Downlink Capacity and Base Station Density in Cellular Networks

Seung Min Yu and Seong-Lyun Kim
School of Electrical and Electronic Engineering, Yonsei University
50 Yonsei-Ro, Seodaemun-Gu, Seoul 120-749, Korea
Email: {smyu, slkim} @ramo.yonsei.ac.kr

***Abstract*—There have been a bulk of analytic results about the performance of cellular networks where base stations are regularly located on a hexagonal or square lattice. This regular model cannot reflect the reality, and tends to overestimate the network performance. Moreover, tractable analysis can be performed only for a fixed location user (e.g., cell center or edge user). In this paper, we use the stochastic geometry approach, where base stations can be modeled as a homogeneous Poisson point process. We also consider the user density, and derive the user outage probability that an arbitrary user is under outage owing to low signal-to-interference-plus-noise ratio or high congestion by multiple users. Using the result, we calculate the density of success transmissions in the downlink cellular network. An interesting observation is that the success transmission density increases with the base station density, but the increasing rate diminishes. This means that the number of base stations installed should be more than n-times to increase the network capacity by a factor of n. Our results will provide a framework for performance analysis of the wireless infrastructure with a high density of access points, which will significantly reduce the burden of network-level simulations.**

I. INTRODUCTION

The capacity of cellular networks has been a classical and important issue for efficient radio resource management [1]. The most improvement of the network capacity has come from reducing the cell size by installing more base stations such as femtocells [2], [3]. We may have a question, "How much does the network capacity increase as we install more base stations?" Unfortunately, answers to the question are not trivial, in particular when it comes to the case of multiple interfering base stations and mobile users. So far, the only attractable approach is to rely on simulations, where various models on radio channels and the spatial distribution of base stations and users are used. In this paper, we tackle the issue to derive closed form formulas for quickly answering the question.

Many previous studies on cellular networks assumed that base stations are positioned regularly and tractable analysis was performed only for a fixed location user (e.g., cell center or edge user) [1], [4]. This regular model tends to overestimate the capacity of cellular networks owing to the perfect geometry of base stations and the neglect of weak interference from outer tier base stations. For this reason, we use the stochastic geometry approach, where base stations can be modeled as a homogeneous Poisson point process (PPP) [5]-[7]. The main advantage of this PPP model is that we can derive the signal-to-interference-plus-noise ratio (SINR) distribution at an arbitrary location considering random channel effects such as fading and shadowing. Moreover, the PPP model reflects random location characteristics of base stations. This randomly located base station scenario exists in heterogeneous networks where a large number of microcell and femtocell base stations are deployed. Particularly, user-deployed femtocells increase the randomness. The stochastic geometry approach has recently got much attention in particular for quantifying the co-channel interference in the wireless network (see [8] and literature therein). It has been applied to CDMA cellular networks [9], cellular networks with multi-cell cooperation [10], femtocells [11], cognitive radio networks [12] and CSMA/CA based wireless multihop networks [13], [14].

In this paper, we derive the *downlink* capacity of a cellular network, as closed form formulas, and evaluate its correctness by means of simulations. The most relevant research to our work is the one by Andrews *et al.* [5]. In that paper, the authors used a PPP modeling for the base station distribution but did not consider the user density. Therefore, their results are useful for calculating the *area* outage probability, i.e., the probability that an arbitrary location is under outage owing to the low SINR. A key observation in [5] is that the area outage probability is independent of the base station density in interference limited cellular networks. This means that the network capacity linearly increases with the base station density. However, the result can be achieved under a assumption that every cell has saturated traffic. This is unreasonable as the number of base stations increases; some of the small cells do not even have any user to serve. Also, even if the user density is sufficiently high for the saturated traffic assumption, each base station can serve only one user in a resource block at a given time, which makes some users be under outage. Therefore, we define and derive the *user* outage probability, the probability that an arbitrary user is under outage considering not only the SINR level but also the user selection.

We assume base stations and mobile users are located with respective densities and radio channels fluctuate according to short-term fading and pathloss. The inter-cell interference is dependent on the frequency reuse factor but here we assume that every channel can be reused in every cell (i.e., the frequency reuse factor is 1). The rest of the paper contains how we derive our results (Propositions 1, 2, 3 and 4).

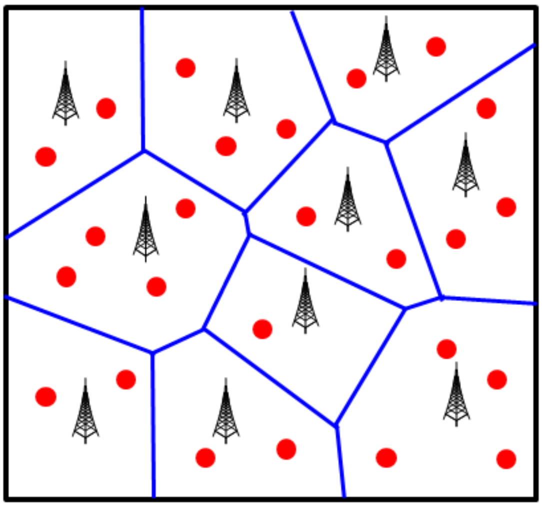


Fig. 1. The base stations and mobile users modeled as Poisson point process. The cell area of each base station forms a Voronoi tessellation.

II. System Model

Consider a downlink cellular network consisting of base stations (BSs) and mobile users (MUs). Many previous studies on cellular networks assumed that BSs are positioned regularly. However, in reality, it is not true and there are some random characteristics. To remedy the model, we apply a homogeneous PPP to the spatial distribution of the BSs such as [5]-[7]. Besides, we consider the density of MUs, where the MUs are randomly distributed according to some independent homogeneous PPP with a different density. One can argue that the MU distribution may not be best modeled as the PPP. However, this is a tractable and reasonable approach as was also used in [15].

The spatial distribution of BSs follows PPP Φ_b with the density λ_b, over which MUs are positioned with PPP Φ_u with the density λ_u. Each MU is served by the nearest BS. This means that the cell area of each BS forms a *Voronoi tessellation* [16] as in Figure 1. We assume that the radio channel attenuation is dependent on pathloss and Rayleigh fading in our analysis (Section IV). Further, we consider log-normal shadowing as well in our simulations (Section V).

We consider only one resource block at a given time and assume that only one MU is scheduled in the resource block. In other words, if there are multiple MUs in the Voronoi cell of a BS, then the BS can serve only one of them in the resource block. The resource block can be interpreted as a time slot (in time division multiple access systems), a sub-carrier (in frequency division multiple access systems) or a scheduled slot (in code division multiple assess systems). We assume that selection probabilities of the MUs within a Voronoi cell are equally likely (i.e., random selection with equal probability) for the fairness. On the other hand, there might be some BSs that do not have any MU to serve. In that case, the BSs will not transmit any signal (i.e., inactive). The inactive probability may increase with the number of BSs.

III. Inactive Base Station Probability and User Selection Probability

In this section, we derive two important probabilities, *inactive BS probability* and *user selection probability*. The inactive BS probability refers to the probability that a randomly chosen BS does not have any MU in its Voronoi cell. This probability will be used for calculating the aggregate inter-cell interference in Section IV. The user selection probability denotes the one that a randomly chosen MU is assigned a resource block at a given time and is served by the nearest BS.

A. Inactive Base Station Probability

At a given time, there can be some BSs that do not have any MU in their Voronoi cells. This happens when the BS density is high, e.g., femtocells. Those BSs are inactive. We start with the probability density function of the size of a typical Voronoi cell, which was derived by the Monte Carlo method [17]:

$$f_X(x) = \frac{3.5^{3.5}}{\Gamma(3.5)} x^{2.5} e^{-3.5x}, \tag{1}$$

where X is a random variable that denotes the size of the typical Voronoi cell normalized by the value $1/\lambda_b$. Using (1), we can derive the probability mass function of the number of MUs in a typical Voronoi cell:

Lemma 1: *Let the random variable N denote the number of MUs in the Voronoi cell of a randomly chosen BS. Then, the probability mass function of N is*

$$P[N=n] = \frac{3.5^{3.5}\Gamma(n+3.5)(\lambda_u/\lambda_b)^n}{\Gamma(3.5)\, n!\,(\lambda_u/\lambda_b+3.5)^{n+3.5}}.$$

Proof: Using the law of total probability and the function (1), the probability mass function of N is given as

$$\begin{aligned} P[N=n] &= \int_0^\infty P[N=n|X=x]\cdot f_X(x)\,dx \\ &= \int_0^\infty \frac{\left(\lambda_u \frac{x}{\lambda_b}\right)^n}{n!} e^{-\lambda_u \frac{x}{\lambda_b}} \cdot f_X(x)\,dx \\ &= \frac{3.5^{3.5}}{\Gamma(3.5)} \frac{(\lambda_u/\lambda_b)^n}{n!} \int_0^\infty x^{n+2.5} e^{-(\lambda_u/\lambda_b+3.5)x} dx \\ &= \frac{3.5^{3.5}}{\Gamma(3.5)} \frac{(\lambda_u/\lambda_b)^n}{n!} L_{x^{n+2.5}}(\lambda_u/\lambda_b+3.5) \\ &= \frac{3.5^{3.5}\Gamma(n+3.5)(\lambda_u/\lambda_b)^n}{\Gamma(3.5)\, n!\,(\lambda_u/\lambda_b+3.5)^{n+3.5}}, \end{aligned}$$

where $L_{f(x)}(s)$ denotes the Laplace transform of $f(x)$. ∎

Using Lemma 1, we derive the inactive BS probability as follows:

Proposition 1: *The probability ($p_{inactive}$) that a randomly chosen BS does not have any MU in its Voronoi cell is*

$$p_{inactive} = P[N=0] = \left(1+3.5^{-1}\lambda_u/\lambda_b\right)^{-3.5}$$

B. User Selection Probability

Now we calculate the probability that a randomly chosen MU is selected for service at a given time. To derive the probability, we need the following property:

Lemma 2: *The probability density function ($f_Y(y)$) of the size of the Voronoi cell to which a randomly chosen MU belongs is*

$$f_Y(y) = \frac{3.5^{4.5}}{\Gamma(4.5)} y^{3.5} e^{-3.5y},$$

where Y is a random variable that denotes the size of the Voronoi cell normalized by the value $1/\lambda_b$.

Proof: Consider a typical Voronoi cell and let $I \in \{0,1\}$ denote the random variable that a randomly chosen MU is located in the Voronoi cell. If the randomly chosen MU is located in the Voronoi cell, then $I = 1$. Otherwise, $I = 0$. Consider the probability $P[I = 1 | X = x]$, where X is a random variable that denotes the size of the typical Voronoi cell as in Equation (1). Using the fact that the probability is proportional to x, we can get the following equations:

$$\begin{aligned} P[I=1|X=x] &= \frac{f_{X,I}(x,1)}{f_X(x)} = cx \\ &\to f_{X,I}(x,1) = cxf_X(x), \end{aligned} \tag{2}$$

where c is a constant value. Note that $f_Y(y) = f_{X|I=1}(y)$ by definition. Therefore, we can derive $f_Y(y)$ as follows:

$$f_Y(y) = f_{X|I=1}(y) = \frac{f_{X,I}(y,1)}{P[I=1]} = \frac{cyf_X(y)}{P[I=1]} = c'yf_X(y),$$

where c' is another constant value. Finally, using the fact that $\int_0^\infty f_Y(y)\,dy = 1$, we get the probability density function in this lemma. ■

The difference between $f_X(x)$ and $f_Y(y)$ comes from the fact that large Voronoi cells have more chance to cover a given fixed point (a randomly chosen MU), which is well explained in [18]. Using Lemma 2, we derive the user selection probability as follows:

Proposition 2: *The probability ($p_{selection}$) that a randomly chosen MU is assigned a resource block at a given time and is served by the nearest BS is*

$$p_{selection} = \frac{1}{\lambda_u/\lambda_b}\left(1 - \left(1 + 3.5^{-1}\lambda_u/\lambda_b\right)^{-3.5}\right).$$

Proof: The user selection probability given the number of the other MUs (i.e., $N' = n$) is equal to $1/(n+1)$, and the location of the other MUs follows the reduced Palm distribution with the PPP Φ_u (Slivnyak's theorem [19]). Therefore, using the law of total probability, $p_{selection}$ is given as

$$\begin{aligned} p_{selection} &= \sum_{n=0}^{\infty} \frac{1}{n+1} \cdot P[N'=n] \\ &= \sum_{n=0}^{\infty} \frac{1}{n+1} \cdot \int_0^\infty P[N'=n|Y=y] \cdot f_Y(y)\,dy \\ &= \int_0^\infty \sum_{n=0}^{\infty} \frac{1}{n+1} \frac{\left(\lambda_u \frac{y}{\lambda_b}\right)^n}{n!} e^{-\lambda_u \frac{y}{\lambda_b}} \cdot f_Y(y)\,dy \\ &= \int_0^\infty \frac{\lambda_b}{\lambda_u} y^{-1} \sum_{k=1}^{\infty} \frac{\left(\lambda_u \frac{y}{\lambda_b}\right)^k}{k!} e^{-\lambda_u \frac{y}{\lambda_b}} \cdot f_Y(y)\,dy \\ &= \int_0^\infty \frac{\lambda_b}{\lambda_u} y^{-1} \left(1 - e^{-\lambda_u \frac{y}{\lambda_b}}\right) \cdot f_Y(y)\,dy \\ &= \frac{3.5^{4.5}}{\Gamma(4.5)} \frac{\lambda_b}{\lambda_u} \int_0^\infty y^{2.5} e^{-3.5y} - y^{2.5} e^{-\left(3.5 + \frac{\lambda_u}{\lambda_b}\right)y} dy \\ &= \frac{3.5^{4.5}}{\Gamma(4.5)} \frac{\lambda_b}{\lambda_u} \left(L_{y^{2.5}}(3.5) - L_{y^{2.5}}\left(3.5 + \frac{\lambda_u}{\lambda_b}\right)\right) \\ &= \frac{1}{\lambda_u/\lambda_b}\left(1 - \left(1 + (3.5)^{-1}\lambda_u/\lambda_b\right)^{-3.5}\right). \end{aligned}$$

■

To verify our analysis, we conduct simulations with 10^5 independent samples of the location of BSs and MUs. We set the user density $\lambda_u = 30$. We numerically calculate the probability density functions of the Voronoi cell size $f_Y(y)$ (Lemma 2), the inactive BS probability $p_{inactive}$ (Proposition 1) and the user selection probability $p_{selection}$ (Proposition 2) in terms of the BS density λ_b. Figure 2 shows the results, which exactly coincide with Equation (1), Lemma 2, and Propositions 1 and 2.

IV. Performance Analysis of Cellular Networks

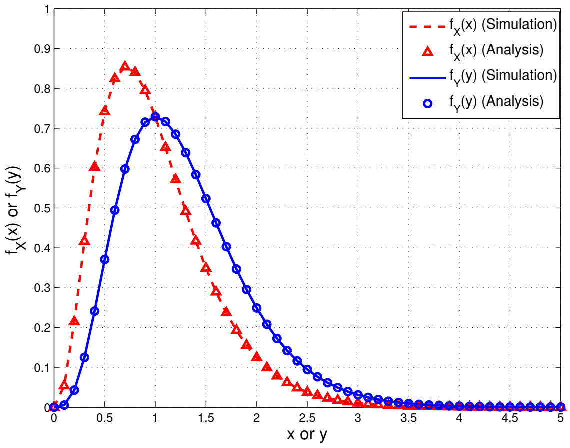
In this section, we analyze the capacity of cellular networks as a function of MU and BS density, and the target service quality. We define *service success probability* and *service capacity* as performance metrics. The service success probability refers to the probability that the cellular network succeeds in serving an arbitrary MU. It is composed with two parts, user selection probability (Proposition 2) and the transmission success probability which is defined in this section. The service capacity refers to the density of MUs with success transmissions.

A. Service Success Probability

Service success probability ($p_{service}$) is defined as

$$p_{service} \triangleq p_{selection} \cdot p_{success}, \tag{3}$$

which means the probability that the cellular network succeeds in serving an arbitrary MU with some target signal-to-



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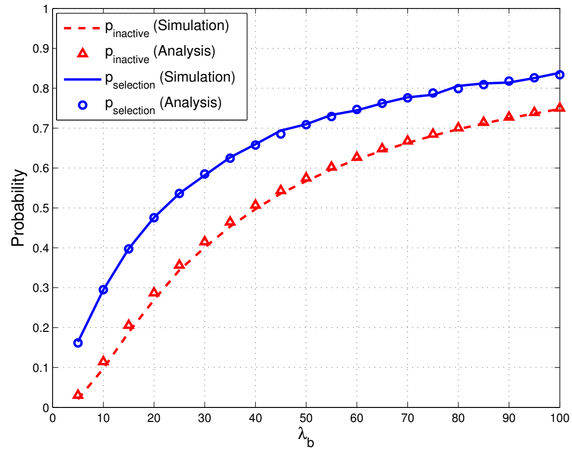


(b)

Fig. 2. (a) The probability density function $f_X(x)$ of the size of a typical Voronoi cell and the probability density function $f_Y(y)$ of the size of the Voronoi cell where a randomly chosen mobile user is located. (b) The inactive base station probability $p_{inactive}$ and the user selection probability $p_{selection}$ as a function of the base station density λ_b (the mobile user density is $\lambda_u = 30$).

interference-noise ratio ($\hat{\gamma}$).[1] The transmission success probability ($p_{success}$) is the one that the MU's received signal to interference-noise ratio (γ) is higher than $\hat{\gamma}$. We derive the transmission success probability in the following lemma:

Lemma 3: *The transmission success probability ($p_{success}$) is*

$$p_{success} = \pi\lambda_b \int_0^\infty e^{-\pi\lambda_b\left(1+\left(1-\left(1+3.5^{-1}\lambda_u/\lambda_b\right)^{-3.5}\right)k\right)x-\frac{\hat{\gamma}\sigma_N^2 x^{\alpha/2}}{s}}\, dx,$$

where σ_N^2 and s denote the noise and the transmitted signal powers, respectively. The value α denotes the pathloss exponent and $k = \hat{\gamma}^{2/\alpha}\int_{\hat{\gamma}^{-2/\alpha}}^{\infty} 1/\left(1+u^{\alpha/2}\right) du$.

Proof: From the result of [5], we get the transmission success probability as follows:

$$p_{success} = \pi\lambda_b \int_0^\infty e^{-\pi(\lambda_b+\lambda_i k)x-\frac{\hat{\gamma}\sigma_N^2 x^{\alpha/2}}{s}}\, dx, \tag{4}$$

where λ_i denotes the density of the BSs interfering with the given MU. Note that λ_i is equal to $\lambda_i = \lambda_b \cdot (1 - p_{inactive})$ by Proposition 1.[2] Then, we get the result of this lemma. ■

The closed form formula ($p_{success}$) can be obtained when the pathloss exponent α is 4. Unfortunately, to the best of our knowledge, the other values of α do not give us such closed form. Using Proposition 2 and Lemma 3, we derive $p_{service}$ in the following proposition:

Proposition 3: *The service success probability ($p_{service}$) is*

$$p_{service} = \frac{\pi\lambda_b^2}{\lambda_u}\left(1-\left(1+3.5^{-1}\lambda_u/\lambda_b\right)^{-3.5}\right) \cdot \int_0^\infty e^{-\pi\lambda_b\left(1+\left(1-\left(1+3.5^{-1}\lambda_u/\lambda_b\right)^{-3.5}\right)k\right)x-\frac{\hat{\gamma}\sigma_N^2 x^{\alpha/2}}{s}}\, dx.$$

If we assume that the noise is negligible (i.e., interference limited system) and $\alpha = 4$, then $p_{service}$ is reduced to the following closed form formula:

$$p_{service} = \frac{1-\left(1+3.5^{-1}\lambda_u/\lambda_b\right)^{-3.5}}{\lambda_u/\lambda_b\left(1+\left(1-\left(1+3.5^{-1}\lambda_u/\lambda_b\right)^{-3.5}\right)k'\right)}, \tag{5}$$

where $k' = \sqrt{\hat{\gamma}}\left(\pi/2 - \arctan\left(1/\sqrt{\hat{\gamma}}\right)\right)$.

B. Service Capacity

Service capacity ($C_{service}$) is defined as

$$C_{service} \triangleq \lambda_u \cdot p_{service}. \tag{6}$$

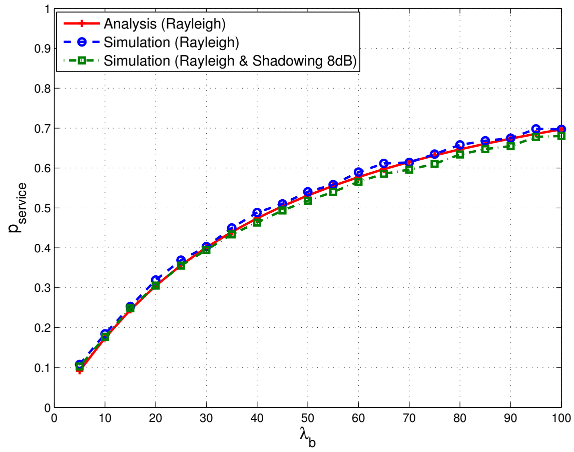
It is interpreted as the density of MUs with success transmissions. Using Proposition 3, we derive $C_{service}$ in the following proposition:

Proposition 4: *The service capacity ($C_{service}$) is*

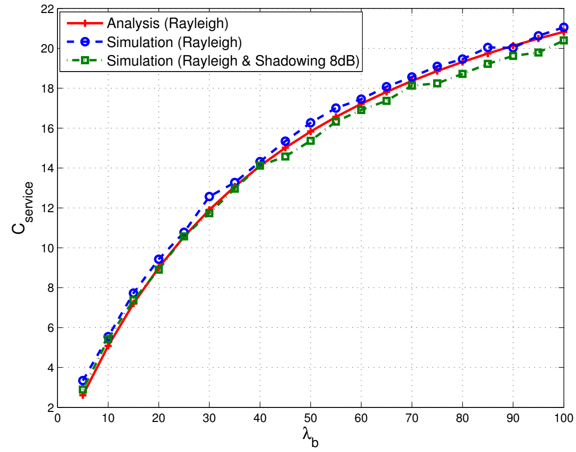
$$C_{service} = \pi\lambda_b^2\left(1-\left(1+3.5^{-1}\lambda_u/\lambda_b\right)^{-3.5}\right) \cdot \int_0^\infty e^{-\pi\lambda_b\left(1+\left(1-\left(1+3.5^{-1}\lambda_u/\lambda_b\right)^{-3.5}\right)k\right)x-\frac{\hat{\gamma}\sigma_N^2 x^{\alpha/2}}{s}}\, dx.$$

[1]The definition of $p_{service}$ is based on the assumption that $p_{selection}$ and $p_{success}$ are independent. Unfortunately, there is dependency between the two. If a MU is selected, then it is more likely to belong to a small cell, and thus interferes are likely to be closer. However, this dependency is negligible, which will be verified by the good match between theoretical and simulation results (Figure 3).

[2]The process of the BSs interfering with the given MU will be a dependent thinning of the initial BS process Φ_b owing to the difference in cell size and shape. For mathematical tractability, however, we assume that it is an independent thinning of Φ_b with the thinning probability $p_{inactive}$ (in an average sense).



(a)



(b)

Fig. 3. Performance metrics as a function of the base station density λ_b: (a) The service success probability $p_{service}$. (b) The service capacity $C_{service}$ (the mobile user density is $\lambda_u = 30$, the pathloss exponent is $\alpha = 4$, the target signal to interference-noise ratio is $\hat{\gamma} = 0$dB, and interference limited system).

Again, if we assume that the noise is negligible and $\alpha = 4$, then $C_{service}$ is reduced to the following closed form formula:

$$C_{service} = \frac{\lambda_b \left(1 - \left(1 + 3.5^{-1}\lambda_u/\lambda_b\right)^{-3.5}\right)}{1 + \left(1 - \left(1 + 3.5^{-1}\lambda_u/\lambda_b\right)^{-3.5}\right) k'}, \tag{7}$$

where k' is given in (5).

To verify Propositions 3 and 4, we conduct simulations with 10^5 independent samples of the location of BSs and MUs. We assume an interference limited system and set the user density $\lambda_u = 30$, the pathloss exponent $\alpha = 4$, the target signal to interference-noise ratio $\hat{\gamma} = 0$dB. We numerically calculate the service probability $p_{service}$ (Proposition 3) and the service capacity $C_{service}$ (Proposition 4). Figure 3 shows the results. In Proposition 3 and 4, we consider pathloss and Rayleigh fading in our channel model. On the other hand, we add the shadow fading in our simulations. Therefore, there is small gap between our analysis and the simulation result as the BS density increases. However, the general shape of the curves exactly match each other.

In Figure 3-(b), we see that the service capacity is a concave function of the number of BSs. In other words, the average quality of service may not increase rapidly with the installation of additional BSs, after some point. This is because some of small cells cannot have any user to serve as the number of BSs increases. Moreover, increase of co-channel interference by a large number of BSs leads to decrease of the marginal capacity.

The numerical results (Figures 3) are based on the pathloss exponent $\alpha = 4$, where the closed form formula is available. For the other cases, we need to calculate the numerical integration part of Propositions 3 and 4. However, this burden is much less than the system level simulations. Figure 4 contains our results where the pathloss component varies between 2 and 4. In the figure, we see that the capacity of networks increases as the pathloss exponent becomes higher. This is due to the fact that the higher pathloss will filter co-channel interference among the cells [20]. On the other hand, we see that the behavior of diminishing the marginal capacity remains the same as in Figure 3-(b).

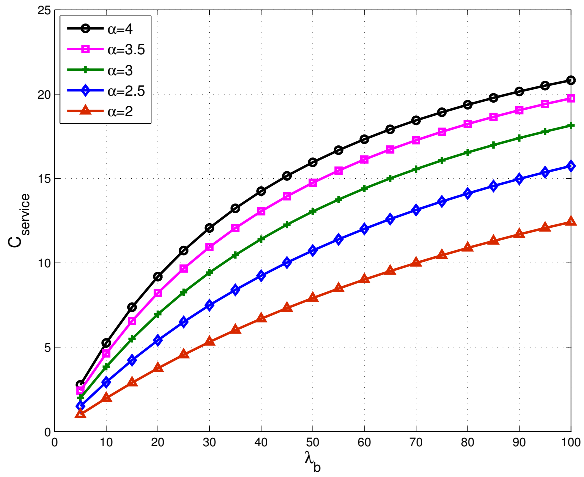


Fig. 4. The service capacity $C_{service}$ for various pathloss exponents α (the mobile user density is $\lambda_u = 30$, the target signal to interference-noise ratio is $\hat{\gamma} = 0$dB, and interference limited system).

C. Asymptotic Cases

To get simpler closed form formulas, we consider two asymptotic cases. The first is the one that the density of BSs is much higher than that of MUs (i.e., $\lambda_b \gg \lambda_u$) like femtocells. In this case, the user selection probability can be approximated to one (i.e., $p_{selection} \approx 1$) and the density of the transmitting BSs can be approximated to that of the MUs (i.e., $\lambda_i \approx \lambda_u$). Therefore, service success probability and service capacity are given as follows:

$$p_{service} \approx \pi\lambda_b \int_0^\infty e^{-\pi(\lambda_b+\lambda_u k)x - \frac{\hat{\gamma}\sigma_N^2 x^{\alpha/2}}{s}} dx,$$

$$C_{service} \approx \pi\lambda_b\lambda_u \int_0^\infty e^{-\pi(\lambda_b+\lambda_u k)x - \frac{\hat{\gamma}\sigma_N^2 x^{\alpha/2}}{s}} dx. \quad (8)$$

Moreover, if we assume that the noise is negligible and $\alpha = 4$, those are reduced to:

$$p_{service} \approx \frac{\lambda_b}{\lambda_b + \lambda_u k'},\ C_{service} \approx \frac{\lambda_b\lambda_u}{\lambda_b + \lambda_u k'}. \quad (9)$$

The second is the case that the density of the MUs is much higher than that of the BSs (i.e., $\lambda_u \gg \lambda_b$). This scenario is for the highly congested area like downtowns. In the case, inactive probability can be approximated to zero (i.e., $p_{inactive} \approx 0$) and the density of the transmitting BSs can be approximated to that of the existing BSs (i.e., $\lambda_i \approx \lambda_b$). Therefore, service success probability and service capacity are given as follows:

$$p_{service} \approx \frac{\pi\lambda_b^2}{\lambda_u} \int_0^\infty e^{-\pi\lambda_b(1+k)x - \frac{\hat{\gamma}\sigma_N^2 x^{\alpha/2}}{s}} dx,$$

$$C_{service} \approx \pi\lambda_b^2 \int_0^\infty e^{-\pi\lambda_b(1+k)x - \frac{\hat{\gamma}\sigma_N^2 x^{\alpha/2}}{s}} dx. \quad (10)$$

Similarly, if we assume that the noise is negligible and $\alpha = 4$, those are reduced to:

$$p_{service} \approx \frac{\lambda_b}{\lambda_u (1 + k')},\ C_{service} \approx \frac{\lambda_b}{1 + k'}. \quad (11)$$

V. CONCLUSIONS

In this paper, we used the stochastic geometry approach and derived useful distributions and probabilities for cellular networks (Propositions 1, 2 and 3). Using these, we calculated the density of success transmissions in the downlink cellular network that was defined as the service capacity (Proposition 4). A key observation is that the success transmission density increases with the base station density, but the increasing rate diminishes. If the MU density is much higher than the BS density (i.e., saturated traffic condition) and the noise is negligible (i.e., interference limited system), however, the success transmission density linearly increases with the BS density (Equation (11)).

The limitation of our current work is as follows: First, we did not consider the shadow fading in the channel model of our analysis. Even though we verified our results using simulations, extension to the shadow fading case seems to be necessary in particular shadowing are correlated [21]. Second, the user selection is equally likely in each base station. However, we may consider more realistic scheduling algorithms into the analysis.

ACKNOWLEDGMENT

This research was supported by the International Research & Development Program of the National Research Foundation of Korea (NRF) funded by the Ministry of Education, Science and Technology (MEST) of Korea (Grant number: 2012K1A3A1A26034281, FY 2012), and the Korea Communications Commission (KCC) under the R&D program supervised by the Korea Communications Agency (KCA) (KCA- 2012-12-911-01-107).

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