The Gaussian Interference Channel revisited as a Non-Cooperative Game with Transmission Cost

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***Abstract*—We consider the *Gaussian interference channel* as a non-cooperative game taking into account the cost of the transmission. We study the conditions of the existence of a pure Nash equilibrium. Particularly, for the many-user case we give sufficient conditions that lead to a Nash equilibrium, and for the two-user case we exhaustively describe the conditions of the existence and the uniqueness of a pure Nash equilibrium and we show the existence of best-response dynamics that converge to one of them.**

I. Introduction

The *Gaussian interference channel* is one of the most fundamental channels in wireless communications [1], but the capacity of this channel still remains an open problem. One way of improving the achievable capacity is the cooperation between the transmitters [2], [3], [4]. However, if such cooperation is not possible then the channel becomes a non-cooperative game and the transmitters become players that selfishly try to optimize their own utility [5].

Many game-theoretic studies have been done for this problem for the case of two users. In [6], the authors study the two-user *Gaussian interference channel* with limited budget of transmission power on a specific bandwidth, providing conditions of existence and uniqueness of a Nash equilibrium and they show conditions of convergence to this. In [7], the authors study the Nash equilibrium region as an extension of the information-theoretic capacity region. In [8], the authors study the two-user *Gaussian interference channel* on a specific channel and they give a pricing mechanism that leads the players to a unique, Pareto-efficient and "fair" Nash equilibrium. In [9], [10] a spectrum sharing problem of two bands is studied. In [11] the authors study a jamming game on many channels with limited budget of transmission power taking into account the transmission cost.

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For many users, in [12] the authors prove the existence of a Nash equilibrium in the *Gaussian interference channel* with many channels and limited budget of transmission power. A spectrum sharing problem with many users and many channels is studied in [13]. In [14] the authors provide an iterative waterfilling algorithm for finding the Nash equilibrium. Finally, in [15] a spectrum sharing problem in an unlicensed band is studied.

Our contribution. None of the above works takes into account the transmission cost, apart from [11] and [8]. Particularly, in [8] the authors give a pricing mechanism leading to a Nash equilibrium for the case of weak interference and this case coincides with one of the cases of our analysis.

In this paper, we study the problem from a game-theoretic point of view taking into account the transmission cost as a linear function of the transmission power, so that the transmitters try to maximize their own rate and simultaneously minimize the cost of the transmission. We give conditions that lead to a Nash equilibrium with M users. Particularly, for the two-user case we prove the existence of a Nash equilibrium under any condition of noise, interference and transmission cost and we show the existence of best-response dynamics that they lead to a Nash equilibrium. We find interesting the striking similarity of our results with the results of the species population competition described by the Lotka-Volterra competition model [16], [17].

II. The model

We consider an M-pair transmitter-receiver *Gaussian interference channel*, where M is an integer greater than one. Any receiver $i \in \{1, \ldots, M\}$ treats the signal of the transmitters different than the transmitter i as interference. Thus, by the *Shannon's formula* [18] we get the link capacities (in nats per channel use):

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$$R_i(p_i, p_{-i}) = \frac{1}{2}\ln\left(1 + \frac{h_{ii}p_i}{\sum\limits_{\substack{j=1\\j\neq i}}^{M} h_{ji}p_j + N_i}\right),$$

where $p_i \in [0, p_{\max}]$ is the power of the transmitter i and $p_{\max}$ is the maximum available power which is assumed for the moment arbitrarily large. Furthermore, p_{-i} are the choices of transmission powers of all transmitters except from transmitter i. The positive constants h_{ji} are the channel coefficients between transmitter j and receiver i and the positive constant N_i is the power of the additive Gaussian white noise (AGWN) at receiver i. By scaling the channel coefficients we have that the link capacities are equal to:

$$R_i(p_i, p_{-i}) = \frac{1}{2}\ln\left(1 + \frac{p_i}{\sum\limits_{\substack{j=1\\j\neq i}}^{M} a_{ji}p_j + n_i}\right),$$

where $a_{ji} = h_{ji}/h_{ii} > 0$, $n_i = N_i/h_{ii} > 0$.

We can consider the transmitters as the players of a non-cooperative game. The pure strategy of every transmitter/player i is the power p_i that she chooses to send her message. We call strategy profile the M-tuple of the assigned transmission powers of all players. In this paper, we only consider pure strategies. We define the payoff of a player i to be the capacity R_i minus a positive cost c_i multiplied by the transmitting power p_i. As we will show below $c_i \leq 1/(2n_i)$, since in the opposite case the best-response strategy of the player i is $p_i = 0$. So, the utility functions of the players are defined as:

$$u_i(p_i, p_{-i}) = \frac{1}{2}\ln\left(1 + \frac{p_i}{\sum\limits_{\substack{j=1\\j\neq i}}^{M} a_{ji}p_j + n_i}\right) - c_i p_i.$$

The utility function of the player i, u_i, is a concave function of p_i for any fixed strategies of the other players, since the second derivative with respect to p_i is negative. In order to find the best-response strategies, we find the roots of the first derivative for p_i of the utility function u_i, for every i, so:

$$du_i/dp_i = \frac{1}{2\left(p_i + \sum\limits_{\substack{j=1\\j\neq i}}^{M} a_{ji}p_j + n_i\right)} - c_i = 0,$$

this implies that

$$p_i + \sum_{\substack{j=1\\j\neq i}}^{M} a_{ji}p_j = \frac{1}{2c_i} - n_i \equiv b_i. \tag{1}$$

So, the best-response strategy of any player i is

$$p_i = \max\{b_i - \sum_{\substack{j=1\\j\neq i}}^{M} a_{ji}p_j, 0\}. \tag{2}$$

We can write up the equation (1) as a linear system

$$\boldsymbol{A^T} \cdot \boldsymbol{p} = \boldsymbol{b}, \tag{3}$$

where

$$\boldsymbol{A} = \begin{pmatrix} 1 & a_{12} & a_{13} & \dots & a_{1M} \\ a_{21} & 1 & a_{23} & \dots & a_{2M} \\ & & \dots & & \\ a_{M1} & a_{M2} & \dots & & 1 \end{pmatrix},$$

$$\boldsymbol{p} = \begin{pmatrix} p_1 \\ p_2 \\ \dots \\ p_M \end{pmatrix}, \text{ and } \boldsymbol{b} = \begin{pmatrix} 1/(2c_1) - n_1 \\ 1/(2c_2) - n_2 \\ \dots \\ 1/(2c_M) - n_M \end{pmatrix}.$$

By equation (1), we can see that for any $i \in \{1, \dots, M\}$, $b_i \geq 0$, which implies that $c_i \leq \frac{1}{2n_i}$. Actually, if $b_i = 0$, we can assume that the player i is not in the game. For the analysis, we assume that $b_i > 0$, for any i. The constant b_i is the power that will be used when the link i is free from interference in order to maximize the utility function u_i. From its definition we see that b_i is higher (and therefore higher capacity can be achieved in the interference-free link i) if the noise power n_i is lower, the link quality h_{ii} is higher and the power cost c_i is lower.

If interference is introduced, the power that maximizes the utility function u_i will be less than b_i. Thus, b_i can be considered as the maximum power that will be used in the link i, let us call it maximum advantageous power. The sum of the power p_i plus the interference power must sum up to b_i and this can be interpreted as a kind of water-filling for the link i. The corresponding maximum capacity $R_{i\max}$ of the link will be $\frac{1}{2}\ln(1 + b_i/n_i) = \frac{1}{2}\ln(1/(2c_i n_i))$.

III. The two-user case

In order to get insight on the problem, we consider a two-pair transmitter-receiver *Gaussian interference channel*, the pair of transmitter 1 (t_1) and receiver 1 (r_1) and the pair of transmitter 2 (t_2) and receiver 2 (r_2). The utility functions of the players are defined as:

$$u_1(p_1, p_2) = \frac{1}{2}\ln\left(1 + \frac{p_1}{a_{21}p_2 + n_1}\right) - c_1 p_1,$$

$$u_2(p_2, p_1) = \frac{1}{2}\ln\left(1 + \frac{p_2}{a_{12}p_1 + n_2}\right) - c_2 p_2,$$

for player 1 and player 2, respectively.

As in (1) the powers that maximize the utilities are:

$$p_1 = b_1 - a_{21}p_2, \tag{4}$$

and

$$p_2 = b_2 - a_{12}p_1. \tag{5}$$

As in (2) the best-response strategies are:

$$p_1 = \max\{b_1 - a_{21}p_2, 0\}, \tag{6}$$

and

$$p_2 = \max\{b_2 - a_{12}p_1, 0\}. \tag{7}$$

A. *Conditions of the existence and the uniqueness of a pure Nash equilibrium*

In this section, we will describe the conditions of the existence and the uniqueness of a pure Nash equilibrium. To do this, we will do case-analysis for all possible scenarios. But first, we give the analytic form of the cross point of the lines described by (4) and (5), if it exists in the first quarter, as a solution of the system that is created by these equations.

Theorem 1: If the lines (4) and (5) cross each other in the first quarter, then the strategy profile $\left(\frac{b_1 - a_{21}b_2}{1-a_{12}a_{21}}, \frac{b_2 - a_{12}b_1}{1-a_{12}a_{21}}\right)$ is a Nash equilibrium.

Proof 1: This is the solution of the system created by the equations (4) and (5), in which both players mutually play best-response strategies. Analytically, for the two-user case the solutions are:

$$p_1 = \frac{\begin{vmatrix} b_1 & a_{21} \\ b_2 & 1 \end{vmatrix}}{\Delta}, p_2 = \frac{\begin{vmatrix} 1 & b_1 \\ a_{12} & b_2 \end{vmatrix}}{\Delta},$$

where $\Delta = \begin{vmatrix} 1 & a_{21} \\ a_{12} & 1 \end{vmatrix}$. If Δ and the numerators are zero, we have an infinite number of solutions.

□

Theorem 2: There always exists at least one pure Nash equilibrium (NE).

Proof 2: In order to prove this theorem we study the existence of the pure Nash equilibrium in six different cases depending on the topology (a_{12}, a_{21}) and the powers (b_1, b_2).

- Case 1: $\{b_1 > a_{21}b_2$ and $b_2 \geq a_{12}b_1\}$, or $\{b_1 = a_{21}b_2$ and $b_2 > a_{12}b_1\}$. From these conditions we conclude that in this case $a_{21} \cdot a_{12} < 1$. We can see that the equation (4) is a line that crosses the axis of p_1 at the point $(b_1, 0)$ and the axis of p_2 at the point $(0, b_1/a_{21})$. The equation (5) is also a line that crosses the axis of p_1 at the point $(b_2/a_{12}, 0)$ and the axis of p_2 at the point $(0, b_2)$. It is easy to see that with two pairs of inequalities there is exactly one cross point of the lines, so a pure Nash equilibrium.

- Case 2: $b_1 < a_{21}b_2$ and $b_2 < a_{12}b_1$. From these two conditions we conclude that $a_{21} \cdot a_{12} > 1$ that is we have strong interference. In this case, there are three Nash equilibria, the cross point of the two lines, the point $(b_1, 0)$ and the point $(0, b_2)$. The point $(b_1, 0)$ is a pure Nash equilibrium, since the best-response strategy for the player 1 when the player 2 plays $p_2 = 0$ is the strategy $p_1 = b_1$, and the best-response strategy of the player 2 when the player 1 plays b_1 is the strategy $p_2 = 0$. A symmetric argument holds for the point $(0, b_2)$.

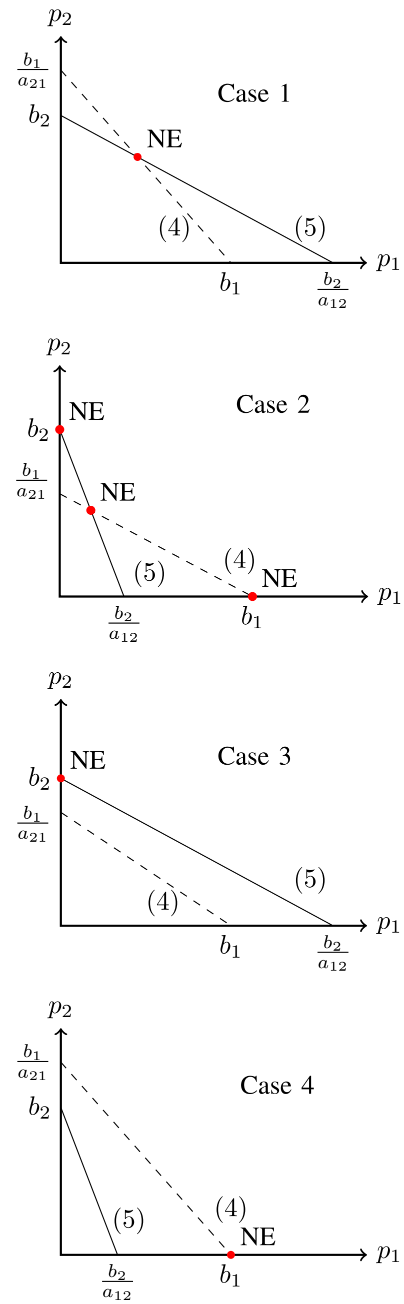

Fig. 1. The first 4 possible cases.

- Case 3: $b_1 < a_{21}b_2$ and $b_2 > a_{12}b_1$. In this case, the point $(0, b_2)$ is a pure Nash equilibrium, since the best-response strategy for the player 2 when the player 1 plays $p_1 = 0$ is the strategy $p_2 = b_2$, and the best-response strategy of the player 1 when the player 2 plays b_2 is the strategy $p_1 = 0$.

- Case 4: $b_1 > a_{21}b_2$ and $b_2 < a_{12}b_1$. In this case, the point $(b_1, 0)$ is a pure Nash equilibrium, since the best-response strategy for the player 1 when the player 2 plays $p_2 = 0$ is the strategy $p_1 = b_1$, and the best-response strategy of the player 2 when the player 1 plays b_1 is the strategy $p_2 = 0$.

- Case 5: $\{b_1 = a_{21}b_2$ and $b_2 < a_{12}b_1\}$, or $\{b_1 < a_{21}b_2$ and $b_2 = a_{12}b_1\}$. From these conditions, in both cases, we conclude that $a_{21} \cdot a_{12} > 1$. In the first case, there are two Nash equilibria, the cross point of the two lines and the point $(b_1, 0)$. The point $(b_1, 0)$ is a pure Nash equilibrium, since the best-response strategy for the player 1 when the player 2 plays $p_2 = 0$ is the strategy $p_1 = b_1$, and the best-response strategy of the player 2 when the player 1 plays b_1 is the strategy $p_2 = 0$. A symmetric argument holds for the case $b_1 < a_{21}b_2$ and $b_2 = a_{12}b_1$.

- Case 6: $b_1 = a_{21}b_2$ and $b_2 = a_{12}b_1$. In this case, the lines (4) and (5) coincide, so we have an infinite number of cross points, so Nash equilibria. It is easy to see that in this case $a_{21} \cdot a_{12} = 1$.

Now, we will give the conditions of uniqueness of a pure Nash equilibrium.

Theorem 3: If $a_{12}b_1 < b_2$ or $a_{21}b_2 < b_1$, then the pure Nash equilibrium is unique.

Proof 3: We can see that in every case there is at least one pure Nash equilibrium and the only case that we have more than one Nash equilibrium is the case $a_{12}b_1 \geq b_2$ and $a_{21}b_2 \geq b_1$. So, the logical negative of this case is $a_{12}b_1 < b_2$ or $a_{21}b_2 < b_1$. $\square$

B. Existence of best-response dynamics

We care about the convergence of a strategy profile to a pure Nash equilibrium. We will graphically show that in any case if the players sequentially play best-response strategies they converge to one of them.

Particularly, in the case 1 where we have $\{b_1 > a_{21}b_2$ and $b_2 \geq a_{12}b_1\}$, or $\{b_1 = a_{21}b_2$ and $b_2 > a_{12}b_1\}$, there are best-response dynamics that converge to the unique Nash equilibrium.

The same holds for the case 2 where $\{b_1 < a_{21}b_2$ and $b_2 < a_{12}b_1\}$. However, in this case we have three Nash equilibria. We call the Nash equilibrium in the cross point as "unstable", since there is a closed area around this that the best-response dynamics can lead to one of Nash equilibria on the axes. For example, from Theorem 1 we know that the cross point is the strategy profile $\left(\frac{b_1 - a_{21}b_2}{1 - a_{12}a_{21}}, \frac{b_2 - a_{12}b_1}{1 - a_{12}a_{21}}\right)$, if the players go to a strategy profile $\left(\frac{b_1 - a_{21}b_2}{1 - a_{12}a_{21}} \pm \varepsilon_1, \frac{b_2 - a_{12}b_1}{1 - a_{12}a_{21}} \pm \varepsilon_2\right)$, for any small positive constants $\varepsilon_1, \varepsilon_2$, then there is the case that the best-response dynamics can lead to one of the Nash equilibria on the axes, depending on which player plays first.

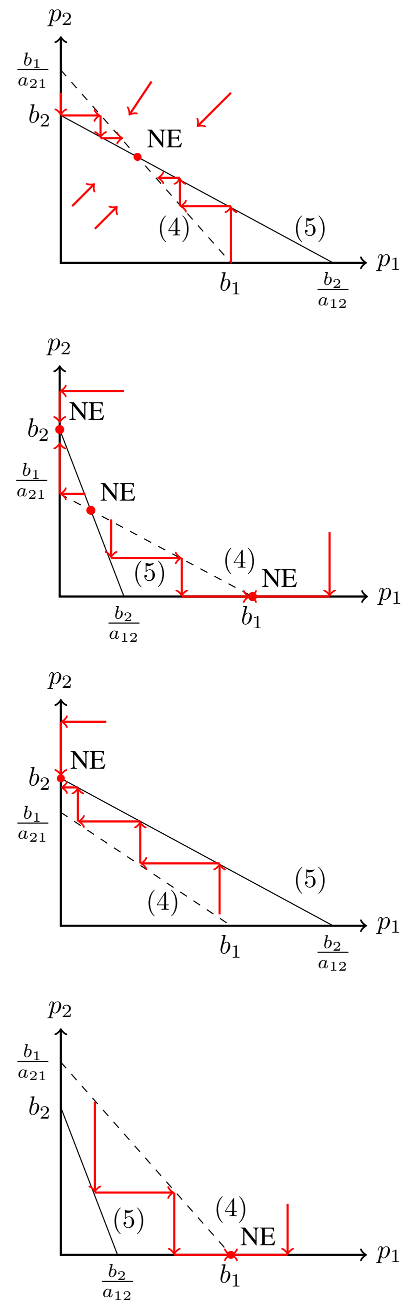

Fig. 2. Best-response dynamics.

In the cases 3 and 4 where $\{b_1 < a_{21}b_2$ and $b_2 > a_{12}b_1\}$, or $\{b_2 < a_{12}b_1$ and $b_1 > a_{21}b_2\}$, there are best-response dynamics that converge to the unique pure Nash equilibrium.

In the case 5, where $\{b_1 = a_{21}b_2$ and $b_2 < a_{12}b_1\}$, or $\{b_1 < a_{21}b_2$ and $b_2 = a_{12}b_1\}$, there are best-response dynamics that converge to a pure Nash equilibrium, but the cross-point as the above cases is "unstable", since there are best-response dynamics in a closed area of the Nash equilibrium that can lead to a different Nash equilibrium

In the case, $b_1 = a_{21}b_2$ and $b_2 = a_{12}b_1$, the two lines (4) and (5) coincide, so any point on the lines is a Nash equilibrium. However, in this case any Nash equilibrium is “unstable”, since for any small neighbourhood of a Nash equilibrium there is the possibility that the best-response dynamics can lead to a different Nash equilibrium.

IV. Many-user case

We generalize the results of the two-user case to the many-user case. In the following Theorem, we give sufficient conditions that lead to a Nash equilibrium.

Theorem 4: If for all i, $b_i > \sum_{\substack{j=1\\j\neq i}}^{M} a_{ji}b_j$, or $1 > \sum_{\substack{j=1\\j\neq i}}^{M} a_{ji}$, then there is a Nash equilibrium equal to $\boldsymbol{p} = (\boldsymbol{A}^T)^{-1}\cdot\boldsymbol{b} > \boldsymbol{b}/2$.

Proof 4: Note that the condition $b_i > \sum_{\substack{j=1\\j\neq i}}^{M} a_{ji}b_j$ means that the interference on link i when the other links operate with the maximum advantageous power b_j is less than the maximum advantageous power b_i of link i and corresponds to the case of $b_1 > a_{21}b_2$ and $b_2 > a_{12}b_1$ of the two-user interference channel (light interference with one Nash equilibrium).

For the proof, we will firstly prove that the matrix $\boldsymbol{A}^T$ is invertible, so there is a Nash equilibrium equal to $p = (\boldsymbol{A}^T)^{-1}\cdot \boldsymbol{b}$. After this, we will prove that the value of the power of every player i, p_i, in the Nash equilibrium is strictly greater than the half of the maximum advantageous power, $\frac{b_i}{2}$.

Let $\boldsymbol{B}$ be a diagonal matrix of size $M\times M$, such that $\boldsymbol{B}_{ii} = b_i$, for any $i \in \{1,\dots,M\}$, and 0 elsewhere. We conclude that the matrix $\boldsymbol{B}$ is invertible, since it is diagonal and $b_i > 0$. Let now $\boldsymbol{A}^T\cdot\boldsymbol{B}$ be the multiplication of two matrices of $\boldsymbol{A}^T$ and $\boldsymbol{B}$. The matrix $\boldsymbol{A}^T\cdot\boldsymbol{B}$ is diagonally dominant, since $b_i > \sum_{\substack{j=1\\j\neq i}}^{M} a_{ji}b_j$, so the inverse $(\boldsymbol{A}^T\cdot\boldsymbol{B})^{-1}$ exists. Since the inverse $(\boldsymbol{A}^T\cdot\boldsymbol{B})^{-1}$ exists, this means that the determinant $det(\boldsymbol{A}^T\cdot\boldsymbol{B}) \neq 0$. However, $det(\boldsymbol{A}^T\cdot\boldsymbol{B}) = det(\boldsymbol{A}^T)\cdot det(\boldsymbol{B}) \neq 0$. But, since $\boldsymbol{B}$ is invertible, the $det(\boldsymbol{B}) \neq 0$. Thus, $det(\boldsymbol{A}^T) \neq 0$, so the matrix $\boldsymbol{A}^T$ is invertible.

For the second condition it is easy to see that if $1 > \sum_{\substack{j=1\\j\neq i}}^{M} a_{ji}$, then the matrix $\boldsymbol{A}^T$ is diagonally dominant, so invertible.

In order to prove that in the Nash equilibrium the power of every player i is $p_i > \frac{b_i}{2}$, we use the fact that since $b_i > \sum_{\substack{j=1\\j\neq i}}^{M} a_{ji}b_j$, this means that $2b_i > \sum_{j=1}^{M} a_{ji}b_j$. Thus,

$$2\boldsymbol{I}\cdot\boldsymbol{b} > \boldsymbol{A}^T\cdot\boldsymbol{b},$$

where $\boldsymbol{I}$ is the identity matrix of size $M\times M$. This means that

$$\begin{aligned}(2\boldsymbol{I}-\boldsymbol{A}^T)\cdot\boldsymbol{b} > 0 \Rightarrow\\ (\boldsymbol{A}^T)^{-1}\cdot(2\boldsymbol{I}-\boldsymbol{A}^T)\cdot\boldsymbol{b} > 0 \Rightarrow 2(\boldsymbol{A}^T)^{-1}\cdot\boldsymbol{b} - \boldsymbol{b} > 0 \Rightarrow\\ (\boldsymbol{A}^T)^{-1}\cdot\boldsymbol{b} > \boldsymbol{b}/2 \Rightarrow \boldsymbol{p} > \boldsymbol{b}/2.\end{aligned}$$

This result, in the best of our knowledge, is also new for the Lotka-Volterra competition model of the species competition [17] and implies that the population of every species in an equilibrium, under the conditions of Theorem 4, is at least half of its carrying capacity.

□

V. The symmetric case

We consider the symmetric case of the model, where $a = a_{ii} = a_{ji}$, $c = c_i$, $n = n_i$ and $b = b_i$, for every $i, j \in \{1,\dots,M\}$. For the symmetric many-user *Gaussian interference channel* we have the following Theorem:

Theorem 5: For the symmetric many-user Gaussian interference channel if $a < 1/(M-1)$, then there is a Nash equilibrium equal to $\boldsymbol{p} = (\boldsymbol{A_{sym}})^{-1}\cdot\boldsymbol{b_{sym}}$, where

$$\boldsymbol{A_{sym}} = \begin{pmatrix} 1 & a & a & \dots & a\\ a & 1 & a & \dots & a\\ & & \dots & & \\ a & a & \dots & & 1\end{pmatrix} > 0, \text{ and}$$

$$\boldsymbol{b_{sym}} = \begin{pmatrix} 1/(2c)-n\\ 1/(2c)-n\\ \dots\\ 1/(2c)-n\end{pmatrix} > 0.$$

Proof 5:
By Theorem 4 we have that:

$$b = \frac{1}{2c} - n > \sum_{\substack{j=1\\j\neq i}}^{M} a\left(\frac{1}{2c}-n\right) = (M-1)a\left(\frac{1}{2c}-n\right)$$

$$\Leftrightarrow a < \frac{1}{M-1}.$$

□

Now, we will analyse the two-user case. In this case the utility functions of the two transmitters are equal to:

$$u_1(p_1,p_2) = \frac{1}{2}\ln\left(1+\frac{p_1}{ap_2+n}\right) - cp_1,$$

$$u_2(p_2,p_1) = \frac{1}{2}\ln\left(1+\frac{p_2}{ap_1+n}\right) - cp_2.$$

The powers that maximize the utilities are:

$$p_1 = b - ap_2, \tag{8}$$

and

$$p_2 = b - ap_1. \tag{9}$$

The best-response strategies are:

$$p_1 = \max\{b - ap_2, 0\},$$

and

$$p_2 = \max\{b - ap_1, 0\}.$$

A. Conditions of the uniqueness of a pure Nash equilibrium

In this section, we will describe the conditions of the uniqueness of a pure Nash equilibrium, since we know the existence by Theorem 2.

Theorem 6: In the two-user symmetric case, if $a < 1$, then the Nash equilibrium is unique.

Proof 6: In order to prove this theorem we study the uniqueness of the pure Nash equilibrium in three different cases.

- Case 1: $ab < b$. In this case, we have weak interference $(a < 1)$. We can see that the equation (8) is a line that crosses the axis of p_1 at the point $(b, 0)$ and the axis of p_2 at the point $\left(0, \frac{b}{a}\right)$. The equation (9) is also a line that crosses the axis of p_1 at the point $(\frac{b}{a}, 0)$ and the axis of p_2 at the point $(0, b)$. It is easy to see that in this case there is an exact cross point of the lines, so a unique pure Nash equilibrium.

- Case 2: $ab > b$. In this case, we have strong interference $(a > 1)$. There are three Nash equilibria, the cross point of the two lines, the point $(b, 0)$ and the point $(0, b)$. The point $(b, 0)$ is a pure Nash equilibrium, since the best-response strategy for the player 1 when the player 2 plays $p_2 = 0$ is the strategy $p_1 = b$, and the best-response strategy of the player 2 when the player 1 plays b is the strategy $p_2 = 0$. A symmetric argument hold for the point $(0, b)$.

- Case 3: $ab = b$. In this case, we have equal interference $(a = 1)$ and we have an infinite number of Nash equilibria. The sum of the powers in every Nash equilibrium is equal with b, so in this case any water-filling strategy that the total sum is equal to b is a Nash equilibrium.

□

Now in the case that the system of the equations (8) and (9) has a unique solution, we will give the strategy profiles of the players.

Theorem 7: The strategy profile $\left(\frac{b}{1+a}, \frac{b}{1+a}\right)$ is a Nash equilibrium.

Proof 7: This is the solution of the system of the equations (8) and (9). If $a = 1$, the lines (8) and (9) coincide, so we have an infinite number of Nash equilibria, but the sum of the powers in the strategy profile $\left(\frac{b}{1+a}, \frac{b}{1+a}\right)$ is equal to b, so this strategy profile is a Nash equilibrium.

□

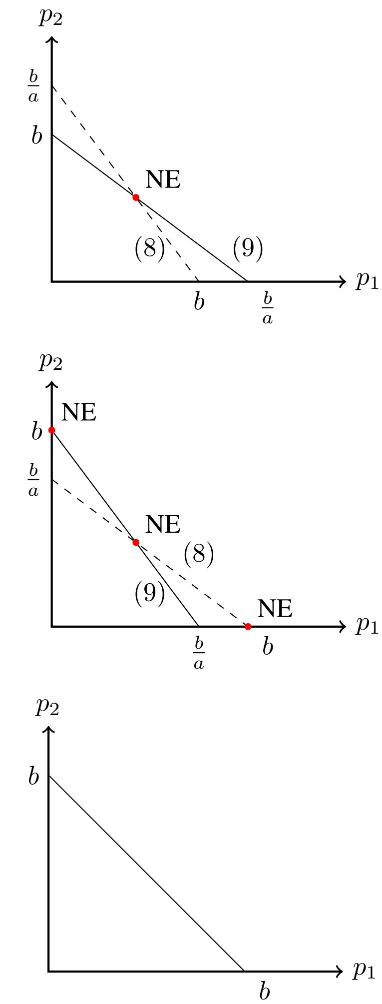

Fig. 3. Two-user symmetric case.

VI. Conclusions

In this work, we study the *Gaussian interference channel* from a game-theoretic perspective. We see how the introduction of the transmission cost affects the strategies of the transmitters. For the two-player case, we exhaustively analyse the possible scenarios, and we show the existence of at least one Nash equilibrium in any case. An important aspect of the model is the existence of best-response dynamics that lead to a Nash equilibrium point, making the Nash equilibria a rational prediction point of our problem. For the many-user case, we give sufficient conditions that lead to a Nash equilibrium, but we believe that most of the results of the two-user case can possibly be extended to the many-user case. Furthermore, it is very interesting the connection of a wireless communication problem with a problem of ecology, the population competition of different species described by the Lotka-Volterra model.

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