A Model-Free Framework for Coverage Evaluation in Device-to-Device Heterogeneous Networks

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***Abstract*—Consider a decentralized device-to-device (D2D) network consisting of K different types of D2D pairs in which the D2D pairs of each specific type form an independent homogeneous Poisson point process (PPP) and the transmitter (TX) of each D2D pair has a unique intended receiver (RX). For this heterogeneous network model, we develop a model-free tractable framework to analyze the coverage probability without any specific model assumptions for channel fading, stochastic transmit power and distance. First we device a novel approach to finding the Laplace transform of the reciprocal of the SIR which is used to characterize the model-free coverage probability of the D2D pair of each type. Our main analytical findings show that the model-free bounds of the coverage probability can be obtained and they reduce to a closed-form result as long as the received signal power has an Erlang distribution. These findings are applied to expound when the randomness of the received signal power benefits/jeopardizes the coverage probability and how to use the distributed stochastic power control to improve the coverage probability of each D2D pair.**

I. Introduction

Consider a large-scale device-to-device (D2D) network in which there are K different types of D2D pairs and the D2D pairs of each specific type form an independent Poisson point process (PPP) with a certain intensity (density). The transmitter (TX) of each D2D pair has a unique intended receiver (RX) away from it by some distance. Namely, such a heterogeneous D2D network consisting of K-type D2D pairs independently scattering on an infinitely large plane $\mathbb{R}^2$. Usually, interference in such a network significantly dominates the transmission performance that is effectively evaluated by the metric of the signal-to-interference power ratio (SIR) at RXs. By assuming all TXs in the network transmit narrow band signals and share the same spectrum bandwidth, the SIR of the RX in a type-k D2D pair, called type-k SIR, can be written as

$$\mathsf{SIR}_k \triangleq \frac{S_k}{I_k} = \frac{P_k H_k R_k^{-\alpha}}{I_k} > \theta, \tag{1}$$

where $k \in \mathcal{K} \triangleq \{1, 2, \ldots, K\}$, I_k denotes the interference power from all interferers in the network, $S_k \triangleq P_k H_k R_k^{-\alpha}$ is the received (desired) signal power, P_k is the transmit power, H_k is the random channel gain, R_k is the (random) distance in the type-k pair, $\alpha > 2$ is the pathloss exponent, and θ is the minimum required SIR for successfully decoding. Note that S_k and I_k are both random variables whose distributions depend upon channel gain, transmit power as well as pathloss models between the type-k RX and TXs. The SIR pertaining to several important transmitting performance metrics, such as coverage probability, ergodic link capacity, network capacity, etc. Understanding the statistical properties of the SIR not only helps us realize how the received random signal powers affect the distribution of the SIR, but also provides us with a crucial clue indicating the interplays of the transmitting policies and behaviors among many different D2D pairs.

Traditionally, the statistical properties of the SIR in a Poisson-distributed wireless network are only analytically accessible in very few special cases. Some prior works have already made a good progress on the analysis of the distribution of the SIR by presuming a specific channel gain model (typically see [1]–[6]). In reference [1], for example, the coverage/outage probability in a single-type Poisson ad hoc network, which is essentially the contemporary cumulative density function (CCDF) of the SIR, was firstly found in closed-form by assuming independent Rayleigh fading channels since the Rayleigh fading channel model gives rise to the tractable Laplace transform of the interference by means of the probability generating function (PGF) of a homogeneous PPP [3], [7], [8]. Although the closed-form Laplace transform of the interference with general channel fading is found in [3], it can only be applied to find the CCDF of the SIR with the exponential-distributed received signal power. In [5], the bounds on the temporally averaged coverage/outage probability are studied specifically for Rayleigh fading channels only due to the tractability in mathematical analysis.

In this work, our first main contribution is to develop the novel analytical framework of tractably analyzing the "model-free" cumulative density function (CDF) of the SIR defined in (1) without assuming any specific channel gain, transmit power and distance models. The main idea behind this framework is to first find the explicitly result of the Laplace transform of the reciprocal of SIR_k with general random models of fading channel gain, transmit power and distance. Then substituting it into the exploited fundamental identity between the CDF of SIR_k and the Laplace transform of the reciprocal of SIR_k. The bounds on the CDF of SIR_k and the type-k coverage probability are characterized and we show that the closed-form type-$k$coverage probability exists if and only if received signal power S_k has an Erlang distribution. Then we apply the model-free result of the type-k coverage probability to show how to design distributed stochastic power control so as to increase the coverage probability of the TXs of each type by exploiting the randomness of the received signal power.

II. System Model

We consider a decentralized interference-limited network on the plane $\mathbb{R}^2$ in which there are K different types of D2D pairs and the D2D pairs of each specific type form an independent, homogeneous and marked Poisson point process (PPP). The TX of each D2D pair has a unique intended RX and the set Φ_k consisting of all the transmitters of the type-k D2D pairs and their marks is expressed as

$$\Phi_k \triangleq \{(X_{k,i}, H_{k,i}, P_{k,i}, R_{k,i}) : X_{k,i} \in \mathbb{R}^2, P_{k,i}, H_{k,i} \in \mathbb{R}_+, R_{k,i} \in [1,\infty), i \in \mathbb{N}\},\ k \in \mathcal{K} \triangleq \{1,2,\dots,K\}, \tag{2}$$

where $X_{k,i}$ denotes the TX of the type-k D2D pair i and its location, $H_{k,i}$ is the fading channel gain from $X_{k,i}$ to the at the origin where a typical RX of a type k D2D pair is located, $P_{k,i}$ is the transmit power of $X_{k,i}$, $R_{k,i}$ is the distance between $X_{k,i}$ and its receiver. Throughout this paper, all random variables (RVs) with different subscripts are independent whereas the RVs of the same class with the same subscript k are i.i.d. In addition, all channel gains $H_{k,i}$'s have unit mean for all $k \in \mathcal{K}$ and $i \in \mathbb{N}_+$. Assume the intensity of all TXs in Φ_k is λ_k.

Assume all TXs adopt the slotted Aloha protocol to access the channel shared in the network. Consider the typical RX of a type-k D2D pair located the origin and it receives the interference given by[1]

$$I_k \triangleq \sum_{k,i:X_{k,i}\in\Phi\setminus X_k} \frac{H_{k,i}P_{k,i}}{\|X_{k,i}\|^{\alpha}}, \tag{3}$$

where $\Phi \triangleq \bigcup_{k=1}^{K}\Phi_k$, $\|X_i - X_j\|$ denotes the Euclidean distance between TXs X_i and X_j, and $\alpha > 2$ is the pathloss exponent. Using this type-k interference I_k, the signal-to-interference power ratio (SIR) at a type-k RX, called type-k SIR as shown in (1), can be explicitly rewritten as follows

$$\mathsf{SIR}_k = \frac{S_k}{I_k} = \frac{P_k H_k R_k^{-\alpha}}{\sum_{k,i:X_{k,i}\in\Phi\setminus X_k} H_{k,i}P_{k,i}\|X_{k,i}\|^{-\alpha}}. \tag{4}$$

Assuming the minimum SIR threshold for successful decoding the received signals in all D2D pairs is θ, the type-k *coverage probability* is defined as

$$p_k(\theta) \triangleq \mathbb{P}\left[\mathsf{SIR}_k > \theta\right], \tag{5}$$

i.e., it is the CCDF of a type-k SIR. Prior works on the coverage/outage probability in Poisson wireless networks are channel-model-dependent and the majority of the prior works find the closed-form coverage probability by assuming exponentially distributed channel gains (i.e., channels undergo Rayleigh fading), whereas the coverage probability for the channel gains without an exponential distribution is generally intractable. As a result, the prior results cannot thoroughly reveal how the coverage probability is impacted as the random models involved in the SIR are changed.

[1]We call this receiver located at the origin "typical receiver" since our following analysis will be based on the location of this typical RX and the statistical results obtained at this receiver are the same as those at all other RXs in the network based on the Slivnyak theorem [5], [7]. Note that all these k homogeneous PPPs are simple in the network, i.e., no more than one node in the network can occupy the same location.

III. Generalized Analytical Framework for Type-k Coverage Probability

In this section, our goal is to characterize the type-k *model-free* coverage probability without presuming any specific random models on channel gain, transmit power and distance, i.e., the received signal power S_k in (4) has an unknown general distribution. We will first study some general results on the distribution of SIR_k and then use them to characterize some general results on the type-k coverage probability.

A. The Generalized Distribution of SIR_k

The Laplace transform of a nonnegative real-valued random variable Z for any $s \in \mathbb{R}_{++}$ is defined as

$$\mathcal{L}_Z(s) \triangleq \mathbb{E}\left[e^{-sZ}\right],\ s > 0. \tag{6}$$

For arbitrary random power-law channel and transmit power models, the reciprocal of SIR_k is shown in the following theorem.

Theorem 1. *The Laplace transform of the reciprocal of* SIR_k *in* (4) *can be explicitly expressed as*

$$\mathcal{L}_{\mathsf{SIR}_k^{-1}}(s) = \int_0^{\infty} s\mathcal{L}_{I_k}\left(\frac{1}{t\mathbb{E}[S_k]}\right) f_{\widehat{S}_k}(st)dt, \tag{7}$$

where $\widehat{S}_k \triangleq S_k/\mathbb{E}[S_k] = P_k H_k R_k^{-\alpha}/\mathbb{E}[P_k H_k R_k^{-\alpha}]$ *is called the type-k received signal power with unit mean and* $\mathcal{L}_{I_k}(\cdot)$ *is given by*

$$\mathcal{L}_{I_k}(s) = \exp\left\{-\pi\Gamma\left(1-\frac{2}{\alpha}\right)s^{\frac{2}{\alpha}}\widetilde{\lambda}\right\}, \tag{8}$$

where $\widetilde{\lambda} = \sum_{k=1}^{K}\widetilde{\lambda}_k$, $\widetilde{\lambda}_k \triangleq \lambda_k\mathbb{E}\left[H_k^{\frac{2}{\alpha}}\right]\mathbb{E}\left[P_k^{\frac{2}{\alpha}}\right]$ *and* $\Gamma(a) = \int_0^{\infty} t^{a-1}e^{-t}dt$ *for* $a > 0$ *is the gamma function. Furthermore, the CDF of* SIR_k *can be shown as*

$$F_{\mathsf{SIR}_k}(\theta) = 1 - \mathcal{L}^{-1}\left\{\int_0^{\infty}\mathcal{L}_{I_k}\left(\frac{1}{t\mathbb{E}[S_k]}\right) f_{\widehat{S}_k}(st)dt\right\}\left(\frac{1}{\theta}\right), \tag{9}$$

where $\theta \in \mathbb{R}_{++}$, $F_Z(\cdot)$ *and* $f_Z(\cdot)$ *denote the CDF and pdf (probability density function) of RV Z, respectively.*

Proof: See Appendix A. □

Theorem 1 demonstrates the model-free expression of the Laplace transform of SIR_k^{-1} as well as the CDF of SIR_k without assuming any specific random channel gain, transmit power and distance models. Although in general the expressions in Theorem 1 cannot be completely found in closed-form, they can be calculated by using the numerical inverse Laplace transform. Nonetheless, as shown in the following corollary, we still can characterize the low-complexity bounds on $F_{\mathsf{SIR}_k}(\theta)$ and the near closed-form of $F_{\mathsf{SIR}_k}(\theta)$ for $\alpha = 4$.

Corollary 1. *The CDF of* SIR_k *in* (9) *can be bounded as shown in the following:*

$$F_{\mathsf{SIR}_k}(\theta)\begin{cases}\leq \min\left\{1,\pi\widetilde{\lambda}\mathbb{E}\left[S_k^{-\frac{2}{\alpha}}\right]\theta^{\frac{2}{\alpha}}\right\}\\ \geq \mathcal{L}^{-1}\left\{\frac{\pi\Gamma(1-\frac{2}{\alpha})\widetilde{\lambda}}{s^{1-\frac{2}{\alpha}}\left(\pi\Gamma(1-\frac{2}{\alpha})\widetilde{\lambda}s^{\frac{2}{\alpha}}+\mathbb{E}\left[S_k^{\frac{2}{\alpha}}\right]\right)}\right\}(\theta^{-1})\end{cases}. \tag{10}$$

In particular, if $\alpha=4$, *then* $F_{\mathsf{SIR}_k}(\theta)$ *can be simply found as*

$$F_{\mathsf{SIR}_k}(\theta)=\mathbb{E}\left[\operatorname{erf}\left(\frac{\pi^{\frac{3}{2}}\widetilde{\lambda}\sqrt{\theta}}{2\sqrt{S_k}}\right)\right] \tag{11}$$

in which $\operatorname{erf}(x)\triangleq\frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt$ *is the error function and* $\widetilde{\lambda}=\sum_{j=1}^{K}\lambda_j\mathbb{E}[\sqrt{P_j}]\mathbb{E}[\sqrt{H_j}]$.

Proof: See Appendix B. □

When $\widetilde{\lambda}\mathbb{E}\left[S_k^{-\frac{2}{\alpha}}\right]\ll 1$ (e.g., the mean of the interference-to-signal power ratio is fairly small), $F_{\mathsf{SIR}_k}(\theta)$ is accurately approximated by the inverse Laplace transform of the Taylor's expansion of the $1-\exp(\cdot)$ term in (30) as

$$F_{\mathsf{SIR}_k}(\theta)\approx\sum_{n=1}^{\lfloor\alpha/2\rfloor}\frac{(-1)^{n+1}\mathbb{E}\left[S_k^{-\frac{2n}{\alpha}}\right]}{\Gamma(1-\frac{2n}{\alpha})}\left[\Gamma\left(1-\frac{2}{\alpha}\right)\pi\theta^{\frac{2}{\alpha}}\widetilde{\lambda}\right]^n, \tag{12}$$

where $\lfloor x\rfloor\triangleq\max\{y\in\mathbb{Z}:y\leq x\}$. Namely, we have $F_{\mathsf{SIR}_k}(\theta)\in\Theta\left(\widetilde{\lambda}\mathbb{E}\left[S_k^{-\frac{2}{\alpha}}\right]\right)$ for a given $\theta>0$ as $\widetilde{\lambda}\mathbb{E}\left[S_k^{-\frac{2}{\alpha}}\right]$ approaches zero. In other words, $F_{\mathsf{SIR}_k}(\theta)$ in (12) is very accurate in this case and the bounds in (10) are very tight since they coverage to each other eventually.

For the case of received signal power S_k having an Erlang distribution, the closed-form result of $F_{\mathsf{SIR}_k}(\theta)$ in Theorem 1 indeed exists, as shown in following corollary.

Corollary 2. *If the type-*k *received signal power* $\widehat{S}_k$ *with unit mean is an Erlang RV (i.e.,* $\widehat{S}_k\sim$ *Erlang*(μ,μ) *where* $\mu\in\mathbb{N}_+$*), then we have*

$$F_{\mathsf{SIR}_k}(\theta)=1-\frac{\frac{d^{\mu-1}}{dv^{\mu-1}}\left[v^{\mu-1}\mathcal{L}_{I_k}\left(\frac{\mu}{v\mathbb{E}[S_k]}\right)\right]\Big|_{v=\theta^{-1}}}{(\mu-1)!}. \tag{13}$$

Proof: Since we assume $\widehat{S}_k\sim\text{Erlang}(\mu,\mu)$, $F_{\mathsf{SIR}_k}(\theta)$ in (9) can be written as

$$F_{\mathsf{SIR}_k}(\theta)=1-\mathcal{L}^{-1}\left\{\frac{\mu^\mu s^{\mu-1}\int_0^\infty\mathcal{L}_{I_k}\left(\frac{1}{t\mathbb{E}[S_k]}\right)\frac{t^{\mu-1}}{e^{\mu st}}\mathrm{d}t}{(\mu-1)!}\right\}\left(\frac{1}{\theta}\right)$$
$$=1-\frac{\mathcal{L}^{-1}\left\{s^{\mu-1}\int_0^\infty\left[\mathcal{L}_{I_k}\left(\frac{\mu}{v\mathbb{E}[S_k]}\right)v^{\mu-1}\right]e^{-sv}\mathrm{d}v\right\}\left(\frac{1}{\theta}\right)}{(\mu-1)!},$$

and using the identity $\mathcal{L}\left\{\frac{\mathrm{d}^\mu}{\mathrm{d}t^\mu}g(t)\right\}(s)=s^\mu\int_0^\infty g(t)e^{-st}\mathrm{d}t$ to simplify $F_{\mathsf{SIR}}(\theta)$ in above yields the result in (13). □

For any particular value of μ, the explicit closed-form expression of $F_{\mathsf{SIR}_k}(\theta)$ can be easily found by carrying out the μth-order derivative in (13). For instance, in the special case of $\mu=1$, i.e., $\widehat{S}_k\sim\exp(1,1)$ is an exponential RV with unit mean and variance[2], $F_{\mathsf{SIR}_k}(\theta)$ in (13) reduce to

$$F_{\mathsf{SIR}_k}(\theta)=1-e^{-\pi\Gamma\left(1-\frac{2}{\alpha}\right)\widetilde{\lambda}(\theta/\mathbb{E}[S_k])^{\frac{2}{\alpha}}}, \tag{14}$$

and this obviously shows that SIR_k has a Weibull distribution with parameters $\frac{2}{\alpha}$ and $\mathbb{E}[S_k]/(\pi\Gamma(1-\frac{2}{\alpha})\widetilde{\lambda})^{\frac{\alpha}{2}}$.

Another case that $F_{\mathsf{SIR}_k}(\theta)$ in Theorem 1 can be found in a simpler form is when S_k does not possess any randomness, as shown in the following corollary.

Corollary 3. *If the received signal power of a type-*k *RX is not a random variable, i.e.,* S_k *in* (4) *is deterministic, the CDF of* SIR_k *in* (9) *reduces to*

$$F_{\mathsf{SIR}_k}(\theta)=1-\mathcal{L}^{-1}\left\{\frac{1}{s}\mathcal{L}_{I_k}\left(\frac{s}{S_k}\right)\right\}(\theta^{-1}). \tag{15}$$

Proof: Notice that $F_{\mathsf{SIR}_k}(\theta)$ in (9) can be rewritten as follows

$$F_{\mathsf{SIR}_k}(\theta)=1-\mathcal{L}^{-1}\left\{\int_0^\infty\frac{1}{s}\mathcal{L}_{I_k}\left(\frac{s}{u\mathbb{E}[S_k]}\right)f_{\widehat{S}_k}(u)\mathrm{d}u\right\}(\theta^{-1})$$
$$=1-\mathcal{L}^{-1}\left\{\mathbb{E}_{S_k}\left[\frac{1}{s}\mathcal{L}_{I_k}\left(\frac{s}{S_k}\right)\right]\right\}(\theta^{-1}).$$

Thus, if S_k is a constant, we readily obtain (15). □

Although the inverse Laplace transforms in (15) in general still cannot be explicitly calculated, they can be evaluated by the numerical inverse Laplace transform for any particular value of θ. For $\pi\widetilde{\lambda}/S_k^{\frac{2}{\alpha}}\ll 1$, the closed-form approximation of $F_{\mathsf{SIR}_k}(\theta)$ also can be inferred from (12) as

$$F_{\mathsf{SIR}_k}(\theta)\approx\sum_{n=1}^{\lfloor\alpha/2\rfloor}\frac{(-1)^{n+1}}{\Gamma(1-\frac{2n}{\alpha})}\left[\Gamma\left(1-\frac{2}{\alpha}\right)\pi\widetilde{\lambda}\left(\frac{\theta}{S_k}\right)^{\frac{2}{\alpha}}\right]^n. \tag{16}$$

Furthermore, for the special case of $\alpha=4$, (15) has a closed-form expression directly obtained from (11) as

$$F_{\mathsf{SIR}_k}(\theta)=\operatorname{erf}\left(\frac{\pi^{\frac{3}{2}}\widetilde{\lambda}}{2}\sqrt{\frac{\theta}{S_k}}\right), \tag{17}$$

where $\widetilde{\lambda}=\sum_{j=1}^{K}\lambda_j\mathbb{E}\left[\sqrt{H_j}\right]\mathbb{E}\left[\sqrt{P_j}\right]$and $S_k=P_kH_kR_k^{-4}$ is a constant.

B. General Results on the type-k coverage probability

In general, the type-k model-free coverage probability $p_k(\theta)$ cannot be derived in an explicit closed form based on (9) if S_k does not have an Erlang distribution. Nonetheless, the bounds on $p_k(\theta)$ can be characterized as shown in the following corollary.

[2]This could happen in the case that the transmit power and distance are constant and the communication channel undergoes Rayleigh fading so that its gain distribution is $\sim\exp(1)$.

Corollary 4. *The type-k coverage probability in* (5) *can be bounded as follows*

$$p_k(\theta)\begin{cases}\geq\left(1-\pi\widetilde{\lambda}\mathbb{E}\left[S_k^{-\frac{2}{\alpha}}\right]\theta^{\frac{2}{\alpha}}\right)^{+}\\ \leq\mathcal{L}^{-1}\left\{\frac{\mathbb{E}\left[S_k^{\frac{2}{\alpha}}\right]}{s\left(\pi\Gamma(1-\frac{2}{\alpha})\widetilde{\lambda}s^{\frac{2}{\alpha}}+\mathbb{E}\left[S_k^{\frac{2}{\alpha}}\right]\right)}\right\}(\theta^{-1})\end{cases}. \quad (18)$$

If $\alpha=4$, $p_k(\theta)$ has a nearly closed-form expression given by

$$p_k(\theta)=\mathbb{E}\left[\mathrm{erfc}\left(\frac{\pi^{\frac{3}{2}}\widetilde{\lambda}}{2}\sqrt{\frac{\theta}{S_k}}\right)\right], \quad (19)$$

where $\widetilde{\lambda}=\sum_{j=1}^{K}\lambda_j\mathbb{E}[\sqrt{P_j}]\mathbb{E}[\sqrt{H_j}]$ and $S_k=P_kH_kR_k^{-4}$.

Proof: The proof is omitted since it is similar to the proof of Corollary 1. □

In addition, using the error function's Maclaurin series (19) can be further written as

$$p_k(\theta)=1-\frac{2}{\sqrt{\pi}}\sum_{n=0}^{\infty}\frac{(-1)^n}{n!(2n+1)}\left(\frac{\pi^{\frac{3}{2}}\widetilde{\lambda}\sqrt{\theta}}{2}\right)^{2n+1}\mathbb{E}\left[\frac{1}{S_k^{n+\frac{1}{2}}}\right] \quad (20)$$

$$\geq\mathrm{erfc}\left(\frac{\pi^{\frac{3}{2}}\widetilde{\lambda}\sqrt{\theta}}{2}\mathbb{E}\left[\frac{1}{\sqrt{S_k}}\right]\right), \quad (21)$$

where the lower bound in (21) is obtained by applying Jensen's inequality to the erfc function with a positive argument that is convex. Although the result in (19) is derived by considering the special case of $\alpha=4$, it is still very important since it is applicable to any random channel gain, transmit power and distance models and able to directly provide some insight into how the randomness of the received signal power affects the coverage probability. Note that $p_k(\theta)$ in (20) reduces to (21) if S_k is constant.

In order to have a more tractable result of $p_k(\theta)$ in practically applicable contexts with a general pathloss exponent, we consider normalized received signal power $\widehat{S}_k$ as a Gamma random variable with mean 1 and variance $1/m_k$, i.e., $\widehat{S}_k\sim\mathrm{Gamma}(m_k,1/m_k)$, for $m_k\in\mathbb{N}_+$. Such a received signal power model is somewhat general because it characterizes the different randomness levels[3] of S_k. According to the results in Corollary 2 and $f_{\widehat{S}_k}(x)=\frac{m_k^{m_k}x^{m_k-1}}{e^{m_kx}\Gamma(m_k)}$, $p_k(\theta)$ without interference cancellation based on (13) for a positive integer m_k can be readily obtained by

$$p_k(\theta)=\frac{\frac{\mathrm{d}^{m_k-1}}{\mathrm{d}v^{m_k-1}}\left[v^{m_k-1}\mathcal{L}_{I_k}\left(\frac{m_k}{v\mathbb{E}[S_k]}\right)\right]\Big|_{v=\theta^{-1}}}{(m_k-1)!}, \quad (22)$$

whose closed-form expression can be explicitly calculated once m_k is designated. For the special case of $\widehat{S}_k\sim\exp(1)$ and Rayleigh fading interference channels, $p_k(\theta)$ in (22) reduces to

$$p_k(\theta)=\exp\left(-\frac{2\pi^2\theta^{\frac{2}{\alpha}}\sum_{j=1}^{K}\lambda_j\mathbb{E}\left[P_j^{\frac{2}{\alpha}}\right]}{\alpha\sin(2\pi/\alpha)(\mathbb{E}[S_k])^{\frac{2}{\alpha}}}\right), \quad (23)$$

which reduces to the seminal result firstly shown in [1] for $K=1$, constant transmit power and distance. The coverage probability in (22) reveals a very important fact that *the closed-form model-free coverage probability exists as long as the received signal power has an Erlang distribution*. This overthrows the traditional impression that the coverage probability only has a closed-form result for constant transmit power, distance and Rayleigh fading channels.

IV. Application of Type-k Model-Free Coverage: Distributed Stochastic Power Control

In this section, we apply the analytical results obtained on the type-k model-free coverage probability to theoretically clarifying an important question, that is, how to do distributed stochastic power control in order to improve the coverage probability. Namely, we would like to investigate how to improve the coverage probability by designing distributed stochastic power control schemes that change the distribution of the received signal power. According to the explicit results of the coverage probability in Section III-B, the key to maximizing the coverage probability of the type-k D2D pairs is how to minimize the term $\widetilde{\lambda}/(\mathbb{E}[S_k])^{\frac{2}{\alpha}}$ by optimally devising distributed power control schemes. Since each TX only has its local information available, we specifically propose the distributed stochastic power control scheme for a type-k D2D pair as follows

$$P_k=\frac{\overline{P}_k\left(H_kR_k^{-\alpha}\right)^{\gamma_k}}{\mathbb{E}\left[H_k^{\gamma_k}\right]\mathbb{E}\left[R_k^{-\alpha\gamma_k}\right]}, \quad (24)$$

where $\overline{P}_k$ is the mean of transmit power P_k, γ_k is the power control exponent needed to be designed. When there is no power control (i.e., constant transmit power is used), $P_k=\overline{P}_k$ (i.e., $\gamma_k=0$). This power control scheme is motivated by the fractional power control in [9] based on the fact that the randomness of the received signal power could improve the coverage probability [10]. We can change γ_k to adjust the randomness of S_k to improve the coverage probability in different network contexts. Therefore, the fundamental problem needed to be firstly studied is how the distributed stochastic power control in (24) changes/benefits the type-k coverage probability. The coverage probability with stochastic power control was essentially intractable in prior works, whereas it becomes much more tractable if using the coverage probability found in Section III-B. The following theorem shows $p_k(\theta)$ with the proposed distributed stochastic power control.

Theorem 2. *Suppose all the type-k TXs adopt the stochastic power control given in* (24). *Let $S_k=\overline{P}_kH_kR_k^{-\alpha}$ here be the received signal power without stochastic power control and the CCDF of S_k has the property $\mathbb{E}[F^c_{S_k}(Z)]\leq F^c_{S_k}(\mathbb{E}[Z])$*

[3] For constant transmit power P_k and distance R_k, $\widehat{S}_k\sim\mathrm{Gamma}(m_k,1/m_k)$ means the communication channel of the type-k RX suffers Nakagami-m_k fading and $H_k\sim\mathrm{Gamma}(m_k,1/m_k)$

for a nonnegative RV Z. For $\gamma_k > -1$, the bounds on $p_k(\theta)$ with stochastic power control are shown as

$$p_k^{pc}(\theta)\begin{cases}\leq F^c_{S_k}\left(\frac{\Gamma(1+\frac{\alpha}{2(1+\gamma_k)})(\theta\mathbb{E}[S_k^{\gamma_k}])^{\frac{1}{1+\gamma_k}}}{\left[\pi\Gamma(1-\frac{2}{\alpha})\widetilde{\lambda}^{pc}\right]^{\frac{\alpha}{2(1+\gamma_k)}}}\right)\\ \geq \left(1-\pi\widetilde{\lambda}^{pc}\left(\mathbb{E}\left[S_k^{\gamma_k}\right]\right)^{\frac{2}{\alpha}}\mathbb{E}\left[S_k^{-\frac{2(1+\gamma_k)}{\alpha}}\right]\theta^{\frac{2}{\alpha}}\right)^+\end{cases}, \tag{25}$$

where superscript "pc" means "power control" and $\widetilde{\lambda}^{pc}$ is

$$\widetilde{\lambda}^{pc} = \sum_{j=1}^{K}\lambda_j\overline{P}_j^{\frac{2}{\alpha}}\mathbb{E}\left[H_j^{\frac{2}{\alpha}}\right]\frac{\mathbb{E}\left[H_j^{\frac{2\gamma_j}{\alpha}}\right]}{(\mathbb{E}[H_j^{\gamma_j}])^{\frac{2}{\alpha}}}\frac{\mathbb{E}\left[R_j^{-2\gamma_j}\right]}{(\mathbb{E}[R_j^{-\alpha\gamma_j}])^{\frac{2}{\alpha}}}, \tag{26}$$

which is smaller than $\widetilde{\lambda} = \sum_{j=1}^{K}\lambda_j\overline{P}_j^{\frac{2}{\alpha}}\mathbb{E}\left[H_j^{\frac{2}{\alpha}}\right]$. Furthermore, if $\alpha = 4$, then $p_k^{pc}(\theta)$ has the following simple identity

$$p_k^{pc}(\theta) = \mathbb{E}\left[\text{erfc}\left(\frac{\pi^{\frac{3}{2}}\widetilde{\lambda}^{pc}}{2}\sqrt{\frac{\theta\mathbb{E}[S_k^{\gamma_k}]}{S_k^{\gamma_k+1}}}\right)\right], \tag{27}$$

where $\widetilde{\lambda}^{pc}$ is given in (26) with $\alpha = 4$ and $S_k = \overline{P}_k H_k R_k^{-4}$.

Proof: Please refer to the proof of Theorem 7 in [10]. □

According to Theorem 2, the distributed stochastic power control scheme with nonzero γ_k can reduce the interference since $\widetilde{\lambda}^{pc} < \widetilde{\lambda}$. This also implies that *the "randomness" of transmit power always results in less interference no matter if the power depends on the channel gain and/or pathloss*. Nonetheless, this does not mean the stochastic power control always benefits the coverage probability since it may not enhance the received signal power without using a proper value of γ_k. To make stochastic power control benefit the type-k coverage probability, this condition $p_k^{pc}(\theta) > p_k(\theta)$ must hold, which poses the constraint on the values of γ_k that are able to improve the type-k coverage probability. Unfortunately, the explicitly constraints on γ_k's for all $k \in \mathcal{K}$ are only tractably found for some special cases.

V. Numerical Results

In this subsection, a few numerical results are provided to validate the success probabilities derived in the previous subsections. We consider the heterogeneous wireless ad hoc network consisting of three disparate types of TXs and the simulation parameters for this heterogeneous network are listed in Table I. In Section III-B, we have shown that the randomness of the received signal power significantly influences the coverage probability, which can be further demonstrated in Fig. 1 for the coverage probabilities without and with channel randomness due to Rayleigh fading. As shown in Fig. 1, we observe an important phenomenon, that is, *channel randomness due to fading does not always weaken the coverage probability under different TX intensities*. In a dense network, channel randomness is usually able to improve the coverage probability since it weakens the interference channels much more than the communication channel. Also, we can exactly find the intensity region in which Rayleigh fading benefits the coverage probability. For example, in the simulation setting here we have $\widetilde{\Lambda}_2 = 2.662\times 10^3\lambda_1$, $p_2(\theta) = \exp(-\sqrt{\pi}\widetilde{\Lambda}_2)$ for Rayleigh fading and $p_2(\theta) = \text{erfc}\left(\widetilde{\Lambda}_2\right)$ for no fading.

TABLE I
Network Parameters for Simulation

Parameter \ TX Type k	Type 1	Type 2	Type 3
Transmit Power P_k (W)	1	0.5	0.05
Intensity λ_k (TXs/m^2)	λ_1	$5\lambda_1$	$10\lambda_1$
Pathloss Exponent α	4		
Transmit Distance R_k (m)	10		
Channel Gain $H_{k,i}$	$\sim\exp(1,1)$		
SIR Threshold θ	1		
Power control exponent γ_k	γ		

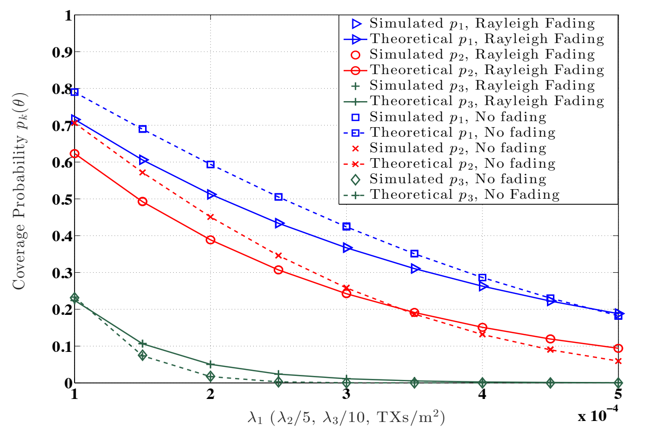

Fig. 1. Simulation results of the coverage probabilities with and without Rayleigh fading.

In Fig. 2, we show the success probabilities when the stochastic power control schemes with $\gamma = -0.5$ and $\gamma = 0.5$ in (24) are adopted. In Fig 2(a) for $\gamma = -0.5$, we observe that stochastic power control (slightly) outperforms no power control in the low intensity region (roughly when $\lambda_1 < 0.0001$), whereas in Fig 2(b) for $\gamma = 0.5$ stochastic power control outperforms no power control in the high intensity region (roughly when $\lambda_1 > 0.0001$). This validates our previous discussion that the power control exponent γ should change based on different TX intensities in order to make stochastic power control work better than no power control, and exploiting more randomness of the received signal power in a dense network (i.e., using a larger power control exponent) achieves a larger coverage probability. In addition, the correctness of $p_k^{pc}(\theta)$ in (27) is validated in Fig. 2 since it is used to provide the theoretical results of p_k^{pc} in the figure that perfectly coincide with their corresponding simulated results.

VI. Conclusions

In prior works, the distribution of the SIR in a pairwise decentralized network was analyzed by presuming some specific random models. Such a model-dependent distribution is unable to provide some insight into how the statistical properties of the SIR are impacted once the random models

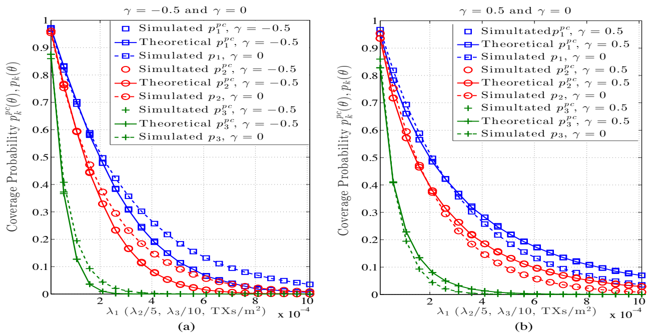

Fig. 2. Simulation results of the coverage probabilities with and without the stochastic power control: (a) $p_k^{pc}(\theta)$ with $\gamma = -0.5$ and $p_k(\theta) = p_k^{pc}(\theta)$ with $\gamma = 0$, (b) $p_k^{pc}(\theta)$ with $\gamma = -0.5$ and $p_k(\theta) = p_k^{pc}(\theta)$ with $\gamma = 0$.

involved in the SIR change. Accordingly, in this paper we introduce a Laplace-transform-based framework of analyzing the distribution of the model-free SIR without using any specific models on he channel gain, transmit power and distance in a network consisting of K different types of D2D pairs. This model-free framework successfully helps us find the general bounds and nearly closed-form result of the type-k coverage probability. We apply the general results of the type-k coverage probability in clarifying the main question regarding how to do distributed stochastic power control in oder to improve the coverage probability. Numerical results validate that the derived coverage probabilities without and with Rayleigh fading channel and the derived coverage probabilities without and with the proposed stochastic power control are all correct.

APPENDIX

A. Proof of Theorem 1

According to (4) and (8), the Laplace transform of the reciprocal of SIR_k in (4) can be expressed as

$$\mathcal{L}_{\mathsf{SIR}_k^{-1}}(s) = \mathbb{E}_{S_k}\left[\mathcal{L}_{I_k}\left(\frac{s/\mathbb{E}[S_k]}{S_k/\mathbb{E}[S_k]}\right)\right] = \int_0^\infty \frac{f_{\widehat{S}_k}(x)\mathrm{d}x}{e^{\pi\Gamma\left(1-\frac{2}{\alpha}\right)\widetilde{\lambda}\left(\frac{\widehat{s}}{x}\right)^{\frac{2}{\alpha}}}}$$
$$= s\int_0^\infty \frac{f_{\widehat{S}_k}(ts)\mathrm{d}t}{e^{\pi\Gamma\left(1-\frac{2}{\alpha}\right)\widetilde{\lambda}(t\mathbb{E}[S_k])^{-\frac{2}{\alpha}}}}, \quad (28)$$

where $\widehat{s} \triangleq s/\mathbb{E}[S_k]$. By the definition of $\mathcal{L}_{\mathsf{SIR}_k^{-1}}(s)$, we have

$$\mathcal{L}_{\mathsf{SIR}_k^{-1}}(s) = \int_0^\infty f_{\mathsf{SIR}_k^{-1}}(t)e^{-st}\mathrm{d}t = \int_0^\infty \frac{\mathrm{d}F_{\mathsf{SIR}_k^{-1}}(t)}{\mathrm{d}t}e^{-st}\mathrm{d}t$$
$$= \int_0^\infty sF_{\mathsf{SIR}_k^{-1}}(t)e^{-st}\mathrm{d}t = \int_0^\infty \frac{sf_{\widehat{S}_k}(ts)\mathrm{d}t}{e^{\pi\Gamma\left(1-\frac{2}{\alpha}\right)\widetilde{\lambda}(t\mathbb{E}[S_k])^{-\frac{2}{\alpha}}}},$$

which indicates

$$\int_0^\infty F_{\mathsf{SIR}_k^{-1}}(t)e^{-st}\mathrm{d}t = \int_0^\infty e^{-\pi\Gamma\left(1-\frac{2}{\alpha}\right)\widetilde{\lambda}(t\mathbb{E}[S_k])^{-\frac{2}{\alpha}}} f_{\widehat{S}_k}(ts)\mathrm{d}t \quad (29)$$

and then taking the inverse Laplace transform of the both sides of (29) yields $F_{\mathsf{SIR}_k^{-1}}(t) = 1 - F_{\mathsf{SIR}_k}\left(t^{-1}\right)$, which is

$$F_{\mathsf{SIR}_k^{-1}}(t) = \mathcal{L}^{-1}\left\{\int_0^\infty \mathcal{L}_{I_k}\left(\frac{1}{t\mathbb{E}[S_k]}\right) f_{\widehat{S}_k}(ts)\mathrm{d}t\right\}(t)$$

and then setting the argument of $F_{\mathsf{SIR}_k}\left(t^{-1}\right)$ as $t^{-1} = \theta$ results in (9).

B. Proof of Corollary 1

The CDF of SIR_k in (9) can be rewritten as

$$F_{\mathsf{SIR}_k}(\theta) = \mathcal{L}^{-1}\left\{\mathbb{E}\left[\frac{1}{s}\left(1 - e^{-\pi\Gamma(1-\frac{2}{\alpha})\widetilde{\lambda}(s/S_k)^{\frac{2}{\alpha}}}\right)\right]\right\}\left(\frac{1}{\theta}\right)$$
$$= \mathbb{E}\left[\mathcal{L}^{-1}\left\{\frac{1}{s}\left(1 - e^{-\pi\Gamma(1-\frac{2}{\alpha})\widetilde{\lambda}(s/S_k)^{\frac{2}{\alpha}}}\right)\right\}(\theta^{-1})\right]. \quad (30)$$

Using the inequality $\frac{x}{1+x} \leq 1 - e^{-x} \leq x$ for $x > 0$, the upper bound on the result in (30) is

$$F_{\mathsf{SIR}_k}(\theta) \leq \mathbb{E}\left[\mathcal{L}^{-1}\left\{\frac{\pi\Gamma(1-\frac{2}{\alpha})\widetilde{\lambda}}{s^{1-\frac{2}{\alpha}}S_k^{\frac{2}{\alpha}}}\right\}\left(\frac{1}{\theta}\right)\right] = \pi\widetilde{\lambda}\mathbb{E}\left[S_k^{-\frac{2}{\alpha}}\right]\theta^{\frac{2}{\alpha}}$$

and

$$F_{\mathsf{SIR}_k}(\theta) \geq \mathcal{L}^{-1}\left\{\mathbb{E}\left[\frac{\pi\Gamma(1-\frac{2}{\alpha})\widetilde{\lambda}}{s^{1-\frac{2}{\alpha}}\left(\pi\Gamma(1-\frac{2}{\alpha})\widetilde{\lambda}s^{\frac{2}{\alpha}} + S_k^{\frac{2}{\alpha}}\right)}\right]\right\}\left(\frac{1}{\theta}\right)$$
$$\geq \mathcal{L}^{-1}\left\{\frac{\pi\Gamma(1-\frac{2}{\alpha})\widetilde{\lambda}}{s^{1-\frac{2}{\alpha}}\left(\pi\Gamma(1-\frac{2}{\alpha})\widetilde{\lambda}s^{\frac{2}{\alpha}} + \mathbb{E}\left[S_k^{\frac{2}{\alpha}}\right]\right)}\right\}(\theta^{-1}),$$

where the second inequality holds due to the convexity of $1/(a+x)$ for $a, x > 0$. Therefore, the upper and lower bounds in (10) is acquired. For $\alpha = 4$, the inverse Laplace transform in (30) can be found in closed-form so that we have

$$F_{\mathsf{SIR}_k}(\theta) = \mathbb{E}\left[\operatorname{erf}\left(\frac{\pi^{\frac{3}{2}}\widetilde{\lambda}\sqrt{\theta}}{2\sqrt{S_k}}\right)\right]. \quad (31)$$

Hence, the result in (11) is obtained.

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