Pricing Competition of Rollover Data Plan

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***Abstract*—Today, many mobile network operators (MNOs) provide data services through a three-part tariff data plan, which involves a fixed subscription fee, a data cap, and a per-unit fee for the data over-usage exceeding the data cap. To increase their market competitiveness, MNOs have been trying to provide more *time flexibility* in the data plans. One of such innovations is the *rollover data plan*, which allows a subscriber to use the unused data of the previous month in the current month. Depending on the consumption priority of the rollover data, different rollover data plans can have different levels of time flexibility. The interactions among multiple MNOs offering rollover data plans, however, are quite complicated and sometimes counter-intuitive. To examine this issue, in this paper we build a simple market model of two MNOs competing to serve the same pool of heterogeneous users. We formulate the market competition as a two-stage game: in Stage I, the MNOs simultaneously decide their pricing strategies of their chosen data mechanisms; In Stage II, users make their subscription decisions among the two MNOs. We characterize the sub-game perfect equilibrium (SPE) of the two-stage game through backward induction. Comparing with a monopoly market where a better time flexibility always improves the MNO's profit, our analysis reveals a rather complicated story in the duopoly market: (i) with a mild competition, the stronger MNO will increase both MNOs' profits by adopting a data plan with a better time flexibility, while the weaker MNO will decrease both MNOs' profits by adopting a data plan with a better time flexibility; (ii) with a fierce competition, any MNO will increase its profit and decrease the competitor's profit by adopting a data plan with a better time flexibility.**

I. Introduction

A. Background and Motivation

Mobile Network Operators (MNOs) provide various mobile data services and profit from these services through the careful design of data plans. A traditional and widely-used data plan is the *three-part tariff*, which involves a monthly one-time subscription fee, a data cap that is free to use (with the paid subscription fee), and a linear price for any data over-usage exceeding the data cap. To maintain their competitiveness in today's telecommunication market, many MNOs are experimenting various innovations on such a three-part tariff data plan. For example, some MNOs have introduced innovative data mechanisms with *time flexibility*. A typical example of such data mechanisms is the *rollover data plan* [1], which allows a subscriber to use the unused data from the previous month in the current month.

The time flexibility in the rollover scheme is attractive to the mobile users, since it helps balance the possible *wasting data* within the data cap and the possible *overage usage* above the data cap when the user cannot accurately estimate his future data demand. However, MNOs have different attitudes towards the rollover data plan. For example, some MNOs are still using the traditional data plans with no time flexibility. Even among those MNOs that have chosen to offer rollover data plans, their plans can be quite different in terms of the level of time flexibility, which are based on the consumption priority between the rollover data and the monthly data cap. For example, AT&T specifies that the rollover data from the previous month will be used only after the current monthly data cap is fully used up [2], while China Mobile specifies that the rollover data from the previous month will be used before consuming the current monthly data cap [3]. These observations motivate us to ask the following two questions in a competitive market:

- Whether an MNO can benefit from a rollover data plan?
- If yes, how such a benefit changes with the time flexibility of the adopted rollover data plan?

As the first step towards addressing the above questions, in this work, we focus on analyzing a simple duopoly market with two MNOs. We will leave the study of the more general oligopoly market in the future work.

B. Related Literature

The optimal design of mobile data plan has been extensively studied in [4]–[10]. Among these existing studies, some of them considered the competitive market and studied the pricing competition among MNOs. For example, Gibbens *et al.* in [4] focused on the Paris Metro pricing scheme and analyzed the competition between two ISPs who may offer multiple service classes. Ma *et al.* in [7] focused on the usage-based scheme and considered the congestion-prone scenario with multiple MNOs. Duan *et al.* in [9] studied the timing problems related to users' subscription change in the competitive market. However, none of these studies considered the time flexibility of mobile data plan.

There are few theoretical studies regarding the rollover data plans with time flexibility. Zheng *et al.* in [1] found that moderately price-sensitive users can benefit from subscribing to the rollover data plan compared with the traditional data plan. Wei *et al.* in [11] analyzed the impact of different rollover period lengths from the MNO's perspective. In our previous work [12]–[15], we studied how to optimize the data plan with time flexibility for a monopoly MNO under the single-cap and multi-cap schemes. However, none of the above works considered the rollover data plan in the competitive market, which motivates our study in this work.

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C. Key Results and Contributions

In this work, we analyze different data mechanisms in a coherent general framework: the traditional data plan and the rollover data plans offered by AT&T and China Mobile. Under such a framework, we study the duopoly competition among two MNOs systematically. The main results and key contributions are summarized as follows:

- *Duopoly Competition for Mobile Data Plan with Time Flexibility:* To the best of our knowlege, this is the first work studying MNOs' duopoly competition considering their data plans with time flexibility.
- *Two-Stage Competition Model:* We formulate the MNOs' competition as a two-stage game, considering heterogeneous MNOs (in terms of Quality-of-Service (QoS) and operational cost) and heterogeneous users (in terms of data valuations).
- *Impact of Time Flexibility:* In a duopoly market with a mild competition, the stronger MNO will increase both MNOs' profits by adopting a data plan with a better time flexibility, while the weaker MNO will decrease both MNOs' profits by adopting a data plan with a better time flexibility. However, in a duopoly market with a fierce competition, any MNO will increase its profit and decrease the competitor's profit by adopting a data plan with a better time flexibility.

The rest of the paper is organized as follows. Section II introduces the system model. Section III studies the two-stage competition model. Section IV presents the numerical results. Finally, we conclude this paper in Section V.

II. System Model

We consider two competitive MNOs who provide mobile data services for heterogeneous users. Each MNO-n ($n = 1, 2$) offers a mobile data plan specified by a tuple $\mathcal{T}_n = \{Q_n, \Pi_n, \pi_n, \kappa_n\}$: a subscriber of MNO-$n$ needs to pay a monthly subscription fee Π_n for the data cap Q_n, and a usage-based overage fee π_n for each unit of data consumption over the data cap. Here $\kappa_n \in \{0, 1, 2\}$[1] denotes three data mechanisms that offer different levels of time flexibility (to be discussed in Section II-A).

In practice, an MNO can offer several data plans with different data caps. The optimization of multi-cap data plan is rather complicated even in the single MNO case, and we have reported some preliminary results recently in [14][15]. To obtain the clear insights of the impact of time flexibility in a competitive market, we assume that Q_n for each MNO-n is fixed (i.e., not part of the optimization decisions).[2]

As shown in Fig. 1, we formulate the system interactions as a two-stage game: in Stage I, MNOs simultaneously determine the corresponding prices $s_1 = \{\Pi_1, \pi_1\}$ and $s_2 = \{\Pi_2, \pi_2\}$; In Stage II, users make their subscription choices.

[1]Here $\kappa_n \in \{0, 1, 2\}$ is a nominal notation (not an ordinal variable) used to label three different data mechanisms.

[2]This is usually the practical case, since most MNOs use integral data caps for simplicity.

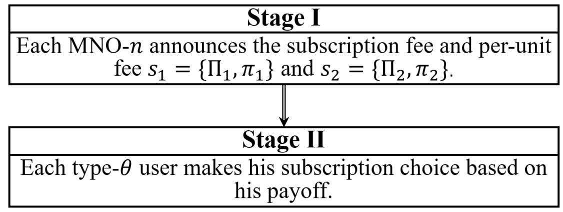


Fig. 1: System model.

Next, we will introduce the data mechanisms with time flexibility in Section II-A, and derive users' payoffs and MNO's profit in Section II-B and Section II-C, respectively. We will also study the monopoly market as a benchmark in Section II-D. Table I summarizes the key notations.

A. Three Data Mechanisms

As mentioned above, we denote the data mechanism choice by $\kappa_n \in \{0, 1, 2\}$. The key differences among the three data mechanisms are the special data and the consumption priority [13]. Specifically, the special data is the rollover data "inherited" from the previous month, which enlarges a subscriber's *effective data cap* within which no overage fee is involved [12]. In other words, at the beginning of a particular month the subscriber may have some special data from the previous month. The consumption priorities of the special data and the current monthly data cap further affect how much the effective data cap can be enlarged.

Here we let τ denote a user's rollover data from the previous month, and $Q_n^e(\tau)$ denote the *effective cap* of the current month under $\mathcal{T}_n = \{Q_n, \Pi_n, \pi_n, \kappa_n\}$. Specifically,

- $\kappa_n = 0$ denotes the traditional data plan without special data, i.e., $\tau = 0$. Thud, the effective cap of each month is $Q_n^e(\tau) = Q_n$.
- $\kappa_n = 1$ denotes the rollover data plan offered by AT&T. The rollover data $\tau \in [0, Q_n]$ from the previous month is consumed *after* the current monthly data cap. Thus, the effective cap of the current month is $Q_n^e(\tau) = \tau + Q_n$;
- $\kappa_n = 2$ denotes the rollover data plan offered by China Mobile. The rollover data $\tau \in [0, Q_n]$ from the previous month is consumed *prior* to the current monthly data cap. Thus, the effective cap of the current month is $Q_n^e(\tau) = \tau + Q_n$;

B. Users' Payoffs

Next we introduce three characterizations of a user: data demand d, data valuation θ, and network substitutability β. Based on these, we derive a user's expected payoff.

First, we model a user's data demand d as a discrete random variable with a probability mass function $f(d)$, a mean value of $\bar{d}$, and a finite integer support $\{0, 1, 2, ..., D\}$. Here the data demand is measured in the minimum data unit, e.g, 1KB or 1MB according to the MNO's billing practice.

Second, we denote θ as a user's utility from consuming one unit of data, i.e., the user's data valuation.

Third, we model a user's change of data consumption behavior after he exceeds the *effective cap*. Although the user will continue consuming data, he will reduce his consumption by relying more heavily on alternative networks (such as office or home Wi-Fi networks). Following [16], we characterize such a behavior by user's network substitutability $\beta \in [0,1]$, which denotes the fraction of overage usage shrink on average. For example, $\beta = 0.8$ means that on average 80% of the user's portion of data demand above the effective cap will be reduced. A larger β represents more overage usage reduction (i.e., a better network substitutability).

In the following, we normalize the total population size to be one and assume that users are homogeneous in the data demand distribution $f(d)$ [3] and network substitutability β [4], but are heterogeneous in the data valuation θ. Hence we can characterize each user according to his type θ, and the distribution of θ of the entire user population has a PDF $h(\theta)$ and CDF $H(\theta)$. Therefore, under MNO-n's data plan $\mathcal{T}_n = \{Q_n, \Pi_n, \pi_n, \kappa_n\}$, *a type-$\theta$ user's payoff* with data demand d and an effective cap $Q_n^e(\tau)$ is

$$\begin{aligned} U_n(\mathcal{T}_n, \theta, d, \tau) = \rho_n\theta(d - \beta\,[d - Q_n^e(\tau)]^+) \\ - \pi_n(1-\beta)\,[d - Q_n^e(\tau)]^+ - \Pi_n, \end{aligned} \tag{1}$$

where $[x]^+ = \max\{0, x\}$, $\rho_n \in (0,1]$ is a utility discount factor that depends on the MNO's quality of services (QoS), $\rho_n\theta(d - \beta[d - Q_n^e(\tau)]^+)$ is the user's utility from data consumption, $\pi_n(1-\beta)[d - Q_n^e(\tau)]^+$ is the user's overage payment due to the data usage exceeding the effective cap, and Π_n is the monthly subscription fee. Here d and τ are two random variables that change in each month, and we will take the expectation over them to get *a type-θ user's expected monthly payoff* under $\mathcal{T}_n$ as follows:

$$\begin{aligned} \bar{U}_n(\mathcal{T}_n, \theta) =& \mathbb{E}_{d,\tau}\big\{U_n\left(\mathcal{T}_n, \theta, d, \tau\right)\big\} \\ =& \rho_n\theta[\bar{d} - \beta A_{\kappa_n}(Q_n)] \\ & - \pi_n(1-\beta)A_{i_n}(Q_n) - \Pi_n, \end{aligned} \tag{2}$$

where $A_{\kappa_n}(Q_n)$ is the user's *expected monthly overage data usage* under $\mathcal{T}_n$, which is given by

$$\begin{aligned} A_{\kappa_n}(Q_n) =& \mathbb{E}_{d,\tau}\big\{[d - Q_n^e(\tau)]^+\big\} \\ =& \sum_\tau\sum_d\,[d - Q_n^e(\tau)]^+\, f(d)p_{\kappa_n}(\tau). \end{aligned} \tag{3}$$

Note that in (2), the difference of the three data mechanisms is entirely captured by $A_{\kappa_n}(\cdot)$. Specifically, $p_{\kappa_n}(\cdot)$ represents the distribution of the subscriber's rollover data under data mechanism κ_n. Furthermore, the *better time flexibility* a data mechanism offers, the *less overage usage* incurred by its subscribers under the same data cap Q, i.e., $A_\kappa(Q)$ is smaller. Our previous analysis in [13] shows that

$$A_0(Q) > A_1(Q) > A_2(Q), \forall\; Q \in (0, D), \tag{4}$$

[3]To make the analysis tractable, here we assume all users follow the same demand distribution $f(d)$. We would like to emphasize that the realized demands of different users in the same month are still different. We will consider heterogeneous demand distributions in the future work.

[4]The market survey on the telecommunication market in mainland China shows that most people would shrink $85\% \sim 95\%$ of the overage usage [13], which means that users do not differ significantly in terms of the network substitutability.

TABLE I: Key notations.

Symbol		Physical Meaning
MNO	Q_n	Monthly data cap of MNO-n.
	Π_n	Monthly subscription fee of MNO-n.
	π_n	Per-unit fee of MNO-n.
	κ_n	Data mechanism of MNO-n.
	ρ_n	Quality of service (QoS) of MNO-n.
	c_n	Marginal cost of MNO-n.
	ψ_n	Cost-quality ratio, i.e., $\psi = c_n/\rho_n$.
	σ_n	Threshold user type of MNO-n.
	$\tilde{\sigma}$	Neutral user type of the two MNOs.
	W_n	Expected Profit of MNO-n.
User	θ	A user's valuation for consuming one unit data.
	β	Users' common network substitutability.
	α_n	Expected data usage of an MNO-n's subscriber.
	$\bar{U}_n$	A user's expected payoff by subscribing to MNO-n.

which means that the subscriber can incur the least overage data consumption under the data mechanism 2, while the most under the data mechanism 0. *Therefore, we say that the data mechanism 2 offers the best time flexibility, while the data mechanism 0 offers the worst.*

Later on, we will study two MNOs' competition given their fixed data caps Q_1 and Q_2. To facilitate our later analysis, here we further define α_n as

$$\alpha_n \triangleq \bar{d} - \beta A_{\kappa_n}(Q_n), \tag{5}$$

which represents a user's *expected monthly data consumption* under MNO-n's data plan $\mathcal{T}_n$. Substitute (5) into (2), we write *a type-θ $\mathcal{T}_n$ subscriber's expected monthly payoff* as

$$\bar{U}_n(\mathcal{T}_n, \theta) = \rho_n\alpha_n\theta - \pi_n\left(\beta^{-1} - 1\right)\left(\bar{d} - \alpha_n\right) - \Pi_n, \tag{6}$$

where $\rho_n\alpha_n$ represents the user's utility increment for per-unit valuation increment under the subscription of MNO-n. In the following analysis, we will directly use (6). To emphasize the dependence on pricing strategy and data mechanism, sometimes we will write $\bar{U}_n(\mathcal{T}_n, \theta)$ as $\bar{U}_n(\kappa_n, s_n, \theta)$.

C. MNOs' Profits in Competition

Next we consider the MNOs' competition in the duopoly market, and derive their profits given their data mechanisms $\kappa = \{\kappa_1, \kappa_2\}$ and the pricing strategies $\boldsymbol{s} = \{s_1, s_2\}$.

The MNO's revenue obtained from a subscriber includes the subscription fee and the overage payment. Therefore, the *expected revenue of MNO-n from a type-θ subscriber* is

$$\bar{R}_n(\kappa_n, s_n, \theta) = \pi_n\left(\beta^{-1} - 1\right)\left(\bar{d} - \alpha_n\right) + \Pi_n, \tag{7}$$

where $\pi_n(\beta^{-1} - 1)(\bar{d} - \alpha_n)$ is the subscriber's expected overage payment, and α_n depends on the data mechanism κ_n. Therefore, the *expected revenue of MNO-n from all its subscribers* is

$$R_n(\boldsymbol{\kappa}, \boldsymbol{s}) = \int_{\Phi_n(\kappa, \boldsymbol{s})} \bar{R}_n(\kappa_n, s_n, \theta)h(\theta)\mathrm{d}\theta, \tag{8}$$

where $\Phi_n(\boldsymbol{\kappa}, \boldsymbol{s}) \subseteq [0, \theta_{\max}]$ denotes the subscribers of MNO-n under data mechanism $\boldsymbol{\kappa}$ and pricing strategy $\boldsymbol{s}$. We will further discuss it in Section III.

As for the MNO's cost, we focus on its operational expenditure (OpEx) that is proportional to the total data consumption of their subscribers. Intuitively, the more data consumed, the MNO needs to use more power resource or invest in more spectrum to guarantee throughput in QoS requirement. Specifically, the *total data consumption of MNO-n's subscribers* is

$$L_n(\kappa, \boldsymbol{s}) = \int_{\Phi_n(\kappa,\boldsymbol{s})} \alpha_n h(\theta) \mathrm{d}\theta, \tag{9}$$

where α_n defined in (5) represents a user's expected data consumption under $\mathcal{T}_n$. We denote c_n as MNO-n's marginal cost from unit data consumption.[5] Accordingly, the *total cost of MNO-n* is

$$C_n(\kappa, \boldsymbol{s}) = L_n(\kappa, \boldsymbol{s}) \cdot c_n. \tag{10}$$

Finally, *MNO-n's profit* is defined as the difference between its revenue and cost, i.e.,

$$W_n(\kappa, \boldsymbol{s}) = R_n(\kappa, \boldsymbol{s}) - C_n(\kappa, \boldsymbol{s}). \tag{11}$$

Now we have formulated MNOs' profits in the duopoly market, and introduced MNOs' two orthogonal characteristics: the QoS parameter ρ_n as in (6) and the marginal cost parameter as in (10). Later in Section III, we will explore the impact of these two parameters on the MNO's pricing decisions.

D. Monopoly Market as Benchmark

Before analyzing the duopoly market, we first analyze a monopoly market with only MNO-n as a benchmark.

In such a monopoly market, the type-θ user will subscribe to MNO-n if and only if he can achieve a non-negative payoff, i.e., $\bar{U}_n(\kappa_n, s_n, \theta) \geq 0$. Therefore, the market share of MNO-n is $\Phi_n = [\sigma_n, \theta_{\max}]$, where σ_n is the threshold user type of MNO-n, defined as follows:

***Definition** 1 (Threshold User Type):* The MNO-n's threshold user type $\sigma_n \in [0, \theta_{\max}]$ corresponds to the user who achieves a zero expected payoff, i.e., $\bar{U}_n(\kappa_n, s_n, \sigma_n) = 0$. Thus, σ_n is

$$\sigma_n(\kappa_n, s_n) = \frac{\pi_n(\beta^{-1} - 1)(\bar{d} - \alpha_n) + \Pi_n}{\rho_n \alpha_n}. \tag{12}$$

Therefore, the MNO-n's expected profit under the data mechanism κ_n and the pricing strategy s_n is

$$\begin{aligned} & W_n(\kappa_n, s_n) \\ =& \rho_n \alpha_n \left[\sigma_n(\kappa_n, s_n) - \frac{c_n}{\rho_n}\right] \Big[1 - H\big(\sigma_n(\kappa_n, s_n)\big)\Big], \end{aligned} \tag{13}$$

where $H(\cdot)$ is the CDF of users' data valuation θ. Note that the profit of MNO-n is negative if $\sigma_n(\kappa_n, s_n) < c_n/\rho_n$. Moreover, the MNO does not have any subscribers if its cost-QoS ratio c_n/ρ_n is greater than users' highest data valuation $\theta_{\max}$. With this observation, we will assume $c_n/\rho_n < \theta_{\max}$ for both $n \in \{1, 2\}$ to avoid the trivial case.

We characterize MNO-n's profit-maximizing pricing strategy and data mechanism in Lemma 1 and Lemma 2, respectively. Here the superscript "MP" means "monopoly".

[5] Such a linear-form cost has been widely used to model an operator's operational cost (e.g., [17]–[19]).

***Lemma** 1:* For a monopoly MNO-n, given the data mechanism κ_n, it maximizes its profit through a pricing strategy s_n^{MP} such that its threshold user type $\sigma_n(\kappa_n, s_n^{\mathrm{MP}}) = \sigma_n^{\mathrm{MP}}(\kappa_n)$, which is the solution to the following equation:

$$\sigma_n^{\mathrm{MP}}(\kappa_n) - \frac{1 - H(\sigma_n^{\mathrm{MP}}(\kappa_n))}{h(\sigma_n^{\mathrm{MP}}(\kappa_n))} = \frac{c_n}{\rho_n}. \tag{14}$$

The value of $\sigma_n^{\mathrm{MP}}(\kappa_n)$ is unique for an arbitrary θ distribution with the increasing failure rate (IFR).[6]

Lemma 1 reveals the trade-off between subscription fee and per-unit fee, i.e., the profit-maximizing subscription fee Π_n^{MP} and per-unit fee π_n^{MP} need to satisfy

$$\pi_n^{\mathrm{MP}} \left(\beta^{-1} - 1\right) \left(\bar{d} - \alpha_n\right) + \Pi_n^{\mathrm{MP}} = \rho_n \alpha_n \sigma_n^{\mathrm{MP}}(\kappa_n). \tag{15}$$

A larger π_n^{MP} would lead to a smaller Π_n^{MP}, and vice versa.

***Lemma** 2:* Under the optimal pricing strategy in Lemma 1, a monopoly MNO-n maximizes its profit by choosing the data mechanism $\kappa_n^{\mathrm{MP}} = 2$.

Lemma 2 indicates that the monopoly MNO should select a data mechanism offering the best time flexibility to maximize its profit, i.e., $\kappa_n^{\mathrm{MP}} = 2$. This conclusion still holds under users' two-dimensional heterogeneity on data valuation and network substitutability, which is studied in our previous work [13].

So far we have derived a monopoly MNO's profit-maximizing pricing strategy and data mechanism. When considering the more realistic duopoly market, will $\kappa_n = 2$ still remain as the optimal (equilibrium) choice for both MNOs? Our analysis next shows that this is not always the case.

III. Duopoly Market

Now we consider the MNOs' duopoly competition and analyze the two-stage game through backward induction.

A. Users' Subscription in Stage II

In Stage II, each user makes his subscription decision given the two MNOs' pricing strategies $\boldsymbol{s} = \{s_1, s_2\}$ and data mechanism $\boldsymbol{\kappa} = \{\kappa_1, \kappa_2\}$. A type-$\theta$ user would subscribe to MNO-n if MNO-n can bring him a larger (among the two MNOs) and non-negative payoff.

Recall that the type-θ user's expected payoff is

$$\bar{U}_n(\kappa_n, s_n, \theta) = \rho_n \alpha_n \theta - \pi_n \left(\beta^{-1} - 1\right) \left(\bar{d} - \alpha_n\right) - \Pi_n, \tag{16}$$

where $\rho_n \alpha_n$ is dependent on MNO's data mechanism κ_n. For notation simplicity, we define ξ as

$$\xi \triangleq \frac{\rho_2 \alpha_2}{\rho_1 \alpha_1}, \tag{17}$$

and suppose that $\xi < 1$ without loss of generality. In other words, MNO-1 has an advantage in terms of $\rho_n \alpha_n$ among the two MNOs (hence we say MNO-1 is "stronger"), i.e., its subscribers' marginal utility change for per-unit valuation increment is larger.

[6] The IFR condition has been widely-used in the literature to capture the user valuation characteristics (see, e.g., [20]–[24]). Many commonly-used distributions, such as the uniform distribution, gamma distribution, and normal distribution, satisfy the IFR condition.

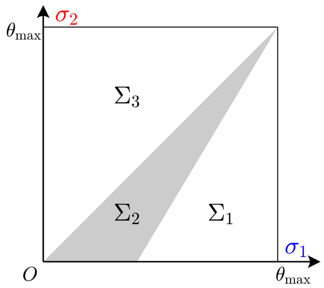
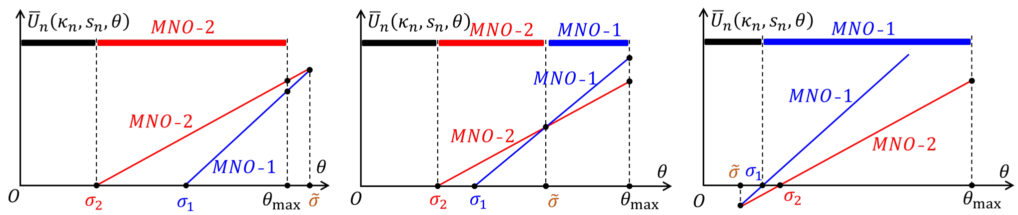


Fig. 2: Partition structure.



(a) Σ_1: MNO-2 surviving (b) Σ_2: Share the market. (c) Σ_3: MNO-1 surviving

Fig. 3: Three market partition modes in the duopoly market.

To facilitate our later analysis, we define the neutral user type $\tilde{\sigma}$ between the two MNOs as follows:

***Definition** 2 (Neutral User Type):* The neutral user type, denoted by $\tilde{\sigma}$, is a user type who achieves the same payoff by subscribing to either MNO, i.e., $\bar{U}_1(\kappa_1, s_1, \tilde{\sigma}) = \bar{U}_2(\kappa_2, s_2, \tilde{\sigma})$. We can derive $\tilde{\sigma}$ as follows

$$\tilde{\sigma}(\sigma_1, \sigma_2) = \frac{\sigma_1 - \xi \cdot \sigma_2}{1 - \xi}, \tag{18}$$

where σ_1 and σ_2 are two MNOs' threshold user types defined in Definition 1.

With the above discussion, we summarize the duopoly market partition in the following lemma.

***Lemma** 3 (Market Partition):* Consider MNOs' threshold user types σ_1 and σ_2 under the data mechanisms $\boldsymbol{\kappa} = \{\kappa_1, \kappa_2\}$ and the pricing strategy $\boldsymbol{s} = \{s_1, s_2\}$. There are three market competition results in the (σ_1, σ_2) plane shown in Fig. 2.

1) Σ_1: MNO-1 has a much larger threshold user type than MNO-2, i.e., $(\sigma_1, \sigma_2) \in \Sigma_1 \triangleq \{(\sigma_1, \sigma_2) : \sigma_1 - \sigma_2 \geq (1-\xi)(\theta_{\max} - \sigma_2)\}$. In this case, MNO-2's market share corresponds to the users with θ in the set of $\Phi_2 = [\sigma_2, \theta_{\max}]$, while MNO-1 has a zero market share $\Phi_1 = \varnothing$, as shown in Fig. 3(a).
2) Σ_2: MNO-1 has a slightly larger threshold user type than MNO-2, i.e., $(\sigma_1, \sigma_2) \in \Sigma_2 \triangleq \{(\sigma_1, \sigma_2) : 0 < \sigma_1 - \sigma_2 < (1-\xi)(\theta_{\max} - \sigma_2)\}$. In this case, MNO-1 has a market share of $\Phi_1 = [\tilde{\sigma}, \theta_{\max}]$, and MNO-2 has a market share of $\Phi_2 = [\sigma_2, \tilde{\sigma}]$, as shown in Fig. 3(b).
3) Σ_3: MNO-1 has a smaller threshold user type than MNO-2, i.e., $(\sigma_1, \sigma_2) \in \Sigma_3 \triangleq \{(\sigma_1, \sigma_2) : \sigma_1 - \sigma_2 \leq 0\}$. In this case, MNO-1 has a market share of $\Phi_1 = [\sigma_1, \theta_{\max}]$, while MNO-2 has a zero market share of $\Phi_2 = \varnothing$, as shown in Fig. 3(c).

Lemma 3 reveals two market competition results, i.e., coexistence (Σ_2) or one-MNO-surviving (Σ_1 and Σ_3), depending on the data mechanism κ and the pricing strategy $\boldsymbol{s}$. Note that the one-MNO-surviving result is different from the monopoly case discussed in Section II-D, since the zero market share MNO in the one-MNO-surviving case might still affect the decision of the surviving MNO. We will further discuss it.

B. MNOs' Pricing Competition in Stage I

In Stage I, the MNOs simultaneously determine the pricing strategies $\boldsymbol{s} = \{s_1, s_2\}$, given their data mechanisms $\boldsymbol{\kappa} = \{\kappa_1, \kappa_2\}$ and the users' subscription responses in Stage II.

As Lemma 1 shows, the market shares of two MNOs are determined by their threshold user types. Equation (12) shows that a given value of threshold user type σ_n corresponds to infinite choices of prices $\{\Pi_n, \pi_n\}$, each representing a different trade-off between the subscription fee and the per-unit fee. Hence a more concise characterization of the MNOs' price competition is the following threshold competition game:

***Game** 1 (Threshold Competition):* Given the data mechanism selection $\boldsymbol{\kappa} = \{\kappa_1, \kappa_2\}$, the two MNOs' threshold competition in Stage II can be modeled as the following game:

- Players: MNO-n for both $n = 1, 2$.
- Strategy: Each MNO-n determines its threshold user type $\sigma_n \in [c_n/\rho_n, \theta_{\max}]$.
- Preference: Each MNO-n obtains a profit $W_n(\boldsymbol{\kappa}, \boldsymbol{\sigma})$, which has been defined in Section II-C.

Next we first study each MNO's best response (denoted by the superscript "$\star$") in *Game 1*, then find the fixed point of the best responses at the equilibrium.

1) Best Response Analysis: Since the two MNOs are not symmetric, i.e., MNO-1 is stronger and MNO-2 is weaker, their best responses are also different.

Before analyzing the weaker MNO-2's best response, we first define three parameters related to the stronger MNO-1: $\theta_1^W \triangleq c_2/\rho_2$, $\theta_1^N \triangleq \xi\sigma_2^{\text{MP}} + (1-\xi)\theta_{\max}$, and θ_1^L satisfying

$$\xi^{-1}\theta_1^L - (\xi^{-1} - 1)\theta_{\max} - \frac{1 - H\left(\xi^{-1}\theta_1^L - (\xi^{-1}-1)\theta_{\max}\right)}{h\left(\xi^{-1}\theta_1^L - (\xi^{-1}-1)\theta_{\max}\right)} = \frac{c_2}{\rho_2}. \tag{19}$$

***Lemma** 4 (Best Response of MNO-2):* Given MNO-1's threshold user type σ_1, MNO-2 maximizes its profit by choosing a threshold user type $\sigma_2^\star(\sigma_1)$ as follows:

- BR2-a: $\sigma_2^\star(\sigma_1) = c_2/\rho_2$, if $\sigma_1 \in [0, \theta_1^W]$;
- BR2-b: $\sigma_2^\star(\sigma_1) - \frac{H(\tilde{\sigma}(\sigma_1, \sigma_2^\star(\sigma_1))) - H(\sigma_2^\star(\sigma_1))}{\frac{\xi}{1-\xi}h(\tilde{\sigma}(\sigma_1, \sigma_2^\star(\sigma_1))) + h(\sigma_2^\star(\sigma_1))} = \frac{c_2}{\rho_2}$, if $\sigma_1 \in (\theta_1^W, \theta_1^L]$;
- BR2-c: $\xi\sigma_2^\star(\sigma_1) + (1-\xi)\theta_{\max} = \sigma_1$, if $\sigma_1 \in (\theta_1^L, \theta_1^N]$;
- BR2-d: $\sigma_2^\star(\sigma_1) = \sigma_2^{\text{MP}}$, if $\sigma_1 \in (\theta_1^N, \theta_{\max}]$;

Recall that $h(\cdot)$ and $H(\cdot)$ represent the PDF and CDF of users' data valuation θ, respectively.

The best response specified in Lemma 4 is applicable to an arbitrary θ distribution satisfying the IFR. For an illustrative purpose, Fig. 4 plots the MNO-2's best response under a uniform distribution of θ, i.e., $h(\theta) = 1/\theta_{\max}$. Specifically, the red line segments (i.e., BR2-a, BR2-b, BR2-c, and BR2-d) denote $\sigma_2^\star(\sigma_1)$. Next we discuss the physical meanings of

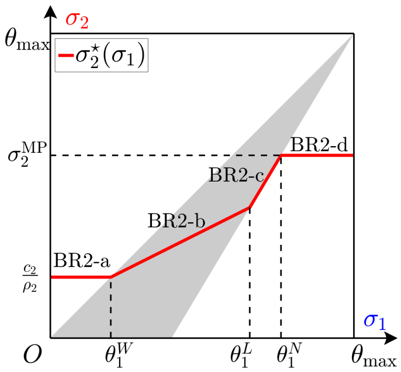
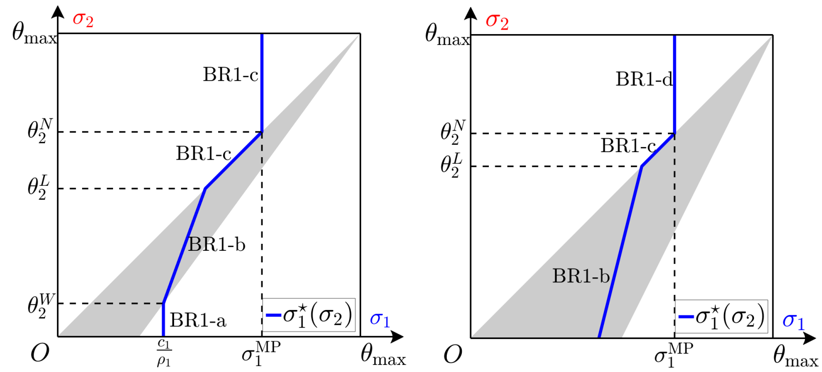


Fig. 4: Illustration of $\sigma_2^\star(\sigma_1)$.



(a) $c_1/\rho_1 > (1-\xi)\theta_{\max}$ (b) $c_1/\rho_1 < (1-\xi)\theta_{\max}$

Fig. 5: Illustration of $\sigma_1^\star(\sigma_2)$.

the four parts of best response in more details. To facilitate later discussion, we refer to θ_1^W, θ_1^L, and θ_1^N as MNO-1's *winning* threshold, *losing* threshold, and *no-influence* threshold, respectively.

- **BR2-a**: MNO-2 gets zero market share, i.e., $(\sigma_1, \sigma_2^\star(\sigma_1)) \in \Sigma_3$, if MNO-1 chooses a threshold user type smaller than its winning threshold, i.e., $\sigma_1 \leq \theta_1^W$.
- **BR2-b**: MNO-2 shares the market with MNO-1, i.e., $(\sigma_1, \sigma_2^\star(\sigma_1)) \in \Sigma_2$, if MNO-1 chooses a threshold user type between its winning and losing threshold, i.e., $\theta_1^W < \sigma_1 < \theta_1^L$.
- **BR2-c**: MNO-2 leaves MNO-1 zero market share, i.e., $(\sigma_1, \sigma_2^\star(\sigma_1)) \in \Sigma_1$, if MNO-1 chooses a threshold user type between its losing and no-influence threshold, i.e., $\theta_1^L \leq \sigma_1 < \theta_1^N$.
- **BR2-d**: MNO-2 leaves zero market to MNO-1 and becomes a monopoly in the market (deciding its threshold user type as in Section II-D without considering the existence of MNO-1), i.e., $\sigma_2^\star(\sigma_1) = \sigma_2^{\text{MP}}$ as defined in (14), if MNO-1 chooses a threshold user type larger than its no-influence threshold, i.e., $\sigma_1 \geq \theta_1^N$.

Next we consider the stronger MNO-1's best response in Lemma 5. Before that, we define three parameters related to the weaker MNO-2: $\theta_2^W \triangleq \xi^{-1}\frac{c_1}{\rho_1} - (\xi^{-1}-1)\theta_{\max}$, $\theta_2^N \triangleq \theta_1^{\text{MP}}$, and θ_2^L satisfying $\theta_2^L - \frac{1-H\left(\theta_2^L\right)}{\frac{1}{1-\xi}h\left(\theta_2^L\right)} = \frac{c_1}{\rho_1}$.

***Lemma** 5 (Best Response of MNO-1):* Given MNO-2's threshold user type σ_2, MNO-1 maximizes its profit by the threshold user type $\sigma_1^\star(\sigma_2)$, which satisfies

- BR1-a: $\sigma_1^\star(\sigma_2) = c_1/\rho_1$, if $\sigma_2 \in [0, \theta_2^W]$;
- BR1-b: $\sigma_1^\star(\sigma_2) - \frac{1-H(\tilde{\sigma}(\sigma_1^\star(\sigma_2),\sigma_2))}{\frac{1}{1-\xi}h(\tilde{\sigma}(\sigma_1^\star(\sigma_2),\sigma_2))} = \frac{c_1}{\rho_1}$, if $\sigma_2 \in (\theta_2^W, \theta_2^L]$;
- BR1-c: $\sigma_1^\star(\sigma_2) = \sigma_2$, if $\sigma_2 \in (\theta_2^L, \theta_2^N]$;
- BR1-d: $\sigma_1^\star(\sigma_2) = \sigma_1^{\text{MP}}$, if $\sigma_2 \in (\theta_2^N, \theta_{\max}]$;

The result in Lemma 5 applies to an arbitrary θ distribution satisfying the IFR. Fig. 5 illustrates the results in Lemma 5 under a uniform distribution of θ. For an easy comparison with Fig. 4, in Fig. 5 we plot the best response $\sigma_1^\star(\sigma_2)$ on the horizontal axis and the variable σ_2 on the vertical axis. We note that Fig. 5 contains two cases (sub-figures):

- Fig. 5(a): MNO-1 has a relatively large cost-QoS ratio, i.e., $c_1/\rho_1 > (1-\xi)\theta_{\max}$, and the corresponding insights are similar to Fig. 4.
- Fig. 5(b): MNO-1 has a small cost-QoS ratio, i.e., $c_1/\rho_1 \leq (1-\xi)\theta_{\max}$, in which case MNO-2's *winning* threshold θ_2^W is always negative. This means that no matter how small the MNO-2's threshold user type σ_2 is, MNO-1 can always get a positive market share, i.e., $(\sigma_1^\star(\sigma_2), \sigma_2) \notin \Sigma_1$ for all σ_2. This is possible as MNO-2 is the weaker one. Fig. 5(a) represents a degenerated case of Fig. 5(b).

2) Equilibrium Analysis: Next we characterize the threshold equilibrium of *Game 1*, denoted by $\boldsymbol{\sigma}^\dagger = \{\sigma_1^\dagger, \sigma_2^\dagger\}$ with a superscript "$\dagger$".

Based on the best response analysis, we note that an MNO's cost-QoS ratio c_n/ρ_n plays a significant role in the best response analysis. Thus, we further define

$$\psi_n \triangleq \frac{c_n}{\rho_n}, \ \forall\, n \in \{1,2\}, \tag{20}$$

which allows us to write θ_1^L, θ_1^N, θ_2^L, and θ_2^N as $\theta_1^L(\psi_2, \xi)$, $\theta_1^N(\psi_2, \xi)$, $\theta_2^L(\psi_1, \xi)$, and $\theta_2^N(\psi_1, \xi)$ to emphasize the dependence relationship.

In the following, we first present five different outcomes of *Game 1* in Theorem 1 in the (ψ_1, ψ_2) plane, then characterize the equilibrium $\sigma^\dagger$ under each kind of outcome.

***Theorem** 1:* Game 1 has five different types of equilibrium based on the values of (ψ_1, ψ_2), as following

1) MNO-1's strong monopoly regime:
$$\Psi_1^{\text{SM}} = \{(\psi_1, \psi_2) : \psi_2 > \theta_2^N(\psi_1, \xi)\}.$$
2) MNO-1's weak monopoly regime:
$$\Psi_1^{\text{WM}} = \{(\psi_1, \psi_2) : \theta_2^L(\psi_1, \xi) < \psi_2 \leq \theta_2^N(\psi_1, \xi)\}.$$
3) Coexistence regime:
$$\Psi^{\text{C}} = \{(\psi_1, \psi_2) : \psi_2 \leq \theta_2^L(\psi_1, \xi), \psi_1 \leq \theta_1^L(\psi_2, \xi)\}.$$
4) MNO-2's weak monopoly regime:
$$\Psi_2^{\text{WM}} = \{(\psi_1, \psi_2) : \theta_1^L(\psi_2, \xi) < \psi_1 \leq \theta_1^N(\psi_2, \xi)\}.$$

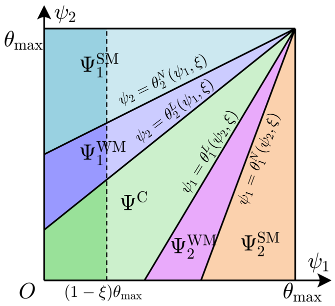
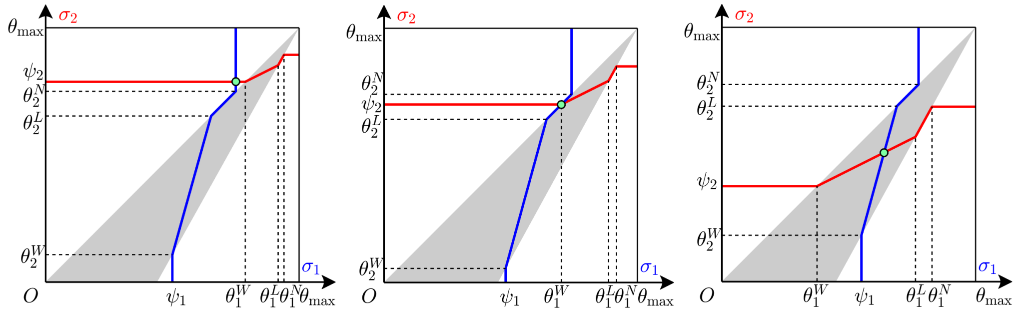


Fig. 6: The equilibrium structure of Game 1.



(a) $(\psi_1, \psi_2) \in \Psi_1^{\mathrm{SM}}$ (b) $(\psi_1, \psi_2) \in \Psi_1^{\mathrm{WM}}$ (c) $(\psi_1, \psi_2) \in \Psi^{\mathrm{C}}$

Fig. 7: Threshold equilibriums under uniformly distributed market.

5) MNO-2's strong monopoly regime:

$$\Psi_2^{\mathrm{SM}} = \{(\psi_1, \psi_2) : \psi_1 > \theta_1^N(\psi_2, \xi)\}.$$

Fig. 6 illustrates Theorem 1 under a uniform distribution of θ. The two axises correspond to the two MNOs' cost-QoS ratio (ψ_1, ψ_2). The four boundary lines correspond to $\psi_2 = \theta_2^N(\psi_1, \xi)$, $\psi_2 = \theta_2^L(\psi_1, \xi)$, $\psi_1 = \theta_1^L(\psi_2, \xi)$, and $\psi_1 = \theta_1^N(\psi_2, \xi)$. The vertical dash line represents the cost-QoS threshold $(1 - \xi)\theta_{\max}$ that separates between Fig. 5(a) and Fig. 5(b).

Due to the space limitation, we illustrate the pricing equilibrium of Ψ_1^{SM}, Ψ_1^{WM}, and Ψ_1^{C} precisely in Fig. 7. Specifically, Fig. 7(a) illustrates an equilibrium corresponding to MNO-1's strong monopoly regime. The green circle represents the equilibrium, where MNO-2 gives up competing for market share, thus, MNO-1 can decide its threshold user type $\sigma_1^\dagger = \sigma_1^{\mathrm{MP}}$ *without* considering the impact of MNO-2. Fig. 7(b) illustrates an equilibrium corresponding to MNO-1's weak monopoly regime. The green circle represents the equilibrium, where MNO-2 still tries to compete for market share, but in vain. However, MNO-1 has to decide its threshold user type *considering* MNO-2, which leads to $\sigma_1^\dagger < \sigma_1^{\mathrm{MP}}$. In Fig. 7(c), the green *circle* denotes the unique equilibrium in the coexistence regime where both MNOs get strictly positive market shares.

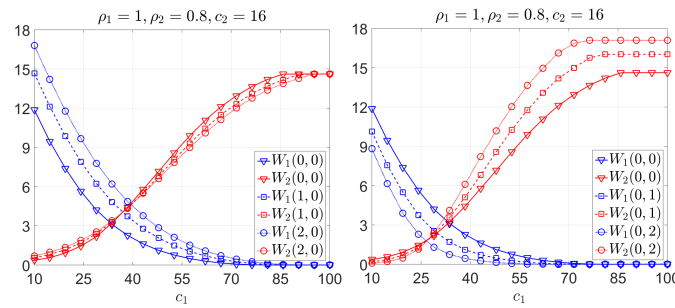
So far we have characterized the equilibrium of *Game 1*. Next we will evaluate the performance at the equilibrium under different data mechanisms.

IV. Numerical Results

Now we evaluate the pricing competition outcome under different data mechanisms using the real data.

We assume that MNO-1 provides better QoS, i.e., $\rho_1 > \rho_2$. The distribution of users' data valuation θ is based on the fitted empirical data from our on-line market survey conducted in [13]. To be more specific, the empirical PDF of θ is well fitted to a gamma distribution with a shape parameter 4.5 and a rate parameter 0.11 (through minimizing the least-squares divergence between the estimated and empirical PDFs).[7] Additionally, our market survey shows that most people would

[7]The gamma distribution also satisfies increasing failure rate (IFR), thus, our previous analysis for Stage I still holds.



(a) $\kappa_1 \in \{0, 1, 2\}$, $\kappa_2 = 0$. (b) $\kappa_1 = 0$, $\kappa_2 \in \{0, 1, 2\}$.

Fig. 8: MNOs' profits at the pricing equilibrium.

shrink $85\% \sim 95\%$ overage usage, thus, we suppose that $\beta = 0.9$ for simplicity. Furthermore, we consider MNOs' 1GB data plan, i.e., $Q_1 = Q_2 = 1$GB. Following the data analysis results in [25], we assume that users' monthly data demand follows a truncated log-normal distribution with mean $\bar{d} = 1$GB over the interval $[0, 10]$.

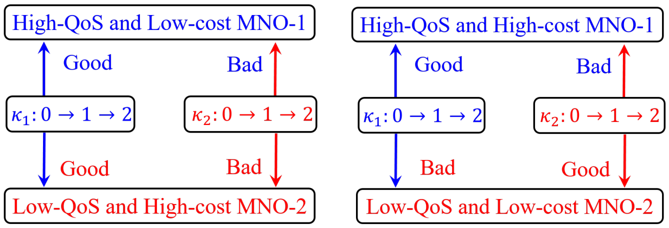
Now we study the impact of one MNO choosing a better time flexibility on itself and the other MNO.

Fig. 8 plots two MNOs' profits $W_n^I(\kappa)$ versus MNO-1's cost c_1, where the blue curves and the red curves represent $W_1^I(\kappa)$ and $W_2^I(\kappa)$, respectively. Here we assume MNO-1 provides a better QoS, i.e., $\rho_1 > \rho_2$.

In Fig. 8(a), we let $\kappa_1 = 0, 1, 2$, while fixing $\kappa_2 = 0$ for MNO-2. Comparing the three blue curves, we find that MNO-1 can increase its profit by offering a better time flexibility. As for MNO-2, however, if MNO-1 upgrades to a better time flexibility, the profit of MNO-2 slightly increases if c_1 is small, but reduces if c_1 is large.

In Fig. 8(b), we let $\kappa_2 = 0, 1, 2$, while fixing $\kappa_1 = 0$ for MNO-1. Comparing the three blue curves, we find that MNO-1's profit is reduced if MNO-2 offers a better time flexibility. As for MNO-2, however, if MNO-2 upgrades to a better time flexibility, its profit slightly reduces if c_1 is small, but improves if c_1 is large.

In the above numerical results, we assume that MNO-1 provides better QoS. Thus, the market competition is mild if c_1 is small, since MNO-1 has an advantage in both QoS and the cost, hence is much stronger than MNO-2. When c_1 is



(a) Mild Competition. (b) Fierce Competition

Fig. 9: Impact of time flexibility in duopoly market.

large, the market competition is fierce, since the two MNOs are comparable (in terms of the QoS and cost). Fig. 9 summarizes the insights obtained from Fig. 8 more clearly as follows:

- In a duopoly market with a *mild competition* as shown in Fig. 9(a), MNO-1 has an absolute advantages (i.e., a higher QoS and a lower cost). (i) if MNO-1 offers a better time flexibility, i.e., $\kappa_1 : 0 \to 1 \to 2$, both MNOs' profits will increase. This is because a better time flexibility enables MNO-1 to increase its price, which will lead to a higher profit and a higher value of threshold user type σ_1. This benefits MNO-2 as well, as MNO-1's market share actually reduces (although MNO-1's profit increases), hence MNO-2 can increase its profit as well. (ii) if MNO-2 offers better time flexibility, i.e., $\kappa_2 : 0 \to 1 \to 2$, both MNOs' profits will reduce. This is because the better time flexibility of MNO-2 makes the competition more fierce, and MNO-1 has to decrease its price to defend its market. As a result, both MNOs lose profits.
- In a duopoly market with a *fierce competition* as shown in Fig. 9(b), MNO-1 has an advantage in QoS but a disadvantage in cost. In this case, two MNOs are comparable. No matter who decides to offer a better time flexibility, it will increase its own profit but reduce the competitor's profit.

V. Conclusions and Future Work

In this paper, we analyze the MNOs' duopoly competition through offering mobile data plans with time flexibility. Our analysis reveals the differences of the time flexibility in monopoly and duopoly markets. Specifically, it is always the optimal choice for a monopoly MNO to offer the best time flexibility to maximize its profit. However, it is more complicated when considering competition. In a duopoly market with a mild competition, the time flexibility added to the stronger MNO can increase both MNOs' profits, while the time flexibility added to the weaker MNO will decrease both MNOs' profits. In a duopoly market with a fierce competition, no matter which MNO adds a better time flexibility, it will increase that MNO own profit but reduce the other's.

As for the future work, we will further explore the impact of heterogeneous user data valuations and network substitutability. We will further understand whether the insights from the duopoly market will carry to the oligopoly market with more than two MNOs.

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