Opportunistic Scheduling in Two-Way Wireless Communication With Energy Harvesting

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***Abstract*—A two-way half-duplex communication model is considered, where two nodes want to exchange a fixed number of bits with each other, and both nodes are powered by energy harvesting (EH) sources. The problem of minimizing the sum of the time required to send the required bits in both the directions is considered. The model also includes the processing cost at each node, that models the power needed for nodes to stay powered on during transmission. In the offline setting, where the EH arrival profile is known non-causally, an iterative algorithm based on alternating maximization is shown to be optimal. In the more realistic setting of causal knowledge of the EH arrival profile, an online algorithm is shown to be optimal in terms of the competitive ratio and the optimal competitive ratio is shown to be 2.**

***Index Terms*—Online algorithms, energy harvesting, two-way communication.**

I. Introduction

Powering communication devices using renewable energy via harvesting energy (EH) from nature has attracted huge interest recently. It not only provides a green option, but also increases the lifetime of nodes. A canonical problem with EH is to minimize the time required to successfully transmit a fixed number of bits [1]. Optimal offline algorithms, where the energy arrival process is known non-causally, have been derived for many communication models for this canonical problem in [1], [6], [3]. The more realistic online case, where energy arrival process is only known causally, is relatively less well studied [5], [3]. Several important extensions of this canonical problem include EH powered receivers [3], multiple access channel [6], [7], [10], [12] etc.

Typically, under the energy harvesting (EH) paradigm, one-way communication is studied, where a transmitter is sending data to a receiver. A more natural communication paradigm is bi-directional or two-way, where two nodes wish to exchange data with each other.

Two-way EH model with processing and decoding costs has been considered in [4], however, with full-duplex capability, i.e., nodes can simultaneously transmit and receive. An optimal online policy for MAC channel with processing cost at each node and common energy source with independent and identically distributed (i.i.d.) energy arrival processes was derived in [13]. With full-duplex capability, the problem decomposes into two separate transmission time completion problems, one for each direction. In this paper, we consider a half-duplex two-way EH model, where the two nodes want to transmit a fixed number of bits to the other node, and the objective is to minimize the sum of the transmission time in each direction. The half-duplex restriction adds a new dimensions to the problem, since it entails finding the optimal choice of time-sharing between the nodes, since the EH profiles at the two nodes can be very different, in addition to efficiently exploiting the arriving energy.

The half-duplex two-way EH model has close connections with the two-user MAC problem [6], [7], however, the main difference is with respect to the strict time-sharing requirement in the half-duplex model compared to MAC, where two users can also transmit simultaneously. For a fixed time sharing schedule, MAC with EH has been studied in [10], however, the general problem considered in this paper, requires an optimization over the time-sharing schedule as well.

In addition to extending the model from one-way to two-way communication, we also include the processing costs at both the nodes, i.e., each node consumes a fixed amount of power to stay powered *on* during transmission, along with the transmission power used to transmit useful data. The processing cost critically changes the structure of the optimal algorithms, since transmitting at a slow rate leads to lot of energy being wasted in just keeping the nodes powered on for transmission. For a single node, energy harvesting model with processing cost has been considered in [11], but for a broadband communication system, modelled as K parallel sub-channels.

Similar to [3], for analytical tractability, we assume that both the nodes know each other's battery states exactly at all times. This can be accomplished with a small amount of feedback. Without this assumption, the problem leads to stochastic control problems with limited information that remain open.

In the first part of the paper, we consider the offline setting for the two-way problem with the processing cost. Similar to [6], we show that an iterative algorithm based on alternating maximization is an optimal algorithm. The basic algorithm and the ensuing proof are, however, different because of the inclusion of the processing cost in the rate function, and more importantly the strict time-sharing requirement, that changes the optimal power transmission profile in each direction. Instead of directly finding an algorithm to minimize the sum of the transmission times, we instead first find the departure

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region, that is the union of the largest pair of bits that can be sent in the two direction in a given amount of time. Finding the departure region is the 'dual' of the transmission time-minimization problem. Once the departure region is found, 'inverting' it directly gives the optimal solution to the transmission completion time-minimization problem.

In the second part of the paper, we consider the more realistic online setting, where the energy arrival profile is only known causally. For the online setting, we consider the commonly used design metric of competitive ratio, that is defined as the ratio of the time taken by any online algorithm and the optimal offline algorithm maximized over all possible energy arrival sequences. We propose a natural extension of the lazy online algorithm for one-way communication [3], and show that it is 2-competitive identical to [3]. Note that this is a worst case guarantee. Moreover, since no online algorithm can be better than 2-competitive even in the one-way communication [3], we conclude that the proposed online algorithm is optimal in terms of the competitive ratio.

II. System Model and Problem Formulation

Consider a model in which two energy harvesting nodes, say Node 1 and Node 2, have a two-way communication link with each other. Node i wishes to communicate B_i bits to the other node. The communication modality is half duplex, and hence the nodes must do time division multiplexing. In addition to the transmission power required to support datarates, each node requires a power P_r for transmission operation, i.e. an extra power P_r is required whenever a node transmits. We assume perfect battery-state co-ordination between the nodes, i.e. each node is aware of both the battery states at all times. The objective is to minimize the completion time by which both the receivers can completely gather the intended bits. The minimum transmission completion time is denoted as T_{min}.

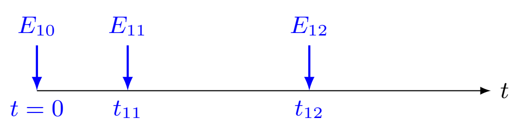

Fig. 1: Arrivals at transmitter

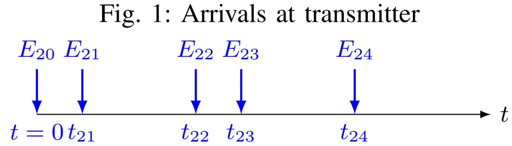

Fig. 2: Arrivals at receiver

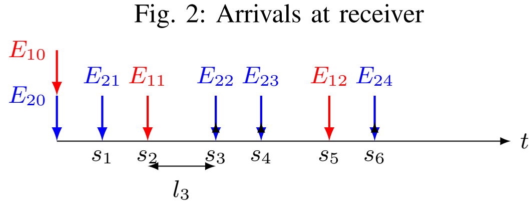

Fig. 3: Total arrivals

Let the energy arrivals at Node i be $\{E_{i0}, E_{i1} \cdots\}$ at time instants $\{0, t_{i1}, t_{i2} \cdots\}$. As shown in Fig 3, let $\{s_1, s_2, \dots\}$ be the arrival instants obtained by combining (superposition) the energy arrivals at both the nodes. We call the interval (s_{j-1}, s_j) as slot j, with $s_0 = 0$. Let $l_j, j \geq 1$ be the duration of slot j. Since the transmissions are in half duplex mode, each slot length $l_j, j \geq 1$ will be shared between the two users. Let $\theta_j \in [0,1]$ be the fraction of slot length l_j allotted to node 1, the remaining is given to node 2. Node k spends energy e_{kj} in slot j. Then the number of bits sent by user k in slot j is denoted as B_{kj}, where,

$$\begin{aligned} B_{1j} &= \theta_j l_j \log\left(1 - P_r + \frac{e_{1j}}{\theta_j l_j}\right), \\ B_{2j} &= (1-\theta_j) l_j \log\left(1 - P_r + \frac{e_{2j}}{(1-\theta_j) l_j}\right). \end{aligned} \tag{1}$$

where P_r in (1) takes care of the processing cost at each node. $B_k(t)$ is the total number of bits sent by user k till time t.

We assume a coordinated offline setting, i.e. both nodes have knowledge of their own and other node's energy arrivals. The transmission completion time-minimization problem can be formulated as follows.

$$\begin{aligned} \min_{e_1, e_2, \theta} \quad & T \\ \text{subject to} \quad & \sum_{i=1}^{n} e_{1i} \leq \sum_{i=0}^{n-1} E_{1i}, \\ & \sum_{i=1}^{n} e_{2i} \leq \sum_{i=1}^{n-1} E_{2i}, \\ & B_1(T) \geq B_1, \\ & B_2(T) \geq B_2, \\ & \theta_i \in [0,1]. \end{aligned} \tag{2}$$

where, $n = \max\{i : \sum_{j=1}^{i} l_j \leq T\}$. It turns out that the above formulation has some connections and applications to the two user multiple access channel (MAC). In particular, under time division multiple access, the users in a MAC equivalently operate in half-duplex. Thus, the formulation in (2) also solves the transmission completion time for a time shared MAC problem where the users have identical link conditions. Notice that the transmission completion time-minimization problem over a general Gaussian MAC (without time-sharing) can be realized by removing the dependence on θ of (2). However, the relation to two-way half duplex is then lost, and the solutions are totally unrelated. More specifically, the Gaussian MAC capacity region for a given set of powers is a pentagon [8] (see Fig 4), whereas under the restriction to time sharing, it becomes a continuous curve. This is depicted in Fig 4 below, where the dotted curve is obtained by time sharing, the sum-rate maximizing point under time-sharing is (b_1, b_2).

The MAC transmission completion time-minimization problem, without restriction to time sharing, was solved in [6]. The solution in [6] makes crucial use of the fact that $B_1(t) + \mu B_2(t)$ can be maximized by operating only at one of the corner points of the pentagon in each slot (see Fig 4). By employing a successive cancellation decoder, a natural decoupling of the

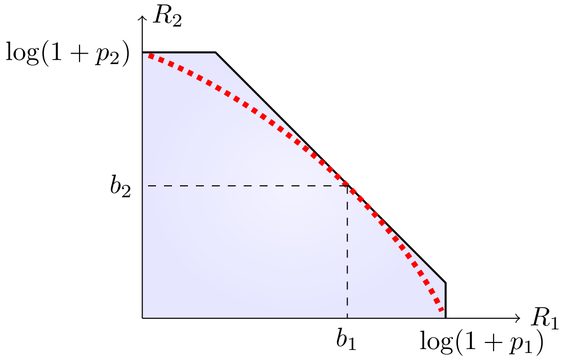

Fig. 4: MAC capacity region with receive powers p_1 and p_2

users can be obtained. However, under time-sharing, the nodes need to choose different operating points on the continuous curve based on the harvesting processes, thus complicating the problem.

The one-way transmission completion time-minimization result provides some useful insights for two-way transmission completion time-minimization problem. We describe the one-way transmission completion time-minimization result in the next section.

III. One-Way Communication

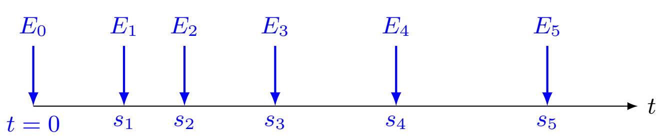

Fig. 5: Energy arrivals at transmitter

Consider a transmitter which needs to transmit B_0 bits to its receiver. Assume that E_0 amount of energy is available at time $t=0$ at the transmitter, and E_i energy is harvested at time $s_i, 1 \leq i \leq k$. Following the earlier notation, we call the interval (s_i, s_{i+1}) as slot $i+1$, and the duration $s_{i+1} - s_i$ as l_{i+1}. The transmitter employs a transmit power of p_i in slot i, which includes a constant processing power P_r to stay *on* during transmission. The transmitter can change its transmission rate depending on the available energy and the remaining bits. The one-way transmission completion time-minimization problem can be cast as,

$$\begin{aligned} \min_{p,l} \quad & T \\ \text{subject to} \quad & \sum_{i=0}^{n} p_{(i+1)} l_{(i+1)} \leq \sum_{i=0}^{n-1} E_i \\ & \sum_{i=1}^{n+1} \log(1 - P_r + p_i) l_i = B_0 \end{aligned} \tag{3}$$

Notice that the rate function $g(p_i) = \log(1 - P_r + p_i)$ gives the AWGN data-rate after deducting the staying ON cost P_r from the transmit power p_i, with $p_i \geq P_r$. The number of bits transmitted can also be written as a function of time, *viz*,

$$f(t) = \log\left(1 + \frac{E - P_r t}{t}\right) t. \tag{4}$$

where E is the energy spent in time t.

Lemma 1. *The rate function $f(t)$ formulated in* (4) *is strictly concave in t for a fixed value of $E > 0$.*

Proof: This can be shown by evaluating the second derivative

$$f''(t) = \frac{-E}{t^2((1 - P_r) + (E/t))} < 0. \tag{5}$$

■

Also, since $f(t)$ is zero at $t = 0$ as well as $t = \frac{E}{P_r}$, there exists some $t^* \in (0, \frac{E}{P_r})$ such that $f(t) \leq f(t^*), \forall t > 0$. The optimal solution to (3) will have the following structural properties.

Lemma 2. *For a given initial energy $E = E_0$, and no further energy arrival, the transmit power which maximizes the datarate or minimizes transmission time* (3) *only depends on P_r which is the power required to stay on during transmission.*

Proof: The maxima of $f(t)$ is found by differentiating,

$$\frac{\partial f(t)}{\partial t} = \log\left(1 + \frac{E}{t} - P_r\right) - \frac{E/t}{1 + E/t - P_r} = 0. \tag{6}$$

Replacing (E/t) by p, we get a value of p which depends only on P_r. ■

The above lemma suggests that unlike for the rate function $r(t) = \log(1 + \frac{E}{t})t$ used in [1] without accounting for *on* power, elongating transmission time as far as possible may not be optimal. For $r(t)$, lowering the power and increasing the transmission time increases the throughput. But for the rate function $f(t)$, which attains a maxima at some t, a longer transmission time may not be optimal. Optimal power for $f(t)$ is obtained by solving (6). Let this optimal power be denoted as p^*.

Lemma 3. *In the optimal policy for* (3)*, the non-zero values of the transmit powers are non-decreasing in slots, i.e. $p_1 \leq p_2 \leq \cdots$.*

Lemma 4. *Under the optimal policy for* (3)*, the non-zero values of the transmission powers remain unchanged in between energy harvests, i.e. power may increase only at energy harvesting instants.*

Remark 1. *The optimal policy may not have continuous transmission, i.e., the transmitter can be OFF for certain durations.*

Lemma 5. *In the optimal policy for* (3)*, whenever the transmit power increases from its last non-zero value, all the energy arrived till that instant is already exhausted.*

Proofs of the above three lemmas are based on concavity of rate function $f(t)$, and follows along similar lines as in [1].

Compared to the prior with $r(t)$ as rate funtion, for $f(t)$ we get the following different result.

Lemma 6. *In the optimal policy for* (3)*, the non-zero transmit powers are always greater than or equal to* p^*.

Proof. We prove this by contradiction. Consider an optimal policy S_1 which transmits with power $p_1 < p^*$ for duration l_1. Now consider another policy S_2 which is same as policy S_1 in all slots except the 1^{st} slot. Instead of p_1, policy S_2 transmits with power p^* for a duration of $\frac{p_1 l_1}{p^*}$. Note that this transmission duration is less than l_1 and hence is feasible. Also the energy spent by both the policies in the 1^{st} slot is same. However, from Lemma 2, the maximum throughput for $E = p_1 l_1$ is attained at $t = \frac{p_1 l_1}{p^*}$. Thus the number of bits transmitted by policy S_2 is greater than policy S_1. Now, by Lemma 3, we have $p_l \geq p^*, l \geq 1$. □

Lemma 5 states that there are specific instants at which the non-zero transmit power strictly increases. In fact, they are aligned with a subset of the energy arrival instants, let us denote the set of such instants as $i_k, k \geq 1$. Characterizing these points is sufficient to specify the optimal policy, which is done in the following theorem. We take $i_0 = 0$.

Theorem 1. *Let the optimal transmission completion time* T_{min} *occur in slot M, i.e., the interval* (s_M, s_{M+1})*. Then, an optimal policy for* (3) *satisfies*

$$\sum_{n=1}^{M+1} g(p_n) l_n = B_0 \tag{7}$$

$$\begin{aligned} i_n &= \underset{i: s_i < T}{\operatorname{argmin}} \left\{ \frac{\sum_{j=i_{n-1}}^{i-1} E_j}{s_i - s_{i_{n-1}}} \right\} \\ p_k &= \max\left\{ p^*, \frac{\sum_{j=i_{n-1}}^{i_n - 1} E_j}{s_{i_n} - s_{i_{n-1}}} \right\}, \forall k \in \{i_{n-1}+1, i_n\} \\ l_k &= \min\left\{ (s_k - s_{k-1}), \frac{E_{k-1}}{p^*} \right\}, \forall k \in \{i_{n-1}+1, i_n\}. \end{aligned} \tag{8}$$

Proof: The proof is given in Appendix A. ■

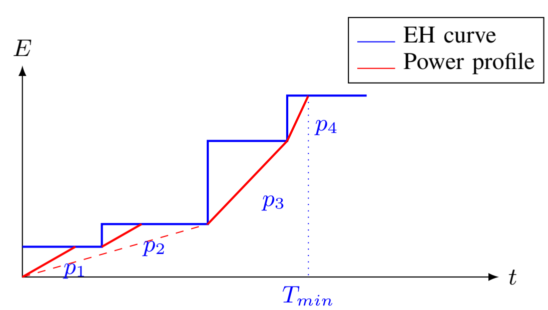

Fig. 6: Optimal power profile for single user

The above theorem concerns with finding the tightest piecewise linear curve that lies below the energy harvesting (EH) curve, this is shown in Fig 6. But if in any slot, the power given by the tightest linear curve is less than p^*, then transmission power for that slot is set at p^*. This is demonstrated in Slots 1 and 2 of Fig 6. Also, observe that the policy obtained by using Theorem 1 may have non-continuous transmissions. Nevertheless, one can defer the transmission times to create a *deferred* policy with the same completion time. By moving all the idle periods to the left, we can have continuous transmissions till completion.

Remark 2. *Among the class of deferred policies, there is a unique optimal policy. The uniqueness of the deferred policy follows along the same lines as Lemma 2 of [3].*

Theorem 1 enables the computation of the minimum completion time (3). For this, we need to first find the index M such that the optimal completion time T_{min} lies between s_M and s_{M+1}. This can be obtained by solving the dual problem of maximizing the throughput in a given time, instead of the original time minimization. The throughput maximization problem can be formulated as,

$$\begin{aligned} \max_{p,l} \quad & \sum_{j=1}^{k} g(p_j) l_j \\ \text{subject to} \quad & \sum_{i=0}^{k-1} p_{(i+1)} l_{(i+1)} \leq \sum_{i=0}^{k-1} E_i \end{aligned} \tag{9}$$

The idea is to compute maximum throughput $B(s_k)$ for every energy arrival instance s_k, till we find an index M, such that, $B(s_M) \leq B_0$ and $B(s_{M+1}) \geq B_0$. This is detailed in Appendix B.

IV. Two-way communication

Now using the results developed for one-way communication, the two-way problem can be solved as follows. Recall that our objective is to solve (2), by choosing appropriate time sharing parameters and transmit powers for the two nodes in each slot. As in the case of a MAC, it is more convenient to compute the weighted throughput till a time instant, and then infer the minimum transmission completion time information [6]. In particular, the rate-pairs possible till time instant T is called the *departure region* $\mathcal{D}(T)$. Let s_i denote the last energy arrival before time T. Then, for a fixed time sharing schedule $\{\theta\}$,

$$\begin{aligned} B_1(T) &= \sum_{j=1}^{i} g(p_{1j}) \theta_j l_j + g(p_{1(j+1)}) \theta_{j+1} (s_{i+1} - T) \\ B_2(T) &= \sum_{j=1}^{i} g(p_{2j}) \bar{\theta}_j l_j + g(p_{2(j+1)}) \bar{\theta}_{j+1} (s_{i+1} - T) \end{aligned} \tag{10}$$

where $\bar{\theta}_j = (1 - \theta_j)$. More formally, if $B_1(T)$ and $B_2(T)$ are the respective amounts of bits transmitted in two directions till time T using a power transmission strategy respecting the energy harvesting constraints, then $D(T)$ can be defined as follows.

Definition 1. For any fixed transmission duration T , the region $\mathcal{D}(T)$ is the union of all supported bit-pairs

$(B_1(T), B_2(T))$ under any feasible time sharing power policy over the duration $[0, T)$.

The departure region for any time $t = T$ can be found as,

$$\max_{e_1,e_2,\theta} \quad B_1(T) + \mu B_2(T), \tag{11}$$

Fig 7 demonstrates $D(T)$ for an example. Observe that every point on the boundary of the outermost curve corresponds to a different value of μ. Since the link is time-shared by the two users in each slot, changing the priority parameter μ leads to a different set of optimal values of $\{\theta_1, \theta_2 \dots\}$.

The departure region as given in (11) till the joint energy arrival s_i, i.e. $D(s_i)$, can be evaluated by,

$$\begin{aligned} &\max_{e_1,e_2,\theta} \left(\sum_{j=1}^{i} g\left(\frac{e_{1j}}{\theta_j l_j}\right)\theta_j l_j + \mu \sum_{j=1}^{i} g\left(\frac{e_{2j}}{\bar{\theta}_j l_j}\right)\bar{\theta}_j l_j \right) \\ &\text{subject to} \quad \sum_{i=1}^{n} e_{1i} \le \sum_{i=0}^{n-1} E_{1i}, \\ &\qquad\qquad\quad \sum_{i=1}^{n} e_{2i} \le \sum_{i=0}^{n-1} E_{2i}. \end{aligned} \tag{12}$$

The following structural results are obvious from the one-way communication results from Section III.

Lemma 7. *For each user, Lemma 3 and Lemma 5 holds true for an optimal policy solving* (12)*.*

The proof follows by observing that in their allotted times, each user essentially operates in a single user fashion. Thus, for a given set of time-fractions θ, the single user conditions have to be met. Notice however that the optimal θ is not only unknown, but also may lack any particular structure. This is the key bottleneck in the above optimization. Fortunately, the optimization turns out to be a convex program, which will be shown to have an efficient computational solution.

Lemma 8. *For a fixed μ and $T = s_i$ for some i, the objective function in* (12) *is jointly concave in e_{1j}, e_{2j} and $\theta_j, 0 \le j \le i$.*

Proof: The Hessian of (12) is negative semi-definite, which is sufficient for joint concavity. See Appendix C for details. ■

Observe that given the powers $(p_{1j}, p_{2j}), 1 \le j \le i$, finding the optimal time sharing parameter θ_j in (12) is indeed possible. This is because the objective is strictly concave in θ_j and hence there is a unique θ_j which maximizes the weighted throughput $B_{1j} + \mu B_{2j}$ as given by (1) for slot $j \in \{1, \cdots, i\}$. The proof for strict concavity of the objective with respect to θ_j is given in Appendix C.

On the other hand, given the time sharing parameter θ, the optimal power allocations can be found by employing an optimal one-way policy over the total allotted duration for each user. By limiting to the class of deferred policies, where the transmitters do not simultaneously idle, we can identify a unique set of power allocations. This paves the way for an alternation maximization framework to solve (12).

Algorithm ATMAX

1: For finding $D(s_i)$, initialize $\theta_j, 1 \le j \le i$ by uniform random values in $(0, 1)$. Note that initializing θ_j to any value in $(0, 1)$ is feasible.
2: Taking $T_1 = \sum_{j=1}^{i} \theta_j l_j$ and $T_2 = \sum_{j=1}^{i} (1 - \theta_j) l_j$, compute the respective optimal single user throughputs $B_1(T_1)$, $B_2(T_2)$ and the power and time duration vector.
3: Using the power and time duration vector from the last step, compute the unique set of θ_j which maximizes $\sum_{j=1}^{i} B_{1j} + \mu B_{2j}$. In particular, θ_j solves

$$\begin{aligned} &\log\left(1 - P_r + \frac{e_{1j}}{\theta_j l_j}\right) - \log\left(1 - P_r + \frac{e_{2j}}{(1-\theta_j) l_j}\right) + \\ &\frac{e_{2j}}{e_{2j} + (1 - P_r)(1 - \theta_j) l_j} - \frac{e_{1j}}{e_{1j} + (1 - P_r)\theta_j l_j} = 0 \end{aligned} \tag{13}$$

θ_j obtained by solving (13) is optimal as it is obtained by setting the derivative of the objective function to 0.
4: Go back to Step 2 if convergence is not achieved.

Convergence can be checked by measuring whether the values of θ_j saturates over consecutive iterations. Since there is a unique maximum in each direction of the alternating search, the procedure converges to the optimal solution [9].

Algorithm ATMAX can compute the maximum departure region $D(T)$ till any time instant T. We can progressively compute the departure region for different arrival instants till s_k where $\mathcal{D}(s_k)$ includes the required bit-rates (B_1, B_2) for the first time. Then, s_{k-1} and s_k are the upper and the lower bounds to the optimal transmission completion time T_{min}. To find T_{min} we can do a linear search in the interval (s_{k-1}, s_k), such that for $t = T_{min}$, (B_1, B_2) lies on the boundary of $\mathcal{D}(T_{min})$.

The optimal departure region for different arrival instants for an example situation is illustrated in Figure 7.

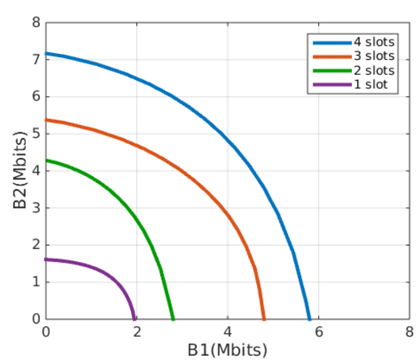

Fig. 7: Departure region for different arrival instants

For the illustration, we consider a band-limited additive white Gaussian noise channel, with bandwidth $W = 1MHz$ and noise power spectral density $N_0 = 10^{-19} W/Hz$, and a pathloss of -110dB. Then, we have $g(p) =$

$W\log\left(1-P_r+\frac{ph}{N_0W}\right)Mbps$. For the energy harvesting process, we assume that at times, $t=[0,2,4]$ we have energy harvested with amounts, $E_1=[6,7,4]mJ$ for the first user. At instants $t=[0,1,3]$, we have energy harvested with amounts $E_2=[8,7,5]mJ$ for the second user. For $P_r=0.5mW$, the departure regions $\mathcal{D}(s_1)$, $\mathcal{D}(s_2)$, $\mathcal{D}(s_3)$, $\mathcal{D}(s_4)$ are shown in Figure 7.

Until now we have considered offline policies in this paper. A more realistic setting is when both the users only have causal information about energy arrivals and make their decisions only dependent on that, this is called the online setting.

V. Online policy for two-way communication

The offline model to solve (2), considered availability of information about energy harvests a-priori at both the nodes. This is to some extent unrealistic assumption of non-causal information. The online model considers more realistic scenario where the two nodes have only causal information about the energy arrivals. With no information about future energy arrivals, our aim is to derive an online algorithm to solve (2). Let σ be the energy arrival sequence at both the nodes, and let $T_{\mathcal{A}}(\sigma)$ and $T_{\text{off}}(\sigma)$ be the total transmission completion time (2) taken by the online algorithm $\mathcal{A}$ and the optimal offline algorithm. We use the competitive ratio as the performance metric for online algorithms, that is defined for an online algorithm $\mathcal{A}$ as

$$\mu_{\mathcal{A}}=\max_{\sigma}\frac{T_{\mathcal{A}}(\sigma)}{T_{\text{off}}(\sigma)}, \tag{14}$$

where the maximum is over all possible energy arrival sequences σ, that can be chosen even adversarily. Thus, by definition, an online algorithm with low competitive ratio has good performance even against adversarial inputs.

Let $\varepsilon_i(t)$ denote the sum of the energy harvested at Node i till time t, $B_{i,rem}(t)$ denote the number of bits remaining to be transmitted for Node i after time t, and $E_{i,rem}(t)$ denote the energy remaining for Node i at time t.

Online Algorithm: Let $T_j, j=1,2$ be the minimum time such that

$$\log\left(1-P_r+\frac{\varepsilon_1(T_j)}{t}\right)t\geq B_j,\ \text{for some } t. \tag{15}$$

i.e. energy available at T_j is sufficient to transmit B_j bits. The algorithm starts transmission only at time $T_{start}=\min_j T_j$ for the node $i=\arg\min_j T_j$ with power P_{i1} which is the solution of,

$$\frac{\varepsilon_i(T_{start})}{P_{i1}}\log(1-P_r+P_{i1})=B_i. \tag{16}$$

Note that the starting condition (15), can be checked using only causal information about energy arrivals. Node i will continue transmission with P_{i1} till it completes its transmission of B_i bits or there is an energy arrival at either of the nodes. In the latter case, at every energy arrival instant s_k, the node which harvested energy that also satisfies (15) with $T_j=s_k$, starts transmission with power P_{ij} which is the solution of,

$$\frac{E_{i,rem}(s_k)}{P_{ij}}\log(1-P_r+P_{ij})=B_{i,rem}(s_k). \tag{17}$$

For example, if $T_1<T_2$, then node 1 starts transmitting first. Then if there is an energy arrival at Node 2 at time $s_k>T_1$, such that (15) is satisfied, then node 2 will start transmitting its bits, until a new energy arrival happens at Node 1 or till it finishes its own B_2 bits.

Theorem 2. *The proposed online algorithm is 2-competitive.*

Proof: The proof is given in Appendix D. ■

Remark 3. *An algorithm that finishes transmission in one direction and only then starts the transmission in second direction is 3-competitive.*

Theorem 3. *The proposed online algorithm is optimal.*

Proof. From [3], we know that even for one-way communication, no online algorithm can have better competitive ratio than $2-\delta$ for any $\delta>0$. □

VI. Conclusion

We have presented optimal communication schemes for a two-way half duplex model sharing an AWGN medium. The alternating maximization framework helps in characterizing the departure region, which enables the computation of the minimum transmission completion time for delivering all the data to the intended receivers. Incidentally, this also solves the problem of a MAC with time-sharing users under energy harvesting, without any additional receiver processing costs.

We are currently working on incorporating receiver processing cost along with the transmitter processing cost to the two-way model. Multi-node system employing relaying is of further interest.

Appendix A
Proof of Theorem 1

We need to prove the necessity and sufficiency of the structure stated in Theorem 1. The proof falls on similar lines as proof of Theorem 1 in [1]. We prove the necessity by contradiction. Consider an optimal policy which follows Lemmas 1, 2, 3, 4 and 5 but does not have a structure given in the theorem. Specifically, the policy is same as described in theorem 1 till the time instant $s_{i_{n-1}}$. However the power policy after this i.e., power p_n is not the smallest average power. We can find another $s_{i'}\leq s_M$ such that

$$p_n>\frac{\sum_{j=i_{n-1}}^{i'-1}E_j}{s_{i'}-s_{i_{n-1}}}. \tag{18}$$

Transmission with power p_n for duration of $(s_{i'}-s_{i_{n-1}})$ is not feasible, as $p_n(s_{i'}-s_{i_{n-1}})$ is greater than the energy harvested till $s_{i'}$ and the energy causality constraint is violated. Such a policy is therefore not feasible.

Now we prove that if a policy X with power vector **p** and time duration vector **l** has the above given structure, then it is optimal. We assume that another policy Y with **p'** and **l'** which has a lower completion time. We assume the policies are same except for slots $i_{n-1}+1$ to i_n. Policy X has power $p_{i_{n-1}+1},\ldots,p_{i_n}$ over these slots, while policy Y has power

${p_{i_{n-1}+1}}', \dots, p'_{i_n}$. In policy X, $p_{i_{n-1}+1} = p_{i_{n-1}+2} = \cdots = p_{i_n}$. With same energy being spent by both policies in these slots, if policy Y allocates different powers in these slots, due to concavity of the rate function, policy X transmits more bits than Y. If policy Y also allocates same power in all slots, but with smaller time duration, even then the throughput of policy X will be higher.

Thus a policy is optimal if and only if it has the structure given in (8).

Appendix B
Transmission Completion Time T for Single User

The algorithm to find maximum throughput till n^{th} arrival is as follows,

Initialize $k = 1$, $i_0 = 0$, $B(s_{i_0}) = 0$

1) Find i_k as mentioned in Theorem 1, which is the last energy arrival instance where all energy was used up.
2) Compute power for the slots $i_{k-1}+1$ to i_k, i.e., $p_{i_{k-1}+1}, \dots, p_{i_k}$, again as in Theorem 1.
3) Update
$$B(s_{i_k}) = B(s_{i_{k-1}}) + \sum_{j=i_{k-1}+1}^{i_k} \log(1 - P_r + p_j) l_j.$$
4) If $i_k = n$, then exit. Else update $k \to k+1$ and go to step 1.

The power policy obtained for maximum throughput till s_{M+1} can be modified to obtain the optimal policy for the transmission completion time.

Notice that for the optimal throughput policy till s_{M+1}, the powers $p_{i_{k-1}+1}, \dots, p_{i_k}$ are equal. Progressively increase each of these powers by the same amount till the following equation
$$\log\left(1 - P_r + \frac{\sum_{j=i_{k-1}}^{i_k-1} E_j}{t_1}\right) t_1 - (B_0 - B_{i_{k-1}}) = 0$$
is satisfied, or the value of i_k changes. In the latter case, repeat the power incrementing step starting from the newly obtained value of i_k.

Appendix C
Proof of Lemma 8

Consider the maximization problem (12). For the i^{th} slot, the objective is,
$$\mathcal{M} = \theta_i \log\left(1 - P_r + \frac{e_{1i}}{l_i\theta_i}\right) + (1-\theta_i)\log\left(1 - P_r + \frac{e_{2i}}{l_i(1-\theta_i)}\right). \tag{19}$$
The second derivative of the objective function with respect to to θ_i, e_{1i} and e_{2i} are,
$$\begin{aligned}
\frac{\mathrm{d}^2\mathcal{M}}{\mathrm{d}\theta_i^2} &= -\frac{{e_{1i}}^2}{\theta_i((1-P_r)l_i\theta_i + e_{1i})^2} \\
&\quad - \frac{{e_{2i}}^2}{(1-\theta_i)((1-P_r)l_i(1-\theta_i) + e_{2i})^2}, \\
\frac{\mathrm{d}^2\mathcal{M}}{\mathrm{d}e_{1i}^2} &= -\frac{\theta_i}{((1-P_r)l_i\theta_i + {e_{1i}}^2)}, \\
\frac{\mathrm{d}^2\mathcal{M}}{\mathrm{d}e_{2i}^2} &= -\frac{(1-\theta_i)}{((1-P_r)l_i(1-\theta_i) + e_{1i})^2}.
\end{aligned} \tag{20}$$
Since e_{1i}, e_{2i} and θ_i are non-negative, the objective is concave in all the three variables, as the derivatives are strictly negative. To prove joint concavity of the objective function, we need to show that the Hessian of the objective function is negative semi-definite. Hessian matrix, in terms of $\theta_i,\ e_{1i},\ e_{2i}$ for (19) is,
$$H = \begin{bmatrix} \alpha & a & b \\ a & \beta & 0 \\ b & 0 & \gamma \end{bmatrix}$$
where, from (20),
$$\alpha = \frac{\mathrm{d}^2\mathcal{M}}{\mathrm{d}\theta_i^2}, \beta = \frac{\mathrm{d}^2\mathcal{M}}{\mathrm{d}e_{1i}^2}, \gamma = \frac{\mathrm{d}^2\mathcal{M}}{\mathrm{d}e_{2i}^2}. \tag{21}$$
$$\begin{aligned}
a &= \frac{\partial^2\mathcal{M}}{\partial\theta_i\partial e_{1i}} = \frac{e_{1i}}{(e_{1i} + (1-P_r)l_i\theta_i)^2}, \\
b &= \frac{\partial^2\mathcal{M}}{\partial\theta_i\partial e_{2i}} = -\frac{e_{2i}}{(e_{2i} + (1-P_r)l_i(1-\theta_i))^2}.
\end{aligned}$$
Observe that the cross term $\frac{\partial^2\mathcal{M}}{\partial e_{1i}\partial e_{2i}} = 0$. The conditions for H to be negative semi-definite are $(-1)^k\Delta_k \geq 0$, where Δ_ks are the k^{th} order principal minors of H.

The first order minors, Δ_1, $\alpha \leq 0, \beta \leq 0, \gamma \leq 0$. The second order minors Δ_2 are $\alpha\beta - a^2 =$
$$\frac{{e_i}^2\theta_i}{(1-\theta_i)(e_{1i} + (1-P_r)l_i\theta_i)^2(e_{2i} + (1-P_r)l_i(1-\theta_i))^2} \geq 0$$
$$\alpha\gamma - b^2 =$$
$$\frac{{e_{1i}}^2(1-\theta_i)}{\theta_i(e_{1i} + (1-P_r)l_i\theta_i)^2(e_{2i} + (1-P_r)l_i(1-\theta_i))^2} \geq 0$$
$$\beta\gamma =$$
$$\frac{\theta_i(1-\theta_i)}{(e_{1i} + (1-P_r)l_i\theta_i)^2(e_{2i} + (1-P_r)l_i(1-\theta_i))^2} \geq 0.$$
and the determinant $\alpha\beta\gamma - a^2\gamma - b^2\beta = 0$. This proves that H is negative semi-definite and hence the objective function is jointly concave in θ_i, e_{1i} and e_{2i}.

Appendix D
Proof of Theorem 2

Consider the optimal offline transmission completion times for both the nodes,
$$T_{1,\text{off}} = \sum_{j=1}^{M+1} \theta_j l_j \text{ and } T_{2,\text{off}} = \sum_{j=1}^{M+1} (1-\theta_j) l_j,$$
where the definition of M, θ and l follow from Section II. For Node i, let $T_{i,s}$ be the time at which Node i begins to transmit its bits for the first time with the proposed online policy.

Claim: $T_{i,s} < T_{i,\text{off}}$ for both $i = 1, 2$.

Proof. We prove this by contradiction. Recall that with the proposed online policy, any node begins to transmit its bits only at the time when a new energy arrival happens. Let that time be $T_{i,s} = s_k$. To prove the contradiction, assume that $T_{i,s} > T_{i,\text{off}}$.

From the definition of the online algorithm,

$$\begin{aligned} \log\left(1 - P_r + \frac{\varepsilon_i(s_{k-1})}{t}\right)t &< B_i \quad \forall\, t \text{ and} \\ \log\left(1 - P_r + \frac{\varepsilon_i(s_k)}{t}\right)t &> B_i \quad \text{for some } t. \end{aligned} \tag{22}$$

For the optimal offline policy, $B_i = \sum_{j=1}^{M+1} \theta_j l_j \log(1 - P_r + p_{ij})$. Let $g(p_{ij}) = \log(1 - P_r + p_{ij})$. Then, by Jenson's inequality,

$$\begin{aligned} \sum_{j=1}^{M+1} \theta_j l_j g(p_{ij}) &\leq (\textstyle\sum_{j=1}^{M+1} l_j) g\left(\frac{\sum_{j=1}^{M+1} p_j l_j \theta_j}{\sum_{j=1}^{M+1} l_j}\right), \\ &\stackrel{(a)}{\leq} (\textstyle\sum_{j=1}^{M+1} l_j) g\left(\frac{\varepsilon_i(T_{i,\text{off}})}{\sum_{j=1}^{M+1} l_j}\right), \\ &\stackrel{(b)}{<} (\textstyle\sum_{j=1}^{M+1} l_j) g\left(\frac{\varepsilon_i(s_{k-1})}{\sum_{j=1}^{M+1} l_j}\right). \end{aligned} \tag{23}$$

(a) follows since $\sum_{j=1}^{M+1} p_j l_j \theta_j \leq \varepsilon_i(T_{i,\text{off}})$, the total energy arrived till then, and (b) follows because of our hypothesis that the optimal offline policy finishes at time $T_{i,\text{off}}$ that is less than s_k. Combining (22) and (23), we get

$$\sum_{j=1}^{M+1} \theta_j l_j \log(1 - P_r + p_{ij}) < B_i.$$

Thus, if $T_{i,s} < T_{i,\text{off}}$, then the transmission of B_i bits is not feasible with the optimal offline policy by time $T_{i,\text{off}}$. $\square$

Let $b_{i,k}$ be the last energy arrival instant at Node i before the optimal offline algorithm finishes its transmission at time $T_{i,\text{off}}$. It is easy to argue that the worst case input for the proposed online algorithm that maximizes its competitive ratio is when the online algorithm can begin its transmission only at time $b_{i,k}$. We consider this scenario henceforth to upper bound the competitive ratio of the proposed online algorithm.

Let the online algorithm transmit with power P_{ik} at $T_{i,off}^{-1}$ [1], and $b_{i,k}$ be the time when the transmission with power P_{ik} started. Using the claim that $T_{i,s} < T_{i,\text{off}}$ for $i = 1, 2$, we know that $P_{ik} > 0$ and $b_{i,k} < T_{i,\text{off}}$ for some i.

Since the optimal offline algorithm can use only the energy that has arrived till time $b_{i,k}$, necessarily the following has to be satisfied

$$B_i \leq T_{i,\text{off}}\, g\left(\frac{\varepsilon_i(b_k)}{T_{i,\text{off}}}\right). \tag{24}$$

Let $T_{i,on}$ be the transmission completion (actual time for which B_i bits are transmitted) time for the online policy of Node i. Then $T_{i,on}$ can be upper bounded by the completion time T'_{on} of a policy which starts from $b_{i,k}$ and does not use any energy arrival after $b_{i,k}$

$$T'_{on}\, g\left(\frac{\varepsilon_i(b_k)}{T'_{on}}\right) = B_i. \tag{25}$$

[1] Just before $T_{i,\text{off}}$

From (24) and (25)),

$$T'_{on}\, g\left(\frac{\varepsilon_i(b_k)}{T'_{on}}\right) \leq T_{i,\text{off}}\, g\left(\frac{\varepsilon_i(b_k)}{T_{i,\text{off}}}\right). \tag{26}$$

Thus, $T_{i,on} \leq T'_{on} \leq T_{i,\text{off}}$. Note that even though the transmission is interspersed for the two nodes, even then the time at which both the nodes finish their transmission is $T_{1,s} + T_{1,on} + T_{2,s} + T_{2,on}$. Hence, the competitive ratio of the proposed online algorithm can be upper bounded by

$$\begin{aligned} \mu = \max_\sigma \left(\frac{T_{\mathcal{A}}}{T_{\text{off}}}\right) &= \frac{T_{1,s} + T_{1,on} + T_{2,s} + T_{2,on}}{T_{1,\text{off}} + T_{2,\text{off}}} \\ &\leq \frac{2(T_{1,\text{off}}) + 2(T_{2,\text{off}})}{T_{1,\text{off}} + T_{2,\text{off}}} = 2. \end{aligned} \tag{27}$$

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