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An application of OWA operators in fuzzy business diagnosis

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ABSTRACT

The paper aims to develop an adjustment index based on OWA operators to enrich the results of diagnostic fuzzy models of business failure. A proposal to verify the diseases prediction accuracy of the models is also added. This allows a reduction of the map of causes or diseases detected in strategic defined areas. At the same time, these key areas can be disaggregated when an alert indicator is identified, and shows which of the causes need special attention. This application of OWA can encourage the development of suitable computer systems for monitoring companies' problems, warn of failures and facilitate decision-making. In addition, taking Vigier and Terceño's 2008 model as a benchmark, causes aggregation operators are introduced to evaluate alternative groupings, and the adjustment measure using approximate solutions is proposed to test the model's prediction.

The empirical estimation and the verification of the improvement proposals in a set of small and medium- sized enterprises (SMEs) in the construction industry are also presented. The functionality and the prediction capacity are thus measured and detected by monitoring key areas that warn about insolvency situations in the firm.

1. Introduction

Business failure prediction has been an important research area for many decades, and the studies in the field have focused on bankruptcy prediction. These failure models compare and classify firms according to quantitative indicators, to predict or distinguish between healthy and unhealthy businesses. A variety of parametric and non-parametric techniques (logit, probit, neural networks, expert systems, rough sets, hybrid systems, genetic algorithms, clusters, survival models, data envelopment analysis, support vector machine, among others [1–5]) have been applied to the business failure process since the pioneering Beaver's univariate model [6] and Altman's multivariate model [7]. Most of these models attempt to improve estimation results without ascertaining the causes of failure.[1]

The traditional methodologies have limitations when dealing with the business failure process because it includes elements of subjectivity and uncertainty. The business diagnosis is performed using qualitative variables and experts' opinions with a high degree of subjectivity and incomplete information. Recently, Cho [15] argued that "*since the business environment is increasingly becoming uncertain and competitive these days, bankruptcy can happen to any business organization*". From this point of view, observing the first signals of failure is therefore essential for an early diagnosis. These considerations open up the possibility of using the tools and methods of fuzzy logic, which enable work with qualitative variables, weak information and measurement of experts' knowledge. Since Zadeh [16], there have been developments in modelling uncertainty, describing human behavior and complex systems distinguished by incomplete information and multiple subjective variables. Fuzzy prediction models are an alternative in this context ([17–29], among others) for overcoming many of the limitations of the traditional models (selection of the dependent variable, selection of the sample, choice of the explanatory variables, and the treatment and inclusion of the sorted errors).[2] Vigier and Terceño [17] and Scarlat [18], among others, have advantages over other prediction models on the basis of diagnostic theory,

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[1] Research of Refs. [8–10] is available for consultation. These models partially analyse the causes of failure and their interrelationships. In other models such as Refs. [11–14], qualitative and external variables are introduced to improve the results.

[2] Revisions by Refs. [1–5] are available for consultation. They discuss the main methodological problems in the traditional models and the new techniques applied.

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which establishes the process of business failure and the formal relationship between causes and symptoms. These models have the capacity to predict insolvency situations, diagnose problems and simulate (or complement) the analyst's task. The formalization of the expert's knowledge reduces discretion and the subjective results.

This research takes [17] as reference, which is based on the estimation of a financial-economic knowledge matrix (R) that arises from the operation between the incidence matrix of symptoms and the incidence matrix of causes. It also proposes the aggregation of matrices and the verification of trends that allows the result of an aggregate matrix with temporary validity and prediction capacity ($\Re$). This matrix is useful for detecting causes (or diseases) in firms. This model is chosen because it has a comprehensive analysis of causes and symptoms compared to the pioneering models of Refs. [30,31]. The recent approaches of Refs. [18–20] taken as Ref. [17], derive the matrix of causes and symptoms based on the difference with the performance matrix. This difference introduces a greater degree of subjectivity in favor of the experts' points of view.

None of these models completely define the symptoms and causes for estimating the knowledge matrix. In addition, they have no evaluation system of diagnosis and no prediction capacity. As a result, a method based on Ref. [17] to group causes through aggregate operators and an adjustment index is proposed to evaluate the estimation results. These contributions enrich the reference model, although they could be applied to any model of fuzzy diagnosis.

2. Advantages and limitations of the model

This section presents the advantages and some of the limitations of Ref. [17]. These limitations show possible contributions that can be made.

2.1. Advantages

a) The difference between causes and symptoms is recognized and formalized. The degree of incidence is therefore determined, i.e. to identify the problems each firm should pay attention to the r_{ij} components in the R matrix. The r_{ij} show the level of incidence of the symptom S_i through the cause C_j.
b) Sample selection. Fuzzy logic tools reduce the limitations of the properties and statistical representation of population. As an essential condition, any of the companies used in the analysis must have been affected by any of the defined diseases without imposing a minimum number of firms. The relationships between causes and symptoms are also estimated, combining the set of firms through time. This minimizes the risk of determining incidence levels between non-significant causes and symptoms (r_{ij}) that are the product of a temporary situation. A sample with a similar size and sector is recommended to homogenize the accepted level of normality in the model's variables.
c) The problem of the independent variables. The cause-effect relationship links the causes with the firms' diseases and the effects with economic- financial ratios. The causes of failure may be found by the model, and moreover, if the ratios of a failure firm are available over time, it is possible to detect the reasons that led to this event. Fuzzy logic overcomes the statistical requirements of independent variables, such as normality and multicollinearity problems. In this methodological conception, to maximizing the available information is recommended in order to estimate possible diseases, as well as explanatory and qualitative variables and independent variables in the modelling.
d) Misclassification Errors. This model provides more information on the firm's situation regarding the binary models that only classify businesses as healthy or unhealthy firms. The incidence degree of each r_{ij} shows the degree of disease of the firms, meaning that measures can be taken to reverse or neutralize problems.
e) Static dimension of business failure. There is no temporal limitation when using fuzzy relations. If incidence level estimations have temporal consistency, they may be considered a continuous system, adding a dynamic perspective to the diagnostic process.

2.2. Limitations

a) Lack of application. Vigier and Terceño [17] is a theoretical model that does not specifically define the causes and symptoms involved in the diagnosis. Furthermore, its simulation requires a large amount of information about firms' performance and it has not been empirically tested, although the internal consistency of the model has been demonstrated.
b) Verification of prediction capacity. The model does not have any mechanism to test its capacity for diagnosis and prediction. There is no guide that evaluates the degree of adjustment of the predictions to the responses provided by experts.
c) Selection of symptoms. For the symptoms, the model mentions that financial economic ratios that arise from the financial statements of companies need to be used, without specifying which ones. A partial study [57] presents the list of symptoms selected for the analysis.
d) Detection of causes. The model only mentions the existence of causes internal and external to the firm. It also identifies the difference and the relationship between symptoms (or effects) and diseases (or causes). The causes are not explicitly defined in a list or handbook that should be considered by the analyst in the diagnosis. In the model mentioned above, there are endogenous and exogenous causes, and objective and subjective causes that can be drawn from the theoretical models. In this regard, Refs. [58,59] proposes the integration of the fuzzy logic with the Balanced Scorecard to determine the causes of failure and in Ref. [60] a complete analysis of possible diseases.

The model therefore fails to verify its capacity to simulate the expert's task, and to judge the economic reasonableness of the relations between the variables. This is only possible with the specific definition of the set of causes and symptoms presented in Refs. [58–60]. This treatment of causes is understood to overcome some of the model's limitations. An improvement of the first two limitations is proposed in this study. To achieve this, a global proposal of treatment of causes through aggregation operators is presented to facilitate business diagnosis by the expert. A formal mechanism to test and verify the model's capacity for diagnosis and prediction is also introduced.

3. Aggregation of causes

When dealing with decision making problems it is necessary to aggregate the available information in order to take decisions. Given P', estimated by the inverse operation between Q and $\Re$; that shows the multiple causes (or diseases) that a firm can undergo, a reduction mechanism of the map of causes through OWA operators is therefore introduced.

3.1. OWA operators

The OWA operator [32] is an aggregation operator that provides a parameterized family of aggregation operators between the minimum and the maximum. $R^n \to R$ has an associated vector W of dimension n, with $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$.

That is,

OWA $(a_1, a_2, \ldots, a_n) = w_j b_j$, where b_j is the jth largest of the a_i.

The OWA operator satisfies the commutative, monotonic and idempotent property and the inequality: $Min(a_1, a_2, \ldots, a_n) \leq h(a_1, a_2, \ldots, a_n) \leq Max(a_1, a_2, \ldots, a_n)$; $\forall$ n-uplas $\in [0,1]^n$ [32–39]. Merigó [40] states that OWA is useful in decision-making under conditions of uncertainty because it considers the degree of optimism of the decision maker. All operations of aggregation between the standard fuzzy intersection and the standard fuzzy union that satisfy the inequality condition are therefore idempotent aggregate operations.

Yager [32] introduces OWA operators. The OWA have subsequently been extended and generalized by many authors in order to develop a more general framework. These include the heavy OWA [41], the generalized OWA-GOWA [42], the prioritized OWA [43], the induced LOWA [44], the induced generalized OWA [45], the uncertain probabilistic OWA-UPOWA [46], the fuzzy probabilistic OWA-FPOWA [47], the probabilistic OWA operator-POWA [48], the induced generalized intuitionist fuzzy ordered weighted averaging- IGIFOWA [49], the ILPOWA using probabilistic information and induced aggregation operators [40], among others.

3.2. Aggregation and monitoring proposal

A dynamic perspective of failure is introduced by Ref. [17]. This allows the evolution of firms to be monitored and attacks the critical issues in their development. Although the model does not mention which factors are used, it provides analytical tools to formalize the relationship between causes and symptoms. Working with multiple qualitative variables is also useful for simulating the firm's operation in a disaggregate way and complements the analyst's task. As a result, in monitoring terms it is useful to concentrate the information in an aggregate number of key areas that represent the disaggregate firm's diseases [58]. The OWA operators are introduced in the original model to concentrate information and easily detect possible diseases in firms. Once a warning indicator is detected in an area of the firm, it is possible to disaggregate this key area into each of the causes or critical factors that generate problems, in order to evaluate and correct the situation. Evaluation of three alternative methods of aggregation between minimum and maximum is proposed in this section (minimum, maximum and average levels of incidence) using OWA operators. Other extensions of the OWA family operators can also be applied to the same end.

Given $\Re = \{\Re_{ij}\}$ with r_{ij} valid and temporary consistent, three matrices are built considering the key areas $A = \{a_1, a_2, \ldots a_m, \ldots, a_w\}$ presented in Ref. [58]. The $\Re^{Mean}$ matrix is obtained by applying the arithmetic mean to the causes within the monitoring area ($\Re^{Mean} = 1/m\sum_{w=1}^{m} r_{ij}$). Meanwhile, the matrices $\Re$ with maximum and minimum membership values ($\Re^{Max}$ y $\Re^{Min}$) are calculated through the maximum ($\Re^{Max} = Max(r_{ij})$) and the minimum ($\Re^{Min} = Min(r_{ij})$) selection of r_{ij} incidence levels within each group of causes. The causes of the aggregate matrix $\Re$ are grouped for these three possible levels of incidence, in order to synthesize the factors that generate diseases. OWA operators therefore warrant the properties for the three fuzzy subsets.

That is,

$$r_{iw}{}^{\mathrm{Mean}} = (1/m)(r_{i1} + r_{i2} + \ldots + r_{ij}),$$

$$r_{iw}{}^{\mathrm{Max}} = Max(\mathrm{r}_{i1}; \mathrm{r}_{i2}; \ldots; \mathrm{r}_{ij}),$$

$$r_{iw}{}^{\mathrm{Min}} = Min(\mathrm{r}_{i1}; \mathrm{r}_{i2}; \ldots; \mathrm{r}_{ij})$$

This grouping allows the prediction of diseases $P' = Q \alpha \Re$; that is, $p'_{hj} = max(min(q_{hi}, r_{ij}))$ for the three possible levels of incidence of causes within each key area. As a result, the membership matrices of causes (or diseases) are estimated in the three levels of incidence (minimum (P'^{Min}), maximum (P'^{Max}) and average (P'^{Mean})).

$$p'_{iw}{}^{\mathrm{Min}} = \Lambda\,[(q_{ih}\,\alpha\,\mathrm{r^{min}}_{\mathrm{h1}}),\ (q_{ih}\,\alpha\,\mathrm{r^{min}}_{\mathrm{h2}}), \ldots, (q_{ih}\,\alpha\,\mathrm{r^{min}}_{\mathrm{hj}}),$$

$$p'_{iw}{}^{\mathrm{Max}} = \Lambda\,[(\mathrm{q}_{ih}\,\alpha\,r^{\mathrm{max}}{}_{h1}),\ (\mathrm{q}_{ih}\,\alpha\,r^{\mathrm{max}}{}_{h2}), \ldots, (\mathrm{q}_{ih}\,\alpha\,r^{\mathrm{max}}{}_{hj}),$$

$$p'_{iw}{}^{\mathrm{Mean}} = \Lambda\,[(q_{ih}\,\alpha\,\mathrm{r^{mean}}_{\mathrm{h1}}),\ (q_{ih}\,\alpha\,\mathrm{r^{mean}}_{\mathrm{h2}}), \ldots, (q_{ih}\,\alpha\,\mathrm{r^{mean}}_{\mathrm{hj}}),$$

The multiple causes identified in the diagnosis or in prediction of firms' situation are grouped into key areas $A = \{a_1, a_2, \ldots, a_w\}$ according to the Balanced Scorecard [58,61]. This facilitates the analyst's task by reducing the information necessary for the analysis. Once a warning indicator in an area of the firm is detected, it is possible to disaggregate this key area into each of the causes or factors that generate problems to evaluate and correct the situation. This option of monitoring through key areas (consistent with the disaggregated estimation of the model) enables continuous and comprehensive tracking of the business areas by fuzzy logic advantages.

4. Adjustment index

As described in Section 1, the model of Vigier and Terceño [17], like other diagnostic fuzzy logic models, does not have a mechanism to verify the estimation results. An index of approximate solutions is therefore introduced to check whether the causes estimated by the model represent the true situation of the firms. This index is also useful for evaluating which of three OWA aggregate methods is more efficient.

To do so, Hamming's distance index, which is useful for comparing two fuzzy sets, is adapted. In other words, the comparison between the original set of causes (P^*) aggregated into minimum ($P^{*Min} = Min(p_{hw})$), maximum ($P^{*Max} = Max(p_{hw})$) and average ($P^{*Mean} = 1/m\sum_{w=1}^{m}(p_{hw})$) incidence levels and the set of estimated causes (P'). P' are also aggregated in the same way as the original causes (P^*).

The adjustment index between P^* and P', that is $\|P* = P'\| = \|P* \subseteq P'\| \wedge \|P' \subseteq P*\|$ is therefore represented by (1):

$$\left[\mathrm{P}^* = P'\right] = 1 - 1/m\sum_{x \in X}^{n} |p^*_{hw} - p'_{hw}| \tag{1}$$

Therefore,

$$[P* = P']^{Min} = [1 - 1/m(|p*_{h1} - p'_{h1}| + |p*_{h2} - p'_{h2}| + \ldots + |p*_{hk} - \mathrm{p'}_{\mathrm{hw}}|)]$$

$$[P* = P']^{Max} = [1 - 1/m(|p*_{h1} - p'_{h1}| + |p*_{h2} - \mathrm{p'}_{\mathrm{h2}}| + \ldots + |p*_{hw} - \mathrm{p'}_{\mathrm{hw}}|)]$$

$$[P* = P']^{Mean} = [1 - 1/m(|p*_{h1} - p'_{h1}| + |p*_{h2} - p'_{h2}| + \ldots + |p*_{hw} - \mathrm{p'}_{\mathrm{hw}}|)]$$

where P^{*Min} selects the minimum degree of incidence within the group of causes for each company, P^{*Max} chooses the maximum degree of incidence within the group of causes for each company, and P^{*Mean} shows the average of the causes within the group or key area of monitoring.

This test is useful for identifying the best mechanism for aggregating causes using OWA operators (by maximum, minimum or average incidence values). It can also be used to estimate the degree

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of adjustment of the predictions to diseases that are present in the companies. In other words, it is helpful for proving the model's ability (or that of any other model) to predict insolvency situations and evaluate the three alternatives for synthesizing causes.

5. Definition of causes and symptoms

One of the weaknesses highlighted in this paper is that in most diagnostic models the causes and symptoms involved in the estimation are not explicitly defined. For this reason, their application and testing is not possible. This limitation is studied in previous works such as Ref. [59] in which a list of 72 causes is proposed, taking as reference the works of Refs. [8–10,30,31,52], DAFO analysis [50] and studies of failure prediction with non-financial variables ([12–14,50,51], etc.). Furthermore, the relationship between causes and the key areas is presented in Refs. [58,61] according to the four perspectives of the balanced scorecard (BSC). A set of key areas for monitoring is defined within each perspective of the BSC. These key areas include many causes or factors that enable diagnosis of firms and which have been partially studied in the theory of business failure. The strategic map of cause-effect relationships according to the BSC is introduced in Ref. [58] and the method for measuring the causes is described in Ref. [59].

The advantage of fuzzy logic is taken into account for the symptoms. 41 ratios are selected, reflecting aspects of profitability, productivity, liquidity, leverage, solvency, financial structure, debt coverage, economic structure, activity, turnover, efficiency and self-financing according to the classification proposed by Refs. [53–55], among others; and the frequency of use in prediction models. The list and classification of symptoms are presented in Ref. [57] and are available at http://fuzzybusinessdiagnosis.blogspot.com.ar.

6. Estimation of the fuzzy model

This section presents the application of the business model of Vigier and Terceño [17] in a sample of small and medium-size companies (SMEs). The construction sector, consisting of two subsectors – construction, and the sale of building materials – is selected. From a base of 98 SMEs registered in this sector in the municipalities of Bahía Blanca and Punta Alta (Argentina), 15 firms – accounting for approximately 15% of the activity in these two cities are selected.[3] This is a very dynamic sector which has a wide variety of (healthy and unhealthy) companies that meet the data availability requirements necessary to estimate the model.

The steps below are followed to apply the model:

1. Data collection for the three periods used in the estimation ($T = \{T_k\}$, where $k = 2008$, 2009 and 2010) by the design of a standardized questionnaire to detect potential causes of diseases in companies using linguistic labels. The data were collected in interviews with 15 experts on SMEs ($E = \{E_h\}$, where $h = 1$, 2, 3,. . .,15), who had a long-standing relationship with the company analyzed. In Argentine SMEs, this role is fulfilled by accounting advisors, managers and business owners.
2. Systematization and information analysis of the interviews with experts and the financial statements.
3. According to the selected ratios ($S = \{S_{ih}\}$, where i = 1, 2,. . ., 41) the cardinal matrix of symptoms for the 15 companies is calculated ($S_{ih} = 41 \times 15$). This matrix is constructed with the estimation of the 41 economic-financial ratios selected for the analysis. The membership matrix of symptoms is also estimated ($Q_{ih} = 41 \times 15$). This is done by ordering the symptoms according to their degree of incidence to generate diseases. The incidence of the symptom is calculated using the cumulative relative frequency ($s_{ih} = |s_{ih}|/h$). A lower membership value represents a healthier situation.
4. Construction of the membership matrix of objectively and subjectively measurable causes ($P_{hj} = 15 \times 72$) according to the handbook of causes detected and the methodology proposed in Ref. [59]. In this case, the causes are valued using linguistic labels that represent the incidence of the cause for the firm.
5. Estimation of the economic-financial knowledge R matrix for the 3 periods ($R_{ij}{}^k = 41 \times 72$; $k = 1, 2, 3$). This is done through the operation between the transposed membership matrix of symptoms and the membership matrix of causes that meets the lower ratio ($R_{ij} = (Q_{ih}{}^{T\alpha} P_{hj})$). The model is explained in detail in Ref. [17].
6. Application of the theoretical filtering method for the treatment of inconsistencies [56]. Two levels of filter ($\phi^* = 0.75$ and $\phi^* = 0.50$) are applied when removing firms. In the first case, firms with lower incidence level are eliminated until $\phi < 0.75$; then when very little variability of each cause is observed in the firms, a higher filtering factor ($\phi^* = 0.50$) is applied, which reduces the error and discards a higher percentage of inconsistent responses. Applying this filtering method does not eliminate firms on a pattern according to the activity within the sector or the size of the firm, but simply removes cases considered "anomalous" for the model.
7. Estimation of the aggregate matrix $\Re$ ($\Re_{ij} = 41 \times 72$) used for forecasting. This involves repeating steps (3)–(6) for the three periods (2008, 2009 and 2010). The calculation of $\Re$ involves aggregating the matrices $R_k = [r_{ij}]_k = (R_{ij}{}^1, R_{ij}{}^2, R_{ij}{}^3)$ and the correction of temporal trends that distort the validity of the results. As mentioned in Ref. [17], if an upward trend is verified in time of r_{ij}, the use of an "average" aggregation procedure would underestimate the true relationship. However, if the trend is decreasing, the average would overestimate the relationship. For this situation, the behavior of each r_{ij} is evaluated in order to determine the aggregation process depending on the trend experienced by each component.

- If $\sum_{k=2}^{t} |\left[r_{ij}\right]_k - \left[r_{ij}\right]_{k-1}| = 0$; $r_{ij}{}^k$ = aggregate r_{ij}
- If $\sum_{k=2}^{t} |\left[r_{ij}\right]_k - \left[r_{ij}\right]_{k-1}| \neq 0$; r_{ij} is determined by the indicator ξ (2), which varies between -1 and 1.

$$\xi = \sum_{k=2}^{t} \left(\left[r_{ij}\right]_k - \left[r_{ij}\right]_{k-1} \right) / \sum_{k=2}^{t} |\left[r_{ij}\right]_k - \left[r_{ij}\right]_{k-1}| \tag{2}$$

If $\xi = 1$, $r_{ij} = Max(Min\ [r_{ij}]_k)$; there is an increasing trend.
If $\xi = -1$, $r_{ij} = Min(Max\ [r_{ij}]_k)$; there is a decreasing trend.
If $-1 < \xi < 1$, $r_{ij} = 1/k \sum_{i=1}^{n} [r_{ij}]_k$; there is no trend.

The aggregation and the correction of the temporal trend are carried out in order to obtain the $\Re$ aggregate matrix ($\Re_{ij} = 41 \times 72$). An increasing trend is observed in 349 r_{ij}, in 337 r_{ij} the trend is decreasing and in the remaining 2266 r_{ij} no trend is observed, with the aggregation performed through the arithmetic mean in this latter case. The estimated r_{ij} coefficients are thus consistent and significant, and explain the true relationship between causes and symptoms according to the model. The disaggregated results and the estimation of all the matrices are available at http://fuzzybusinessdiagnosis.blogspot.com.ar/.

7. Empirical verification of the aggregating proposal

This is carried out by applying the following steps:

i) The proposed OWA minimum, maximum and mean aggregation operators are applied to P to obtain the aggregated matrices that

[3] If the firms constituted in formal societies are considered in isolation, as they are the only ones required to submit economic-financial statements, the percentage of representation is around 30%.

reflect these three categories (P^{*Min}; P^{*Max}; P^{*Mean}). Appendix A shows the three matrices (P^*).

For example, taking as reference the causes' membership matrix of firm 1, the causes are grouped ($p_1, p_{2,\ldots}, p_{10}, \ldots, p_{15}$) in a single cause ($p^*_1$) that reflects the problems related to entrepreneurial learning.

That is,

$$P*_{11}{}^{Mean} = (1/15)(0.25 + 1.00 + 0.50 + 0.20 + 0.20 + 0.50 + 0.86 + 0.33 + 0.29 + 0.43 + 0.29 + 0.36 + 1.00 + 0.20 + 0.43) = 0.46$$

$$P*_{11}{}^{Max} = \text{Max}(0.25;\ 1.00;\ 0.50;\ 0.20;\ 0.20;\ 0.50;\ 0.86;\ 0.33;\ 0.29;\ 0.43;\ 0.29;\ 0.36;\ 1.00;\ 0.20;\ 0.43) = 1.00$$

$$P*_{11}{}^{Min} = \text{Min}(0.25;\ 1.00;\ 0.50;\ 0.20;\ 0.20;\ 0.50;\ 0.86;\ 0.33;\ 0.29;\ 0.43;\ 0.29;\ 0.36;\ 1.00;\ 0.20;\ 0.43) = 0.20$$

ii) OWA operators are applied to $\Re$ to estimate companies' diseases which are grouped into key areas ($\Re^{Min} = Min(r_{ij})$; $\Re^{Max} = Max(r_{ij})$; $\Re^{Mean} = 1/m\sum_{w=1}^{m}(r_{ij})$). Through $P' = Q_{1\alpha}\ \Re$; being; $p'_{hj} = max(min(q_{hi}, r_{ij}))$ three new matrices of causes in minimum (P'^{Min}), maximum (P'^{Max}) and mean (P'^{Mean}) membership values are estimated (see Appendix B).

For example, the membership level of the cause $p'_{11}{}^{Max}$ for the aggregate is calculated through the operation of the first row of Q matrix with the first column of $\Re^{Max}$ matrix, which is $p'_{11} = Q\,(1 \times 41)\ \alpha\ \Re^{Max}\,(41 \times 1) = p'_{11}(1 \times 1)$.

$$p'_{11}{}^{Max} = \Lambda\,[(0.60\,\alpha\,0.50),\ (0.67\,\alpha\,0.50),\ \ldots,\ (0.80\,\alpha\,0.60),\ \ldots,\ (0.80\,\alpha\,0.57)]$$

$$p'_{11}{}^{Max} = Max\,[(0.50),\ (0.50),\ \ldots,\ (0.60),\ \ldots,\ (0.57)] = 0.60$$

Likewise, $p'_{11}{}^{Mean}$ and $p'_{11}{}^{Min}$ are estimated by operating in the first row of Q matrix with the first column of $\Re^{Max}$.

$$p'_{11}{}^{Mean} = \Lambda\,[(0.60\,\alpha\,0.29),\ (0.67\,\alpha\,0.28),\ \ldots,\ (0.93\,\alpha\,0.28),\ \ldots,\ (0.80\,\alpha\,0.29)]$$

$$p'_{11}{}^{Mean} = Max\,[(0.29),\ (0.28),\ \ldots,\ (0.28),\ \ldots,\ (0.29)] = 0.34$$

And,

$$p'_{11}{}^{Min} = \Lambda\,[(0.60\,\alpha\,0.14),\ (0.67\,\alpha\,0.14),\ \ldots,\ (0.13\,\alpha\,0.14),\ \ldots,\ (0.80\,\alpha\,0.14)]$$

$$p'_{11}{}^{Mean} = Max\,[(0.14),\ (0.14),\ \ldots,\ (0.14),\ \ldots,\ (0.14)] = 0.14$$

iii) Based on the Hamming Distance, the adjustment index of grouping causes, is estimated by comparing the two fuzzy subsets P^* and P'. The reduction of causes in monitoring key areas is thereby verified and its degree of adaptation to the answers given by the experts is evaluated.

Table 1
Degree of fit considering several periods [$P^* = P'$].

Firm	Min. (P'^{Min})	Max. (P'^{Max})	Mean (P'^{Mean})
1	0.94	0.86	0.91
2	0.98	0.82	0.93
3	0.97	0.86	0.94
4	0.93	0.89	0.93
5	0.95	0.89	0.92
6	0.92	0.84	0.88
7	0.91	0.80	0.88
8	0.92	0.82	0.87
9	0.96	0.85	0.88
10	0.91	0.82	0.87
11	0.88	0.80	0.90
12	0.91	0.85	0.87
13	0.90	0.86	0.91
14	0.93	0.89	0.89
15	0.95	0.90	0.92
Mean	0.93	0.85	0.90

For example, for firm 1:

$$[P* = P']^{Min} = 1 - 1/14\,(|0.20 - 0.14| + |0.12 - 0.13| + |0.57 - 0.56| + |0.20 - 0.20| + |0.13 - 0.19| + |0.20 - 0.40| + |0.20 - 0.60| + |0.13 - 0.13| + |0.20 - 0.26| + |0.43 - 0.43| + |0.20 - 0.20| + |0.20 - 0.20| + |0.20 - 0.14| + |0.27 - 0.24|) = 0.94$$

$$[P* = P']^{Max} = 1 - 1/14\,(|1.00 - 0.60| + |0.57 - 0.57| + |0.83 - 0.61| + |0.67 - 0.67| + |1.00 - 0.93| + |1.00 - 0.62| + |1.00 - 0.80| + |1.00 - 1.00| + |0.37 - 0.31| + |0.83 - 0.73| + |0.50 - 0.51| + |1.00 - 0.93| + |1.00 - 0.67| + |0.86 - 0.80|) = 0.86$$

$$[P* = P']^{Mean} = 1 - 1/14\,(|0.46 - 0.34| + |0.27 - 0.35| + |0.71 - 0.57| + |0.46 - 0.47| + |0.43 - 0.47| + |0.70 - 0.51| + |0.60 - 0.70| + |0.47 - 0.47| + |0.29 - 0.26| + |0.61 - 0.58| + |0.29 - 0.37| + |0.72 - 0.61| + |0.64 - 0.35| + |0.55 - 0.49|) = 0.91$$

This analysis concludes that the best approximation is obtained with minimum incidence rates, which reflect a higher adjustment index, with a degree of 93% (see Table 1). The best adjustment with minimal incidence rates is consistent with the least adverse risk theories in terms of warnings and the analysis of results. Furthermore, the properties of the minimum t-norm are validated over other decision rules.

The adjustment index is applied to the results estimated for the last period and to the aggregate one, and a similar degree of adjustment is identified (93%). The incidence relationships therefore have almost no changes in short periods (3 years). As a result, the R matrix (compared with $\Re$) is temporally valid for diagnosis and forecast in the medium term.

The performance of firms is also checked five years after the diseases' estimation. The classification of firms is based on the diseases presented in Ref. [60]. According to this empirical estimation one of the firms (Company 5) identified as sick had four applications bankruptcy between 2011 and 2014, and finally the bankruptcy was imposed by a judge in July 2015. Company 13 has lawsuits due to failure to pay taxes between 2011 and 2014. Companies 3, 8, 9 and 12 present many problems related with labor procedures in the period 2012–2015, confirming the initial diagnosis in the estimation. The healthier companies confirm their situation, not presenting critical problems. Only one of the firms (company 7) identified as moderately sick have few labor problems. In this case the estimation identified some problems in the key area of labor quality. These results confirm the validity and fit of the model's diagnosis and prediction (this information is taken from http://receptorias.scba.gov.ar/busqueda.php).

8. Conclusions

This paper proposes an enrichment of the fuzzy models of business failure, and particularly that of Vigier and Terceño [17] by introducing a mechanism grouping causes by aggregation operators and an adjustment index using approximate solutions to test the model's functionality.

This approach provides a synthesis of firms' diseases and facilitates the expert's task. It also has an advantage in predicting disease since it involves a reduction of the general map of firms' diseases into key areas. These causes are identified using the Balanced Scorecard and are grouped according to OWA operators (minimum, maximum and average). As a result, once a warning indicator in an area of the firm has been detected, it is possible to disaggregate this key area into multiple factors that generate problems, in order to evaluate and correct the situation. This methodology of grouping causes by OWA is useful in identifying the greatest incidence factors for comprehensive and continuous monitoring of the company. This application of OWA can encourage the development of suitable computer systems for monitoring companies' problems, warn of failures and facilitate decision-making.

Furthermore, the adjustment measure, using approximate solutions, is useful for evaluating the best way of grouping and testing the model's ability to predict diseases. According to the empirical estimation, the superiority of the minimum t- norm is validated over other decision rules.

An empirical estimation with the Vigier and Terceño [17] diagnostic model is also presented in order to verify its performance and capacity to diagnose and predict future situations. This entails a precise definition of the set of causes and symptoms, for which there is as yet no agreement in the literature. A set of consistent and significant incidence levels (r_{ij}) that explain the true relationship between symptoms and diseases (effects and causes) close to the firm's real conditions is estimated. The $\Re$ matrix is therefore useful for simulating the expert's knowledge when forecasting.

In short, an advance in the analysis of causes of business failure is proposed in this paper. This is achieved by synthesizing the importance of the causes in key areas through OWA operators. A methodology to evaluate the adjustment of the aggregation or grouping of causes is also introduced. Finally, the proposal to improve the empirical estimation of the model is applied, tested and checked with performance during these years.

Appendix A.

Grouped membership matrix of causes (P^{*Max})

Areas	Learning and Growth				Business process			Customers perspective				Finance		
	1	2	3	4	5	6	7	8	9	10	11	12	13	14
E1	1.00	0.57	0.83	0.67	1.00	1.00	1.00	1.00	0.37	0.83	0.50	1.00	1.00	0.86
E2	0.83	0.57	0.75	0.33	1.00	0.71	0.60	1.00	0.31	0.57	0.33	1.00	0.36	0.67
E3	0.73	0.53	0.79	0.83	0.73	0.71	0.80	1.00	0.40	0.71	0.75	1.00	0.83	0.71
E4	0.83	0.29	0.67	0.83	0.87	0.71	0.80	0.80	0.42	0.57	0.70	1.00	0.83	1.00
E5	0.71	0.57	0.83	0.50	0.73	0.64	0.80	0.79	0.37	0.73	0.63	1.00	0.67	0.72
E6	0.80	0.57	0.60	0.20	0.67	1.00	0.80	0.71	0.48	0.90	0.81	0.80	0.86	0.71
E7	0.80	0.57	0.60	0.20	0.63	1.00	0.80	0.71	0.48	0.90	0.81	0.80	0.86	0.80
E8	0.80	0.57	0.60	0.20	0.63	1.00	1.00	0.71	0.48	0.90	0.81	0.80	0.86	0.80
E9	1.00	0.63	0.80	0.67	1.00	0.71	1.00	1.00	0.40	1.00	0.75	0.80	1.00	1.00
E10	1.00	0.61	0.72	0.20	0.80	0.71	0.80	0.63	1.00	0.73	0.75	1.00	0.67	1.00
E11	1.00	0.57	1.50	0.83	0.93	0.79	0.80	0.80	0.47	0.86	0.58	0.81	0.83	1.00
E12	1.00	0.63	1.00	0.83	0.93	1.00	0.80	0.80	0.60	0.59	0.75	0.88	0.65	1.00
E13	1.00	0.63	0.80	1.00	0.80	0.71	0.80	1.00	0.43	0.83	0.75	1.00	0.83	0.69
E14	1.00	0.63	0.75	1.00	0.87	0.60	0.80	0.80	0.40	0.83	0.75	0.88	0.83	1.00
E15	0.80	0.57	0.80	0.67	0.80	0.80	0.80	0.80	0.48	0.83	0.61	1.00	0.65	1.00

Grouped membership matrix of causes (P^{*Min})

Areas	Learning and Growth				Business process			Customers perspective				Finance		
	1	2	3	4	5	6	7	8	9	10	11	12	13	14
E1	0.20	0.12	0.57	0.20	0.13	0.20	0.20	0.13	0.20	0.43	0.20	0.20	0.20	0.27
E2	0.14	0.13	0.57	0.17	0.07	0.40	0.60	0.13	0.20	0.39	0.20	0.20	0.14	0.27
E3	0.14	0.13	0.60	0.17	0.07	0.53	0.60	0.13	0.26	0.29	0.20	0.22	0.14	0.17
E4	0.14	0.13	0.14	0.20	0.13	0.40	0.40	0.25	0.40	0.43	0.20	0.20	0.14	0.27
E5	0.14	0.13	0.71	0.33	0.25	0.53	0.60	0.13	0.20	0.57	0.20	0.20	0.14	0.20
E6	0.14	0.13	0.56	0.17	0.25	0.47	0.80	0.20	0.40	0.43	0.13	0.11	0.55	0.27
E7	0.14	0.13	0.56	0.17	0.25	0.47	0.80	0.20	0.40	0.43	0.13	0.11	0.55	0.27
E8	0.14	0.13	0.56	0.17	0.25	0.47	0.80	0.20	0.40	0.43	0.13	0.11	0.55	0.27
E9	0.14	0.14	0.43	0.17	0.13	0.40	0.80	0.13	0.37	0.43	0.20	0.20	0.14	0.27
E10	0.14	0.38	0.57	0.17	0.25	0.20	0.60	0.25	0.42	0.57	0.38	0.33	0.14	0.27
E11	0.14	0.24	0.14	0.20	0.14	0.20	0.20	0.38	0.40	0.29	0.20	0.33	0.14	0.24
E12	0.20	0.57	0.71	0.20	0.14	0.40	0.60	0.13	0.52	0.57	0.20	0.20	0.14	0.27
E13	0.17	0.53	0.29	0.20	0.14	0.40	0.60	0.29	0.40	0.57	0.20	0.40	0.14	0.27
E14	0.20	0.53	0.57	0.40	0.25	0.40	0.60	0.13	0.32	0.43	0.20	0.20	0.14	0.27
E15	0.20	0.25	0.57	0.20	0.25	0.40	0.60	0.13	0.40	0.57	0.20	0.33	0.14	0.27

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Grouped membership matrix of causes (P^{*Mean})

Areas	Learning and Growth				Business process			Customers perspective				Finance		
	1	2	3	4	5	6	7	8	9	10	11	12	13	14
E1	0.46	0.27	0.71	0.46	0.43	0.70	0.60	0.47	0.29	0.61	0.29	0.72	0.64	0.55
E2	0.29	0.40	0.67	0.23	0.46	0.50	0.60	0.50	0.25	0.46	0.26	0.49	0.26	0.47
E3	0.35	0.36	0.65	0.40	0.46	0.61	0.75	0.47	0.33	0.48	0.40	0.78	0.41	0.41
E4	0.45	0.19	0.52	0.51	0.46	0.54	0.65	0.54	0.41	0.48	0.38	0.59	0.48	0.58
E5	0.46	0.36	0.80	0.41	0.50	0.59	0.65	0.53	0.29	0.62	0.39	0.43	0.51	0.59
E6	0.39	0.39	0.57	0.18	0.45	0.66	0.80	0.45	0.44	0.63	0.35	0.41	0.71	0.52
E7	0.39	0.39	0.57	0.18	0.47	0.66	0.80	0.45	0.44	0.63	0.35	0.41	0.71	0.55
E8	0.39	0.39	0.57	0.18	0.47	0.66	0.85	0.45	0.44	0.63	0.35	0.41	0.71	0.55
E9	0.51	0.41	0.64	0.34	0.53	0.55	0.90	0.56	0.38	0.67	0.43	0.46	0.50	0.57
E10	0.42	0.52	0.65	0.18	0.51	0.53	0.75	0.51	0.71	0.62	0.61	0.55	0.41	0.59
E11	0.45	0.35	0.94	0.62	0.40	0.48	0.60	0.62	0.43	0.51	0.35	0.63	0.58	0.56
E12	0.59	0.59	0.85	0.62	0.43	0.70	0.70	0.57	0.56	0.58	0.43	0.69	0.42	0.56
E13	0.41	0.57	0.65	0.62	0.42	0.57	0.70	0.59	0.41	0.66	0.52	0.77	0.38	0.45
E14	0.48	0.57	0.67	0.80	0.52	0.53	0.75	0.47	0.36	0.61	0.43	0.61	0.49	0.57
E15	0.44	0.39	0.63	0.46	0.48	0.61	0.65	0.52	0.44	0.66	0.45	0.59	0.42	0.60

Appendix B.

Estimated membership matrix of diseases (P'^{Max})

Areas	Learning and Growth					Business process			Customers perspective			Finance		
	1	2	3	4	5	6	7	8	9	10	11	12	13	14
E1	0.57	0.57	0.61	0.67	0.87	0.80	0.80	0.93	0.31	0.83	0.51	1.00	0.83	0.67
E2	0.57	0.57	0.60	0.60	1.00	0.60	0.60	0.60	0.40	0.73	0.60	0.93	0.67	1.00
E3	0.67	0.43	0.61	0.80	1.00	0.60	0.80	0.80	0.27	0.59	0.63	0.80	0.55	1.00
E4	0.67	0.57	0.61	0.73	0.78	0.60	0.80	0.73	0.40	0.73	0.51	0.93	0.73	0.80
E5	0.83	0.57	0.60	0.67	0.63	0.60	0.80	0.80	0.40	0.73	0.63	1.00	0.67	0.92
E6	0.73	0.57	0.60	0.83	0.63	0.80	0.80	0.80	0.40	0.80	0.63	0.87	0.67	0.67
E7	0.57	0.57	0.60	0.67	0.63	0.80	0.80	1.00	0.40	0.83	0.63	1.00	0.83	0.73
E8	0.83	0.57	0.60	0.83	0.63	0.80	0.80	0.80	0.40	0.80	0.63	1.00	0.67	0.80
E9	0.67	0.57	0.61	1.00	1.00	0.60	0.80	0.87	0.33	0.83	0.63	1.00	0.83	1.00
E10	0.57	0.57	0.60	0.80	1.00	0.60	0.80	0.60	0.40	0.73	0.63	1.00	0.67	1.00
E11	0.67	0.57	0.61	0.67	0.89	0.67	0.67	0.67	0.40	0.67	0.51	0.80	0.67	0.89
E12	0.80	0.57	0.61	0.83	1.00	0.60	0.80	0.80	0.40	0.59	0.63	0.93	0.53	1.00
E13	0.67	0.57	0.61	0.93	1.00	0.60	0.80	0.80	0.40	0.83	0.63	0.93	0.83	1.00
E14	0.67	0.57	0.61	0.87	1.00	0.60	0.80	0.80	0.31	0.60	0.63	0.93	0.60	1.00
E15	0.83	0.57	0.60	0.67	0.80	0.80	0.80	0.80	0.31	0.83	0.51	1.00	0.50	1.00

Estimated membership matrix of diseases (P'^{Min})

Areas	Learning and Growth				Business process			Customers perspective				Finance		
	1	2	3	4	5	6	7	8	9	10	11	12	13	14
E1	0.14	0.13	0.56	0.20	0.14	0.40	0.60	0.13	0.20	0.43	0.20	0.20	0.14	0.24
E2	0.14	0.13	0.56	0.20	0.14	0.40	0.60	0.13	0.26	0.43	0.20	0.20	0.14	0.24
E3	0.14	0.13	0.57	0.20	0.14	0.40	0.60	0.13	0.26	0.29	0.20	0.20	0.14	0.17
E4	0.14	0.13	0.57	0.20	0.14	0.40	0.60	0.13	0.26	0.43	0.20	0.20	0.14	0.24
E5	0.14	0.13	0.57	0.20	0.14	0.40	0.60	0.13	0.26	0.43	0.20	0.20	0.14	0.24
E6	0.14	0.13	0.56	0.20	0.14	0.40	0.60	0.13	0.26	0.43	0.20	0.20	0.14	0.24
E7	0.14	0.13	0.56	0.20	0.14	0.40	0.60	0.13	0.26	0.43	0.20	0.20	0.14	0.24
E8	0.14	0.13	0.56	0.20	0.14	0.40	0.60	0.13	0.26	0.43	0.20	0.20	0.14	0.24
E9	0.14	0.13	0.57	0.20	0.14	0.40	0.60	0.13	0.26	0.43	0.20	0.20	0.14	0.24
E10	0.14	0.13	0.56	0.20	0.14	0.40	0.60	0.13	0.26	0.43	0.20	0.20	0.14	0.24
E11	0.14	0.13	0.57	0.20	0.14	0.33	0.60	0.13	0.26	0.29	0.20	0.20	0.14	0.24
E12	0.14	0.13	0.56	0.20	0.14	0.40	0.60	0.13	0.26	0.29	0.20	0.20	0.14	0.24
E13	0.14	0.13	0.57	0.20	0.14	0.40	0.60	0.13	0.26	0.43	0.20	0.20	0.14	0.24
E14	0.14	0.13	0.57	0.20	0.14	0.40	0.60	0.13	0.20	0.33	0.20	0.20	0.14	0.24
E15	0.14	0.13	0.56	0.20	0.14	0.40	0.60	0.13	0.20	0.43	0.20	0.20	0.14	0.24

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Estimated membership matrix of diseases (P'^{Mean})

Areas	Learning and Growth					Business process			Customers perspective			Finance		
	1	2	3	4	5	6	7	8	9	10	11	12	13	14
E1	0.33	0.33	0.57	0.47	0.51	0.53	0.70	0.42	0.25	0.58	0.37	0.64	0.38	0.49
E2	0.32	0.33	0.57	0.51	0.47	0.51	0.60	0.42	0.33	0.58	0.35	0.65	0.38	0.49
E3	0.33	0.33	0.60	0.57	0.51	0.51	0.70	0.42	0.27	0.53	0.37	0.64	0.35	0.49
E4	0.32	0.33	0.60	0.57	0.47	0.53	0.65	0.42	0.33	0.58	0.33	0.61	0.38	0.49
E5	0.33	0.33	0.60	0.57	0.51	0.51	0.65	0.42	0.33	0.58	0.35	0.65	0.38	0.49
E6	0.33	0.33	0.57	0.51	0.41	0.53	0.70	0.42	0.33	0.58	0.35	0.65	0.38	0.49
E7	0.32	0.33	0.57	0.57	0.41	0.53	0.70	0.42	0.33	0.58	0.37	0.65	0.38	0.49
E8	0.33	0.33	0.57	0.57	0.40	0.53	0.70	0.42	0.33	0.58	0.37	0.65	0.35	0.49
E9	0.33	0.33	0.60	0.57	0.51	0.53	0.70	0.42	0.33	0.58	0.37	0.64	0.38	0.49
E10	0.32	0.33	0.57	0.53	0.47	0.51	0.70	0.42	0.33	0.58	0.37	0.61	0.38	0.49
E11	0.33	0.33	0.60	0.57	0.51	0.53	0.65	0.42	0.33	0.56	0.35	0.54	0.38	0.49
E12	0.33	0.33	0.57	0.57	0.51	0.53	0.70	0.42	0.33	0.53	0.37	0.64	0.36	0.49
E13	0.33	0.33	0.60	0.57	0.51	0.51	0.70	0.42	0.33	0.58	0.37	0.64	0.38	0.49
E14	0.33	0.33	0.60	0.57	0.51	0.51	0.70	0.42	0.25	0.56	0.37	0.64	0.36	0.49
E15	0.33	0.33	0.57	0.46	0.47	0.53	0.65	0.42	0.25	0.58	0.37	0.65	0.35	0.49

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