = L_a + L_0 \cdot 10^{-D_{\rm max}}$$
 , $L_{\rm max} = L_a + L_0 \cdot 10^{-D_{\rm min}}$

and the j values will correspondingly range from $j_{min} = j(L_{min})$ to $j_{max} = j(L_{max})$.

If this span of j values is represented by an N-bit P-Value, ranging from 0 for j_{min} to 2^{N} -1 for j_{max} , the j values will correspond to P-Values as follows:

$$j(p) = j_{\min} + \frac{p}{2^N - 1} \cdot (j_{\max} - j_{\min})$$

and the corresponding L values will be L(j(p)).

Finally, converting the L(j(p)) values to densities results in:

$$D(p) = -Log_{10}(\frac{L(j(p)) - L_a}{L_0}).$$

Note: Typical values for the parameters used in transmissive hardcopy printing are $L_0 = 2000 \text{ cd/m}^2$ $L_a = 10 \text{ cd/m}^2$.

7.3 REFLECTIVE HARDCOPY PRINTERS

For reflective hardcopy printing, the relationship between luminance, L, and the printed optical density, D, is:

$$L = L_0 \cdot 10^{-D}$$

where:

L₀ is the maximum luminance obtainable from diffuse reflection of the illumination that is present.

If film is to be printed with a density ranging from D_{min} to D_{max} , the final luminance will range between:

$$L_{\rm min} = L_0 \cdot 10^{-D_{\rm max}}$$
 , $L_{\rm max} = L_0 \cdot 10^{-D_{\rm min}}$

and the j values will correspondingly range from $j_{min} = j(L_{min})$ to $j_{max} = j(L_{max})$.

If this span of j values is represented by an N-bit P-Value, ranging from 0 for j_{min} to 2^N -1 for j_{max} , the j values will correspond to P-Values as follows:

$$j(p) = j_{\min} + \frac{p}{2^N - 1} \cdot (j_{\max} - j_{\min})$$

and the corresponding L values will be L(j(p)).

Finally, converting the L(j(p)) values to densities results in

$$D(p) = -Log_{10}(\frac{L(j(p))}{L_0}).$$

Note: Typical values for the parameters used in reflective hardcopy printing are $L_0 = 150 \text{ cd/m}^2$.

8 References

- 1) Barten, P.G.J., Physical model for the Contrast Sensitivity of the human eye. Proc. SPIE 1666, 57-72 (1992)
- 2) Barten, P.G.J., Spatio-temporal model for the Contrast Sensitivity of the human eye and its temporal aspects. Proc. SPIE 1913-01 (1993)

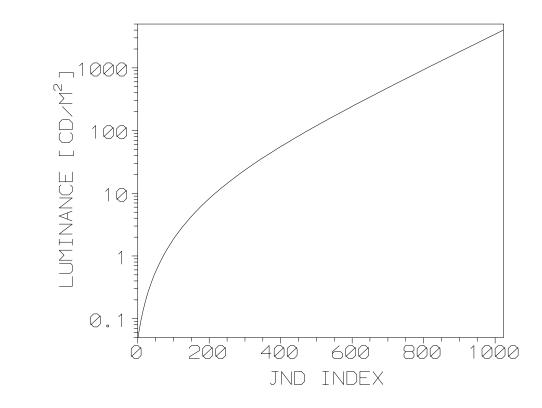


Fig. 7-1. The Grayscale Standard Display Function presented as logarithm-of-Luminance versus JND-Index

Annex A (INFORMATIVE) A DERIVATION OF THE GRAYSCALE STANDARD DISPLAY FUNCTION

A.1. RATIONALE FOR SELECTING THE GRAYSCALE STANDARD DISPLAY FUNCTION

In choosing the Grayscale Standard Display Function, it was considered mandatory to have only one continuous, monotonically behaving mathematical function for the entire Luminance Range of interest. Correspondingly, for simplicity of implementing the Grayscale Standard Display Function, it was felt to be useful to define it by only one table of data pairs. As a secondary objective, it was considered desirable that the Grayscale Standard Display Function provide similarity in grayscale rendition on Display Systems of different Luminance Range and that good use of the available DDLs of a Display System was facilitated.

Perceptual linearization was thought to be a useful concept for arriving at a Grayscale Standard Display Function for meeting the above secondary objectives; however, it is not considered an objective by itself. Apart from the fact that is probably an elusive goal to perceptually linearize all types of medical images under various viewing conditions by one mathematical function, medical images are mostly presented by application-specific Display Functions that assign contrast non-uniformly according to clinical needs.

Intuitively, one would assume that perceptually linearized images on different Display Systems will be judged to be similar. To achieve perceptual linearization, a model of the human visual system response was required and the Barten model [A1] was chosen.

Early experiments showed that an appealing degree of contrast equalization and similarity could be obtained with a Display Function derived from Bartenís model of human visual system response. The employed images were square patterns, the SMPTE pattern, and the Briggsí pattern [A2].

It was wished to relate DDLs of a Display System to some perceptually linear scale, primarily, to gain efficient utilization of the available input levels. If digitization levels lead to luminance or optical density levels that are perceptually indistinguishable, they are wasted. If they are too far apart, the observer may see contours. Hence, the concept of perceptual linearization was retained, not as a goal for the Grayscale Standard Display Function, but to obtain a concept for a measure of how well these objectives have been met.

Perceptual linearization is realizable, in a strict sense, only for rather simple images like square patterns or gratings in a uniform surrounding. Nevertheless, the concept of a perceptually linearized Display Function derived from experiments with simple test patterns has been successfully applied to complex images as described in the literature [A3-A8]. While it was clearly recognized that perceptual linearization can never be achieved for all details or spatial frequencies and object sizes at once, perceptual linearization for frequencies and object sizes near the peak of human Contrast Sensitivity seemed to do a ireasonable jobî also in complex images.

Limited (unpublished) experiments have indicated that perceptual linearization for a particular detail in a complex image with a wide Luminance Range and heterogeneous surround required Display Functions which are rather strongly bent in the dark regions of the image and that such Display Functions for a low-luminance and a high-luminance display system would not be part of a continuous, monotonic function. This experience may underly the considerations of the CIELab curve [A9] proposed by other standards groups.

Other experiments and observations with computed radiographs seemed to suggest that similarity could also be obtained between grayscale renditions on Display Systems of different Luminance when the same application-specific function is combined with log-linear Characteristic Curves of the Display Systems. Thus similarity, if not contrast equalization, could be gained by a straight, luminance-independent shape for the Display Function.

While it might have been equally sensible to choose the rather simple log-linear Display Function as a standard, this was not done for the following reason, among others.

For high-resolution Display Systems with high intrinsic video bandwidth, digitization resolution is limited to 8 or 10 bits because of technology and other constraints. The more a Grayscale Standard Display Function deviates from the Characteristic Curve of a Display System, the poorer the utilization of DDLs typically is from a perception point of view. The Characteristic Curve of CRT Display Systems has a convex curvature with respect to a log-linear straight line. It differs much less from Display Functions derived from human vision models and the concept of perceptual linearization than from a log-linear Display Function.

When using application-specific display processes which cause the resultant Display Function to deviate strongly from the Grayscale Standard Display Function, the function conceivably does not provide good similarity. In this case, other functions may yield better similarity.

In summary, a Display Function was derived from Bartenís model of the human visual system to gain a single continuous mathematical function which in its curvature falls between a log-linear response and a Display Function that may yield perceptual linearization in complex scenery with a wide luminance range within the image. Other models of human contrast sensitivity may potentially provide a better function, but were not evaluated. The notion of perceptual linearization was chosen to meet the secondary objectives of the Grayscale Standard Display Function, but not as an explicit goal of the Grayscale Standard Display Function itself. It is recognized that better functions may exist to meet these objectives. It is believed that almost any single mathematically defined Standard Function will greatly improve image presentations on Display Systems in communication networks.

A.2. DETAILS OF THE BARTEN MODEL

Barten's model considers neural noise, lateral inhibition, photon noise, external noise, limited integration capability, the optical modulation transfer function, orientation, and temporal filtering. Neuron noise represents the upper limit of Contrast Sensitivity at high spatial frequencies. Low spatial frequencies appear to be attenuated by lateral inhibition in the ganglion cells which seems to be caused by the subtraction of a spatially low-pass filtered signal from the original. Photon noise is defined by the fluctuations of the photon flux h, the pupil diameter d, and quantum detection efficiency η of the eye. At low light levels, the Contrast Sensitivity is proportional to the square-root of Luminance according to the de Vries-Rose law. The temporal integration capability in the model used here is simply represented by a time constant of T = 0.1 sec. Temporal filtering effects are not included. Next to the temporal integration capability, the eye also has limited spatial integration capability: There is a maximum angular size $X_{\mbox{\footnotesize E}}$ x

Y_E as well as a maximum number of cycles N_E over which the eye can integrate information in the presence of various noise sources. The optical modulation transfer function

$$M_{opt}(u) = e^{-\pi^2 \cdot \sigma^2 \cdot u^2}$$
 $\sigma = \sqrt{\sigma_0^2 + (C_{sph} \cdot d^3)^2}$ (A1)

(u, spatial frequency in c/deg) is derived from a Gaussian point-spread function including the optical properties of the eye-lens, stray light from the optical media, diffusion in the retina, and the discrete nature of the receptor elements as well as from the spherical aberration, C_{sph} , which is the main pupil-diameter-dependent component. σ_0 is the value of σ at small pupil sizes. External noise may stem from Display System noise and image noise. Contrast sensitivity varies approximately sinusoidally with the orientation of the test pattern with equal maximum sensitivity at 0 and 90 deg and minimal sensitivity at 45 deg. The difference in Contrast Sensitivity is only present at high spatial frequencies. The effect is modeled by a variation in integration capability.

The combination of these effects yields the equation for contrast as a function of spatial frequency:

$$S(u) = \frac{1}{k} \sqrt{\frac{T}{2}} \frac{M_{opt}(u)}{\sqrt{\left(\frac{1}{\eta h I_{L}} + \frac{\Phi_{0}}{(1-F(u))^{2}} + \Phi_{ext}(u)\right) \cdot \left(\frac{1}{X_{0}^{2}} + \frac{1}{X_{E}^{2}} + \left(\frac{u}{N_{E}}\right)^{2}\right)}}$$
(A2)

The effect of noise appears in the first parenthesis within the square-root as a noise contrast related to the variances of photon (first term), filtered neuron (second term), and external noise. The Illuminance, $I_L = \pi/4 \text{ d}^2 L$, of the eye is expressed in trolands [td], d is the pupil diameter in mm, and L the Luminance of the Target in cd/m². The pupil diameter is determined by the formula of de Groot and Gebhard:

$$d = 4.6 - 2.8 \cdot \tanh(0.4 \cdot \text{Log}_{10}(0.625 \cdot \text{L})) \tag{A3}$$

The term $(1 - F(u))^2 = 1 - \exp(-u^2/u_0^2)$ describes the low frequency attenuation of neuron noise due to lateral inhibition $(u_0 = 8 \text{ c/deg})$. Equation (A2) represents the simplified case of square targets, $X_0 = Y_0$ [deg]. Φ_{ext} is the contrast variance corresponding to external noise. k = 3.3, $\eta = 0.025$, $h = 357 \cdot 3600$ photons/td sec deg²; the contrast variance corresponding to the neuron noise $\Phi_0 = 3 \cdot 10^{-8} \text{ sec}$ deg², $X_E = 12 \text{ deg}$, $N_E = 15 \text{ cycles}$ (at 0 and 90 deg and $N_E = 7.5 \text{ cycles}$ at 45 deg for frequencies above 2 c/deg), $\sigma_0 = 0.0133 \text{ deg}$, $C_{sph} = 0.0001 \text{ deg/mm}^3$ [A1]. Equation (A2) provides a good fit of experimental data for $10^{-4} \le L \le 10^3 \text{ cd/m}^2$, $0.5 \le X_0 \le 60 \text{ deg}$, $0.2 \le u \le 50 \text{ c/deg}$.

After inserting all constants, Equation (A2) reduces to

$$S(L) = \frac{q_1 \cdot M_{opt}(L)}{\sqrt{\frac{q_2}{d^2 L} + q_3}}$$
 (A4)

with $q_1 = 0.1183034375$, $q_2 = 3.962774805 \cdot 10^{-5}$, and $q_3 = 1.356243499 \cdot 10^{-7}$.

When viewed from 250 mm distance, the Standard Target has a size of about 8.7 mm x 8.7 mm and the spatial frequency of the grid equals about 0.92 line pairs per millimeter.

The Grayscale Standard Display Function is obtained by computing the Threshold Modulation $\mathbf{S_j}$ as a function of mean grating Luminance and then stacking these values on top of each other. The mean Luminance of the next higher level is calculated by adding the peak-to-peak modulation to the mean Luminance $\mathbf{L_i}$ of the previous level:

$$L_{j+1} = L_j \cdot \frac{1 + S_j}{1 - S_j}$$
 (A5)

Thus, in PS 3.14, the peak-to-peak Threshold Modulation is called a just-noticeable Luminance difference.

When a Display System conforms with the Grayscale Standard Display Function, it is perceptually linearized when observing the Standard Target: If a Display System had infinitely fine digitization resolution, equal increments in P-Value would produce equally perceivable contrast steps and, under certain conditions, just-noticeable Luminance differences (displayed one at a time) for the Standard Target (the grating with sinusoidal modulation of 4 c/degree over a 2 degree x 2 degree area, embedded in a uniform background with a Luminance equal to the mean target Luminance).

The display of the Standard Target at different Luminance levels one at a time is an academic display situation. An image containing different Luminance levels with different targets and Luminance distributions at the same time is in general not perceptually linearized. It is once more emphasized that the concept of perceptual linearization of Display Systems for the Standard Target served as a logical means for deriving a continuous mathematical function and for meeting the secondary goals of the Grayscale Standard Display Function. The function may represent a compromise between perceptual linearization of complex images by strongly-bent Display Functions and gaining similarity of grayscale perception within an image on Display Systems of different Luminance by a log-linear Display Function.

The Characteristic Curve of the Display System is measured and represented by {Luminance, DDL}-pairs $L_m = F(D_m)$. A discrete transformation may be performed that maps the previously used DDLs, D_{input} , to D_{output} according to Equations (A6) and (A7) such that the available ensemble of discrete Luminance levels is used to approximate the Grayscale Standard Display Function L = G(j). The transformation is illustrated in Fig. A1. By such an operation, conformance with the Grayscale Standard Display Function may be reached.

$$D_{\text{output}} = s \cdot F^{-1}[G(j)] \tag{A6}$$

s is a scale factor for accommodating different input and output digitization resolutions.

The index j (which in general will be a non-integer number) of the Standard Luminance Levels is determined from the starting index j_0 of the Standard Luminance level at the minimum Luminance of the Display System (including ambient light), the number of Standard JNDs, N_{JND} , over the Luminance Range of the Display System, the digitization resolution DR, and the DDLs, D_{input} , of the Display System:

$$I = I_0 + \frac{N_{JND}}{DR} \cdot D_{input}$$
 (A7)

A detailed example for executing such a transformation is given in Annex D.

A.3. REFERENCES

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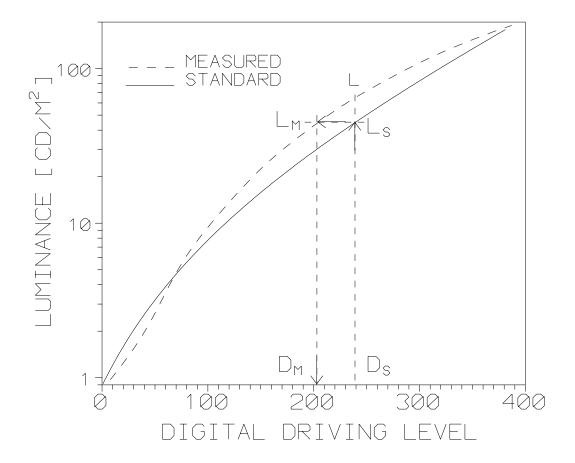


Fig. A-1. Illustration for determining the transform that changes the Characteristic Curve of a Display System to a Display Function that approximates the Grayscale Standard Display Function

Annex B (INFORMATIVE) TABLE OF THE GRAYSCALE STANDARD DISPLAY FUNCTION

The Grayscale Standard Display Function based on the Barten model was introduced in Section 7 and details are presented in Annex A above. This annex presents the Grayscale Standard Display Function as a table of values for Luminance as a function of the Just-Noticeable Difference Index for integer values of the Just-Noticeable Difference Index.

Table B-1
Grayscale Standard Display Function: Luminance versus JND Index

	Grayscale .	Jianuaru L	Jispiay Fullci		inance versu	5 JND IIIU	
JND	L[cd/m ²]	JND	L[cd/m ²]	JND	L[cd/m ²]	JND	L[cd/m ²]
1	0.0500	2	0.0547	3	0.0594	4	0.0643
5	0.0696	6	0.0750	7	0.0807	8	0.0866
9	0.0927	10	0.0991	11	0.1056	12	0.1124
13	0.1194	14	0.1267	15	0.1342	16	0.1419
17	0.1498	18	0.1580	19	0.1664	20	0.1750
21	0.1839	22	0.1931	23	0.2025	24	0.2121
25	0.2220	26	0.2321	27	0.2425	28	0.2532
29	0.2641	30	0.2752	31	0.2867	32	0.2984
33	0.3104	34	0.3226	35	0.3351	36	0.3479
37	0.3610	38	0.3744	39	0.3880	40	0.4019
41	0.4161	42	0.4306	43	0.4454	44	0.4605
45	0.4759	46	0.4916	47	0.5076	48	0.5239
49	0.5405	50	0.5574	51	0.5746	52	0.5921
53	0.6100	54	0.6281	55	0.6466	56	0.6654
57	0.6846	58	0.7040	59	0.7238	60	0.7440
61	0.7644	62	0.7852	63	0.8064	64	0.8278
65	0.8497	66	0.8718	67	0.8944	68	0.9172
69	0.9405	70	0.9640	71	0.9880	72	1.0123
73	1.0370	74	1.0620	75	1.0874	76	1.1132
77	1.1394	78	1.1659	79	1.1928	80	1.2201
81	1.2478	82	1.2759	83	1.3044	84	1.3332
85	1.3625	86	1.3921	87	1.4222	88	1.4527
89	1.4835	90	1.5148	91	1.5465	92	1.5786
93	1.6111	94	1.6441	95	1.6775	96	1.7113
97	1.7455	98	1.7802	99	1.8153	100	1.8508
101	1.8868	102	1.9233	103	1.9601	104	1.9975
105	2.0352	106	2.0735	107	2.1122	108	2.1514
109	2.1910	110	2.2311	111	2.2717	112	2.3127
113	2.3543	114	2.3963	115	2.4388	116	2.4817

121 2.7041 122 2.7501 123 2.7966 124 2.4 3.4 3.4 3.4 3.4 3.4 3.4 3.4 3.4 3.4 3.4 3.4 3.4 3.4 3.3 3.5 3.9 133 3.6 131 140 3.6 131 140 3.6 131 140 3.6 131 140 3.6 3.6 131 140 3.6 3.6 131 140 3.6 3.6 134 3.4 3.8 3.5 147 4.0754 148 4.2 4.4 4.0155 147 4.0754 148 4.2 4.4 4.149 4.016 4.2 4.4 4.2 4.4 4.4 4.4 <th></th>	
125 2.8912 126 2.9392 127 2.9878 128 3.0 129 3.0866 130 3.1367 131 3.1875 132 3.3 137 3.5033 138 3.5579 139 3.6131 140 3.4 141 3.7252 142 3.7820 143 3.8395 144 3.3 145 3.9563 146 4.0155 147 4.0754 148 4.5 149 4.1969 150 4.2586 151 4.3209 152 4.3 153 4.4473 154 4.5115 155 4.5763 156 4.4 157 4.7078 158 4.7745 159 4.8419 160 4.3 161 4.9785 162 5.0479 163 5.1179 164 5. 165 5.2599 166 5.3319 167 5.4046 168 5. 173 5.8555 174 <td< td=""><td>6587</td></td<>	6587
129 3.0866 130 3.1367 131 3.1875 132 3.3 133 3.2905 134 3.3429 135 3.3958 136 3.4 137 3.5033 138 3.5579 139 3.6131 140 3.4 1441 3.7252 142 3.7820 143 3.8395 144 3.4 1449 4.1969 150 4.2586 151 4.0754 148 4.5 149 4.1969 150 4.2586 151 4.3209 152 4.3 157 4.7078 158 4.7745 159 4.8419 160 4.3 161 4.9785 162 5.0479 163 5.1179 164 5. 165 5.2599 166 5.3319 167 5.4046 168 5.4 169 5.5521 170 5.6269 171 5.7024 172 5. 173 5.8555 174 <	3436
133 3.2905 134 3.3429 135 3.3958 136 3.4 137 3.5033 138 3.5579 139 3.6131 140 3.4 141 3.7252 142 3.7820 143 3.8395 144 3.8 145 3.9563 146 4.0155 147 4.0754 148 4. 149 4.1969 150 4.2586 151 4.3209 152 4. 153 4.4473 154 4.5115 155 4.5763 156 4. 157 4.7078 158 4.7745 159 4.8419 160 4. 161 4.9785 162 5.0479 163 5.1179 164 5. 169 5.5521 170 5.6269 171 5.7024 172 5. 177 6.1702 178 6.2508 179 6.3320 180 6. 181 6.4968 182 6.58	369
137 3.5033 138 3.5579 139 3.6131 140 3.6131 141 3.7252 142 3.7820 143 3.8395 144 3.8395 145 3.9563 146 4.0155 147 4.0754 148 4.31 149 4.1969 150 4.2586 151 4.3209 152 4.3 153 4.4473 154 4.5115 155 4.5763 156 4.4 157 4.7078 158 4.7745 159 4.8419 160 4.3 161 4.9785 162 5.0479 163 5.1179 164 5.3 165 5.2599 166 5.3319 167 5.4046 168 5.4 173 5.8555 174 5.9331 175 6.0114 176 6.3 181 6.4968 182 6.5803 183 6.6645 184 6.3 189 7.1863 190	2387
141 3.7252 142 3.7820 143 3.8395 144 3.8 149 4.1969 150 4.2586 151 4.3209 152 4.3 153 4.4473 154 4.5115 155 4.5763 156 4.6 157 4.7078 158 4.7745 159 4.8419 160 4.9 161 4.9785 162 5.0479 163 5.1179 164 5. 165 5.2599 166 5.3319 167 5.4046 168 5. 169 5.5521 170 5.6269 171 5.7024 172 5. 173 5.8555 174 5.9331 175 6.0114 176 6.6 181 6.4968 182 6.5803 183 6.6645 184 6. 189 7.1863 190 7.2760 191 7.3665 192 7. 197 7.9266 198 8.	1493
145 3.9563 146 4.0155 147 4.0754 148 4.5 149 4.1969 150 4.2586 151 4.3209 152 4.3 153 4.4473 154 4.5115 155 4.5763 156 4.4 157 4.7078 158 4.7745 159 4.8419 160 4.9 161 4.9785 162 5.0479 163 5.1179 164 5. 165 5.2599 166 5.3319 167 5.4046 168 5. 169 5.5521 170 5.6269 171 5.7024 172 5. 173 5.8555 174 5.9331 175 6.0114 176 6. 181 6.4968 182 6.5803 183 6.6645 184 6. 189 7.1863 190 7.2760 191 7.3665 192 7. 197 7.9266 198 8.0	688
149 4.1969 150 4.2586 151 4.3209 152 4.3 153 4.4473 154 4.5115 155 4.5763 156 4.3 157 4.7078 158 4.7745 159 4.8419 160 4.9 161 4.9785 162 5.0479 163 5.1179 164 5. 165 5.2599 166 5.3319 167 5.4046 168 5. 169 5.5521 170 5.6269 171 5.7024 172 5. 173 5.8555 174 5.9331 175 6.0114 176 6.0 177 6.1702 178 6.2508 179 6.3320 180 6.4 181 6.4968 182 6.5803 183 6.6645 184 6. 189 7.1863 190 7.2760 191 7.3665 192 7. 193 7.5500 194 7.	3976
153 4.4473 154 4.5115 155 4.5763 156 4.8 157 4.7078 158 4.7745 159 4.8419 160 4.8 161 4.9785 162 5.0479 163 5.1179 164 5. 165 5.2599 166 5.3319 167 5.4046 168 5. 169 5.5521 170 5.6269 171 5.7024 172 5. 173 5.8555 174 5.9331 175 6.0114 176 6.6 181 6.4968 182 6.5803 183 6.6645 184 6. 189 7.1863 190 7.2760 191 7.3665 192 7. 193 7.5500 194 7.6429 195 7.7366 196 7.3 201 8.3167 202 8.4163 203 8.5168 204 8.3 209 9.1382 210 9.	1358
157 4.7078 158 4.7745 159 4.8419 160 4.8 161 4.9785 162 5.0479 163 5.1179 164 5. 165 5.2599 166 5.3319 167 5.4046 168 5. 169 5.5521 170 5.6269 171 5.7024 172 5. 173 5.8555 174 5.9331 175 6.0114 176 6.6 181 6.4968 182 6.5803 183 6.6645 184 6. 185 6.8354 186 6.9219 187 7.0093 188 7. 189 7.1863 190 7.2760 191 7.3665 192 7. 197 7.9266 198 8.0229 199 8.1199 200 8.3 201 8.3167 202 8.4163 203 8.5168 204 8. 209 9.1382 210 9.24	3838
161 4.9785 162 5.0479 163 5.1179 164 5.1179 164 5.1179 164 5.1179 164 5.1179 164 5.1179 164 5.1179 164 5.1179 166 5.3319 167 5.4046 168 5.21 170 5.6269 171 5.7024 172 5.1179 <td>6417</td>	6417
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185 6.8354 186 6.9219 187 7.0093 188 7.0093 189 7.1863 190 7.2760 191 7.3665 192 7.4 193 7.5500 194 7.6429 195 7.7366 196 7.8 197 7.9266 198 8.0229 199 8.1199 200 8.3 201 8.3167 202 8.4163 203 8.5168 204 8.6 205 8.7204 206 8.8235 207 8.9275 208 9.6 209 9.1382 210 9.2449 211 9.3525 212 9.6 213 9.5705 214 9.6809 215 9.7922 216 9.9 217 10.0176 218 10.1318 219 10.2469 220 10 225 10.9579 226 11.0799 227 11.2028 228 11 229 11.4518 230	1140
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193 7.5500 194 7.6429 195 7.7366 196 7.8 197 7.9266 198 8.0229 199 8.1199 200 8.3 201 8.3167 202 8.4163 203 8.5168 204 8.6 205 8.7204 206 8.8235 207 8.9275 208 9.6 209 9.1382 210 9.2449 211 9.3525 212 9.4 213 9.5705 214 9.6809 215 9.7922 216 9.9 221 10.4800 222 10.5980 223 10.7169 224 10. 225 10.9579 226 11.0799 227 11.2028 228 11. 229 11.4518 230 11.5779 231 11.7050 232 11.	974
197 7.9266 201 8.3167 205 8.7204 209 9.1382 213 9.5705 217 10.0176 221 10.4800 225 10.9579 229 11.4518 198 8.0229 199 8.1199 203 8.5168 203 8.5168 207 8.9275 208 9.6809 215 9.7922 216 9.9 221 10.4800 222 10.5980 223 10.7169 227 11.2028 228 11. 229 11.4518 230 11.5779 231 11.7050 232 11.7050	1578
201 8.3167 205 8.7204 209 9.1382 213 9.5705 217 10.0176 221 10.4800 225 10.9579 229 11.4518 202 8.4163 203 8.5168 207 8.9275 207 8.9275 208 9.0 211 9.3525 215 9.7922 215 9.7922 219 10.2469 223 10.7169 227 11.2028 228 11. 230 11.5779 231 11.7050 232 11.7050	3312
205 8.7204 209 9.1382 213 9.5705 217 10.0176 221 10.4800 225 10.9579 229 11.4518 206 8.8235 207 8.9275 211 9.3525 211 9.3525 215 9.7922 216 9.9 219 10.2469 223 10.7169 224 10. 227 11.2028 231 11.7050 232 11.7050	2179
209 9.1382 210 9.2449 211 9.3525 212 9.4 213 9.5705 214 9.6809 215 9.7922 216 9.9 217 10.0176 218 10.1318 219 10.2469 220 10. 221 10.4800 222 10.5980 223 10.7169 224 10. 225 10.9579 226 11.0799 227 11.2028 228 11. 229 11.4518 230 11.5779 231 11.7050 232 11.	3182
213 9.5705 217 10.0176 221 10.4800 225 10.9579 229 11.4518 214 9.6809 215 9.7922 219 10.2469 221 10.4800 222 10.5980 223 10.7169 227 11.2028 228 11. 231 11.7050 232 11.	324
217 10.0176 218 10.1318 219 10.2469 220 10. 221 10.4800 222 10.5980 223 10.7169 224 10. 225 10.9579 226 11.0799 227 11.2028 228 11. 229 11.4518 230 11.5779 231 11.7050 232 11.	1611
221 10.4800 222 10.5980 223 10.7169 224 10. 225 10.9579 226 11.0799 227 11.2028 228 11. 229 11.4518 230 11.5779 231 11.7050 232 11.	0044
225 10.9579 226 11.0799 227 11.2028 228 11. 229 11.4518 230 11.5779 231 11.7050 232 11.	3629
229 11.4518 230 11.5779 231 11.7050 232 11.	8369
	3268
233 11.9622 234 12.0925 235 12.2237 236 12.	8331
	3561
237 12.4895 238 12.6240 239 12.7596 240 12.	8963
241 13.0341 242 13.1730 243 13.3130 244 13.	4542
245 13.5965 246 13.7399 247 13.8844 248 14.	0302
249 14.1770 250 14.3251 251 14.4743 252 14.	6247
253 14.7763 254 14.9291 255 15.0831 256 15.	2384
257 15.3948 258 15.5525 259 15.7114 260 15.	8716
261 16.0330 262 16.1957 263 16.3596 264 16.	5249
265 16.6914 266 16.8592 267 17.0283 268 17.	1987
269 17.3705 270 17.5436 271 17.7180 272 17.	8938
273 18.0709 274 18.2494 275 18.4293 276 18.	6105

277	18.7931	278	18.9772	279	19.1626	280	19.3495
281	19.5378	282	19.7275	283	19.9187	284	20.1113
285	20.3054	286	20.5009	287	20.6980	288	20.8965
289	21.0966	290	21.2981	291	21.5012	292	21.7058
293	21.9120	294	22.1197	295	22.3289	296	22.5398
297	22.7522	298	22.9662	299	23.1818	300	23.3990
301	23.6179	302	23.8383	303	24.0605	304	24.2842
305	24.5097	306	24.7368	307	24.9656	308	25.1961
309	25.4283	310	25.6622	311	25.8979	312	26.1353
313	26.3744	314	26.6153	315	26.8580	316	27.1025
317	27.3488	318	27.5969	319	27.8468	320	28.0985
321	28.3521	322	28.6075	323	28.8648	324	29.1240
325	29.3851	326	29.6481	327	29.9130	328	30.1798
329	30.4486	330	30.7193	331	30.9920	332	31.2667
333	31.5434	334	31.8220	335	32.1027	336	32.3854
337	32.6702	338	32.9570	339	33.2459	340	33.5369
341	33.8300	342	34.1251	343	34.4224	344	34.7219
345	35.0235	346	35.3272	347	35.6332	348	35.9413
349	36.2516	350	36.5642	351	36.8790	352	37.1960
353	37.5153	354	37.8369	355	38.1608	356	38.4870
357	38.8155	358	39.1463	359	39.4795	360	39.8151
361	40.1530	362	40.4933	363	40.8361	364	41.1813
365	41.5289	366	41.8790	367	42.2316	368	42.5866
369	42.9442	370	43.3043	371	43.6669	372	44.0321
373	44.3998	374	44.7702	375	45.1431	376	45.5187
377	45.8969	378	46.2778	379	46.6613	380	47.0475
381	47.4365	382	47.8281	383	48.2225	384	48.6197
385	49.0196	386	49.4224	387	49.8279	388	50.2363
389	50.6475	390	51.0616	391	51.4786	392	51.8985
393	52.3213	394	52.7470	395	53.1757	396	53.6074
397	54.0421	398	54.4798	399	54.9205	400	55.3643
401	55.8112	402	56.2611	403	56.7142	404	57.1704
405	57.6298	406	58.0923	407	58.5581	408	59.0270
409	59.4992	410	59.9747	411	60.4534	412	60.9354
413	61.4208	414	61.9094	415	62.4015	416	62.8969
417	63.3958	418	63.8980	419	64.4037	420	64.9129
421	65.4256	422	65.9418	423	66.4615	424	66.9848
425	67.5117	426	68.0422	427	68.5763	428	69.1140
429	69.6555	430	70.2006	431	70.7495	432	71.3021
433	71.8585	434	72.4187	435	72.9827	436	73.5505
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437	74.1222	438	74.6978] [439	75.2773	' Г	440	75.8608
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441	76.4482	442	77.0396		443	77.6351	۱ <u> </u>	444	78.2346
445	78.8381	446	79.4458		447	80.0576	۱	448	80.6735
449	81.2936	450	81.9179		451 455	82.5464	۱	452	83.1792
453	83.8163	454	84.4577		455	85.1034	۱	456	85.7535
457	86.4079	458	87.0668		459	87.7302	۱	460	88.3980
461	89.0703	462	89.7472		463	90.4286	۱	464	91.1147
465	91.8053	466	92.5006		467	93.2006	۱	468	93.9053
469	94.6147	470	95.3289		471	96.0480	۱	472	96.7718
473	97.5005	474	98.2341		475	98.9726	۱	476	99.7161
477	100.4646	478	101.2181		479	101.9767	۱	480	102.7403
481	103.5091	482	104.2830		483	105.0621	۱	484	105.8464
485	106.6359	486	107.4308		487	108.2309	۱	488	109.0364
489	109.8473	490	110.6637		491	111.4854		492	112.3127
493	113.1455	494	113.9838		495	114.8278		496	115.6773
497	116.5326	498	117.3935		499	118.2602		500	119.1326
501	120.0109	502	120.8950		503	121.7850		504	122.6809
505	123.5828	506	124.4907		507	125.4047		508	126.3247
509	127.2508	510	128.1831		511	129.1215		512	130.0662
513	131.0172	514	131.9745		515	132.9381		516	133.9082
517	134.8847	518	135.8676		519	136.8571	ا [520	137.8531
521	138.8557	522	139.8650		523	140.8810		524	141.9037
525	142.9331	526	143.9694		527	145.0125	[528	146.0625
529	147.1195	530	148.1835		531	149.2545		532	150.3326
533	151.4178	534	152.5101		535	153.6097	[536	154.7166
537	155.8307	538	156.9523		539	158.0812	[540	159.2175
541	160.3614	542	161.5128		543	162.6718	[544	163.8384
545	165.0128	546	166.1948		547	167.3847		548	168.5824
549	169.7880	550	171.0015		551	172.2230		552	173.4526
553	174.6902	554	175.9360		555	177.1900		556	178.4522
557	179.7227	558	181.0016		559	182.2889		560	183.5846
561	184.8889	562	186.2017		563	187.5232		564	188.8533
565	190.1921	566	191.5398		567	192.8963		568	194.2617
569	195.6360	570	197.0194		571	198.4119		572	199.8134
573	201.2242	574	202.6442		575	204.0735		576	205.5122
577	206.9603	578	208.4179		579	209.8851		580	211.3618
581	212.8482	582	214.3444		583	215.8503		584	217.3661
585	218.8919	586	220.4276		587	221.9733		588	223.5292
589	225.0952	590	226.6715		591	228.2581	۱ 	592	229.8550
593	231.4624	594	233.0803		595	234.7088		596	236.3479

601 244.7054 602 246.4097 603 248.1252 604 249.8 605 251.5899 606 253.3392 607 255.0999 608 256.8 609 258.6559 610 260.4512 611 262.2583 612 264.0 617 273.3509 618 275.2420 619 277.1455 620 279.0 621 280.9897 622 282.9306 623 284.8841 624 286.8 625 288.8294 626 290.8213 627 292.8262 628 294.6 633 305.1324 634 307.2304 635 309.3420 636 311.4 637 313.6065 638 315.7595 639 317.9266 640 320.1 641 322.3030 642 324.5126 643 326.7365 644 328.9 653 349.7858 654 352.1738 655 354.5773 656 53.54.5773 <									
605 251.5899 606 253.3392 607 255.0999 608 256.85 609 258.6559 610 260.4512 611 262.2583 612 264.0 613 265.9079 614 267.7506 615 269.6052 616 271.4 617 273.3509 618 275.2420 619 277.1455 620 279.0 621 280.9897 622 282.9306 623 284.8841 624 286.8 629 296.8752 630 298.9195 631 300.9770 632 303.0 633 305.1324 634 307.2304 635 309.3420 636 331.36065 638 315.7595 639 317.9266 640 320.1 641 322.3030 642 324.5126 643 326.7365 644 328.9 645 331.278 646 333.4953 647 335.7776 648 338.0 653 349.7858 <t< td=""><td>597</td><td>237.9977</td><td>598</td><td>239.6583</td><td>599</td><td>241.3297</td><td>Ī</td><td>600</td><td>243.0120</td></t<>	597	237.9977	598	239.6583	599	241.3297	Ī	600	243.0120
609 258.6559 610 260.4512 611 262.2583 612 264.0 613 265.9079 614 267.7506 615 269.6052 616 271.4 617 273.3509 618 275.2420 619 277.1455 620 279.0 621 280.9897 622 282.9306 623 284.8841 624 286.8 629 296.8752 630 298.9195 631 300.9770 632 303.1 633 305.1324 634 307.2304 635 309.3420 636 311.4 637 313.6065 638 315.7595 639 317.9266 640 320.3 641 322.3030 642 324.5126 643 326.7365 644 328.9 649 340.3867 650 342.7137 651 345.0558 652 347.4 653 349.7858 654 352.1738 655 354.5773 656 356.9 66	601	244.7054	602	246.4097	603	248.1252		604	249.8519
613 265.9079 614 267.7506 615 269.6052 616 271.4 617 273.3509 618 275.2420 619 277.1455 620 279.0 621 280.9897 622 282.9306 623 284.8841 624 286.8 625 288.8294 626 290.8213 627 292.8262 628 294.8 629 296.8752 630 298.9195 631 300.9770 632 303.0 633 305.1324 634 307.2304 635 309.3420 636 311.4 637 313.6065 638 315.7595 639 317.9266 640 320.1 641 322.3030 642 324.5126 643 326.7365 644 328.9 645 331.278 646 333.4953 657 345.958 654 352.1738 655 354.5773 656 652 347.4 653 349.7858 654 352.17	605	251.5899	606	253.3392	607	255.0999		608	256.8721
617 273.3509 618 275.2420 619 277.1455 620 279.0 621 280.9897 622 282.9306 623 284.8841 624 286.8 625 288.8294 626 290.8213 627 292.8262 628 294.8 629 296.8752 630 298.9195 631 300.9770 632 303.0 633 305.1324 634 307.2304 635 309.3420 636 311.4 637 313.6065 638 315.7595 639 317.9266 640 320.1 641 322.3030 642 324.5126 643 326.7365 644 328.9 645 331.2278 646 333.4953 647 335.7776 648 338.0 653 349.7858 654 352.1738 655 354.5733 656 652 347.4 661 369.3294 662 371.8442 663 374.3754 664 376.	609	258.6559	610	260.4512	611	262.2583		612	264.0772
621 280.9897 622 282.9306 623 284.8841 624 286.8 625 288.8294 626 290.8213 627 292.8262 628 294.8 629 296.8752 630 298.9195 631 300.9770 632 303.0 633 305.1324 634 307.2304 635 309.3420 636 311.4 637 313.6065 638 315.7595 639 317.9266 640 320.1 641 322.3030 642 324.5126 643 326.7365 644 328.9 645 331.2278 646 333.4953 647 335.7776 648 338.0 649 340.3867 650 342.7137 651 345.0558 652 347.4 653 349.7858 654 352.1738 655 354.3734 666 360.3594 662 371.8442 663 374.3754 666 367.934 666 382.0676 667	613	265.9079	614	267.7506	615	269.6052		616	271.4720
625 288.8294 626 290.8213 627 292.8262 628 294.8 629 296.8752 630 298.9195 631 300.9770 632 303.0 633 305.1324 634 307.2304 635 309.3420 636 311.4 637 313.6065 638 315.7595 639 317.9266 640 320.1 641 322.3030 642 324.5126 643 326.7365 644 328.9 649 340.3867 650 342.7137 651 345.0558 652 347.4 653 349.7858 654 352.1738 655 354.5773 656 356.9 667 359.4312 658 361.8818 659 364.3483 660 366.8 665 379.4869 666 382.0676 667 384.6650 668 387.2 673 400.6069 674 403.3245 675 406.0596 676 408.8	617	273.3509	618	275.2420	619	277.1455		620	279.0614
629 296.8752 630 298.9195 631 300.9770 632 303.0 633 305.1324 634 307.2304 635 309.3420 636 311.4 637 313.6065 638 315.7595 639 317.9266 640 320.1 641 322.3030 642 324.5126 643 326.7365 644 328.9 649 340.3867 650 342.7137 651 345.0558 652 347.4 653 349.7858 654 352.1738 655 354.5773 656 356.9 667 359.4312 658 361.8818 659 364.3483 660 366.8 661 369.3294 662 371.8442 663 374.3754 664 376.9 665 379.4869 666 382.0676 667 384.6650 668 387.2 673 400.6069 674 403.3245 675 406.0596 676 408.8	621	280.9897	622	282.9306	623	284.8841		624	286.8504
633 305.1324 634 307.2304 635 309.3420 636 311.4 637 313.6065 638 315.7595 639 317.9266 640 320.1 641 322.3030 642 324.5126 643 326.7365 644 328.9 649 340.3867 650 342.7137 651 345.0558 652 347.4 653 349.7858 654 352.1738 655 354.5773 656 356.9 661 369.3294 662 371.8442 663 374.3754 664 376.9 665 379.4869 666 382.0676 667 384.6650 668 387.2 673 400.6069 674 403.3245 675 406.0596 676 408.8 681 422.8468 682 425.7085 683 428.5886 684 431.4 683 458.4361 694 461.5282 695 464.6402 696 467.7	625	288.8294	626	290.8213	627	292.8262		628	294.8442
637 313.6065 638 315.7595 639 317.9266 640 320.1 641 322.3030 642 324.5126 643 326.7365 644 328.9 645 331.2278 646 333.4953 647 335.7776 648 338.0 649 340.3867 650 342.7137 651 345.0558 652 347.4 653 349.7858 654 352.1738 655 354.5773 656 366.8 667 359.4312 658 361.8818 659 364.3483 660 366.8 661 369.3294 662 371.8442 663 374.3754 664 376.9 665 379.4869 666 382.0676 667 384.6650 668 387.2 673 400.6069 674 403.3245 675 406.0596 676 408.8 681 422.8468 682 425.7085 683 428.5886 684 431.4	629	296.8752	630	298.9195	631	300.9770		632	303.0480
641 322.3030 642 324.5126 643 326.7365 644 328.9 649 340.3667 650 342.7137 651 345.0558 652 347.4 653 349.7858 654 352.1738 655 354.5773 656 356.9 667 359.4312 658 361.8818 659 364.3483 660 366.8 661 369.3294 662 371.8442 663 374.3754 664 376.9 669 389.9105 670 392.5587 671 395.2241 672 397.9 673 400.6069 674 403.3245 675 406.0596 676 408.8 681 422.8468 682 425.7085 683 428.5886 684 431.4 689 446.2655 690 449.2788 691 452.3116 692 455.3 697 470.9249 698 474.0977 699 477.2911 700 480.5	633	305.1324	634	307.2304	635	309.3420		636	311.4673
645 331.2278 646 333.4953 647 335.7776 648 338.0 649 340.3867 650 342.7137 651 345.0558 652 347.4 653 349.7858 654 352.1738 655 354.5773 656 366.9 667 359.4312 658 361.8818 659 364.3483 660 366.8 661 369.3294 662 371.8442 663 374.3754 664 376.9 665 379.4869 666 382.0676 667 384.6650 668 387.2 669 389.9105 670 392.5587 671 395.2241 672 397.9 673 400.6069 674 403.3245 675 406.0596 676 408.8 687 422.8468 682 425.7085 683 428.5886 684 431.4 685 434.4051 686 437.3415 687 440.2970 688 443.2	637	313.6065	638	315.7595	639	317.9266		640	320.1077
649 340.3867 650 342.7137 651 345.0558 652 347.4 653 349.7858 654 352.1738 655 354.5773 656 356.9 657 359.4312 658 361.8818 659 364.3483 660 366.8 661 369.3294 662 371.8442 663 374.3754 664 376.9 665 379.4869 666 382.0676 667 384.6650 668 387.2 669 389.9105 670 392.5587 671 395.2241 672 397.9 673 400.6069 674 403.3245 675 406.0596 676 408.8 677 411.5833 678 414.3719 679 417.1787 680 420.0 681 422.8468 682 425.7085 683 428.5886 684 431.4 685 434.4051 686 437.3415 687 440.2970 688 445.32	641	322.3030	642	324.5126	643	326.7365		644	328.9749
653 349.7858 654 352.1738 655 354.5773 656 356.9 657 359.4312 658 361.8818 659 364.3483 660 366.8 661 369.3294 662 371.8442 663 374.3754 664 376.9 665 379.4869 666 382.0676 667 384.6650 668 387.2 669 389.9105 670 392.5587 671 395.2241 672 397.9 673 400.6069 674 403.3245 675 406.0596 676 408.8 677 411.5833 678 414.3719 679 417.1787 680 420.0 681 422.8468 682 425.7085 683 428.5886 684 431.4 685 434.4051 686 437.3415 687 440.2970 688 443.2 693 458.4361 694 461.5282 695 464.6402 696 467.7	645	331.2278	646	333.4953	647	335.7776		648	338.0747
657 359.4312 658 361.8818 659 364.3483 660 366.8 661 369.3294 662 371.8442 663 374.3754 664 376.9 665 379.4869 666 382.0676 667 384.6650 668 387.2 669 389.9105 670 392.5587 671 395.2241 672 397.9 673 400.6069 674 403.3245 675 406.0596 676 408.8 681 422.8468 682 425.7085 683 428.5886 684 431.4 685 434.4051 686 437.3415 687 440.2970 688 443.2 689 446.2655 690 449.2788 691 452.3116 692 455.3 697 470.9249 698 474.0977 699 477.2911 700 480.5 705 496.8900 706 500.2308 707 503.5932 708 506.9	649	340.3867	650	342.7137	651	345.0558		652	347.4131
661 369.3294 662 371.8442 663 374.3754 664 376.9 665 379.4869 666 382.0676 667 384.6650 668 387.2 669 389.9105 670 392.5587 671 395.2241 672 397.9 673 400.6069 674 403.3245 675 406.0596 676 408.8 677 411.5833 678 414.3719 679 417.1787 680 420.0 681 422.8468 682 425.7085 683 428.5886 684 431.4 685 434.4051 686 437.3415 687 440.2970 688 443.2 689 446.2655 690 449.2788 691 452.3116 692 455.3 693 458.4361 694 461.5282 695 464.6402 696 467.7 701 483.7400 702 486.9958 703 490.2726 704 493.5	653	349.7858	654	352.1738	655	354.5773		656	356.9964
665 379.4869 666 382.0676 667 384.6650 668 387.2 669 389.9105 670 392.5587 671 395.2241 672 397.9 673 400.6069 674 403.3245 675 406.0596 676 408.8 677 411.5833 678 414.3719 679 417.1787 680 420.0 681 422.8468 682 425.7085 683 428.5886 684 431.4 685 434.4051 686 437.3415 687 440.2970 688 443.2 689 446.2655 690 449.2788 691 452.3116 692 455.3 697 470.9249 698 474.0977 699 477.2911 700 480.5 705 496.8900 706 500.2308 707 503.5932 708 506.9 713 524.2294 714 527.7471 715 531.2874 716 534.8	657	359.4312	658	361.8818	659	364.3483		660	366.8308
669 389.9105 670 392.5587 671 395.2241 672 397.9 673 400.6069 674 403.3245 675 406.0596 676 408.8 677 411.5833 678 414.3719 679 417.1787 680 420.0 681 422.8468 682 425.7085 683 428.5886 684 431.4 685 434.4051 686 437.3415 687 440.2970 688 443.2 689 446.2655 690 449.2788 691 452.3116 692 455.3 697 470.9249 698 474.0977 699 477.2911 700 480.5 705 496.8900 706 500.2308 707 503.5932 708 506.9 713 524.2294 714 527.7471 715 531.2874 716 534.8 721 553.0155 722 556.7192 723 560.4469 724 564.1	661	369.3294	662	371.8442	663	374.3754		664	376.9229
673 400.6069 674 403.3245 675 406.0596 676 408.8 677 411.5833 678 414.3719 679 417.1787 680 420.0 681 422.8468 682 425.7085 683 428.5886 684 431.4 685 434.4051 686 437.3415 687 440.2970 688 443.2 689 446.2655 690 449.2788 691 452.3116 692 455.3 693 458.4361 694 461.5282 695 464.6402 696 467.7 697 470.9249 698 474.0977 699 477.2911 700 480.5 701 483.7400 702 486.9958 703 490.2726 704 493.5 705 496.8900 706 500.2308 707 503.5932 708 506.9 713 524.2294 714 527.7471 715 531.2874 716 534.8	665	379.4869	666	382.0676	667	384.6650		668	387.2793
677 411.5833 678 414.3719 679 417.1787 680 420.00 681 422.8468 682 425.7085 683 428.5886 684 431.4 685 434.4051 686 437.3415 687 440.2970 688 443.2 689 446.2655 690 449.2788 691 452.3116 692 455.3 697 470.9249 698 474.0977 699 477.2911 700 480.5 701 483.7400 702 486.9958 703 490.2726 704 493.5 705 496.8900 706 500.2308 707 503.5932 708 506.9 709 510.3835 710 513.8116 711 517.2619 712 520.7 713 524.2294 714 527.7471 715 531.2874 716 534.8 721 553.0155 722 556.7192 723 560.4469 724 564.1	669	389.9105	670	392.5587	671	395.2241		672	397.9068
681 422.8468 682 425.7085 683 428.5886 684 431.4 685 434.4051 686 437.3415 687 440.2970 688 443.2 689 446.2655 690 449.2788 691 452.3116 692 455.3 693 458.4361 694 461.5282 695 464.6402 696 467.7 697 470.9249 698 474.0977 699 477.2911 700 480.5 701 483.7400 702 486.9958 703 490.2726 704 493.5 705 496.8900 706 500.2308 707 503.5932 708 506.9 709 510.3835 710 513.8116 711 517.2619 712 520.7 713 524.2294 714 527.7471 715 531.2874 716 534.8 721 553.0155 722 556.7192 723 560.4469 724 564.1	673	400.6069	674	403.3245	675	406.0596		676	408.8125
685 434.4051 686 437.3415 687 440.2970 688 443.2 689 446.2655 690 449.2788 691 452.3116 692 455.3 693 458.4361 694 461.5282 695 464.6402 696 467.7 697 470.9249 698 474.0977 699 477.2911 700 480.5 701 483.7400 702 486.9958 703 490.2726 704 493.5 705 496.8900 706 500.2308 707 503.5932 708 506.9 709 510.3835 710 513.8116 711 517.2619 712 520.7 713 524.2294 714 527.7471 715 531.2874 716 534.8 717 538.4370 718 542.0465 719 545.6793 720 549.3 721 553.0155 722 556.7192 727 575.6000 728 579.4	677	411.5833	678	414.3719	679	417.1787		680	420.0036
689 446.2655 690 449.2788 691 452.3116 692 455.3 693 458.4361 694 461.5282 695 464.6402 696 467.7 697 470.9249 698 474.0977 699 477.2911 700 480.5 705 496.8900 706 500.2308 707 503.5932 708 506.9 709 510.3835 710 513.8116 711 517.2619 712 520.7 713 524.2294 714 527.7471 715 531.2874 716 534.8 717 538.4370 718 542.0465 719 545.6793 720 549.3 721 553.0155 722 556.7192 723 560.4469 724 564.1 729 583.3242 730 587.2238 731 591.1486 732 595.0 731 615.2357 738 619.3415 739 623.4738 740 627.6	681	422.8468	682	425.7085	683	428.5886		684	431.4875
693 458.4361 694 461.5282 695 464.6402 696 467.7 697 470.9249 698 474.0977 699 477.2911 700 480.5 701 483.7400 702 486.9958 703 490.2726 704 493.5 705 496.8900 706 500.2308 707 503.5932 708 506.9 709 510.3835 710 513.8116 711 517.2619 712 520.7 713 524.2294 714 527.7471 715 531.2874 716 534.8 717 538.4370 718 542.0465 719 545.6793 720 549.3 721 553.0155 722 556.7192 723 560.4469 724 564.1 729 583.3242 730 587.2238 731 591.1486 732 595.0 733 599.0744 734 603.0758 735 607.1030 736 611.1	685	434.4051	686	437.3415	687	440.2970		688	443.2717
697 470.9249 698 474.0977 699 477.2911 700 480.5 701 483.7400 702 486.9958 703 490.2726 704 493.5 705 496.8900 706 500.2308 707 503.5932 708 506.9 709 510.3835 710 513.8116 711 517.2619 712 520.7 713 524.2294 714 527.7471 715 531.2874 716 534.8 717 538.4370 718 542.0465 719 545.6793 720 549.3 721 553.0155 722 556.7192 723 560.4469 724 564.1 729 583.3242 730 587.2238 731 591.1486 732 595.0 733 599.0744 734 603.0758 735 607.1030 736 611.1 737 615.2357 738 619.3415 743 640.2717 744 644.5	689	446.2655	690	449.2788	691	452.3116		692	455.3640
701 483.7400 702 486.9958 703 490.2726 704 493.5 705 496.8900 706 500.2308 707 503.5932 708 506.9 709 510.3835 710 513.8116 711 517.2619 712 520.7 713 524.2294 714 527.7471 715 531.2874 716 534.8 717 538.4370 718 542.0465 719 545.6793 720 549.3 721 553.0155 722 556.7192 723 560.4469 724 564.1 729 583.3242 730 587.2238 731 591.1486 732 595.0 733 599.0744 734 603.0758 735 607.1030 736 611.1 741 631.8187 742 636.0316 743 640.2717 744 644.5 745 648.8343 746 653.1571 747 657.5079 748 661.8 <td>693</td> <td>458.4361</td> <td>694</td> <td>461.5282</td> <td>695</td> <td>464.6402</td> <td></td> <td>696</td> <td>467.7724</td>	693	458.4361	694	461.5282	695	464.6402		696	467.7724
705 496.8900 706 500.2308 707 503.5932 708 506.9 709 510.3835 710 513.8116 711 517.2619 712 520.7 713 524.2294 714 527.7471 715 531.2874 716 534.8 717 538.4370 718 542.0465 719 545.6793 720 549.3 721 553.0155 722 556.7192 723 560.4469 724 564.1 729 583.3242 730 587.2238 731 591.1486 732 595.0 737 615.2357 738 619.3415 739 623.4738 740 627.6 741 631.8187 742 636.0316 743 640.2717 744 644.5 745 648.8343 746 653.1571 747 657.5079 748 661.8	697	470.9249	698	474.0977	699	477.2911		700	480.5052
709 510.3835 710 513.8116 711 517.2619 712 520.7 713 524.2294 714 527.7471 715 531.2874 716 534.8 717 538.4370 718 542.0465 719 545.6793 720 549.3 721 553.0155 722 556.7192 723 560.4469 724 564.1 725 567.9746 726 571.7750 727 575.6000 728 579.4 729 583.3242 730 587.2238 731 591.1486 732 595.0 733 599.0744 734 603.0758 735 607.1030 736 611.1 741 631.8187 742 636.0316 743 640.2717 744 644.5 745 648.8343 746 653.1571 747 657.5079 748 661.8	701	483.7400	702	486.9958	703	490.2726		704	493.5706
713 524.2294 714 527.7471 715 531.2874 716 534.8 717 538.4370 718 542.0465 719 545.6793 720 549.3 721 553.0155 722 556.7192 723 560.4469 724 564.1 729 583.3242 730 587.2238 731 591.1486 732 595.0 737 615.2357 738 619.3415 739 623.4738 740 627.6 741 631.8187 742 636.0316 743 640.2717 744 644.5 745 648.8343 746 653.1571 747 657.5079 748 661.8	705	496.8900	706	500.2308	707	503.5932		708	506.9774
717 538.4370 718 542.0465 719 545.6793 720 549.3 721 553.0155 722 556.7192 723 560.4469 724 564.1 725 567.9746 726 571.7750 727 575.6000 728 579.4 729 583.3242 730 587.2238 731 591.1486 732 595.0 733 599.0744 734 603.0758 735 607.1030 736 611.1 737 615.2357 738 619.3415 739 623.4738 740 627.6 741 631.8187 742 636.0316 743 640.2717 744 644.5 745 648.8343 746 653.1571 747 657.5079 748 661.8	709	510.3835	710	513.8116	711	517.2619		712	520.7344
721 553.0155 722 556.7192 723 560.4469 724 564.1 725 567.9746 726 571.7750 727 575.6000 728 579.4 729 583.3242 730 587.2238 731 591.1486 732 595.0 737 615.2357 738 619.3415 739 623.4738 740 627.6 741 631.8187 742 636.0316 743 640.2717 744 644.5 745 648.8343 746 653.1571 747 657.5079 748 661.8	713	524.2294	714	527.7471	715	531.2874	Ī	716	534.8507
725 567.9746 726 571.7750 727 575.6000 728 579.4 729 583.3242 730 587.2238 731 591.1486 732 595.0 737 615.2357 738 619.3415 739 623.4738 740 627.6 741 631.8187 742 636.0316 743 640.2717 744 644.5 745 648.8343 746 653.1571 747 657.5079 748 661.8	717	538.4370	718	542.0465	719	545.6793		720	549.3356
729 583.3242 730 587.2238 731 591.1486 732 595.0 733 599.0744 734 603.0758 735 607.1030 736 611.1 737 615.2357 738 619.3415 739 623.4738 740 627.6 741 631.8187 742 636.0316 743 640.2717 744 644.5 745 648.8343 746 653.1571 747 657.5079 748 661.8	721	553.0155	722	556.7192	723	560.4469		724	564.1986
733 599.0744 734 603.0758 735 607.1030 736 611.1 737 615.2357 738 619.3415 739 623.4738 740 627.6 741 631.8187 742 636.0316 743 640.2717 744 644.5 745 648.8343 746 653.1571 747 657.5079 748 661.8	725	567.9746	726	571.7750	727	575.6000		728	579.4497
737 615.2357 738 619.3415 739 623.4738 740 627.6 741 631.8187 742 636.0316 743 640.2717 744 644.5 745 648.8343 746 653.1571 747 657.5079 748 661.8	729	583.3242	730	587.2238	731	591.1486		732	595.0988
741 631.8187 742 636.0316 743 640.2717 744 644.5 745 648.8343 746 653.1571 747 657.5079 748 661.8	733	599.0744	734	603.0758	735	607.1030		736	611.1563
745 648.8343 746 653.1571 747 657.5079 748 661.8	737	615.2357	738	619.3415	739	623.4738		740	627.6328
	741	631.8187	742	636.0316	743	640.2717	Ī	744	644.5392
749 666.2939 750 670.7295 751 675.1937 752 679.6	745	648.8343	746	653.1571	747	657.5079		748	661.8867
	749	666.2939	750	670.7295	751	675.1937		752	679.6868
753 684.2089 754 688.7602 755 693.3409 756 697.9	753	684.2089	754	688.7602	755	693.3409		756	697.9512

757	702.5913	758	707.2613		759	711.9615	Ţ	760	716.6921
761	702.3913	762	726.2450		763	711.9613	ŀ	764	735.9217
765	740.8070	766	745.7238		767	750.6723	-	768	755.6529
769	740.6676	770	765.7106		771	770.7882	-	772	775.8986
773	781.0420	774	786.2187		775	791.4287	ŀ	776	796.6724
777	801.9500	778	807.2616	-	779	812.6075	F	780	817.9880
781	823.4031	782	828.8533		783	834.3386	-	784	839.8594
785	845.4158	786	851.0081		787	856.6365	-	788	862.3012
789	868.0025	790	873.7407		791	879.5158	-	792	885.3283
793	891.1783	794	897.0661		795	902.9919	-	796	908.9559
797	914.9585	798	920.9998		799	927.0801	-	800	933.1997
801	939.3588	802	945.5577		803	951.7966	F	804	958.0758
805	964.3956	806	970.7561		807	977.1578	-	808	983.6008
809	990.0853	810	996.6118		811	1003.1800	-	812	1009.7910
813	1016.4450	814	1023.1420		815	1029.8820	F	816	1036.6650
817	1043.4930	818	1050.3640		819	1057.2800	F	820	1064.2400
821	1071.2460	822	1078.2960		823	1085.3920	F	824	1092.5340
825	1099.7220	826	1106.9570		827	1114.2380	F	828	1121.5670
829	1128.9420	830	1136.3660		831	1143.8370	F	832	1151.3570
833	1158.9250	834	1166.5420		835	1174.2080	F	836	1181.9240
837	1189.6890	838	1197.5050		839	1205.3710	-	840	1213.2890
841	1221.2570	842	1229.2770		843	1237.3480	F	844	1245.4720
845	1253.6480	846	1261.8770		847	1270.1600	F	848	1278.4950
849	1286.8850	850	1295.3290		851	1303.8270	F	852	1312.3810
853	1320.9900	854	1329.6540		855	1338.3740	F	856	1347.1510
857	1355.9840	858	1364.8750		859	1373.8230	F	860	1382.8290
861	1391.8930	862	1401.0160		863	1410.1970	ľ	864	1419.4380
865	1428.7390	866	1438.1000		867	1447.5220		868	1457.0040
869	1466.5480	870	1476.1530		871	1485.8210		872	1495.5510
873	1505.3440	874	1515.2010		875	1525.1210	Ī	876	1535.1050
877	1545.1540	878	1555.2680		879	1565.4470	Ī	880	1575.6930
881	1586.0040	882	1596.3820		883	1606.8280	Ī	884	1617.3410
885	1627.9220	886	1638.5710		887	1649.2900	Ī	888	1660.0780
889	1670.9350	890	1681.8630		891	1692.8620	Ī	892	1703.9310
893	1715.0730	894	1726.2860		895	1737.5730	Ī	896	1748.9320
897	1760.3650	898	1771.8720		899	1783.4530	ſ	900	1795.1090
901	1806.8410	902	1818.6490		903	1830.5330	Ī	904	1842.4940
905	1854.5330	906	1866.6500		907	1878.8450		908	1891.1190
909	1903.4730	910	1915.9060		911	1928.4200		912	1941.0160
913	1953.6930	914	1966.4520		915	1979.2940		916	1992.2190

917	2005.2270	Ī	918	2018.3200	919	2031.4980	Ī	920	2044.7620
921	2058.1110		922	2071.5470	923	2085.0700		924	2098.6800
925	2112.3790		926	2126.1670	927	2140.0440		928	2154.0110
929	2168.0690		930	2182.2170	931	2196.4580		932	2210.7910
933	2225.2170		934	2239.7360	935	2254.3500	Ī	936	2269.0580
937	2283.8620		938	2298.7620	939	2313.7590		940	2328.8530
941	2344.0450		942	2359.3350	943	2374.7250	Ī	944	2390.2140
945	2405.8040		946	2421.4960	947	2437.2890		948	2453.1850
949	2469.1840		950	2485.2860	951	2501.4940		952	2517.8060
953	2534.2250		954	2550.7500	955	2567.3820		956	2584.1230
957	2600.9720		958	2617.9310	959	2634.9990	Ī	960	2652.1790
961	2669.4710		962	2686.8740	963	2704.3910		964	2722.0220
965	2739.7670		966	2757.6270	967	2775.6040		968	2793.6970
969	2811.9080		970	2830.2380	971	2848.6870		972	2867.2550
973	2885.9440		974	2904.7550	975	2923.6880		976	2942.7450
977	2961.9250		978	2981.2300	979	3000.6600		980	3020.2170
981	3039.9020		982	3059.7140	983	3079.6550		984	3099.7260
985	3119.9270		986	3140.2600	987	3160.7260		988	3181.3240
989	3202.0570		990	3222.9240	991	3243.9280		992	3265.0680
993	3286.3460		994	3307.7620	995	3329.3180		996	3351.0140
997	3372.8520		998	3394.8310	999	3416.9540		1000	3439.2210
1001	3461.6330		1002	3484.1910	1003	3506.8970		1004	3529.7500
1005	3552.7520		1006	3575.9030	1007	3599.2060		1008	3622.6610
1009	3646.2680		1010	3670.0300	1011	3693.9460		1012	3718.0180
1013	3742.2480		1014	3766.6350	1015	3791.1810		1016	3815.8880
1017	3840.7550		1018	3865.7850	1019	3890.9780		1020	3916.3350
1021	3941.8580		1022	3967.5470	1023	3993.4040			

Annex C (INFORMATIVE) MEASURING THE ACCURACY WITH WHICH A DISPLAY SYSTEM MATCHES THE GRAYSCALE STANDARD DISPLAY FUNCTION

C.1. GENERAL CONSIDERATIONS REGARDING CONFORMANCE AND METRICS

To demonstrate conformance with the Grayscale Standard Display Function is a much more complex task than, for example, validating the responses of a totally digital system to DICOM messages.

Display systems ultimately produce analog output, either directly as Luminances or indirectly as optical densities. For some Display Systems, this analog output can be affected by various imperfections in addition to whatever imperfections exist in the Display System's Display Function which is to be validated. For example, there may be spatial non-uniformities in the final presented image (e.g., arising from film, printing, or processing non-uniformities in the case of a hardcopy printer) which are measurable but are at low spatial frequencies which do not ordinarily pose an image quality problem in diagnostic radiology.

It is worth noting that CRTs and light-boxes also introduce their own spatial non-uniformities. These non-uniformities are outside the scope of the Grayscale Standard Display Function and the measurement procedures described here. But because of them, even a test image which is perfectly presented in terms of the Grayscale Standard Display Function will be less than perfectly perceived on a real CRT or a real light-box.

Furthermore, the question "How close (to the Grayscale Standard Display Function) is close enough?" is currently unanswered, since the answer depends on psychophysical studies not yet done to determine what difference in Display Function is "just noticeable" when two nearly identical image presentations (e.g., two nearly identical films placed on equivalent side-by-side light-boxes) are presented to an observer.

Furthermore, the evaluation of a given Display System could be based either on visual tests (e.g., assessing the perceived contrast of many low-contrast targets in one or more test images) or by quantitative analysis based on measured data obtained from instruments (e.g., photometers or densitometers).

Even the quantitative approach could be addressed in different ways. One could, for example, simply superimpose plots of measured and theoretical analog output (i.e., Luminance or optical density) vs. P-Value, perhaps along with "error bars" indicating the expected uncertainty (non-repeatable variations) in the measured output. As a mathematically more elegant alternative, all the measured data points could be used as input to a statistical mathematical analysis which could attempt to determine the underlying Display Function of the Display System, yielding one or more quantitative values (metrics) which define how well the Display System conforms with the Grayscale Standard Display Function.

In what follows in this and the following annexes, an example of the latter type of metric analysis is used, in which measured data is analyzed using a "FIT" test which is intended to validate the shape of the Characteristic Curve and a "LUM" test which is intended to show the degree of scatter from the ideal Grayscale Standard Display Function. This approach has been applied, for example, to quantitatively demonstrate how improvements were successfully made to the Display Function of certain Display Systems.

Before proceeding with the description of the methodology of this specific metric approach, it should be noted that it is offered as one possible approach, not necessarily as the most appropriate approach for evaluating all Display Systems. In particular, the following notes should be considered before selecting or interpreting results from any particular metric approach.

1) There may be practical issues which limit the number of P-Values which can be meaningfully used in the analysis. For example, it may be practical to measure all 256 possible Luminances from a fixed position on the screen of an 8-bit video monitor, but it may be impractical to meaningfully measure all

4096 densities theoretically printable by a 12-bit film printer. One reason for the impracticality is the limited accuracy of densitometers (or even film digitizers). A second reason is that the film density measurements, unlike the CRT photometer measurements, are obtained from different locations on the display area, so any spatial non-uniformity which is present in the film affects the hardcopy measurement. Current hardcopy printers and densitometers both have absolute optical density accuracy limitations which are significantly worse than the change which would be caused by a change in just the least significant bit of a 12-bit P-Value. In general, selecting a larger number of P-Values allows, in principle, more localized aberrations from the Grayscale Standard Display Function to be "caught", but the signal-to-noise ratio (or significance) of each of these will be decreased.

- 2) If the measurement data for a particular Display System has significant "noise" (as indicated by limited repeatability in the data when multiple sets of measurements are taken), it may be desirable to apply a statistical analysis technique which goes beyond the "FIT" and "LUM" metric by explicitly utilizing the known standard deviations in the input data set, along with the data itself, to prevent the fitting technique from over-reacting to noise. See, for example, the section "General Linear Least Squares" in Reference C1 and the chapter "Least-Squares Fit to a Polynomial" in Reference C2. If measurement noise is not explicitly taken into account in the analysis, the metric's returned root-mean-square error of the data points relative to the fit could be misleadingly high, since it would include the combined effect of errors due to incorrectness in the Display Function and errors due to measurement noise.
- 3) If possible, the sensitivity and specificity of the metric being considered should be checked against visual tests. For example, a digital test pattern with many low-contrast steps at many ambient Luminances could be printed on a "laboratory standard" Grayscale Standard Display Function printer and also printed on a printer being evaluated. The resultant films could then be placed side-by-side on light-boxes for comparison by a human observer. A good metric technique should detect as sensitively and repeatably as the human observer the existence of deviations (of any shape) from the Grayscale Standard Display Function. For example, if a Display System has a Characteristic Curve which, for even a very short interval of DDL values, is too contrasty, too flat, or (worse yet) non-monotonic, the metric should be able to detect and respond to that anomaly as strongly as the human observer does.
- 4) Finally, in addition to the experimentally encountered non-repeatabilities in the data from a Display System, there may be reason to consider additional possible causes of variations. For example, varying the ordering of P-Values in a test pattern (temporally for CRTs, spatially for printers) might affect the results. For printers, switching to different media might affect the results. A higher confidence can be placed in the results obtained from any metric if the results are stable in the presence of any or all such changes.

C.2. METHODOLOGY

Step (1)

The Characteristic Curve of the test Display System should be determined with as many measurements as practical (see Sections D.1, D.2, and D.3). Using the Grayscale Standard Display Function, the fractional number of JNDs are calculated for each Luminance interval between equally spaced P-Value steps. The JNDs/Luminance interval may be calculated directly, or iteratively. For example, if only a few JNDs belong to every Luminance interval, a linear interpolation may be performed. After transformation of the grayscale response of the Display System, the Luminance Levels for every P-Value are Li and the corresponding Standard Luminance Levels are Lj; dj specifies the JNDs /Luminance-Interval on the Grayscale Standard Display Function for the given number of P-Values. Then, the JNDs/Luminance interval for the transformed Display Function are

$$r = d_i (L_{i+1} - L_i)(L_{i+1} + L_i) / ((L_{i+1} + L_i)(L_{i+1} - L_i))$$
 (C1)

Additionally, an iterative method can be used to calculate the number of JNDs per Luminance interval, requiring only the Grayscale Standard Display Function that defines a JND step in Luminance given a Luminance value. This is done by simply counting the number of complete JND steps in the Luminance interval, and then the remaining fractional step. Start at the Luminance low end of the interval, and calculate from the Grayscale Standard Display Function the Luminance step required for one JND step.

Then continue stepping from the low Luminance value to the high Luminance value in single JND steps, until the Luminance value of the upper end of the Luminance Range is passed. Calculate the fraction portion of one JND that this last step represents. the total number of completed integer JND steps plus the fractional portion of the last uncompleted step is the fractional number of JND steps in the Luminance interval.

Plot the number of JNDs per Luminance interval (vertical axis) versus the index of the Luminance interval (horizontal axis). This curve is referred to as the *Luminance intervals vs JNDs* curve. An example of a plot of Luminance intervals vs JNDs is shown in figure C-1. The plot is matched very well by a horizontal line when a linear regression is applied.

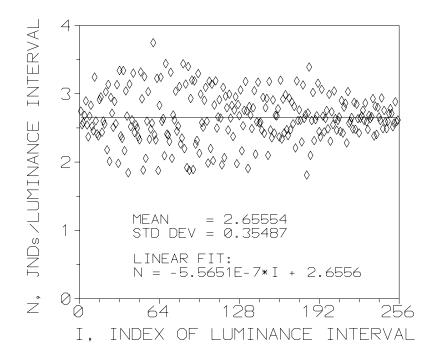


Figure C-1. Illustration for the LUM and FIT conformance measures

The JNDs/Luminance interval data are evaluated by two statistical measures [C4]. The first assesses the global match of the test Display Function with the Grayscale Standard Display Function. The second measure locally analyses the approximation of the Grayscale Standard Display Function to the test Display Function.

Step (2)

Two related measures of a regression analysis are applied after normal multiple linear regression assumptions are verified for the data [C3]. The first measure, named the *FIT* test, attempts to match the Luminance-Intervals-vs-JNDs curve of the test Luminance distribution with different order polynomial fits. The Grayscale Standard Display Function is characterized by exactly one JND per Luminance interval over the entire Luminance Range. Therefore, ideally, the data of JNDs/Luminance intervals vs index of the Luminance interval are best fit by a horizontal line of a constant number of JNDs/Luminance interval, indicating that both the local and global means of JNDs/Luminance interval are constant over the given Luminance Range. If the curve is better matched by a higher-order curve, the distribution is not closely approximating the Grayscale Standard Display Function. The regression analysis should test comparisons through third-order curves.

The second measure, the Luminance uniformity metric (LUM), analyzes whether the size of Luminance steps are uniform in perceptual size (i.e. JNDs) across the Luminance Range. This is measured by the Root Mean Square Error (RMSE) of the curve fit by a horizontal line of the JNDs/Luminance interval. The smaller the RMSE of the JNDs/Luminance interval, the more closely the test Display Function approximates the Grayscale Standard Display Function on a microscopic scale.

Both the FIT and LUM measures can be conveniently calculated on standard statistical packages.

Assuming the test Luminance distribution passes the FIT test, then the measure of quality of the distribution is determined by the single quantitative measurement (LUM) of the standard deviation of the JNDs/Luminance interval from their mean. Clinical practice is expected to determine the tolerances for the FIT and LUM values.

An important factor in reaching a close approximation of a test Display Function to the Grayscale Standard Display Function is the number of discrete output levels of the Display System. For instance, the LUM measure can be improved by using only a subset of the available DDLs while maintaining the full available output digitization resolution at the cost of decreasing contrast resolution.

While the LUM is influenced by the choice of the number of discrete output gray levels in the Grayscale Standard Display Function, the appropriate number of output levels is determined by the clinical application, including possible gray scale image processing that may occur independently of the Grayscale Standard Display Function standardization. Thus, PS 3.14 does not prescribe a certain number of gray levels of output. However, in general, the larger the number of distinguishable gray levels available, the higher the possible image quality because the contrast resolution is increased. It is recommended that the number of necessary output driving levels for the transformed Display Function be determined prior to standardization of the Display System (based on clinical applications of the Display System), so that this information can be used when calculating the transformation in order to avoid using gray scale distributions with fewer output levels than needed.

C.3. REFERENCES

- [C1] Press, William H, et al., Numerical Recipes in C, Cambridge University Press, 1988, Section "General Linear Least Squares"
- [C2] Bevington, Phillip R., Data Reduction and Error Analysis for the Physical Sciences, McGraw-Hill, 1969, the chapter "Least-Squares Fit to a Polynomial".
- [C3] Kleinbaum DG, Kupper LL, Muller KE, Applied Regression Analysis and Other Multivariable Methods, Duxbury Press, 2nd Edition, pp 45-49, 1987.
- [C4] Hemminger, B., Muller, K., "Performance Metric for evaluating conformance of medical image displays with the ACR/NEMA display function standard", SPIE Medical Imaging 1997, editor Yongmin Kim, vol 3031-25, 1997.

Annex D (INFORMATIVE) ILLUSTRATIONS FOR ACHIEVING CONFORMANCE WITH THE GRAYSCALE STANDARD DISPLAY FUNCTION

The following sections illustrate how conformance with the Grayscale Standard Display Function may be achieved for emissive (soft-copy) Display Systems as well as systems producing image presentations (hard-copies) on transmissive and reflective media. Each section contains four sub-sections on 1) a procedure for measuring the system Characteristic Curve, 2) the application of the Grayscale Standard Display Function to the Luminance Range of the Display System, 3) the implementation of the Grayscale Standard Display Function, and 4) the application of the conformance metrics as proposed in Annex C.

It is emphasized that there are different ways to configure a Display System or to change its performance so that it conforms to the Grayscale Standard Display Function. In fact, conceivably, a Display System may calibrate itself automatically to maintain conformance with the Standard. Hence, the following three illustrations are truly only examples.

Luminance of any Display System, hard-copy or softcopy, may be measured with a photometer. The photometer should have the following characteristics:

- be accurate to within 3% or less of the absolute Luminance level across its full range of operation;
- have a relative accuracy of at least two times the least significant digit at any Luminance level in its range of operation;
- maintain this accuracy at Luminance levels that are one-tenth of the minimum measured Luminance of the Display System;
- have an acceptance angle that is small enough to incorporate only the measurement field without overlapping the surrounding background.

Note: The photometer may be of the type that attaches directly to the display face (with a suction cup) or of the type that is held away from the display face. If of the latter type, the photometer should be well baffled to exclude extraneous light sources, including light from the background area of the test pattern.

For a film Display System the photometer may be appropriately used to measure the background Illuminance and the Luminance of the light-box on which the film will be displayed. The Luminance characteristics of the film Display System may be measured directly with the photometer or indirectly using measured optical density of the film and the values for the measured background Illuminance and the light-box Luminance.

D.1 EMISSIVE DISPLAY SYSTEMS

D.1.1 Measuring the system Characteristic Curve

Before the characteristic Luminance response of the emissive Display System is measured, it is allowed to warm up as recommended by the manufacturer and is adjusted such that it conforms to the manufacturer's performance specifications. In particular, adjustment procedures for setting the black and white levels of the display should be obtained from the Display System manufacturer. The goal is to maximize the dynamic Luminance Range of the display without introducing artifacts, resulting in the highest possible number of Just-Noticeable Differences (JNDs).

Note: A simple test that the system is set up properly can be performed by viewing the 5% and 95% squares in the SMPTE pattern. The perceived contrast between the 5% square and its 0% surrounding should be equal to the perceived contrast between the 95% square and a white square.

Measurement of the Characteristic Curve of the Display System may be accomplished using a test pattern (Fig. D.1-1) consisting of:

a square measurement field comprising 10% of the total number of pixels displayed by the system positioned in the center of the display;

a full-screen uniform background of 20% of maximum Luminance surrounding the target.

Note: With a measurement field of 10% of the total number of displayed pixels and a surrounding set to 20% of maximum Luminance, internal light scatter in the monitor causes the Luminance Range to be typically comparable to that found in radiographs, such as a thorax radiograph, when displayed on the CRT monitor.

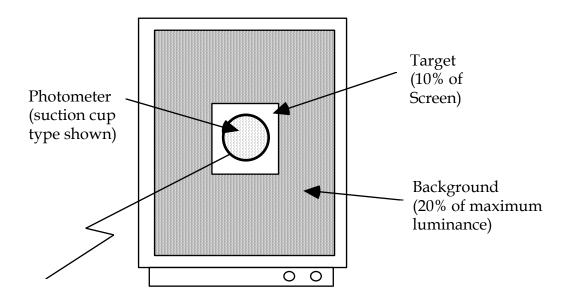


Figure D.1-1. The test pattern will be a variable intensity square in the center of a low Luminance background area.

- Notes: 1. For example, on a 5-megapixel Display System with a matrix of 2048 by 2560 pixels, the target would be a square with 724 pixels on each side.
 - 2. Ideally, the test pattern should fill the entire screen. Under certain windowed operating environments, it may be difficult to eliminate certain user-interface objects from the display, in particular, menu bars at the top of the screen. In this case, the background should fill as much of the screen as possible.

The Characteristic Curve of the Display System may be determined by

- turning off all ambient lighting (necessary only when a suction cup photometer is used or when a handheld photometer casts a shadow on the display screen);
- displaying the above test pattern;
- setting the DDL for the measurement field to a sequence of different values, starting with 0 and increasing at each step until the maximum DDL is reached;
- using a photometer to measure and record the Luminance of the measurement field at each command value.

As discussed in Annex C, the number and distribution of DDLs at which measurements are taken must be sufficient to accurately model the Characteristic Curve of the Display System over the entire Luminance Range.

- Notes: 1. If a handheld photometer is used, it should be placed at a distance from the display screen so that Luminance is measured in the center of the measurement field, without overlapping the surrounding background. This distance can be calculated using the acceptance angle specification provided by the photometer manufacturer.
 - 2. The exact number and distribution of DDLs should be based both on the characteristics of the Display System and on the mathematical technique used to interpolate the Characteristic Curve of the system. It is recommended that at least 64 different command values be used in the procedure.
 - 3. Successive Luminance measurements should be spaced in time such that the Display System always reaches a steady state. It may be particularly important to allow the system to settle before taking the initial measurement at DDL 0.

As stated in the normative section, the effect of ambient light on the apparent Characteristic Curve must always be included when configuring a Display System to conform with the Grayscale Standard Display Function.

If a handheld photometer that does not cast a shadow on the display screen is used to measure the Characteristic Curve, then the Luminance produced by the display plus the effect of ambient light may be measured simultaneously.

When a suction cup photometer is used to take the Luminance measurements or when a handheld photometer casts a shadow on the display screen, all ambient lighting should be turned off while measuring the Characteristic Curve. The effect of ambient light is determined separately: The Display System is turned off, the ambient light is turned on, and the Luminance produced by scattering of ambient light at the display screen is measured by placing the photometer at a distance from the display screen so that its acceptance angle includes a major portion of the screen and that the measurement is not affected by direct illumination from areas outside the display screen. The Luminance related to ambient light is added to the previously measured Luminance levels produced by the Display System to determine the effective Characteristic Curve of the system.

Changes in ambient lighting conditions may require recalibration of the display subsystem in order to Note: maintain conformance to this standard.

In the following, an example for measurements and transformation of a Display Function is presented. The Display System for this example is a CRT monitor with display controller. It is assumed that the display controller allows a transformation of the DDLs with 8-bit input precision and 10-bit output precision.

The Luminance is measured with a photometer with a narrow (1⁰) acceptance angle. The ambient light level was adjusted as low as possible. No localized highlights were visible.

- 1) The maximum Luminance was measured when setting the DDL for the measurement field to the value that yielded the highest Luminance and the DDL of the surrounding to the middle DDL range. From this measurement, the Luminance - 20% of the maximum Luminance - for the surrounding of the measurement field was calculated.
- 2) The ambient light was turned off. With the photometer centered on the measurement field of the test pattern of Fig. D.1-1, the Luminance was measured when varying the input level D_m in increments of 1 from 0 to 255. The transformation operator of the hypothetical display controller linearly mapped 8 bits on the input to 10 bits on the output. The measured data represent the Characteristic Curve L = $F(D_m)$ for the given operating conditions and this test pattern.
- 3) Next, the CRT was turned off and the ambient light turned on. The photometer was placed on the center axis of the CRT sufficiently far away so that it did not cast a shadow on the CRT face and its

aperture intercepted light scattered from a major portion of the CRT face. The measured Luminance of 0.3 cd/m² produced by the ambient light on the CRT face was added to the measured Luminance values of the Characteristic Curve without ambient light. The result is listed in Table D.1-1 and plotted in Fig. D.1-2.

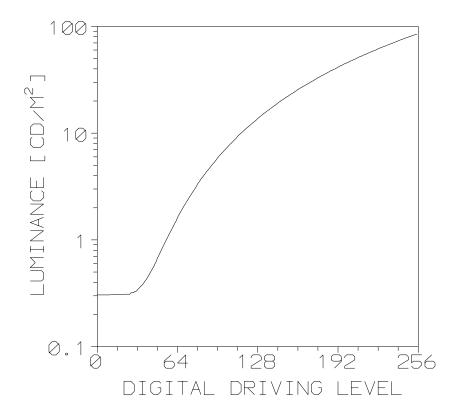


Figure D.1-2. Measured Characteristic Curve with Ambient Light of an emissive Display System

Table D.1-1
Measured Characteristic Curve plus Ambient Light

	1	Wicasui	Cu Onaracter	13	lic Oui ve	pius Ambien		igiit	
DDL	Luminance	DDL	Luminance		DDL	Luminance		DDL	Luminance
0	0.305	1	0.305		2	0.305		3	0.305
4	0.305	5	0.305		6	0.305		7	0.305
8	0.305	9	0.305		10	0.305		11	0.307
12	0.307	13	0.307		14	0.307		15	0.307
16	0.307	17	0.307		18	0.307		19	0.307
20	0.307	21	0.307		22	0.310		23	0.310
24	0.310	25	0.310		26	0.310		27	0.320
28	0.320	29	0.320		30	0.330		31	0.330
32	0.340	33	0.350		34	0.360		35	0.370
36	0.380	37	0.392		38	0.410		39	0.424
40	0.442	41	0.464		42	0.486		43	0.512
44	0.534	45	0.562		46	0.594		47	0.626
48	0.674	49	0.710		50	0.750		51	0.796
52	0.842	53	0.888		54	0.938		55	0.994
56	1.048	57	1.108		58	1.168		59	1.232
60	1.294	61	1.366		62	1.438		63	1.512
64	1.620	65	1.702		66	1.788		67	1.876
68	1.960	69	2.056		70	2.154		71	2.248
72	2.350	73	2.456		74	2.564		75	2.670
76	2.790	77	2.908		78	3.022		79	3.146
80	3.328	81	3.460		82	3.584		83	3.732
84	3.870	85	4.006		86	4.156		87	4.310
88	4.456	89	4.608		90	4.766		91	4.944
92	5.104	93	5.268		94	5.444		95	5.630
96	5.864	97	6.050		98	6.238		99	6.438
100	6.610	101	6.820		102	7.024		103	7.224
104	7.428	105	7.644		106	7.872		107	8.066
108	8.298	109	8.528		110	8.752		111	8.982
112	9.330	113	9.574		114	9.796		115	10.060
116	10.314	117	10.560		118	10.820		119	11.080
120	11.340	121	11.620		122	11.880		123	12.180
124	12.460	125	12.700		126	13.020		127	13.300
128	13.720	129	14.020		130	14.360		131	14.640
132	14.940	133	15.300		134	15.600		135	15.900
136	16.240	137	16.560		138	16.920		139	17.220
140	17.600	141	17.940		142	18.240		143	18.640
144	19.120	145	19.460		146	19.800		147	20.260
148	20.560	149	20.920		150	21.360		151	21.760
	L		l .	l	<u> </u>	i .	J		<u>i </u>

152	22.060	153	22.520	154	22.960	155	23.300
156	23.700	157	24.080	158	24.600	159	24.980
160	25.520	161	26.040	162	26.480	163	26.700
164	27.380	165	27.620	166	28.040	167	28.580
168	28.980	169	29.400	170	29.840	171	30.540
172	30.800	173	31.380	174	31.880	175	32.400
176	33.060	177	33.400	178	34.040	179	34.400
180	34.840	181	35.360	182	35.900	183	36.400
184	37.060	185	37.400	186	38.300	187	38.420
188	39.160	189	39.760	190	39.980	191	40.840
192	41.540	193	41.900	194	42.800	195	43.060
196	43.620	197	44.520	198	44.620	199	45.500
200	46.100	201	46.380	202	47.400	203	47.600
204	48.320	205	49.060	206	49.380	207	50.320
208	50.920	209	51.600	210	52.420	211	52.680
212	53.520	213	54.220	214	54.620	215	55.420
216	56.100	217	56.600	218	57.400	219	57.820
220	58.660	221	59.320	222	59.800	223	60.720
224	61.520	225	62.240	226	63.040	227	63.480
228	64.460	229	65.020	230	65.500	231	66.500
232	66.960	233	67.840	234	68.600	235	68.980
236	70.040	237	70.520	238	71.420	239	72.180
240	72.900	241	73.980	242	74.580	243	75.320
244	76.200	245	76.540	246	77.720	247	78.220
248	79.200	249	79.880	250	80.420	251	81.560
252	81.960	253	83.140	254	83.720	255	84.340

D.1.2 Application of the Standard Formula

The section of the Grayscale Standard Display Function for the Luminance Range of the CRT monitor Display System is shown in Figure D.1-3. Minimum and maximum Luminance levels correspond to JND indices of $JND_{min} = 32.54$ and $JND_{max} = 453.85$, respectively. Thus, there are theoretically about 420 just-noticeable Luminance differences for the Standard Target (see Normative Section 6). Obviously, with 8-bit input digitization resolution, at best 256 noticeable Luminance increments can be realized.

D.1.3 Implementation of the Standard

The measured Characteristic Curve is interpolated for the available output levels D_{output} , in this case, yielding 1024 Luminance levels $L_{I,m}$. The Grayscale Standard Display Function is also interpolated between JND_{min} and JND_{max} (($JND = [JND_{max} - JND_{min}]/1023 = [453.85 - 32.54]/1023$) yielding 1024 Standard Luminance levels $L_{I,STD}$. Interpolations can be performed by a variety of techniques. Here, a cubic spline technique was employed.

For every $L_{I,STD}$, the closest $L_{J,m}$ is determined. The data pair I,J defines the transformation between D_{input} and D_{output} (Table D.1-2) by which the Luminance response of the Display System is made to approximate the Grayscale Standard Display Function.

Table D.1-2 Look-Up Table for Calibrating Display System

Input	Output	Input	Output	עו	Input	Output	Input	Output
0	0	1	118		2	131	3	140
4	148	5	153		6	160	7	164
8	169	9	173		10	178	11	182
12	185	13	189		14	191	15	194
16	198	17	201		18	204	19	207
20	210	21	214		22	217	23	219
24	222	25	225		26	228	27	231
28	234	29	237		30	240	31	243
32	245	33	248		34	251	35	253
36	255	37	257		38	260	39	263
40	265	41	268		42	271	43	274
44	276	45	279		46	282	47	284
48	287	49	290		50	292	51	295
52	298	53	301		54	303	55	306
56	308	57	311		58	314	59	317
60	319	61	320		62	323	63	326
64	329	65	331		66	334	67	336
68	339	69	342		70	345	71	347
72	350	73	353		74	356	75	359
76	361	77	364		78	367	79	370
80	372	81	375		82	378	83	381
84	383	85	385		86	388	87	391
88	393	89	396		90	399	91	402
92	405	93	407		94	410	95	413
96	416	97	419		98	422	99	425
100	428	101	431		102	434	103	437
104	440	105	443		106	445	107	448
108	450	109	452		110	456	111	459
112	462	113	465		114	468	115	471
116	474	117	477		118	480	119	483
120	486	121	490		122	492	123	495
124	499	125	502		126	505	127	509
128	511	129	513		130	516	131	519

132	522	133	526	134	529	135	532
136	535	137	539	138	542	139	545
140	549	141	552	142	555	143	559
144	562	145	565	146	569	147	572
148	575	149	578	150	581	151	585
152	588	153	591	154	595	155	599
156	602	157	605	158	609	159	613
160	616	161	619	162	623	163	627
164	631	165	633	166	637	167	640
168	643	169	646	170	650	171	655
172	657	173	663	174	666	175	669
176	674	177	678	178	682	179	684
180	688	181	693	182	696	183	700
184	703	185	706	186	711	187	714
188	719	189	723	190	727	191	731
192	735	193	738	194	743	195	745
196	752	197	754	198	758	199	764
200	766	201	769	202	775	203	777
204	783	205	787	206	789	207	796
208	799	209	805	210	808	211	811
212	818	213	821	214	827	215	830
216	834	217	838	218	841	219	848
220	851	221	856	222	861	223	864
224	870	225	874	226	880	227	883
228	889	229	893	230	897	231	901
232	905	233	911	234	915	235	922
236	925	237	931	238	935	239	941
240	945	241	951	242	955	243	960
244	964	245	969	246	975	247	979
248	985	249	991	250	995	251	1002
252	1006	253	1012	254	1016	255	1023

D.1.4 Measures of Conformance

The FIT and the LUM metrics proposed in Annex C are applied to determine the macroscopic and microscopic approximation of the $L_{J,m}$ to the $L_{J,STD}$. Figure D.1-3 shows the perceptually linearized Display Function superimposed on the Grayscale Standard Display Function and Figure D.1-4 summarizes the results of the two metrics. A good global fit was achieved as demonstrated by the nearly horizontal-line fit as best fit obtained with the FIT metric. The RMSE is acceptable. All 255 P-Value intervals lead to JNDs on the transformed Display Function for the Standard Target.

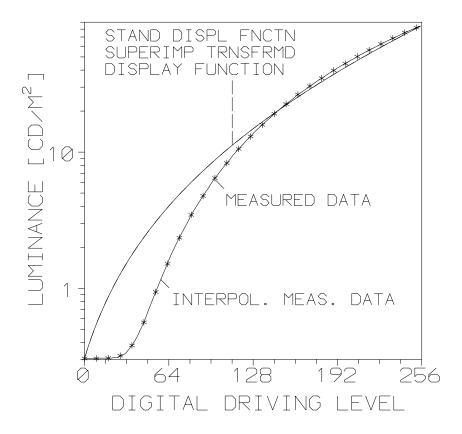


Figure D.1-3. Measured and interpolated Characteristic Curve, Grayscale Standard Display Function and transformed Display Function of an emissive Display System. The transformed Display Function for this Display System matches the Grayscale Standard Display Function and the two curves are superimposed and indistinguishable.

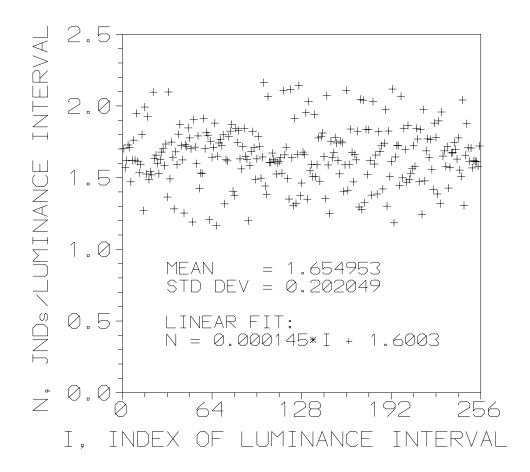


Figure D.1-4. LUM and FIT measures of conformance for a the transformed Display Function of an emissive Display System

D.2 TRANSPARENT HARDCOPY DEVICES

D.2.1 Measuring the system Characteristic Curve

A transparent hardcopy device is exemplified by a laser printer (including processor) which prints (exposes and processes) one or more images on a sheet of transparent film (typically a 14" x 17" film). This film is eventually placed over a high Luminance light-box in a darkened room for viewing.

The Characteristic Curve for such a transparent hardcopy device is obtained by printing a test image consisting of a pattern of n bars, each bar having a specific numeric value (DDL). The optical density of each printed bar is then measured, using a transmission densitometer, for each of the printed bars.

To accurately define a printer's Characteristic Curve, it is desirable that n be as large as possible (to capture as many points as possible on the Characteristic Curve). However, the limitations on absolute quantitative repeatability imposed by the printer, processor, or media technologies may dictate that a much smaller value of n be used (to prevent a conformance metric which is sensitive to differences from becoming unstable and meaningless, as the density differences between adjacent bars become "in the noise" as the number of bars becomes large).

One example of a test image is a pattern of 32 approximately equal-height bars, spanning the usable printable region of the film, having 32 approximately equi-spaced DDLs as follows:

Density Step 0	
Density Step 1	
Density Step 2	
Density Step 3	
Density Step 4	
Density Step 5	
Density Step 6	
Density Step 7	
Density Step 8	
Density Step 9	
Density Step 10	
Density Step 11	
Density Step 12	
Density Step 13	
Density Step 14	
Density Step 15	
Density Step 16	
Density Step 17	
Density Step 18	
Density Step 19	
Density Step 20	
Density Step 21	
Density Step 22	
Density Step 23	
Density Step 24	
Density Step 25	
Density Step 26	
Density Step 27	
Density Step 28	
Density Step 29	
Density Step 30	
 Density Step 31	

Figure D.2-1. Layout of a Test Pattern for Transparent Hardcopy Media

To define a test pattern with n DDLs for a printer with an N-bit input, the DDL of step # i can be set to

$$DDL_{i} = (2^{N}-1) \frac{i}{n-1}$$
 (D.2-1)

rounded to the nearest integer.

The tabulated values of DDL_i and the corresponding measured optical densities OD_i constitute a Characteristic Curve of the printer.

D.2.2 Application of the Grayscale Standard Display Function

The films which are produced by transparent hardcopy printers are often brought to a variety of locations, where they may be viewed on different light-boxes and under a variety of viewing conditions. Accordingly, the approach of PS 3.14 is to define, for hardcopy transparent printers, what densities (rather than Luminances) should be produced, and to provide here a method of applying the Grayscale Standard Display Function to the transparent hardcopy case, based on parameters which are typical of the expected range of light-box Luminances and other viewing parameters.

The specific parameters which are used in the following example are as follows:

 L_0 (Luminance of light-box with no film present): 2000 cd/m²

La (ambient room light reflected by film): 10 cd/m²

D_{min} (minimum optical density obtainable on film): 0.20

D_{max} (maximum optical density desirable on film): 3.00.

The process of constructing a table of desired OD values from the Grayscale Standard Display Function begins with defining the Luminance Range and the corresponding range of the Just-Noticeable Difference Index, j. The minimum and maximum Luminance values are given respectively by

$$L_{min} = L_a + L_0 10^{-D_{max}} = 12.0 \text{ cd/m}^2$$
 (D.2-2)

$$L_{\text{max}} = L_a + L_0 10^{-\text{Dmin}} = 1271.9 \text{ cd/m}^2$$
 (D.2-3)

Next, calculate the corresponding Just-Noticeable Difference Index values, j_{min} and j_{max} . For the current example, we obtain

$$j_{min} = 233.32$$
 (D.2-4)

$$j_{\text{max}} = 848.75$$
 (D.2-5)

This gives us the range of j-values which the printer should cover. The printer should map its minimum input (P-Value = 0) to j_{min} and the corresponding L_{min} . It should map its maximum input (P-Value = 2^{N} -1 where N is the number of input bits) to j_{max} and the corresponding L_{max} . At any intermediate input it should map its input proportionately:

$$j (PV) = j_{min} + (j_{max}-j_{min}) \frac{PV}{2^{N}-1}$$
 (D.2-6)

and target values for the Luminance given by the Standard's formula: L(j(P-Value)). This "targeting" consists of producing an optical density OD for this P-Value which will give the desired Luminance L(j(P-Value)) under the conditions of L_0 and L_a previously defined. The required density can thus be calculated as follows:

OD(PV) =
$$-\log_{10} \frac{(L(j(PV)) - L_a)}{L_0}$$
 (D.2-7)

D.2.3 Implementation of the Grayscale Standard Display Function

Carrying this example into the even more specific case of a printer with an 8-bit input leads to the following table, which defines the OD's to be generated for each of the 256 possible P-Values.

Table D.2-1 Optical Densities for Each P-Value for an 8-Bit Printer

P-Value	Optical Density (OD)	P-Value	Optical Density (OD)	P-Value	Optical Density (OD)	P-Value	Optical Density (OD)
0	3.000	1	2.936	2	2.880	3	2.828
4	2.782	5	2.739	6	2.700	7	2.662
8	2.628	9	2.595	10	2.564	11	2.534
12	2.506	13	2.479	14	2.454	15	2.429

16	2.405	17	2.382	18	2.360	19	2.338
20	2.317	21	2.297	22	2.277	23	2.258
24	2.239	25	2.221	26	2.203	27	2.185
28	2.168	29	2.152	30	2.135	31	2.119
32	2.103	33	2.088	34	2.073	35	2.058
36	2.043	37	2.028	38	2.014	39	2.000
40	1.986	41	1.973	42	1.959	43	1.946
44	1.933	45	1.920	46	1.907	47	1.894
48	1.882	49	1.870	50	1.857	51	1.845
52	1.833	53	1.821	54	1.810	55	1.798
56	1.787	57	1.775	58	1.764	59	1.753
60	1.742	61	1.731	62	1.720	63	1.709
64	1.698	65	1.688	66	1.677	67	1.667
68	1.656	69	1.646	70	1.636	71	1.626
72	1.616	73	1.605	74	1.595	75	1.586
76	1.576	77	1.566	78	1.556	79	1.547
80	1.537	81	1.527	82	1.518	83	1.508
84	1.499	85	1.490	86	1.480	87	1.471
88	1.462	89	1.453	90	1.444	91	1.434
92	1.425	93	1.416	94	1.407	95	1.398
96	1.390	97	1.381	98	1.372	99	1.363
100	1.354	101	1.346	102	1.337	103	1.328
104	1.320	105	1.311	106	1.303	107	1.294
108	1.286	109	1.277	110	1.269	111	1.260
112	1.252	113	1.244	114	1.235	115	1.227
116	1.219	117	1.211	118	1.202	119	1.194
120	1.186	121	1.178	122	1.170	123	1.162
124	1.154	125	1.146	126	1.138	127	1.130
128	1.122	129	1.114	130	1.106	131	1.098
132	1.090	133	1.082	134	1.074	135	1.066
136	1.058	137	1.051	138	1.043	139	1.035
140	1.027	141	1.020	142	1.012	143	1.004
144	0.996	145	0.989	146	0.981	147	0.973
148	0.966	149	0.958	150	0.951	151	0.943
152	0.935	153	0.928	154	0.920	155	0.913
156	0.905	157	0.898	158	0.890	159	0.883
160	0.875	161	0.868	162	0.860	163	0.853
164	0.845	165	0.838	166	0.831	167	0.823
168	0.816	169	0.808	170	0.801	171	0.794
172	0.786	173	0.779	174	0.772	175	0.764

4=0	0	Ī	4	0.750	4=0	0.740	4=0	0.505
176	0.757	ļ	177	0.750	178	0.742	179	0.735
180	0.728		181	0.721	182	0.713	183	0.706
184	0.699		185	0.692	186	0.684	187	0.677
188	0.670		189	0.663	190	0.656	191	0.648
192	0.641		193	0.634	194	0.627	195	0.620
196	0.613		197	0.606	198	0.598	199	0.591
200	0.584		201	0.577	202	0.570	203	0.563
204	0.556		205	0.549	206	0.542	207	0.534
208	0.527		209	0.520	210	0.513	211	0.506
212	0.499		213	0.492	214	0.485	215	0.478
216	0.471		217	0.464	218	0.457	219	0.450
220	0.443		221	0.436	222	0.429	223	0.422
224	0.415		225	0.408	226	0.401	227	0.394
228	0.387		229	0.380	230	0.373	231	0.366
232	0.359		233	0.352	234	0.345	235	0.338
236	0.331		237	0.324	238	0.317	239	0.311
240	0.304		241	0.297	242	0.290	243	0.283
244	0.276		245	0.269	246	0.262	247	0.255
248	0.248		249	0.241	250	0.234	251	0.228
252	0.221		253	0.214	254	0.207	255	0.200

Plotting these values gives the curve of Figure D.2-3.

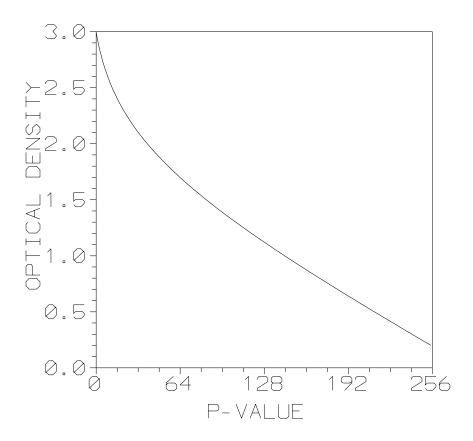


Figure D.2-3. Plot of OD vs P-Value for an 8-Bit Printer

D.2.4. Measures of Conformance

As an example, a bar pattern with 32 optical densities was printed on transmissive media (film). Beforehand, the printer had been set up to print over a density range from 0.2 (Dmin) to 3.0 (Dmax) and had been pre-configured by the manufacturer to use the Grayscale Standard Display Function, converted by the manufacturer into the table of target density values vs. P-Values described earlier.

The test pattern which was used for this was an 8-bit image consisting essentially of 32 horizontal bars. The 32 P-Values used for the bars were as follows: 0, 8, 16, 25, 33, 41, 49, 58, 66, 74, 82, 90,99, 107, 115, 123, 132, 140, 148, 156, 165, 173, 181, 189, 197, 206, 214,222, 230, 239, 247, 255.

For a given film, the 32 bars' optical densities were measured (near the middle of the film), converted to Luminances (using the standard parameters of light-box Luminance and reflected ambient light described earlier), and converted to Just-Noticeable Difference Indices by mathematically computing j(L) from L(j), where L(j) is the Grayscale Standard Display Function of Luminance L as a function of the Just-Noticeable Difference Index j. For each of the 31 intervals between consecutive measured values, a calculated value of "JNDs per increment in P-Values" was obtained by dividing the difference in Just-Noticeable Difference Index by the difference in P-Values for that interval. (In these calculations, density, L, and j are all floating-point variables. No rounding to integer values is done, so no truncation error is introduced.)

In this example, the film's data could be reasonably well fit by a horizontal straight line. That is, the calculated "JNDs per increment in P-Values was essentially constant at 2.4. A mathematical fit yielded a

slight non-zero slope (specifically, dropping from 2.5 to 2.3 as the P-Value went from 0 to 255), but the 0.2 total difference was considerably smaller than the noise which was present in the 31 individual values of "JNDs per increment in P-Value" so is of doubtful significance. (The "noise" referred to here consists of the random, non-repeatable variations which are seen if a new set of measured data (e.g., from a second print of the same test pattern) is compared with a previous set of measurements.)

No visual tests were done to see if a slope that small could be detected by a human observer in side-byside film comparisons.

Incidentally, if one considers just the 32 original absolute measured densities (rather than differential values based on small differences), one finds, in this case, quite reasonable agreement between the target and measured optical densities (within the manufacturer's norms for density accuracy, at a given density). But if one uses any metric which is based on differential information over small intervals, the results must be considered more cautiously, since they can be strongly affected by (and may be dominated by) various imperfections which are independent of a device's "true" (or averaged over many cases) characteristic behavior.

D.3 REFLECTIVE DISPLAY SYSTEMS

This last example illustrates how conformance with the Grayscale Standard Display Function may be achieved for a thermal-dye-transfer paper printer/office-light system. The thermal-dye-transfer printer produces black-and-white grayscale prints on a semi-glossy 8-inch x 10-inch heavy-gauge paper. The print is illuminated uniformly by fluorescent lamps so that the minimum reflective density produces a

Luminance of 150 cd/m². The hypothetical transformation operator is assumed to have equal input and output digitization resolution of 8 bits.

D.3.1. Measuring the system Characteristic Curve

A print with a 64-step grayscale tablet was printed for DDLs 4, 8, 12, ...,248, 252, 255. The reflection optical densities (from 0.08 to 2.80) were measured with a densitometer. The Luminance levels corresponding to the measured optical densities and illumination conditions are plotted in Fig. D.3-1.

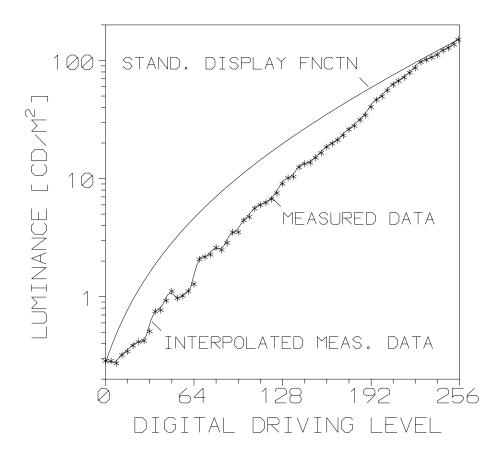


Figure D.3-1 Measured and interpolated Characteristic Curve and Grayscale Standard Display

Function for a printer producing reflective hard-copies

D.3.2. Application of the Grayscale Standard Display Function

This last example illustrates how conformance with the Grayscale Standard Display Function may be achieved for a thermal-dye-transfer paper printer/office-light system. The thermal-dye-transfer printer produces black-and-white grayscale prints on a semi-glossy 8-inch x 10-inch heavy-gauge paper. The print is illuminated uniformly by fluorescent lamps so that the minimum reflective density produces a

Luminance of 150 ${\rm cd/m}^2$. The hypothetical transformation operator is assumed to have equal input and output digitization resolution of 8 bits.

D.3.3 Implementation of the Grayscale Standard Display Function

The measured Characteristic Curve is interpolated for the available DDLs yielding 256 Luminance levels $L_{I,m}$. The Grayscale Standard Display Function is also interpolated between JND_{min} and JND_{max} (*DJND* = [JND_{max} - JND_{min}]/255) yielding 256 Standard Luminance levels $L_{I,STD}$.

For every $L_{I,STD}$, the closest $L_{J,m}$ is determined. The data pair I,J defines the transformation between D_{input} and D_{output} (Table D.3-1 and Fig. D.3-2) by which the Luminance response of the Display System is made to approximates the Grayscale Standard Display Function.

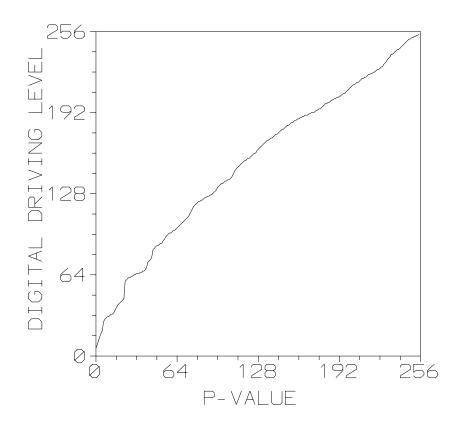


Figure D.3-2 Transformation for modifying the Characteristic Curve of the printer to a Display Function that approximates the Grayscale Standard Display Function

Table D.3-1 Look-Up Table for Calibrating Reflection Hardcopy System

				5		пагисору э	, ~		1
P-Value	DDL	P-Value	DDL		P-Value	DDL		P-Value	DDL
0	6	1	9		2	12		3	15
4	18	5	20		6	27		7	29
8	30	9	31		10	31		11	32
12	33	13	33		14	34		15	36
16	38	17	40		18	41		19	42
20	43	21	44		22	45		23	59
24	60	25	61		26	62		27	62
28	63	29	63		30	64		31	64
32	65	33	65		34	65		35	66
36	66	37	67		38	67		39	68
40	70	41	74		42	75		43	76
44	78	45	84		46	85		47	86
48	87	49	87		50	88		51	89
52	89	53	91		54	92		55	94
56	95	57	96		58	97		59	97
60	98	61	99		62	99		63	100
64	101	65	102		66	103		67	104
68	105	69	106		70	107		71	108
72	109	73	110		74	112		75	114
76	116	77	118		78	119		79	120
80	121	81	122		82	122		83	123
84	123	85	124		86	125		87	125
88	126	89	126		90	127		91	127
92	128	93	129		94	130		95	131
96	133	97	134		98	135		99	136
100	136	101	137		102	138		103	138
104	139	105	139		106	140		107	141
108	143	109	145		110	147		111	148
112	149	113	150		114	151		115	152
116	153	117	154		118	154		119	155
120	156	121	156		122	157		123	158
124	159	125	160		126	160		127	162
128	163	129	164		130	165		131	166
132	167	133	168		134	169		135	170
136	170	137	171		138	172		139	172
140	173	141	174		142	175		143	175
144	176	145	177		146	178		147	179
148	179	149	180		150	181		151	182

182	153	183	154	184	155	184
185	157	186	158	186	159	187
187	161	188	162	188	163	189
189	165	190	166	190	167	190
191	169	191	170	192	171	192
192	173	193	174	194	175	194
195	177	195	178	196	179	197
198	181	199	182	199	183	200
200	185	201	186	202	187	202
203	189	203	190	204	191	204
205	193	205	194	206	195	207
207	197	208	198	209	199	210
211	201	212	202	213	203	214
214	205	215	206	216	207	216
217	209	218	210	219	211	219
220	213	220	214	221	215	222
222	217	223	218	223	219	224
224	221	225	222	226	223	226
227	225	228	226	228	227	230
231	229	232	230	234	231	235
236	233	238	234	238	235	239
240	237	241	238	242	239	242
243	241	244	242	245	243	246
247	245	248	246	249	247	250
250	249	251	250	251	251	252
252	253	253	254	253	255	254
	185 187 189 191 192 195 198 200 203 205 207 211 214 217 220 222 224 227 231 236 240 243 247 250	185 157 187 161 189 165 191 169 192 173 195 177 198 181 200 185 203 189 205 193 207 197 211 201 214 205 220 213 222 217 224 221 227 225 231 229 236 233 240 237 243 241 247 245 250 249	185 157 186 187 161 188 189 165 190 191 169 191 192 173 193 195 177 195 198 181 199 200 185 201 203 189 203 205 193 205 207 197 208 211 201 212 214 205 215 217 209 218 220 213 220 221 223 224 221 225 228 231 229 232 236 233 238 240 237 241 243 241 244 247 245 248 250 249 251	185 157 186 158 187 161 188 162 189 165 190 166 191 169 191 170 192 173 193 174 195 177 195 178 198 181 199 182 200 185 201 186 203 189 203 190 205 193 205 194 207 197 208 198 211 201 212 202 214 205 215 206 217 209 218 210 220 213 220 214 222 217 223 218 224 221 225 222 227 225 228 226 231 229 232 230 236 233 238 234	185 157 186 158 186 187 161 188 162 188 189 165 190 166 190 191 169 191 170 192 192 173 193 174 194 195 177 195 178 196 198 181 199 182 199 200 185 201 186 202 203 189 203 190 204 205 193 205 194 206 207 197 208 198 209 211 201 212 202 213 214 205 215 206 216 217 209 218 210 219 220 213 220 214 221 222 227 225 228 226 228 231 <	185 157 186 158 186 159 159 159 159 159 159 159 159 159 163 159 163 163 163 163 163 163 166 190 167 167 191 169 191 170 192 171 171 192 171 172 172 173 193 174 194 175 178 196 179 179 178 196 179 179 183 200 185 201 186 202 187 189 203 189 203 190 204 191 191 192 193 205 194 206 195 193 205 194 206 195 199 199 199 199 211 201 212 202 213 203 203 204 207 203 218 209 199 211 201 219 211

D.3.4 Measures of Conformance

The FIT and LUM metrics as proposed in Annex C are applied to determine the macroscopic and microscopic approximation of the $L_{J,m}$ to the $L_{J,STD}$. Fig. D.3-3 shows the perceptually linearized Display Function superimposed on the Grayscale Standard Display Function and Figure D.3-4 summarizes the results of the two metrics. FIT provides as best fit of the JNDs/Luminance interval a straight line almost perfectly parallel to the horizontal axis indicating good global fit of the transformed Display Function with the Grayscale Standard Display Function. The RMSE computed by LUM is relatively large indicating more pronounced local deviations from the Grayscale Standard Display Function as, for example, with the soft-copy Display System illustrated in Section D.1. At least in part, the larger RMSE is due to the fact that the input and output digitization resolution for the transform are equal. The transformation table (Table D.3-1) and Fig. D.3-2 show that several P-Values lead to the same Luminance levels on the transformed Display Function. In fact, only 205 of the 255 Luminance intervals lead to JNDs for the Standard Target.

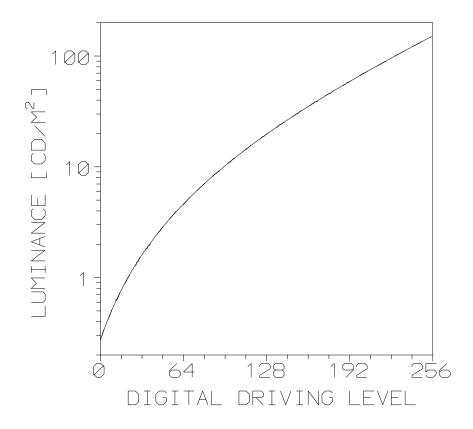


Figure D.3-3. Transformed Display Function and superimposed Grayscale Standard Display Function for a reflective hard-copy Display System. The transformed Display Function for this Display System matches the Grayscale Standard Display Function and the two curves are superimposed and indistinguishable.

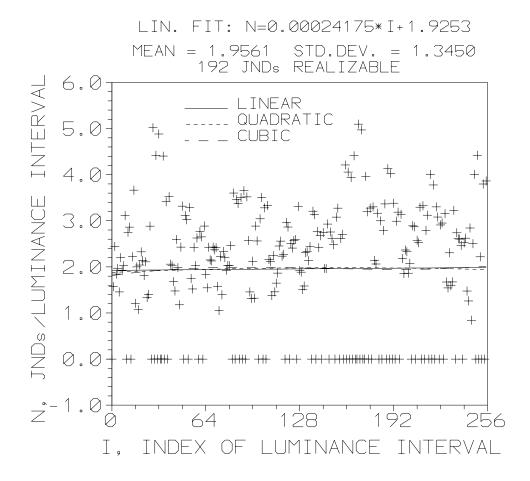


Figure D.3-4 Measures of conformance for a reflective hard-copy Display System with equal input and output digitization resolution of 8 bits

Annex E (INFORMATIVE) REALIZABLE JND RANGE OF A DISPLAY UNDER AMBIENT LIGHT

Dynamic range is an often used measures of the information content that can be presented by a Display System. However, there are many definitions of dynamic range, and most such definitions do not take into account real world conditions that affect the actual amount of information which can be conveyed by a gray scale pixel. For example, Poynton [E1] refers to the *contrast ratio* of a gray scale display device as the ratio of display intensity between the brightest white and the darkest black of the particular display device in question. However, this definition of dynamic range applies to ideal viewing conditions. Real world conditions such as veiling glare, noise, spatial frequency content of the image, power supply saturation, and ambient lighting in a cathode ray tube (CRT) based viewing situation can degrade the measured dynamic range of the system significantly [E2, E3]. Because of all of these variables dynamic range is an ill-defined concept for a Display System.

Note: Veiling Glare is the phenomenon wherein internal light reflections inside the CRT creates a "background lighting" thus reducing the contrast range of the CRT device.

The methods used to determine the degree to which the Display Function of a Display System approximates the Grayscale Standard Display Function can also be used to define two measures which might better characterize the potential capabilities of a Display System to convey information content. Two measures, the theoretically achievable JNDs and the realized JNDs, are useful for comparing Display Systems [E4].

The number of theoretically achievable JNDs is simply the number of JNDs predicted by the visual model given the Luminance Range of the Display System used. The number of theoretically achievable JNDs of a Display System may be found from Table B-1 of Annex B by counting the number of JNDs in the table which fall between the measured minimum and maximum Luminance of the Display System.

This number of JNDs may not actually be achievable due to resolution limitations of other portions of the Display System, in particular, the quantization resolution given by the finite number of bits per pixel driving the Display System. For example, Table B-1 of Annex B may show that a particular Display System is capable of delivering 352 JNDs. However, if only 8 bits per pixel are presented to the Display System, the number of JNDs achievable cannot exceed $2^8 = 256$ JNDs because of the quantizing effect. In actual fact, the number of JNDs realized in a Display System will always be smaller than or equal to the lower of the theoretically achievable JNDs and the quantization limit. This is because some of the quantized values input to the display may not line up with the input value required to achieve the next JND.

The more useful number of realized JNDs, describes how many JNDs are actually achieved given the specifics of the Display System (i.e. the number of gray levels of contrast resolution and the distribution of Luminance values). This definition gives a measure of the information which can actually be conveyed by the system to a human observer, in essence, an informational dynamic range. This number is calculated beginning at the minimum Luminance of the Display System, and then stepping one JND in Luminance from the current Luminance value, and choosing the smallest increment in DDL value that achieves a step at least that large. Repeating this through all the available DDLs will produce a sequence of steps, all at least 1 JND apart, and the length of this sequence of steps is then the number of realizable JNDs of the Display System.

The methods of PS 3.14 cannot precisely duplicate all of the real world sources of degradation in a Display System. However, this uniform method of determining the realizable number of JNDs should give a measure of the actual performance of a particular Display System which would be experienced by a human observer when using the Display System in a real world situation such as the viewing of radiological images in medicine.

References

- [E1] Poynton, C. "Frequently Asked Questions about Gamma", Internet ftp://ftp.inforamp.net/pub/users/poynton/doc/colour/gammaFAQ.pdf
- [E2] Roehrig, H., Blume, H., Ji, T. and Browne, M.; "Performance Tests and Quality Control of Cathode Ray Tube Displays"; J. Digital Imaging, Vol. 3, No. 3, August 1990; pp. 134-145.
- [E3] Gray, J.; "Use of the SMPTE Test Pattern in Picture Archiving and Communication Systems"; J. Digital Imaging, Vol. 5, No. 1, February 1992; pp. 54-58.
- [E4] Hemminger, B., Muller, K., "Performance Metric for evaluating conformance of medical image displays with the ACR/NEMA display function standard", SPIE Medical Imaging 1997, editor Yongmin Kim, vol 3031-25, 1997.