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STOCHASTIC SYSTEMS

On the Problem of Maximizing the Probability of Successful Passing of a Time-Limited Test

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Abstract—The problem of finding the optimal sequence of performing a set of tasks in a time-limited test is considered. That is, a task group is allocated for mandatory initial execution in the test, the remaining tasks are performed during the remaining time until the end of the test. For each correctly completed task of the test, the subject is awarded a certain number of points. The proposed criterion is the probability that the total number of points scored for the test exceeds a certain level, which is a fixed parameter, while simultaneously fulfilling the time limit of the test. Random parameters are the user's response time to each test task. The correctness of the user's answer to the task is modeled by a random variable with a Bernoulli distribution. The resulting stochastic bilinear programming problem boils down to a deterministic integer problem of mathematical programming.

Keywords: time-constrained test, maximum likelihood estimation, integer mathematical programming

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1. INTRODUCTION

In the paradigm of computerized adaptive testing [1–6] the problem of constructing an optimal test passing strategy is related to the formation of an individual learning trajectory. This problem seems to be relevant in various fields of the educational process: preparation for passing the Unified State Exam (USE) by applicants, passing regular tests in the learning management system (LMS), checking the residual knowledge of students, etc. As a rule, such forms of testing are limited in time, and the structure of the test is known in advance with an accuracy of the types of tasks, or sections of the course, to test the knowledge of which these tasks are aimed. At the same time, there are considerable statistics of the performance of such tasks by the subjects during the training both by the type of tasks and by the individual user. Often, for example, in the context of using LMS in the educational process, the collection and storage of this information are automated, as is done in the CLASS.NET LMS of the Moscow Aviation Institute [7, 8]. This allows us to reasonably use in the problem under consideration mathematical models of random parameters taken into account, for example, the time spent by the subject on solving a task of a certain type. Models of the response time of the user to the test task are widely represented in the literature. Van der Linden proposed a lognormal time model [1], and in [9, 10] gamma distribution and discrete distribution were used as models. The frequency of the correct solution of the problems of a certain type by the subject, obtained on the basis of data on the work of the subject in the learning process, can serve as a good estimate of the parameter of the Bernoulli distribution, modeling the correct solution of the corresponding task in the test by the subject. All tasks of the test, as a rule, are characterized by a certain number of points that the subject scores, having solved them correctly. The total number of points scored during testing characterizes the quality of the subject's preparation and is the basis for his assessment. The achievement by the subject of a certain total score for the test can serve as a certain target indicator for him. As a strategy of the subject in the presence of the above-mentioned random parameters in the problem, a set of test tasks can be used, which should be performed first.

The literature presents quite widely the problems of constructing various tests in order to check the level of knowledge of the subjects, including in probabilistic or quantile statement [3, 4, 6, 9, 10]. However, the authors are not aware of publications in which the problems of constructing an optimal strategy for the subject to pass the test would be considered.

The paper formulates the problem of finding the optimal strategy of the subject (in the above sense) according to the criterion of maximizing the probability of gaining a total score for the test above a certain level chosen by the subject. This takes into account the probabilistic constraint associated with the fact that the time spent by the subject on completing the test should not exceed the total fixed testing time.

This problem of stochastic bilinear programming based on the generalized minimax approach [11] according to the technique proposed in [12] is reduced to an integer mathematical programming problem. The initial data for estimating the parameters of the probabilistic models used in the problem are taken from the statistics of the work of users of the MAI CLASS.NET LMS. The paper also discusses the results of a numerical experiment and the dependence of the optimal value of the criterion on the total score, which the subject seeks to exceed.

2. DISTRIBUTIONS OF RANDOM PARAMETERS USED IN THE WORK

Two vector random parameters are used in the problem under consideration. One of them is the vector $X = \mathrm{col}\,(X_1, \ldots, X_n)$, the ith coordinate of which model the correctness of the solution of the ith test task. It is assumed that X_i are independent random variables having a Bernoulli distribution, with parameters $p_i, i = 1, \ldots, n$, estimated by the frequency of correct answers of the subject to similar tasks of the ith type during preparation for testing or in the learning process. Equality $X_i = 1$ models the correctness of the solution of the ith task, and equality to zero — the opposite event. Another random parameter is the vector $T = \mathrm{col}(T_1, \ldots, T_n)$, the coordinates of which characterize the time spent by the subject on solving the task of the ith type. Random variables $T_i,\ i = 1, \ldots, n$ are also assumed to be independent. However, it would be reckless to assume independence between the values of X and T, therefore for each value of X_i (0 or 1) its own distribution of the random variable T_i is estimated also on the basis of statistics of solving tasks of a similar type by the subject. Continuous distributions of the user's response time to the task (Van der Linden [1], Gamma distributions [9]) do not allow finding an exact solution to the problem in a probabilistic statement, therefore a discretized response time model with three values is used in the work, modeling situations of quick solution, standard solution and solution with difficulties. The technique for constructing a discrete distribution law of the response time of the subject to the test tasks can be different: from discretization in various ways of continuous distribution models (for example, the Van der Linden model [1]), the parameters of which are determined based on statistical data on the time of solving the problems of the corresponding class by the subject, to using the initial distribution histogram, built according to the same statistical data. In this work, for each task of the test, based on the available statistical data obtained from the CLASS.NET system [8], a variation series of the response time of the subject to similar tasks is constructed, which is divided into three equal parts at a distance between the maximum and minimum elements. For each part, the sample mean is calculated, which is used as the corresponding possible value of the random variable of interest to us. The probabilities of the obtained three possible values are assumed to be equal to the frequencies of the elements of the sample used falling into the corresponding selected ranges. Thus, the general vector of random variables has a discrete distribution with the number of implementations $D = 2^n 3^n$. The probabilities of each implementation can be found using the formula for multiplying probabilities, and using the conditional distribution of the response time of the subject to the test tasks under the conditions of its correct or incorrect solution.

3. STATEMENT OF THE PROBLEM AND METHOD OF ITS SOLUTION

It is required to define the strategy of the testee while performing a time-limited test consisting of n tasks. The strategy is defined by a vector of Boolean variables $u \in \{0,1\}^n$, where

$$u_i \triangleq \begin{cases} 1, & \text{if the testee tries to solve the } i\text{th task of the test,} \\ 0 & \text{otherwise,} \end{cases} \qquad i = \overline{1,n}.$$

For each ith task of the test, b_i points are awarded. To successfully pass the test, it is necessary to score at least φ points. The time for completing the test is limited to $\overline{T}$. The probability of successfully passing the test while simultaneously fulfilling the time limit for its completion was chosen as the optimization criterion.

Let us consider the probability function

$$P_{\varphi,\overline{T}}(u) \triangleq P\left\{\sum_{i=1}^{n} u_i X_i b_i \geqslant \varphi,\ \sum_{i=1}^{n} u_i T_i \leqslant \overline{T}\right\}.$$

In it, the values φ and $\overline{T}$ play the role of parameters. To ensure the reasonableness of the problem statement, we impose restrictions on the specified parameters: $0 < \varphi \leqslant \sum_{i=1}^{n} b_i$ and $\sum_{i=1}^{n} T_i^{\min} \leqslant \overline{T}$, where $T_i^{\min}$ is the minimum time for the testee to solve the ith task. Then the problem of finding the optimal strategy for the testee can be formulated as follows:

$$P_{\varphi,\overline{T}}(u) \to \max_{u \in \{0,1\}^n}. \tag{1}$$

This problem is a stochastic programming problem with Boolean variables.

As mentioned above, the number of all possible realizations of the random parameter vector $\mathrm{col}(X^\top, T^\top)$ is $D = 2^n 3^n$. Let us consider the vector $\delta \in \{0,1\}^D$, each νth coordinate of which corresponds to one of the realizations $\mathrm{col}((x^\nu)^\top, (t^\nu)^\top)$ of the vector $(X^\top, T^\top)^\top$ and can take values 0 or 1. Let $\Upsilon \triangleq e^T b$, where $e = \mathrm{col}(1, \ldots, 1) \in R^n$, i.e. $\Upsilon = \sum_{i=1}^{n} b_i$ — the maximum number of points that can be scored for the test. Let $p_\nu = P(\mathrm{col}(X^\top, T^\top) = \mathrm{col}((x^\nu)^\top, (t^\nu)^\top))$, $\nu = \overline{1,D}$. Then, similarly to the method proposed in [12] for solving the problem of minimizing the quantile function based on the confidence method [11], the stochastic programming problem (1) can be reduced to a deterministic optimization problem with Boolean variables:

$$\sum_{\nu=1}^{D} p_\nu \delta_\nu \to \max_{\substack{u \in \{0,1\}^n \\ \delta \in \{0,1\}^D}} \tag{2}$$

under the constraints

$$\varphi - \Upsilon - \delta_\nu \left(\sum_{i=1}^{n} u_i x_i^\nu\, b_i - \Upsilon\right) \leqslant 0, \quad \nu = \overline{1,D}, \tag{3}$$

$$\delta_\nu u^T t^\nu - \overline{T} \leqslant 0, \quad \nu = \overline{1,D}. \tag{4}$$

In the problem considered above, the optimal value of the vector δ determines the type of the optimal confidence set (in terms of the confidence method [11]) as the implementations of the random parameter vector corresponding to ones in the optimal vector δ. The total probability measure of such implementations is maximized in the problem under consideration, and the constraints on the test execution time and the number of points scored during the test are satisfied, while for the remaining implementations corresponding to zero values of the coordinates of the optimal vector δ, these constraints in the original problem may not be satisfied, and constraints (3) and (4) are satisfied by construction.

Problem (2)–(4), constructed strictly according to the technique of [12], is a bilinear programming problem (with a bilinear system of constraints), which, together with the Boolean nature of the variables and the large dimensionality, makes it difficult to solve. However, the structure of the problem under consideration makes it possible to rewrite it as a linear programming problem (LPP), which will possibly allow the use of special methods for solving LPPs with Boolean variables implemented in modern applied optimization software packages. The form of this LPP is as follows:

$$\sum_{\nu=1}^{D} p_\nu \delta_\nu \to \max_{\substack{u\in\{0,1\}^n \\ \delta\in\{0,1\}^D}} \tag{5}$$

under the constraints

$$\delta_\nu \varphi \leqslant \sum_{i=1}^{n} u_i x_i^\nu \, b_i, \quad \nu = \overline{1, D}, \tag{6}$$

$$\sum_{i=1}^{n} u_i b_i \geqslant \varphi, \quad \sum_{i=1}^{n} u_i T_i^{\min} \leqslant \overline{T}.$$

$$u^T t^\nu \leqslant \delta_\nu \overline{T} + (1-\delta_\nu) T^{MAX}, \ \nu = \overline{1, D}, \tag{7}$$

where $T^{MAX} = \sum_{i=1}^{n} T_i^{\max}$, and $T_i^{\max}$ is the maximum of all possible realizations of the random variable T_i, $i = \overline{1, n}$.

If the dimensions of the problems (2)–(4), (5)–(7) allow to solve them using standard procedures from known libraries of optimization programs, then the solution can be found with their help. However, these problems contain an additional vector of optimization variables $\delta \in \{0,1\}^D$ of large dimension, which, taking into account the large number of constraints, makes them difficult to solve by exhaustive search of all possible values of Boolean optimization variables and requires the development of special solution methods that take into account the structure of the problem. Next, we consider an algorithm for solving the original problem. The efficiency of its application in comparison with standard library procedures for solving problems (2)–(4), (5)–(7) will be discussed in the section concerning the results of a numerical experiment.

4. ALGORITHM FOR SOLVING THE ORIGINAL PROBLEM

Step 0.

Of all 2^n strategies $u \in \{0,1\}^n$ we choose N, forming the set $\underline{U}$, for the elements of which the following conditions are satisfied

$$\sum_{i=1}^{n} u_i b_i \geqslant \varphi, \ \sum_{i=1}^{n} u_i T_i^{\min} \leqslant \overline{T}.$$

The point is that in this way we filter out strategies that are obviously unsuitable in terms of the total time or the number of points even in the most optimistic case, when all the tasks selected for solving the problem are solved correctly and in the minimum possible time.

We renumber all elements of the set $\underline{U}$. Thus, a number from 1 to N uniquely defines an element of the set. By u^m we will mean the mth element of the set $\underline{U}$. Let $m := 1$, $P^* := 0$, and $u^* := (0, \ldots, 0)^\top$.

At this step, the external loop for enumerating all N selected optimization strategies is initialized.

Step 1.

If $m > N$, then go to Step 5. Otherwise $P_m := 0$.

The auxiliary parameter P_m is used below to calculate the probability of fulfilling the constraints at $u = u^m$.

Step 2.

Suppose that the vector u^m contains exactly K ones. Suppose that the nonzero components of the vector u^m are the components with numbers $i_1, \ldots, i_K$. Consider the subvector $\mathrm{col}(X_{i_1}, \ldots, X_{i_K})$ of the random vector X. Let $J := 2^K$, and $j := 1$.

At this step, the cycle of enumerating all possible realizations $\mathrm{col}(x^j_{i_1}, \ldots, x^j_{i_K})$, $j = \overline{1, 2^K}$ is initialized.

Step 3.

If $j > J$ and $P_m > P^*$, then we assume $P^* := P_m$, $u^* := u^m$, $m := m + 1$ and proceed to Step 1.

If $j > J$ and $P_m \leqslant P^*$, then we assume $m := m + 1$ and proceed to Step 1.

Otherwise, if for the realization $\mathrm{col}(x^j_{i_1}, \ldots, x^j_{i_K})$ the condition

$$\sum_{i \in \{i_1, i_2, \ldots, i_K\}} u^m_i x^j_i \geqslant \varphi,$$

is satisfied, then we assume $L := 3^K$, $l := 1$ and proceed to Step 4. If the specified condition is not satisfied, then we assume $j := j + 1$ and proceed to the beginning of Step 3.

At this step, the cycle of enumerating all possible realizations $\mathrm{col}(t^l_{i_1}, \ldots, t^l_{i_K})$, $l = \overline{1, L}$ of the subvector $\mathrm{col}(T_{i_1}, \ldots, T_{i_K})$ of the random vector T is initialized.

Step 4.

If $l > L$, then we assume $j := j + 1$ and proceed to Step 3. Otherwise, if for the realization $\mathrm{col}(t^l_{i_1}, \ldots, t^l_{i_K})$ the condition

$$\sum_{i \in \{i_1, i_2, \ldots, i_K\}} u^m_i t^l_i \leqslant \overline{T},$$

is satisfied, then we assume $P_m := P_m + \prod_{i \in \{i_1, i_2, \ldots, i_K\}} P(T_i = t^l_i | X_i = x^j_i) P(X_i = x^j_i)$.

We assume $l := l + 1$ and proceed to the beginning of Step 4.

Step 5. We assume the optimal value of the criterion to be equal to P^*, and the optimal value of the strategy to be equal to u^*.

Note that in all nested cycles considered in the algorithm, there is a significant reduction in the required volume of enumeration of possible values of optimization variables. The volume of full enumeration can be reduced by an order of magnitude depending on the selected values of the task parameters φ and $\overline{T}$.

5. RESULTS OF NUMERICAL EXPERIMENT

The initial distributions for solving the problem were obtained based on the analysis functioning of the MAI CLASS.NET learning management system [8]. We will assume the number of tasks in the test is $n = 10$. Estimates of the parameters of the initial distributions are given in Tables 1–6.

Table 1. Probability of correct solution of test tasks

Task number in the test	Probability of correct solution	Number of points for the task
1	0.9	1
2	0.91	1
3	0.95	1
4	0.97	1
5	0.90	1
6	0.8	2
7	0.65	3
8	0.75	2
9	0.5	4
10	0.55	3

Table 2. Conditional distribution of the test subject response time for a test task in case of its incorrect solution

Task number in the test	Conditional distribution of response time			
1	t_1^j	60	100	160
	$P(T_1 = t_1^j \mid X_1 = 0)$	0.3	0.55	0.15
2	t_2^j	70	130	250
	$P(T_2 = t_2^j \mid X_2 = 0)$	0.25	0.6	0.15
3	t_3^j	60	150	270
	$P(T_3 = t_3^j \mid X_3 = 0)$	0.2	0.45	0.35
4	t_4^j	100	200	350
	$P(T_4 = t_4^j \mid X_4 = 0)$	0.15	0.6	0.25
5	t_5^j	75	140	210
	$P(T_5 = t_5^j \mid X_5 = 0)$	0.2	0.45	0.35
6	t_6^j	190	290	400
	$P(T_6 = t_6^j \mid X_6 = 0)$	0.1	0.65	0.25
7	t_7^j	310	380	450
	$P(T_7 = t_7^j \mid X_7 = 0)$	0.2	0.4	0.4
8	t_8^j	180	250	320
	$P(T_8 = t_8^j \mid X_8 = 0)$	0.1	0.3	0.6
9	t_9^j	360	480	600
	$P(T_9 = t_9^j \mid X_9 = 0)$	0.1	0.3	0.6
10	t_{10}^j	320	400	470
	$P(T_{10} = t_{10}^j \mid X_{10} = 0)$	0.3	0.55	0.15

Consider the value $b^{\max} = \sum_{i=1}^{n} b_i$. As a result of the proposed algorithm, the dependences of optimal solutions on the values of the problem parameters φ and $\overline{T}$ were obtained, see Tables 4 and 5.

The calculations were performed on a computer ASUS X550LC (Intel Core i5 2.3 GHz, 8Gb RAM). The linear programming problem was solved by the IBM Cplex package from the Python library. As can be seen, the values of the problem parameters significantly affect the speed of the

Table 3. Conditional distribution of the test subject response time for a test task in case of its correct solution

Task number in the test	Conditional distribution of response time			
1	t_i^j	60	180	300
	$P(T_1 = t_1^j \mid X_1 = 1)$	0.3	0.5	0.2
2	t_i^j	75	190	330
	$P(T_2 = t_2^j \mid X_2 = 1)$	0.15	0.6	0.25
3	t_i^j	60	120	250
	$P(T_3 = t_3^j \mid X_3 = 1)$	0.15	0.35	0.5
4	t_i^j	130	200	350
	$P(T_4 = t_4^j \mid X_4 = 1)$	0.1	0.3	0.6
5	t_i^j	75	140	210
	$P(T_5 = t_5^j \mid X_5 = 1)$	0.2	0.45	0.35
6	t_i^j	200	275	380
	$P(T_6 = t_6^j \mid X_6 = 1)$	0.2	0.35	0.45
7	t_i^j	310	380	450
	$P(T_7 = t_7^j \mid X_7 = 1)$	0.2	0.4	0.4
8	t_i^j	200	290	370
	$P(T_8 = t_8^j \mid X_8 = 1)$	0.25	0.4	0.35
9	t_i^j	380	470	650
	$P(T_9 = t_9^j \mid X_9 = 1)$	0.1	0.25	0.65
10	t_i^j	150	275	500
	$P(T_{10} = t_{10}^j \mid X_{10} = 1)$	0.3	0.55	0.15

Table 4. Dependence of the optimal solution on the parameter φ at $\overline{T} = 0.8\, T^{\max}$

φ	Optimal strategy	Opt. value of the criterion	Calculation time (sec)	Number of investigated strategies
$0.4b^{\max}$	1. 1. 1. 1. 1. 1. 1. 1. 0. 1.	0.9241	29.9	727
$0.5b^{\max}$	1. 1. 1. 0. 1. 1. 1. 1. 1. 1.	0.7874	18.2	511
$0.6b^{\max}$	1. 1. 1. 0. 1. 1. 1. 1. 1. 1.	0.5777	8.9	295
$0.7b^{\max}$	1. 1. 1. 0. 1. 1. 1. 1. 1. 1.	0.3830	3.2	131
$0.8b^{\max}$	1. 1. 1. 1. 1. 1. 1. 1. 1. 1.	0.2028	0.9	39
$0.9b^{\max}$	1. 1. 1. 1. 1. 1. 1. 1. 1. 1.	0.0816	0.1	5

author's algorithm. Thus, increasing the desired number of points for the test leads to a decrease in the probability of achieving this result and a decrease in the calculation time using the proposed algorithm due to a decrease in the volume of enumeration of admissible optimization strategies u. Comparative analysis of the running time of the algorithms shows its significant growth with an increase in the number of tasks in the test, see Table 6. All algorithms with the same problem parameters received coinciding solutions for all values of n. At $n \geqslant 7$, it was not possible to solve the LP problem due to problems with insufficient memory for storing the matrix of the constraint system. The most effective was the authors' algorithm, which for large n exceeded by an order of magnitude the running time of other considered algorithms.

Table 5. Dependence of the optimal solution on the parameter $\overline{T}$ at $\varphi = 0.6\ b^{\max}$

$\overline{T}$	Optimal strategy	Opt. value of the criterion	Calculation time (sec)	Number of investigated strategies
$0.4T^{\max}$	0. 0. 0. 0. 0. 1. 1. 0. 1. 1.	0.0633	1.8	273
$0.5T^{\max}$	0. 0. 1. 0. 1. 1. 1. 1. 0. 1.	0.1733	9.5	296
$0.6T^{\max}$	1. 1. 1. 0. 1. 1. 1. 1. 0. 1.	0.2786	8.7	296
$0.7T^{\max}$	1. 1. 1. 1. 1. 1. 1. 1. 0. 1.	0.5049	9.0	296
$0.8T^{\max}$	1. 1. 1. 0. 1. 1. 1. 1. 1. 1.	0.5777	8.9	296
$0.9T^{\max}$	1. 1. 1. 1. 1. 1. 1. 1. 1. 1.	0.7213	8.3	296

Table 6. Algorithms running time (s) for different values of n

n	LPP	Full enumeration	Authors' algorithm
1	0.0010	0.0340	0.0010
2	0.0010	0.0400	0.0016
3	0.0102	0.0580	0.0020
4	0.0202	0.0800	0.0029
5	0.0628	0.2000	0.0077
6	0.0991	0.6100	0.0117
7	–	2.1900	0.0636
8	–	9.3800	0.1600
9	–	48.1000	2.1500
10	–	3010.0000	9.1100

6. CONCLUSION

This article considers the problem of stochastic programming to search for an optimal strategy for passing a time-limited test according to the criterion of the maximum probability of the testee overcoming a certain number of points scored for the test, taking into account the time limit for completing the test. For the testee, the probability of a correct solution to each test task is considered known. The time spent by the testee on solving each task is also random.

The considered problem of stochastic programming with a probabilistic quality criterion is reduced to a deterministic problem of large dimensionality, for which standard optimization procedures from known program libraries can be used. In addition, an algorithm is proposed for a directed search possible values of a discrete optimization strategy, reducing time costs for solving the problem. The conducted numerical experiment confirmed the effectiveness of using the developed algorithm for solving the original problem in comparison with the use of standard optimization procedures for its deterministic equivalents. This efficiency, determined by the difference in the time of calculating the optimal strategy in various ways, increases with an increase in the number of tasks in the test. Numerical experiment also showed a significant dependence of the optimal solution and the time of its calculation on the parameters tasks, which justifies the relevance of further improvement her solution algorithm.

The results obtained in the work can be extended to the quantile formulation of the problem under consideration, when the testee seeks to maximize the number of points scored for the test while maintaining the selected probability level of fulfilling all the constraints of the problem. This is a separate study, the results of which the authors plan to publish.

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