Optimization and Learning Algorithms for Stochastic and Adversarial Power Control

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***Abstract*—Power control in wireless networks is a well-studied problem. However, recently it has been demonstrated that significant throughput gains can be achieved using data-driven online learning algorithms, supported by a cloud computing infrastructure. In this paper, we provide theoretical guarantees for such algorithms. In particular, we consider two variants of the problem: one which emphasizes long-term throughput and the other which emphasizes robust short-term throughput. The first problem reduces to solving a convex optimization problem with noisy, stochastic measurements while the second one is an online optimization problem where an adversary chooses the reward functions. We provide stochastic and online gradient descent methods customized for the power control problem and establish their convergence analysis. We show that in both cases, the total regret over a time horizon T grows sublinearly at rate $O(\sqrt{T})$ for suitable choices of algorithms and algorithm parameters.**

***Index Terms*—Stochastic Gradient Descent, Online Convex Optimization, Resource Allocation, Power Control.**

I. Introduction

Power control for utility maximization in wireless networks and cellular communication systems has been intensively studied. There is an extensive body of work on this topic, from the early papers [1] and [2] which tackle the problem in CDMA cellular networks, to the more recent work such as [3], [4], [5] and [6] which optimize utility measures jointly for both the transmission rate and power.

Our motivation comes from a recent paper [7], where the authors tackle the uplink power control problem in a 5000-cell LTE system in a live major service provider's network, using a learning-based algorithm. Somewhat surprisingly, the authors show that such an approach yields throughput gains (such as the doubling of throughput for 80% of the users) that typically can only be obtained by adding additional antennas. Unlike much of the prior work, [7] emphasizes the importance of data in the form of channel state measurements and the fact that the measurements are noisy samples of an underlying stochastic process. The main challenge that makes the power control problem in modern wireless networks such as the LTE systems unique is that the primary source of interference in such systems is inter-cell interference. Moreover, the interference pattern keeps changing every time slot, thereby making it important to use fast learning algorithms.

In this paper, we aim to solve the problem of maximizing the cumulative utility of different users in such a wireless network, using online learning-based power control. We consider the problem in two different scenarios. First, we consider the problem of average log Signal-to-Noise-Ratio-based (SINR-based) utility maximization. Average log SINR is an approximation to long-term throughput, used for tractability reasons, but real-life implementations show its efficacy in addressing long-term throughput (see [7]). In this scenario, we consider a similar problem formulation as in [7], i.e., we consider a LTE-like wireless network, with several base stations (each base station determines a unique cell) and several users within each base station. We consider the utility of each user in the network to be a function of the expected value of the logarithmic value of its SINR. As discussed in [7], this formulation is effective in optimizing the performance of a wireless network in the long-term, while having the advantage of being mathematically tractable.

The second problem that we consider is that of robust short-term performance, in a similar LTE-like wireless network setup as in the previous case. In this scenario, at any time instant, we measure the utility of each user as a function of the user's instantaneous throughput. This allows us to model the short-term performance of the network. We formulate the problem as an online convex optimization problem, and use regret as the metric to quantify the short-term performance of the resulting power control policy. Note that the primary difference between the two problem scenarios is the following: while the first case focuses on improving the utility in the long-term by assuming it to be a function of expected logarithmic SINR, the latter case focuses on achieving robust performance in the short-term by assuming utility to be instantaneous.

For both the aforementioned problem scenarios, we further consider two different cases based on the restrictions imposed on the power control policy, including the General Power Control (GPC) policy, i.e., when the power assigned to each user is allowed to be different, and the Fractional Power Control (FPC) policy, as used in practical LTE-based systems (see [8], [9], [10] and [11] for recent work on FPC). In FPC, the power for a user i in a cell c is given by $Q_c h_{ic}^{-\alpha_c}$, where Q_c is the nominal transmit power for cell c, h_{ic} is the channel gain for user i within cell c, and $0 \leq \alpha_c \leq 1$ is the fractional path loss compensation factor for cell c.

Our main contributions in this paper are the following:

1) We formulate the average SINR-based optimal power

control problem as a convex optimization problem. Subsequently, we present a stochastic gradient descent-based (SGD-based) algorithm to compute the optimal power control policy, for both General Power Control and Fractional Power Control (see Sections II and III).

2) We prove that the SGD-based algorithm converges to the optimal power control policy. We also present guarantees on its convergence rate. Moreover, we believe that the proof for the learning-based power control algorithm presented in [7] has a gap, since they do not show that the dual variables in their optimization problem lie in a bounded set. We fix this gap in our proof. Moreover, in [7], only asymptotic convergence analysis is provided, whereas we provide explicit finite-time guarantees on the rate of convergence as well as the regret (see Sections II-B and III-B).

3) For the robust short-term performance problem, we present an online gradient descent-based algorithm, for both General Power Control and Fractional Power Control. We also provide guarantees on the regret achieved by the resulting power control policy (see Section IV).

II. SINR-BASED GENERAL POWER CONTROL

A. Problem Formulation

In order to formalize the optimization problem, let us introduce some notation first. Let there be a total of n users and m base stations in the network. Let γ_i denote the expected logarithmic value of SINR for user i, and let $U_i(\gamma_i)$ denote the utility function for user i. Let j_i denote the base station (or cell) to which user i belongs to, and let $h_{ik}(t) \geq 0$ denote the random channel gain for user i's transmissions to cell k, at time t. We assume that all channel gains are drawn i.i.d. at each time slot t, from their respective distributions. Let $X_i(t)$ be a binary random variable denoting whether user i is active or not, at any time slot t. Let $X_i(t) = 0$ if the user is inactive and $X_i(t) = 1$ if the user is active. We assume that $X_i(t)$ is drawn i.i.d. at each time slot t. Since $X_i(t)$ and $h_{ik}(t)$ are drawn i.i.d. at each time slot, we drop their dependence on t and denote them by X_i and h_{ik} respectively. Let $P = (P_1, P_2, ..., P_n)$ be the vector of power assignments to various users. Formally, the optimization problem we aim to solve is:

$$\begin{aligned} &\max_{\gamma,P} \sum_i U_i(\gamma_i) \\ &\text{s. t. } \gamma_i = \mathbb{E}\left[\log\left(\frac{h_{ij_i}P_i}{\sum_{k\neq i} h_{kj_i}X_kP_k + \sigma_{j_i}^2}\right)\right], \forall i \\ &0 < P_i \leq e^M,\ \forall i. \end{aligned} \tag{1}$$

where $\sigma_{j_i}^2$ is the noise variance in cell j_i, e^M is the maximum power that can be assigned to each user, and the expectation is with respect to the random channel gains and the "on/off" random variables: $X_i, \forall i$. We assume that $\forall i$, the utility function $U_i(\gamma_i)$ is a concave and increasing function of γ_i, with $\lim_{\gamma_i \to -\infty} U_i(\gamma_i) = -\infty$. Moreover, we assume that $\forall i$, $U_i(\gamma_i)$ is well-defined for $\gamma_i \in (-\infty, \infty)$ and is continuously differentiable. Note that these assumptions are fairly general, and hence allow for several different utility functions, for instance: $U_i(\gamma_i) = \log(\log(1 + e^{\gamma_i}))$. Under the above assumptions, it can be shown that the optimization problem given by (1) is a non-convex optimization problem, and hence it is not amenable to the large number of efficient off-the-shelf algorithms available to solve convex optimization problems. As in [7] (also see [12]), we will transform the above optimization problem into a convex optimization problem, allowing us to exploit the extensive body of work available for solving convex optimization problems efficiently.

Consider the following standard variable transformation:

$$P_i = e^{q_i}, \forall i,$$

Since $P_i > 0, \forall i$, the above transformation is bijective. With the above transformation, we can rewrite (1) as:

$$\begin{aligned} &\max_{\gamma,q} \sum_i U_i(\gamma_i) \\ &\text{s. t. } \gamma_i = \mathbb{E}\left[\log\left(\frac{h_{ij_i}e^{q_i}}{\sum_{k\neq i} h_{kj_i}X_ke^{q_k} + \sigma_{j_i}^2}\right)\right], \forall i \\ &q_i \leq M,\ \forall i. \end{aligned} \tag{2}$$

Note that we assumed the objective function above to be a concave function in γ. Also, the second constraint is an affine inequality, therefore the feasible set corresponding to it will be a convex set. Therefore, in order to obtain a convex optimization problem, all we need to do is to transform the first constraint (the equality constraint), so that the resulting constraint has a convex feasible set. To this end, we observe that if we change the "equal to" operator in the first constraint to a "less than equal to" operator, the solution to the optimization problem will remain the same, as we assumed $U_i(\gamma_i)$ to be an increasing function of γ_i (the maximum will always be achieved at the highest value possible). Therefore:

$$\begin{aligned} &\gamma_i \leq \mathbb{E}\left[\log\left(\frac{h_{ij_i}e^{q_i}}{\sum_{k\neq i} h_{kj_i}X_ke^{q_k} + \sigma_{j_i}^2}\right)\right] \\ &\gamma_i \leq \mathbb{E}[\log h_{ij_i} + q_i - \log(\sum_{k\neq i} h_{kj_i}X_ke^{q_k} + \sigma_{j_i}^2)] \\ &\Rightarrow \gamma_i - \big(\mathbb{E}[\log h_{ij_i} + q_i - \log(\sum_{k\neq i} h_{kj_i}X_ke^{q_k} + \sigma_{j_i}^2)]\big) \leq 0 \end{aligned}$$

Since log-sum-of-exponentials is a convex function, the feasible set for the above constraints will be convex, $\forall i$. Hence, we get the following convex optimization problem:

$$\begin{aligned} &\min_{\gamma,q} -\sum_i U_i(\gamma_i) \\ &\text{s. t. } \gamma_i - \big(\mathbb{E}[\log h_{ij_i} + q_i - \log(\sum_{k\neq i} h_{kj_i}X_ke^{q_k} + \sigma_{j_i}^2)]\big) \\ &\qquad \leq 0, \forall i \\ &q_i \leq M,\ \forall i. \end{aligned} \tag{3}$$

For ease of notation, we will assume that $\sigma_{j_i}^2 = \sigma^2, \forall i$, i.e., the noise variance in all the cells is the same. Note that this assumption does not affect the results in this paper in any way.

B. Algorithm and Analysis

In this subsection, we first prove that there exists a solution to the optimization problem in (3). Subsequently, we prove certain properties for the primal-dual optimization problem corresponding to (3), in order to design an algorithm based on the stochastic gradient descent algorithm (see [13]). We eventually present a SGD-based algorithm to solve (3), and also provide theoretical guarantees on its performance. Let us begin with proving the existence of a solution to (3).

Lemma 1. *For the optimization problem in* (3)*, there exists a non-empty, closed and bounded set of optimal solutions.*

Proof. Let us denote the variables γ and q jointly as $s = (\gamma, q)$. Note that under the constraints of (3), $||s|| \to \infty$ implies $q_i \to -\infty$ or $\gamma_i \to -\infty$, for at least some i. Also, note that $q_i \to -\infty \Rightarrow \gamma_i \to -\infty$. Therefore, $||s|| \to \infty$ implies $\gamma_i \to -\infty$, for some i. Moreover, we know that $\lim_{\gamma_i \to -\infty} \sum_j U_j(\gamma_j) = -\infty, \forall i$, since we assume that $\lim_{\gamma_i \to -\infty} U_i(\gamma_i) = -\infty, \forall i$. Let $\mathbb{C}$ denote the feasible set of (3). Therefore, we have:

$$\lim_{||s|| \to \infty, s \in \mathbb{C}} -\sum_i U_i(\gamma_i) = \infty.$$

Hence, the objective function in (3) is weakly coercive with respect to the set $\mathbb{C}$. Using Weierstrass' Theorem (see Theorem 4.7 in [14]), we get the lemma. $\square$

The Lagrangian function corresponding to the optimization problem in (3) is:

$$\begin{aligned} &L(\gamma, q, \lambda, \mu) \\ &\quad = -\sum_i U_i(\gamma_i) + \sum_i \lambda_i \big(\gamma_i - (\mathbb{E}[\log h_{ij_i} + q_i \\ &\qquad - \log(\sum_{k \neq i} h_{kj_i} X_k e^{q_k} + \sigma_{j_i}^2)])\big) + \sum_i \mu_i (q_i - M). \end{aligned} \tag{4}$$

where $\lambda = (\lambda_1, \lambda_2, ..., \lambda_n)$ and $\mu = (\mu_1, \mu_2, ..., \mu_n)$ are vectors representing the Lagrange multipliers for the inequality constraints in (3). We now show that strong duality holds for the primal and dual optimization problems characterized by (4).

Lemma 2. *For the primal and dual optimization problems characterized by the Lagrangian function in* (4)*, strong duality holds, i.e.:*

$$\begin{aligned} L^* &= \min_{\gamma, q} \max_{\lambda \geq 0, \mu \geq 0} L(\gamma, q, \lambda, \mu) \\ &= \max_{\lambda \geq 0, \mu \geq 0} \min_{\gamma, q} L(\gamma, q, \lambda, \mu). \end{aligned} \tag{5}$$

where L^* denotes the (optimal) value of the objective function in (3)*.*

Proof. Let us consider the point $q_i = \frac{M}{2} < M, \forall i$. Subsequently, consider $\gamma_i < \mathbb{E}[\log h_{ij_i} + \frac{M}{2} - \log(\sum_{k \neq i} h_{kj_i} X_k e^{\frac{M}{2}} + \sigma^2)], \forall i$. Clearly, the above values of q and λ are feasible for (3). Moreover, for these values, the inequality constraints hold with strict inequality. Therefore, using Slater's conditions, we conclude the lemma. $\square$

Let γ^*, q^*, λ^* and μ^* be the solution to (5). In order to use the stochastic gradient descent algorithm from [13], we need to prove certain properties of the Lagrangian function in (4), and its optimal solution: γ^*, q^*, λ^* and μ^*. We start with the following lemma:

Lemma 3. $(\gamma^*, q^*, \lambda^*, \mu^*) \in \xi = \xi(M, h', \sigma', X')$*, where $\xi(M, h', \sigma', X')$ is a non-empty bounded closed convex set, and can be computed from* (14)*,* (15)*,* (16)*,* (17) *and* (18) *as a function of M, h', σ', and X', where M is the upper bound on the variable $q_i, \forall i$, h' is the vector comprising upper and lower bounds on the random channel gains for all the users in the network, σ' is the vector comprising upper and lower bounds on the noise variance in all the cells, and X' is the vector comprising upper and lower bounds on the probability that a user is "on" at any time slot, i.e., the upper and lower bounds on $\mathbb{E}[X_i], \forall i$.*

Proof. Using the KKT stationarity conditions from optimization theory, we know that if γ^*, q^* solve (3), $\exists\ \lambda^* \geq 0$ and $\mu^* \geq 0$ such that:

$$-U_i'(\gamma_i^*) + \lambda_i^* = 0, \forall i, \tag{6a}$$

$$-\lambda_i^* + \sum_{k \neq i} \lambda_k^* \mathbb{E}[\frac{X_i h_{ij_k} e^{q_i^*}}{\sum_{l \neq k} h_{lj_k} X_l e^{q_l^*} + \sigma^2}] + \mu_i^* = 0, \forall i. \tag{6b}$$

Adding (6b) for all i, we get:

$$\begin{aligned} &\sum_i \lambda_i^* = \sum_i \mu_i^* + \sum_k \lambda_k^* \mathbb{E}[\frac{\sum_{l \neq k} h_{lj_k} X_l e^{q_l^*}}{\sum_{l \neq k} h_{lj_k} X_l e^{q_l^*} + \sigma^2}], \\ &\Rightarrow \sum_i \mu_i^* = \sum_i \lambda_i^* \mathbb{E}[\frac{\sigma^2}{\sum_{l \neq i} h_{lj_i} X_l e^{q_l^*} + \sigma^2}]. \end{aligned} \tag{7}$$

Since the channel gains are positive and $e^{q_i^*} \in [0, e^M], \forall i$, we have:

$$\sum_i \lambda_i^* \mathbb{E}[\frac{\sigma^2}{\sum_{l \neq i} h_{lj_i} X_l e^M + \sigma^2}] \leq \sum_i \mu_i^* \leq \sum_i \lambda_i^*. \tag{8}$$

Combining the fact that $\forall i$, the utility function U_i is an increasing function with (6a), we observe that $\lambda_i^* > 0$. Combining this fact with (8), we get:

$$\sum_i \mu_i^* > 0. \tag{9}$$

Since $\mu_i^* \geq 0, \forall i$, therefore, $\exists\ m \in \{1, 2, ..., n\}$, such that $\mu_m^* > 0$. Now, by complementary slackness: $\mu_m^*(q_m^* - M) = 0$. Therefore, $q_m^* = M$. Using $q_m^* = M$ in (6b):

$$\lambda_m^* = \mu_m^* + \sum_{k \neq m} \lambda_k^* \mathbb{E}[\frac{X_m h_{mj_k} e^M}{\sum_{l \neq k} h_{lj_k} X_l e^{q_l^*} + \sigma^2}]. \tag{10}$$

Since all the terms in the above expression are non-negative, therefore:

$$\begin{aligned} &\lambda_k^* \mathbb{E}[\frac{X_m h_{mj_k} e^M}{\sum_{l \neq k} h_{lj_k} X_l e^{q_l^*} + \sigma^2}] \leq \lambda_m^*, \forall k \neq m, \\ &\Rightarrow \lambda_k^* \mathbb{E}[\frac{X_m h_{mj_k} e^M}{\sum_{l \neq k} h_{lj_k} X_l e^{M} + \sigma^2}] \leq \lambda_m^*, \forall k \neq m. \end{aligned} \tag{11}$$

where the last step follows from the fact that $e^{q_i^*} \in [0, e^M], \forall i$. Now, we observe that the inequality constraints on γ_i (for all i) in (3) will hold with equality (at the solution γ_i^*) as the utility function $U_i(\gamma_i)$ is an increasing function of γ_i. Therefore:

$$\begin{aligned} \gamma_m^* &= \mathbb{E}[\log(\frac{h_{mj_m} e^M}{\sum_{k \neq m} h_{kj_m} X_k e^{q_k^*} + \sigma^2})] \\ &\geq \mathbb{E}[\log(\frac{h_{mj_m} e^M}{\sum_{k \neq m} h_{kj_m} X_k e^{M} + \sigma^2})]. \end{aligned} \tag{12}$$

where the last step follows from the fact that $e^{q_i^*} \in [0, e^M], \forall i$. Combining (12) with (6a), we get:

$$\lambda_m^* \leq U_m'(\mathbb{E}[\log(\frac{h_{mj_m} e^M}{\sum_{k \neq m} h_{kj_m} X_k e^{M} + \sigma^2})]). \tag{13}$$

where the inequality follows from the fact that $U_i'(\gamma_i)$ is a decreasing function of γ_i, since $U_i(\gamma_i)$ is a concave function. Combining (13) and (11):

$$\begin{aligned} 0 \leq \lambda_i^* \leq C = \max_l \big\{ & U_l'\big(\mathbb{E}[\log(\frac{h_{lj_l} e^M}{\sum_{k \neq l} h_{kj_l} X_k e^{M} + \sigma^2})]\big) \\ & \times \big[\mathbb{E}[\frac{h_{lj_i} X_l e^M}{\sum_{k \neq i} h_{kj_i} X_k e^{M} + \sigma^2}]\big]^{-1} \big\}. \end{aligned} \tag{14}$$

Since $\lambda^*, \mu^* \geq 0$, combining (14) with (6b):

$$0 \leq \mu_i^* \leq C, \tag{15}$$

where C is the constant defined in (14). Also, since the first set of inequality constraints in (3) hold with equality at γ^* and q^*, we have:

$$\gamma_i^* = \mathbb{E}[\log(\frac{h_{ij_i} e^{q_i^*}}{\sum_{k \neq i} h_{kj_i} X_k e^{q_k^*} + \sigma^2})] \leq \mathbb{E}[\log \frac{h_{ij_i} e^M}{\sigma^2}], \forall i. \tag{16}$$

where the last step follows from the fact that $e^{q_i^*} \in [0, e^M], \forall i$. Also, using (6a) and (14) along with the fact that, $\forall i$, $U_i'(\gamma_i)$ is a decreasing function of γ_i, we get:

$$\gamma_i^* \geq U_i^{'-1}(C), \tag{17}$$

where C is the constant defined in (14). Combining (16) and (17):

$$\begin{aligned} &\mathbb{E}[\log(\frac{h_{ij_i} X_i e^{q_i^*}}{\sum_{k \neq i} h_{kj_i} X_k e^{q_k^*} + \sigma^2})] = \gamma_i^* \\ &\Rightarrow q_i^* \geq \gamma_i^* - \mathbb{E}[\log(\frac{h_{ij_i} X_i}{\sigma^2})] \\ &\Rightarrow q_i^* \geq U_i^{'-1}(C) - \mathbb{E}[\log(\frac{h_{ij_i} X_i}{\sigma^2})], \end{aligned} \tag{18}$$

where C is the constant defined in (14). Combining (14), (15), (16), (17) and (18), along with the fact that $q_i^* \leq M, \forall i$, we conclude that $(\gamma^*, q^*, \lambda^*, \mu^*) \in \xi$, where ξ is a non-empty bounded closed convex set. □

Remark 1. *Observe that the set ξ can be computed with loose upper and lower bounds on random channel gains, "on" probabilities for the users and the noise variance in each cell. Any reasonable and non-trivial bounds on these quantities will allow the computation of a non-empty bounded closed convex set ξ, to which the primal and dual optimal variables should belong to. Note that we do not need to compute the smallest such set. This remark is also applicable to the FPC case discussed in the next section.*

Next, we have the following lemma:

Lemma 4. *$L(\gamma, q, \lambda, \mu)$ is Lipschitz continuous on the set ξ.*

Proof. Note that L is a continuously differentiable function of γ, q, λ, μ. Moreover, we proved in Lemma 1 that ξ is a non-empty bounded closed convex set. The lemma follows directly by combining the above two statements. □

We now present the SGD-based general power control algorithm SGPC-SINR, as applicable to our problem. For ease of notation, let $z = (\gamma, q, \lambda, \mu)$. At each time instant t, we have $X(t) = (X_1(t), X_2(t), ..., X_n(t))$, i.e., i.i.d. samples of the random variables $X_i, \forall i$ (i.e., we know which users are active or not). We also have the random channel gains $h(t) = \{h_{ik}(t), \forall i, k\}$. Let us use these values to compute the random gradient estimate of the Lagrangian:

$$\nabla L(z, h(t), X(t)) = \begin{bmatrix} \big(-U_i'(\gamma_i) + \lambda_i\big)_{i=1}^n \\ \big(-\lambda_i + \sum_{k \neq i} \frac{\lambda_k X_i(t) h_{ij_k}(t) e^{q_i}}{\sum_{l \neq k} h_{lj_k}(t) X_l(t) e^{q_l} + \sigma^2} + \mu_i\big)_{i=1}^n \\ \big(\gamma_i - \log h_{ij_i}(t) - q_i + \log(\sum_{k \neq i} h_{kj_i}(t) X_k(t) e^{q_k} + \sigma^2)\big)_{i=1}^n \\ \big(q_i - M\big)_{i=1}^n \end{bmatrix} \tag{19}$$

Since we need to minimize the Lagrangian function with respect to the primal variables and maximize it with respect to the dual variables, the corresponding random gradient estimate to plug in a descent step is:

$$\nabla \tilde{L}(z, h(t) X(t)) = \begin{bmatrix} \big(-U_i'(\gamma_i) + \lambda_i\big)_{i=1}^n \\ \big(-\lambda_i + \sum_{k \neq i} \frac{\lambda_k X_i(t) h_{ij_k}(t) e^{q_i}}{\sum_{l \neq k} h_{lj_k}(t) X_l(t) e^{q_l} + \sigma^2} + \mu_i\big)_{i=1}^n \\ -\big(\gamma_i - \log h_{ij_i}(t) - q_i + \log(\sum_{k \neq i} h_{kj_i}(t) X_k(t) e^{q_k} + \sigma^2)\big)_{i=1}^n \\ -\big(q_i - M\big)_{i=1}^n \end{bmatrix} \tag{20}$$

Note that since L is a continuously differentiable function and we showed in Lemma 3 that the variable z can be effectively restricted to the set ξ, we have: $\mathbb{E}[||\nabla L(z, h(t), X(t))||_2^2] \leq G^2, \forall z \in \xi$, where $G = G(\xi)$ is a finite constant. Also, let

$D_\xi = \sqrt{2}\max_{z\in\xi}||z||_2$. D_ξ quantifies the size of the set ξ. Now, the SGPC-SINR algorithm is presented as Algorithm 1.

Algorithm 1 SGD-based GPC for SINR-based utility maximization (SGPC-SINR)[1]

initialize $z(0) = (\gamma(0), q(0), \lambda(0), \mu(0)) \in \xi$, constant $\theta > 0$.
for $t = 1, 2, \dots$:
1) Compute $\nabla\tilde{L}(z(t-1), h(t), X(t)), \eta_t = \frac{\theta D_\xi}{G\sqrt{t}}$.
2) $z(t) = \prod_\xi \big(z(t-1) - \eta_t\nabla\tilde{L}(z(t-1), h(t), X(t))\big)$.
3) Output $\tilde{z}(t) = \frac{\sum_{i=1}^t \eta_i z(i)}{\sum_{i=1}^t \eta_i}$.

end for

Theorem 1. *After T iterations of the SGPC-SINR algorithm, we have:*

$$\mathbb{E}[\epsilon(T)] = O(\frac{\max\{\theta,\theta^{-1}\}D_\xi G}{\sqrt{T}})$$

where $\epsilon(t) = \big[\max_{\lambda,\mu} L(\tilde{\gamma}(t), \tilde{q}(t), \lambda, \mu) - L^*\big] + \big[L^* - \min_{\gamma,q} L(\gamma, q, \tilde{\lambda}(t), \tilde{\mu}(t))\big]$.

Proof. Follows from equation (3.15) in Section 3.1 of [13]. $\square$

Corollary 1. *The SGPC-SINR algorithm solution converges to the optimal value of* (5), *i.e.:*

$$\lim_{T\to\infty} \mathbb{E}[L(\tilde{z}(T)) - L^*] = 0.$$

Proof. Using the definition of $\epsilon(T)$ from Theorem 1, we have:

$$\begin{aligned}
\Rightarrow L(\tilde{z}(T)) - L^* &\le \max_{\lambda,\mu} L(\tilde{\gamma}(T), \tilde{q}(T), \lambda, \mu) - L^*,\\
&\le \epsilon(T),\\
\Rightarrow \mathbb{E}[L(\tilde{z}(T)) - L^*] &\le \mathbb{E}[\epsilon(T)],\\
\Rightarrow \lim_{T\to\infty} \mathbb{E}[L(\tilde{z}(T)) - L^*] &\le \lim_{T\to\infty} \mathbb{E}[\epsilon(T)] = 0.
\end{aligned}$$

where the last step follows from the result in Theorem 1. $\square$

The following remarks apply to both this section and the next.

Remark 2. *The implementation of the power control algorithm presented in this section requires the knowledge of various channel gains. Some of these channel gains may not be directly available; in practice, one may be able to estimate a distribution of these channel gains and sample from this distribution when some channel gains are not directly measured. See [7] for details.*

Remark 3. *Note that Theorem 1 implies that the total expected regret up to time t given by*

$$\sum_{t=1}^{T} (\mathbb{E}[L(\tilde{z}(t)) - L^*]) = O\big(\sqrt{T}\big).$$

[1] $\prod_\xi$ denotes the projection operator for set ξ. See the text for the definitions of remaining quantities. Note that $\prod_\xi$ can be computed efficiently from (14), (15), (16), (17) and (18). Moreover, it can also be computed in parallel for different variables, since the bounds on each variable are independent of each other.

III. SINR-BASED FRACTIONAL POWER CONTROL (FPC)

A. Problem Formulation

As discussed in the introduction section, Fractional Power Control (FPC) is the method of power control used in LTE systems. As before, let us consider a user i, belonging to the cell (or base station) j_i. Let the channel gain for the user i for transmissions to cell k be denoted by h_{ik}. In FPC, the power assigned to user i, P_i is given as follows:

$$P_i = Q_{j_i} h_{ij_i}^{-\alpha_{j_i}} \tag{21}$$

where $0 \le Q_{j_i}$ is the nominal transmit power for cell j_i and $0 \le \alpha_{j_i} \le 1$ is the path loss compensation factor for cell j_i such that $Q_{j_i} h_{ij_i}^{-\alpha_{j_i}} \le e^M$ (e^M is the maximum power allowed).

Note that the major advantage of using FPC over GPC is that using FPC leads to a huge reduction in the number of free parameters. In GPC, each user can have its own power assignment, leading to a large number of parameters to be optimized. On the other hand, in FPC, each cell has only two parameters that need to be optimized, thereby drastically reducing the number of free parameters. For instance, in a typical LTE network with about 5,000 cells with each cell having around 50 users, GPC will have 250,000 free parameters whereas FPC will have only 100. Although FPC leads to a drastic reduction in the number of free parameters, note that the performance of GPC will always be better than FPC as it is more general. Setting $\pi_{j_i} = \log Q_{j_i}$, plugging (21) into (1) and following a similar process as in Section II-A, we get the following optimization problem for FPC:

$$\begin{aligned}
&\min_{\gamma,\pi,\alpha} - \sum_i U_i(\gamma_i)\\
\text{s. t. } &\gamma_i - \mathbb{E}[(1-\alpha_{j_i})\log h_{ij_i} + \pi_{j_i}\\
&\qquad - \log(\sum_{k\ne i} h_{kj_i} X_k (e^{\pi_{j_k} - \alpha_{j_k}\log h_{kj_k}}) + \sigma^2)] \le 0, \forall i,\\
&\pi_{j_i} - \alpha_{j_i}\log h_{ij_i} \le M,\ \forall i,\\
&-\alpha_{j_i} \le 0,\ \forall i,\\
&\alpha_{j_i} \le 1,\ \forall i,
\end{aligned} \tag{22}$$

where we have assumed $\sigma_{j_i}^2 = \sigma^2, \forall i$, as before. With the same assumptions on the utility functions as in the GPC section (see Section II-A for details), we can show that the above optimization problem is a convex optimization problem. The proof relies on the fact that the composition of an increasing log-sum-of-exponentials function with a convex function results in a convex function. We omit the details here due to space constraints.

B. Algorithm and Analysis

Similar to the GPC case, in this subsection, we first present a lemma that proves that there exists a solution to the optimization problem in (22). Subsequently, we prove certain properties for the primal-dual optimization problem corresponding to (22), in order to design an algorithm based on the stochastic gradient descent algorithm. We eventually present a SGD-based algorithm to solve (22), and also provide theoretical guarantees on its performance. All proofs in this section are omitted due to space limitations since they are similar to those in the previous section.

Lemma 5. *For the optimization problem in* (22)*, there exists a non-empty, closed and bounded set of optimal solutions.*

Let $s = (\gamma, \pi, \alpha)$ denote the vector of primal variables and $r = (\lambda, \mu, \beta, \zeta)$ denote the vector of Lagrange multipliers associated with the first, second, third and fourth set of constraints in 22 respectively. Also, let $\mathbb{B}$ denote the set of all cells/base stations. Then, the Lagrangian function corresponding to the optimization problem in (22) is:

$$\begin{aligned} &L(s,r) \\ &= -\sum_i U_i(\gamma_i) + \sum_i \lambda_i\big(\gamma_i - \mathbb{E}[(1-\alpha_{j_i})\log h_{ij_i} + \pi_{j_i} \\ &\quad - \log(\sum_{k\neq i} h_{kj_i} X_k (e^{\pi_{j_k} - \alpha_{j_k}\log h_{kj_k}}) + \sigma^2)]\big) \\ &\quad + \sum_i \mu_i(\pi_{j_i} - \alpha_{j_i}\log h_{ij_i} - M) + \\ &\quad \sum_{j_i\in\mathbb{B}} \beta_{j_i}(-\alpha_{j_i}) + \sum_{j_i\in\mathbb{B}} \zeta_{j_i}(\alpha_{j_i} - 1). \end{aligned} \tag{23}$$

We now show that strong duality holds for the primal and dual optimization problems characterized by (23).

Lemma 6. *For the primal and dual optimization problems characterized by the Lagrangian function in* (23)*, strong duality holds, i.e.:*

$$\begin{aligned} L^* &= \min_s \max_{r\geq 0} L(s,r) \\ &= \max_{r\geq 0} \min_s L(s,r). \end{aligned} \tag{24}$$

where L^* denotes the (optimal) value of the objective function in (22)*.*

Let s^* and r^* be the solution to (24). As in the GPC case, in order to use the stochastic gradient descent algorithm from [13], we need to prove certain properties of the Lagrangian function in (23), and its optimal solution: s^* and r^*.

Lemma 7. $(s^*, r^*) \in \xi = \xi(M, h', \sigma', X')$*, where $\xi(M, h', \sigma', X')$ is a non-empty bounded closed convex set and as in Lemma 3, is an efficiently computable function of M, h', σ', and X', where e^M is the maximum power for each user, h' is the vector comprising upper and lower bounds on the random channel gains for all the users in the network, σ' is the vector comprising upper and lower bounds on the noise variance in all the cells, and X' is the vector comprising upper and lower bounds on the probability that a user is "on" at any time slot, i.e., the upper and lower bounds on $\mathbb{E}[X_i], \forall i$.*

Note that the exact bounds on the set ξ in this case can be obtained similar to the bounds (14), (15), (16), (17) and (18) in the proof of Lemma 3. We omit this derivation due to space constraints.

Lemma 8. *$L(s,r)$ is Lipschitz continuous on the set ξ.*

We now present the SGD-based general power control algorithm SFPC-SINR, as applicable to the SINR-based FPC problem. For ease of notation, let $z = (s,r)$. At each time instant t, we have $X(t) = (X_1(t), X_2(t), ..., X_n(t))$, i.e., i.i.d. samples of the random variables $X_i, \forall i$ (i.e., we know which users are active or not). As before, we also have the random channel gains $h(t) = \{h_{ik}(t), \forall i, k\}$. Also, for each cell $c \in \mathbb{B}$, let $\mathbb{F}_c$ denote the set of users belonging to that cell. As in the previous section, let us compute the random gradient estimate of the Lagrangian to plug into a descent step:

$$\nabla \tilde{L}(z, h(t), X(t)) = \begin{bmatrix} \big(\nabla_{\gamma_i} L(z,h(t),X(t))\big)_{i=1}^n \\ \big(\nabla_{\pi_c} L(z,h(t),X(t))\big)_{c\in\mathbb{B}} \\ \big(\nabla_{\alpha_c} L(z,h(t),X(t))\big)_{c\in\mathbb{B}} \\ \big(-\nabla_{\lambda_i} L(z,h(t),X(t))\big)_{i=1}^n \\ \big(-\nabla_{\mu_i} L(z,h(t),X(t))\big)_{i=1}^n \\ \big(-\nabla_{\beta_c} L(z,h(t),X(t))\big)_{c\in\mathbb{B}} \\ \big(-\nabla_{\zeta_c} L(z,h(t),X(t))\big)_{c\in\mathbb{B}}, \end{bmatrix} \tag{25}$$

where the components are given by:

$$\nabla_{\gamma_i} L(z,h(t),X(t)) = -U_i'(\gamma_i) + \lambda_i,$$

$$\begin{aligned} &\nabla_{\pi_c} L(z,h(t),X(t)) \\ &= -\sum_{i\in\mathbb{F}_c} \lambda_i + \mu_c \\ &\quad + \sum_k \frac{\sum_{m\in\mathbb{F}_c, m\neq k} \lambda_m X_m(t) h_{mj_k}(t) e^{\pi_c - \alpha_c \log h_{mc}(t)}}{\sum_{l\neq k} h_{lj_k}(t) X_l(t) (e^{\pi_{j_l} - \alpha_{j_l}\log h_{lj_l}(t)}) + \sigma^2}, \end{aligned}$$

$$\begin{aligned} &\nabla_{\alpha_c} L(z,h(t),X(t)) \\ &= \sum_{i\in\mathbb{F}_c} \lambda_i \log h_{ic}(t) - \beta_c + \zeta_c + \mu_c \\ &+ \sum_k \frac{\sum\limits_{\substack{m\in\mathbb{F}_c \\ m\neq k}} \lambda_m X_m(t) h_{mj_k}(t) e^{\pi_c - \alpha_c\log h_{mc}(t)}(-\log h_{mc}(t))}{\sum\limits_{l\neq k} h_{lj_k}(t) X_l(t)(e^{\pi_{j_l} - \alpha_{j_l}\log h_{lj_l}(t)}) + \sigma^2}, \end{aligned}$$

$$\begin{aligned} &\nabla_{\lambda_i} L(z,h(t),X(t)) \\ &= \gamma_i - \big[(1-\alpha_{j_i})\log h_{ij_i}(t) + \pi_{j_i} \\ &\quad - \log(\sum_{k\neq i} h_{kj_i}(t) X_k(t)(e^{\pi_{j_k} - \alpha_{j_k}\log h_{kj_k}(t)}) + \sigma^2)\big], \end{aligned}$$

$$\nabla_{\mu_i} L(z,h(t),X(t)) = \pi_{j_i} - \alpha_{j_i}\log h_{ij_i}(t) - \log M,$$

$$\nabla_{\beta_c} L(z,h(t),X(t)) = -\alpha_c, \nabla_{\zeta_c} L(z,h(t),X(t)) = \alpha_c - 1.$$

As in the previous section, note that since L is a continuously differentiable function and we showed in Lemma 7 that the variable z can be effectively restricted to the set ξ, we have: $\mathbb{E}[||\nabla L(z,h(t),X(t))||_2^2] \leq G^2, \forall z \in \xi$, where $G = G(\xi)$ is a finite constant. Also, let $D_\xi = \sqrt{2}\max_{z\in\xi} ||z||_2$. Now, the SFPC-SINR algorithm is presented as Algorithm 2.

Algorithm 2 SGD-based FPC for SINR-based utility maximization (SFPC-SINR)

initialize $z(0) = (s(0), r(0)) \in \xi$, constant $\theta > 0$.
for $t = 1, 2, \dots$:

1) Compute $\nabla \tilde{L}(z(t-1), h(t), X(t)), \eta_t = \frac{\theta D_\xi}{G\sqrt{t}}$.
2) $z(t) = \prod_\xi \left(z(t-1) - \eta_t \nabla \tilde{L}(z(t-1), h(t), X(t))\right)$.
3) Output $\tilde{z}(t) = \frac{\sum_{i=1}^t \eta_i z(i)}{\sum_{i=1}^t \eta_i}$.

end for

Theorem 2. *After T iterations of the SFPC-SINR algorithm, we have:*

$$\mathbb{E}[\epsilon(T)] = O(\frac{\max\{\theta, \theta^{-1}\} D_\xi G}{\sqrt{T}})$$

where $\epsilon(t) = \left[\max_r L(\tilde{s}(t), r) - L^*\right] + \left[L^* - \min_s L(s, \tilde{r}(t))\right]$

Corollary 2. *The SFPC-SINR algorithm solution converges to the optimal value of* (24)*, i.e.:*

$$\lim_{T \to \infty} \mathbb{E}[L(\tilde{z}(T)) - L^*] = 0.$$

IV. Robust short-term performance

A. Problem Formulation

In the average SINR-based utility maximization, the focus was on achieving good long-term performance. In this section, we consider the problem of providing good short-term performance, which is further robust to channel state statistics. Therefore, we consider a formulation in which the utilities for the users are functions of their instantaneous throughputs and we do not make any assumptions on the statistics of the channel states. In particular, we allow the channel states to be even chosen by an adversary. As is standard in the machine learning literature, the quality of an online algorithm is measured by comparing its performance with the performance of an algorithm (called the expert) which knows the future channel states but is only allowed to choose a time-independent power vector.

As in previous sections, let j_i denote the cell/base station to which user i belongs to. Let $h_{ic}(t)$ denote the channel gain (arbitrary) for user i for transmissions to cell c, at time t. Also, let $P_i(t)$ be the power assigned to user i at time t and let $P(t) = (P_1(t), P_2(t), ..., P_n(t))$. Moreover, let $\gamma_i(P(t), t)$ denote the instantaneous logarithmic SINR for user i at time t, i.e.:

$$\gamma_i(P(t), t) = \log\left(\frac{h_{ij_i}(t) P_i(t)}{\sum_{k \neq i} h_{kj_i}(t) X_k(t) P_k(t) + \sigma^2}\right) \quad (26)$$

where $X_j(t)$ is an arbitrary binary variable denoting whether user j is "on/off" at time t, and σ^2 is the noise variance (assumed to be the same in all cells). We assume that there are known lower and upper bounds on the channel gains, i.e., $h_{\min} \leq h_{ik}(t) \leq h_{\max}, \forall i, k, t$. Also, we assume that there is an upper bound $P_{\max}$ on the power assigned to any user. Moreover, as in [7], we assume that there is a minimum decodable SINR $\delta_{\min}$ such that $h_{\min} P_{\max} \geq \delta_{\min}\{(n-1) h_{\max} P_{\max} + \sigma^2\}$ holds. This allows the problem to be feasible and also ensures that the $P_i(t) \in [P_{\min}, P_{\max}]$, where $P_{\min}$ is a function of $\delta_{\min}, h_{\min}$ and $h_{\max}$.

The instantaneous throughput $\tau_i(t)$ for user i at time t is simply: $\tau_i(t) = \log(1 + e^{\gamma_i(P(t), t)})$. Using the popular log-utility function, the cumulative instantaneous utility of all the users at time t is $\sum_i w_i \log(\log(1 + e^{\gamma_i(P(t), t)}))$, where $w_i > 0$ is the weight factor associated with the utility function of user i. Therefore, to optimize the short-term performance of the network for T time slots, an expert needs to solve the following optimization problem to compute the best time-independent power assignment $P^* = (P_1^*, P_2^*, ..., P_n^*)$ in hindsight:

$$\min_P \sum_{t=1}^{T} \sum_i -w_i \log(\log(1 + e^{\gamma_i(P, t)})). \quad (27)$$

We want to design an algorithm that performs as close to the above expert as possible by doing online power control. Observe that $-\log\log(1 + e^{\gamma_i(P(t), t)})$ is a convex and decreasing function of $\gamma_i(P(t), t)$. Therefore, as long as $\gamma_i(P(t), t)$ is a concave function of the power control parameters $P(t)$, the overall composition of functions will be a convex function and the resulting problem will be an online convex optimization problem. Clearly, $\gamma_i(P(t), t)$ computed in (26), is not a concave function. But, as we observed in previous sections, it is easy to convert $\gamma_i(P(t), t)$ to a concave function of the power control parameters with suitable transformations.

1) General Power Control (GPC): For GPC, let us consider the same transformation as in Section II, i.e., $P_i(t) = e^{q_i(t)}$. Let $z(t) = (\{q_j(t)\}_{j=1}^n)$. Plugging this transformation in (26):

$$\begin{aligned}\gamma_i(P(t), t) &= \gamma_i'(z(t), t) \\ &= \log h_{ij_i}(t) + q_i(t) - \log(\sum_{k \neq i} h_{kj_i}(t) X_k(t) e^{q_k(t)} + \sigma^2)\end{aligned}$$

Clearly, the above function is a concave function of $z(t)$ as negative of log-sum-of-exponentials is a concave function. Also, observe that $z(t) \in \xi(\delta_{\min}, h_{\min}, h_{\max}) = \xi$, where ξ is a computable non-empty closed convex bounded set, since the power for each user has to lie in $[P_{\min}, P_{\max}]$.

2) Fractional Power Control (FPC): For FPC, we have $P_i(t) = e^{\pi_{j_i}(t) - \alpha_{j_i}(t) \log h_{ij_i}(t)}$. Plugging this in (26), it can be verified that $\gamma_i(P(t), t) = \gamma_i'(z(t), t)$ becomes a concave function of $z(t) = (\{\pi_c(t)\}_{c \in \mathbb{B}}, \{\alpha_c(t)\}_{c \in \mathbb{B}})$. Again, observe that $z(t) \in \xi(\delta_{\min}, h_{\min}, h_{\max}) = \xi$, where ξ is a computable non-empty closed convex bounded set, since the power for each user has to lie in $[P_{\min}, P_{\max}]$.

Using the above transformations, for both FPC and GPC, we can get an online convex optimization problem with respect to the power control parameters $z(t)$. Now, in order to measure the performance of any algorithm (producing $z(t)$) as compared to the best expert in hindsight, a popular metric is regret (see [15] for an introduction to online convex optimization). Regret is defined as:

$$\mathcal{R}(T) = \sum_{t=1}^{T} \sum_{i} -w_i \log(\log(1 + e^{\gamma_i'(z(t),t)})) - \big\{ \min_{z} \sum_{t=1}^{T} \sum_{i} -w_i \log(\log(1 + e^{\gamma_i'(z,t)})) \big\}. \tag{28}$$

In the next subsection, we present an online gradient descent-based algorithm and obtain theoretical bounds on the regret defined above.

B. Algorithm and Analysis

We use the online gradient descent (Algorithm 6 in [15]) to design an efficient online power control algorithm. Let $f_t(z) = \sum_i -w_i \log(\log(1 + e^{\gamma_i'(z,t)}))$. Let D_ξ denote the diameter of the set ξ, i.e., $D_\xi = \max_{x,y\in\xi} ||x-y||_2$. Note that since $f_t(z)$ is continuously differentiable on the set ξ, and the set ξ is closed, bounded and convex, it implies that $f_t(z)$ is a G-Lipschitz function of z, where $G = G(\xi)$ is a function of the set ξ. Now, we present our algorithm OPC-RSTP (short for Online Power Control for Robust Short-term Performance) as Algorithm 3. We now present the main result on the regret achieved by OPC-RSTP.

Algorithm 3 Online Power Control for Robust Short-term Performance (OPC-RSTP)

initialize $z(0) \in \xi$.
for $t = 1, 2, \dots :$
1) Compute $\nabla f_t(z(t-1)), \eta_t = \frac{D_\xi}{G\sqrt{t}}$.
2) $z(t) = \prod_\xi \big(z(t-1) - \eta_t \nabla f_t(z(t-1))\big)$.
3) Output $z(t)$.

end for

Theorem 3. *For the OPC-RSTP algorithm, for any $T \geq 1$, we have:*

$$\mathcal{R}(T) \leq \frac{3}{2} G D_\xi \sqrt{T}.$$

Proof. Follows from Theorem 3.1 in [15]. □

Note that $o(T)$ regret implies that the average performance of OPC-RSTP is asymptotically as good as the best expert in hindsight. Therefore, OPC-RSTP is both online and efficient.

V. Conclusion

In this paper, we consider the optimal power control problem in modern wireless networks. We present learning-based algorithms SGPC-SINR and SFPC-SINR for the problem of SINR-based utility maximization, using general power control and fractional power control respectively. We also provide theoretical guarantees on the performance of these two algorithms. Moreover, we present OPC-RSTP, an online algorithm to achieve robust short-term performance for both general power control and fractional power control. Again, we provide a theoretical guarantee on OPC-RSTP's performance.

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