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GRA Method for Probabilistic Linguistic Multiple Attribute Group Decision Making with Incomplete Weight Information and Its Application to Waste Incineration Plants Location Problem

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ABSTRACT

In this essay, we investigate the probabilistic linguistic multiple attribute group decision making (PL-MAGDM) with incomplete weight information. In this method, the linguistic representation developed recently is converted into probabilistic linguistic information. For deriving the weight information of the attribute, an optimization model is built on the basis of the fundamental idea of grey relational analysis (GRA), by which the attribute weights can be decided. Then, the optimal alternative is chosen through calculating largest relative relational degree from the probabilistic linguistic positive ideal solution (PLPIS) which considers both the largest grey relational coefficient (GRC) from the PLPIS and the smallest GRC form probabilistic linguistic negative ideal solution (PLNIS). In the end, a case study concerning waste incineration plants location problem is given to demonstrate the merits of the developed methods.

1. INTRODUCTION

The grey relational analysis (GRA) method was initially designed by Deng [1] for tackling the MADM issue. In addition, GRA method is one of the very popular and useful tools to analyze diverse relationships among the discrete information and make decisions in different situations [2–4]. The major merits of the GRA method are that the analyzed results are depended upon the original data, the calculating process are simple and straightforward and, finally, it is one of the optimal methods to make decisions under diverse business environment [5–7]. Kung and Wen [5] employed GRA to study the grey MADM issue for venture capital enterprises. Tan, Chen and Wu [8] studied the green design alternatives and GRA connected with AHP. Chiang [9] extended GRA for dependent criteria MADM issue. Malek, Ebrahimnejad and Tavakkoli-Moghaddam [10] proposed an improved hybrid GRA method for green resilient supply. Alptekin, Alptekin and Sarac [11] assessed the low carbon development with GRA model in some countries. Zhu, Yuan and Ye [12] aimed at discerning the multi-timescales of carbon market through GRA method and empirical mode decomposition (EMD). Yazdani, Kahraman, Zarate and Onar [13] provided a good platform to ease decision process through the integration of quality function deployment (QFD) and GRA in showing main supply chain drivers under fuzzy setting. Chen [14] connected intuitionistic fuzzy GRA techniques with entropy-based TOPSIS for selecting building materials supplier. Wei [15] tackled dynamic hybrid MADM issue based on the GRA.

In real life, the decision makers (DMs) always employ linguistic terms rather than using the exact numbers because of the complex socioeconomic environment and fuzzy human beings' thinking [16–18]. Under this situations, Zadeh [19] proposed fuzzy linguistic method to depict the qualitative assessment information. In certain environments, DMs may be possible to be uncertain about some diverse linguistic terms when they depict the objects. Thus, Rodriguez, Martinez and Herrera [20] devised the tool of hesitant fuzzy linguistic term set (HFLTS) to snatch the hesitancy degree in the context of the linguistic. Gou, Xu and Liao [21] expanded the entropy and cross-entropy measures to HFLTS for MADM. Wei [22] gave the generalized dice similarity measures for MADM with HFLTS. Liao, Xu and Zeng [23] provided the VIKOR method for qualitative MADM with HFLTS. However, at certain times, the DMs may intend to express the linguistic term "medium" to the linguistic term "bad," that's to say, these two linguistic terms' probabilities are not same [24]. It can be easily found that the HFLTS can't describe this sort of complicated qualitative

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information. To overcome this drawback and exactly describe this kind of complicated qualitative information, Pang, Wang and Xu [25] developed the probabilistic linguistic term set (PLTS), which permits DMs to express the preference information of themselves as a collection of several possible linguistic terms combined with probabilities. Lin, Chen, Liao and Xu [26] proposed the ELECTRE II method to deal with PLTSs for edge computing. Feng, Liu and Wei [27] constructed possibility degree comparison of probabilistic linguistic QUALIFLEX method. Bai, Zhang, Qian and Wu [28] gave comparative method and proposed a more efficient mean to tackle PLTSs. Chen, Wang and Wang [29] employed the probabilistic linguistic MULTIMOORA for cloud-based ERP system selection. Lin, Chen, Liao and Xu [26] developed the ELECTRE II method to deal with PLTSs for edge computing. Liu and Teng [30] proposed some Muirhead mean operators for PLTSs. Liu and Teng [31] defined the probabilistic linguistic TODIM method. Liu and Li [32] designed the generalized maclaurin symmetric mean aggregation operators for probabilistic linguistic information. Liu and Li [32] gave the bidirectional projection method for probabilistic linguistic multi-criteria group decision making based on power average operator. Liang, Kobina and Quan [33] designed the probabilistic linguistic grey relational analysis (PL-GRA) for MAGDM based on geometric Bonferroni mean [34–37]. Lu, Wei, Wu and Wei [38] designed the TOPSIS method for probabilistic linguistic MAGDM with entropy weight foro supplier selection of new agricultural machinery products.

But the PL-GRA [33] method have two shortcomings: (1) this method think that the optimal alternative is chosen through calculating largest relative relational degree from the probabilistic linguistic positive ideal solution (PLPIS) which considers both the largest grey relational coefficient (GRC) from the PLPIS which neglects the smallest GRC form probabilistic linguistic negative ideal solution (PLNIS); (2) this method can't tackled the probabilistic linguistic MADM (PL-MADM) or PL-MAGDM issues with incomplete weight information. As a supplement to it, we shall develop a novel GRA-based method for PL-MAGDM in this study and then apply it to select the waste incineration plants sites. The innovativeness of the paper can be summarized as follows: (1) an optimization model is built to derive the weight information of the attribute on the basis of the fundamental idea of conventional GRA method under PLTSs; (2) the PL-GRA method is proposed to solve the probabilistic linguistic MAGDM problems with incomplete weight information; (3) a case study for selecting the waste incineration plants sites is supplied to show the developed approach and (4) some comparative studies are provided with the existing methods.

The remainder of this paper is arranged in the following way: Section 2 gives some fundamental ideas connected to PLTSs. In Section 3, the GRA approach is proposed for PL-MAGDM with incomplete weight information. In Section 4, given a numerical example a case study concerning waste incineration plants location problem to illustrate the developed PL-GRA method and some comparative analysis is provided. In the end, this study makes some conclusions in Section 5.

2. PRELIMINARIES

As an essential and useful tool, The HFLTS [20] is utilized to tackle the hesitancy in the context of the linguistic. Zhao, Xu and Ren [39] designed some distance measures for HFLTS. Wei, Zhao and Tang [40] devised some operations on HFLTSs and possibility degree formulas for comparing HFLTSs. In order to strengthen the modeling capability of HFLTSs, Pang, Wang and Xu [25] proposed the definition of PLTSs to link each linguistic term with a probability value.

Definition 1. [25] Let $L = \{l_\alpha | \alpha = -\theta, \cdots, -2, -1, 0, 1, 2, \cdots \theta\}$ be an LTS, the linguistic terms l_α can express the equivalent information to β is derived by the transformation function g:

$$g : [l_{-\theta}, l_\theta] \to [0,1], \ g(l_\alpha) = \frac{\alpha + \theta}{2\theta} = \beta \tag{1}$$

At the same time, β can be expressed the equivalent information to the linguistic terms l_α, β is derived by the transformation function g^{-1}:

$$g^{-1} : [0,1] \to [l_{-\theta}, l_\theta], \ g^{-1}(\beta) = l_{(2\beta-1)\theta} = l_\alpha \tag{2}$$

Definition 2. Given an LTS $L = \left\{l_j | j = -\theta, \cdots, -2, -1, 0, 1, 2, \cdots \theta\right\}$, a PLTS L is devised as

$$L(p) = \left\{ l^{(\phi)}\left(p^{(\phi)}\right) | l^{(\phi)} \in L, p^{(\phi)} \geq 0, \phi = 1, 2, \cdots, \#L(p), \sum_{\phi=1}^{\#L(p)} p^{(\phi)} \leq 1 \right\} \tag{3}$$

where $l^{(\phi)}\left(p^{(\phi)}\right)$ is the ϕth linguistic term $l^{(\phi)}$ linked with the probability $p^{(\phi)}$, and $\#L(p)$ is the number of all different linguistic terms in $L(p)$. The linguistic term $l^{(\phi)}$ in $L(p)$ is listed in ascending rank.

In order to facile computation, Pang, Wang and Xu [25] normalized the PLTS $L(p)$ as $\tilde{L}(p) = \left\{ l^{(\phi)}\left(\tilde{p}^{(\phi)}\right) | l^{(\phi)} \in L, \tilde{p}^{(\phi)} \geq 0, \phi = 1, 2, \cdots, \#L(\tilde{p}), \sum_{\phi=1}^{\#L(\tilde{p})} \tilde{p}^{(\phi)} = 1 \right\}$, where $\tilde{p}^{(\phi)} = p^{(\phi)} \Big/ \sum_{\phi=1}^{\#L(p)} p^{(\phi)}$ for all $\phi = 1, 2, \cdots, \#L(\tilde{p})$.

Definition 3. Let $L = \{l_\alpha | \alpha = -\theta, \cdots, -1, 0, 1, \cdots \theta\}$ be an LTS, $\tilde{L}_1(\tilde{p}) = \left\{ l_1^{(\phi)}\left(\tilde{p}_1^{(\phi)}\right) | \phi = 1, 2, \cdots, \#\tilde{L}_1(\tilde{p}) \right\}$ and $\tilde{L}_2(\tilde{p}) = \left\{ l_2^{(\phi)}\left(\tilde{p}_2^{(\phi)}\right) | \phi = 1, 2, \cdots, \#\tilde{L}_2(\tilde{p}) \right\}$ be two PLTSs, where $\#\tilde{L}_1(\tilde{p})$ and $\#\tilde{L}_2(\tilde{p})$ are the numbers of LTS in $\tilde{L}_1(\tilde{p})$ and $\tilde{L}_2(\tilde{p})$, respectively. If $\#\tilde{L}_1(\tilde{p}) > \#\tilde{L}_2(\tilde{p})$, then add $\#\tilde{L}_1(\tilde{p}) - \#\tilde{L}_2(\tilde{p})$ linguistic terms to $\tilde{L}_2(\tilde{p})$. What's more, the newly added linguistic terms should be the smallest linguistic term in $\tilde{L}_2(\tilde{p})$ and the corresponding probabilities of newly added linguistic terms should be zero.

Definition 4. For a PLTS $\tilde{L}(\tilde{p}) = \left\{ l^{(\phi)}(\tilde{p}^{(\phi)}) | \phi = 1, 2, \cdots, \#\tilde{L}(\tilde{p}) \right\}$, the score $s\left(\tilde{L}(\tilde{p})\right)$ and deviation degree $\sigma\left(\tilde{L}(\tilde{p})\right)$ of $\tilde{L}(\tilde{p})$ is illustrated as follows:

$$s\left(\tilde{L}(\tilde{p})\right) = \sum_{\phi=1}^{\#\tilde{L}(\tilde{p})} g\left(l^{(\phi)}\right) \tilde{p}^{(\phi)} \Bigg/ \sum_{\phi=1}^{\#\tilde{L}(\tilde{p})} \tilde{p}^{(\phi)} \tag{4}$$

$$\sigma\left(\tilde{L}\left(\tilde{p}\right)\right)=\sqrt{\sum_{\phi=1}^{\#\tilde{L}(\tilde{p})}\left(g\left(l^{(\phi)}\right)\tilde{p}^{(\phi)}-s\left(\tilde{L}\left(\tilde{p}\right)\right)\right)^{2}\Big/\sum_{\phi=1}^{\#\tilde{L}(\tilde{p})}\tilde{p}^{(\phi)}} \tag{5}$$

By using the Eqs. (4) and (5), the order of two PLTSs is defined in the following: (1) if $s\left(\tilde{L}_1\left(\tilde{p}\right)\right)>s\left(\tilde{L}_2\left(\tilde{p}\right)\right)$, then $\tilde{L}_1\left(\tilde{p}\right)>\tilde{L}_2\left(\tilde{p}\right)$; (2) if $s\left(\tilde{L}_1\left(\tilde{p}\right)\right)=s\left(\tilde{L}_2\left(\tilde{p}\right)\right)$, then if $\sigma\left(\tilde{L}_1\left(\tilde{p}\right)\right)=\sigma\left(\tilde{L}_2\left(\tilde{p}\right)\right)$, then $\tilde{L}_1\left(\tilde{p}\right)=\tilde{L}_2\left(\tilde{p}\right)$; if $\sigma\left(\tilde{L}_1\left(\tilde{p}\right)\right)<\sigma\left(\tilde{L}_2\left(\tilde{p}\right)\right)$, then, $\tilde{L}_1\left(\tilde{p}\right)>\tilde{L}_2\left(\tilde{p}\right)$.

Definition 5. [41] Let $L=\{l_\alpha|\alpha=\theta,\cdots,-1,0,1,\cdots\theta\}$ be an LTS. And let $\tilde{L}_1\left(\tilde{p}\right)=\left\{l_1^{(\phi)}\left(\tilde{p}_1^{(\phi)}\right)|\phi=1,2,\cdots,\#\tilde{L}_1\left(\tilde{p}\right)\right\}$ and $\tilde{L}_2\left(\tilde{p}\right)=\left\{l_2^{(\phi)}\left(\tilde{p}_2^{(\phi)}\right)|\phi=1,2,\cdots,\#\tilde{L}_2\left(\tilde{p}\right)\right\}$ be two PLTSs with $\#\tilde{L}_1\left(\tilde{p}\right)=\#\tilde{L}_2\left(\tilde{p}\right)$, then the Hamming distance $d\left(\tilde{L}_1\left(\tilde{p}\right),\tilde{L}_2\left(\tilde{p}\right)\right)$ between $\tilde{L}_1\left(\tilde{p}\right)$ and $\tilde{L}_2\left(\tilde{p}\right)$ is given as follows:

$$d\left(\tilde{L}_1\left(\tilde{p}\right),\tilde{L}_2\left(\tilde{p}\right)\right)= \tag{6}$$

$$\sqrt{\sum_{\phi=1}^{\#\tilde{L}_1(\tilde{p})}\left(\tilde{p}_1^{(\phi)}g\left(l_1^{(\phi)}\right)-\tilde{p}_2^{(\phi)}g\left(l_2^{(\phi)}\right)\right)\Big/\#\tilde{L}_1\left(\tilde{p}\right)}$$

3. GRA METHOD FOR PROBABILISTIC LINGUISTIC MAGDM WITH INCOMPLETE WEIGHT INFORMATION

In such section, we propose a novel PL-GRA method to tackle the MAGDM issues with incomplete weight information. Assume that $A=\{A_1,A_2,\cdots,A_m\}$ is a collection of potential alternatives, $G=\{G_1,G_2,\cdots,G_n\}$ is the collection of chosen attributes, and $E=\{E_1,E_2,\cdots,E_q\}$ is the collection of qualified experts. Suppose that $L_k=\left(l_{ij}^{(k)}\right)_{m\times n}$ is the group decision making matrix, where $l_{ij}^{k}\left(i=1,2,\cdots,m,j=1,2,\cdots,n,k=1,2,\cdots,q\right)$ is linguistic variables, given by the DM $E_k\in E$, for the alternative $A_i\in A$ with regard to the attribute $G_j\in G$, $w=(w_1,w_2,\cdots,w_n)$ is the weight vector of the attributes $G_j\left(j=1,2,\cdots,n\right)$, where $w_j\in[0,1]$, $\sum_{j=1}^{n}w_j=1$. H is a sort of partially known weight information, which can be listed by the following forms, for $i\neq j$ [42–44]: Case 1. A weak ranking: $w_i\geq w_j$; Case 2. A strict ranking: $w_i-w_j\geq\alpha_i$, $\alpha_i>0$; Case 3. A ranking of differences: $w_i-w_j\geq w_k-w_l$, for $j\neq k\neq l$; Case 4. A ranking with multiples: $w_i\geq\beta_i w_j$, $0\leq\beta_i\leq1$; Case 5. An interval form: $\alpha_i\leq w_i\leq\alpha_i+\varepsilon_i$, $0\leq\alpha_i<\alpha_i+\varepsilon_i\leq1$.

Subsequently, we shall use PL-GRA to tackle MAGDM issues with incomplete weight information. The detailed calculating steps of the proposed method are presented in the following:

Step 1. Shift cost attributes into beneficial attributes. If $L=\{l_\alpha|\alpha=-\theta,\cdots,-1,0,1,\cdots\theta\}$ is an LTS, then the cost attribute value is l_α, then the corresponding beneficial attribute value is $l_{-\alpha}$.

Step 2. Switch the linguistic variables $l_{ij}^{k}(i=1,2,\cdots,m,j=1,2,\cdots,n,k=1,2,\cdots,q)$ into probabilistic linguistic information $L_{ij}\left(p\right)$, and build the probabilistic linguistic decision matrix $L=\left(L_{ij}\left(p\right)\right)_{m\times n}$, $L_{ij}\left(\tilde{p}\right)=\left\{l_{ij}{}^{(\phi)}\left(p_{ij}{}^{(\phi)}\right)|\phi=1,2,\cdots,\#L_{ij}\left(p\right)\right\}$ $\left(i=1,2,\cdots,m,j=1,2,\cdots,n\right)$. Thus, probabilistic linguistic information for the alternative $A_i\in A$ with regard to the all the attribute G can be expressed as $PLA_i=\left(l_{i1}{}^{(\phi)}\left(p_{i1}{}^{(\phi)}\right),l_{i2}{}^{(\phi)}\left(p_{i2}{}^{(\phi)}\right),\cdots,l_{in}{}^{(\phi)}\left(p_{in}{}^{(\phi)}\right)\right)$, $\phi=1,2,\cdots,\#L_{ij}\left(p\right)$.

Step 3. Derive the normalized assessing matrix $\tilde{L}=\left(\tilde{L}_{ij}\left(\tilde{p}\right)\right)_{m\times n}$ with PLTSs, $\tilde{L}_{ij}\left(\tilde{p}\right)=\left\{l_{ij}{}^{(\phi)}\left(\tilde{p}_{ij}{}^{(\phi)}\right)|\phi=1,2,\cdots,\#L_{ij}\left(\tilde{p}\right)\right\}$ $\left(i=1,2,\cdots,m,j=1,2,\cdots,n\right)$.

Step 4. Giving the definition of PLPIS and PLNIS as follows:

$$PLPIS^{+}=(PLPIS_1,PLPIS_2,\cdots,PLPIS_n) \tag{7}$$

$$PLNIS^{+}=(PLNIS_1,PLNIS_2,\cdots,PLNIS_n) \tag{8}$$

where

$$PLPIS_j=\left\{pl_j^{(\phi)}\left(pp_j^{(\phi)}\right)|\phi=1,2,\cdots,\#L_{ij}\left(\tilde{p}\right)\right\}=\left\{\max_i s\left(L_{ij}\left(\tilde{p}\right)\right)\right\} \tag{9}$$

$$PLNIS_j=\left\{nl_j^{(\phi)}\left(np_j^{(\phi)}\right)|\phi=1,2,\cdots,\#L_{ij}\left(\tilde{p}\right)\right\}=\left\{\min_i s\left(L_{ij}\left(\tilde{p}\right)\right)\right\} \tag{10}$$

Step 5. Computing the corresponding GRC of each alternative from PLPIS and PLNIS by utilizing the following equation, respectively:

$$PLPIS\left(\xi_{ij}\right)=\frac{\min\limits_{1\leq i\leq m}\min\limits_{1\leq j\leq n}d\left(PLA_{ij},PLPIS_j\right)+\rho\max\limits_{1\leq i\leq m}\max\limits_{1\leq j\leq n}d\left(PLA_{ij},PLPIS_j\right)}{d\left(PLA_{ij},PLPIS_j\right)+\rho\max\limits_{1\leq i\leq m}\max\limits_{1\leq j\leq n}d\left(PLA_{ij},PLPIS_j\right)} \tag{11}$$
$$i=1,2,\cdots,m,j=1,2,\cdots,n,$$

$$PLNIS\left(\xi_{ij}\right)=\frac{\min\limits_{1\le i\le m}\min\limits_{1\le j\le n} d\left(PLA_{ij},PLNIS_j\right)+\rho\max\limits_{1\le i\le m}\max\limits_{1\le j\le n} d\left(PLA_{ij},PLNIS_j\right)}{d\left(PLA_{ij},PLNIS_j\right)+\rho\max\limits_{1\le i\le m}\max\limits_{1\le j\le n} d\left(PLA_{ij},PLNIS_j\right)} \quad i=1,2,\cdots,m, j=1,2,\cdots,n, \tag{12}$$

$$d\left(PLA_{ij},PLPIS_j\right)=\left(\sum_{\phi=1}^{\#L_{ij}(\tilde{p})}\left|p_{ij}^{(\phi)}g\left(l_{ij}^{(\phi)}\right)-\tilde{p}_j^{(\phi)}g\left(pl_j^{(\phi)}\right)\right|\right)\Big/\#L_{ij}\left(\tilde{p}\right) \tag{13}$$

$$d\left(PLA_{ij},PLNIS_j\right)=\left(\sum_{\phi=1}^{\#L_{ij}(\tilde{p})}\left|p_{ij}^{(\phi)}g\left(l_{ij}^{(\phi)}\right)-\tilde{p}_j^{(\phi)}g\left(nl_j^{(\phi)}\right)\right|\right)\Big/\#L_{ij}\left(\tilde{p}\right) \tag{14}$$

Step 6. Calculating the degree of GRC of all possible alternatives from PLPIS and PLNIS, respectively:

$$PLPIS\left(\xi_i\right)=\sum_{j=1}^{n} w_j PLPIS\left(\xi_{ij}\right), i=1,2,\cdots,m \tag{15}$$

$$PLNIS\left(\xi_i\right)=\sum_{j=1}^{n} w_j PLNIS\left(\xi_{ij}\right), i=1,2,\cdots,m \tag{16}$$

The fundamental idea of GRA method is that the optimal alternative is supposed to possess the "largest degree of GRC" from PLPIS and "smallest degree of GRC" from PLNIS. Evidently, the larger $PLPIS\left(\xi_i\right)$ along with smaller $PLNIS\left(\xi_i\right)$, the better alternative AL_i is. But the attribute weights' information is incompletely known. So, to derive the $PLPIS\left(\xi_i\right)$ and $PLNIS\left(\xi_i\right)$, we can build the multiple objective optimization models (MOOM-1) in the following:

$$\begin{cases}\max PLPIS\left(\xi_i\right)=\sum\limits_{j=1}^{n} w_j PLPIS\left(\xi_{ij}\right)\\ \min PLNIS\left(\xi_i\right)=\sum\limits_{j=1}^{n} w_j PLNIS\left(\xi_{ij}\right)\\ subject\ to:\ w\in H, i=1,2,\cdots,m.\end{cases}$$

Due to each alternative is non-inferior, for all the alternatives, there is no preference relation. Besides, the above multiple objective optimization models (MOOM-1) might be aggregated with equal weights into the following single objective optimization model (SOOM-1):

$$\begin{cases}\min PLPIS\left(\xi\right)=\sum\limits_{i=1}^{m} w_j\left(PLNIS\left(\xi_i\right)-PLPIS\left(\xi_i\right)\right)\\ =\sum\limits_{i=1}^{m}\sum\limits_{j=1}^{n} w_j\left(PLNIS\left(\xi_{ij}\right)-PLPIS\left(\xi_{ij}\right)\right)\\ =\sum\limits_{i=1}^{m}\sum\limits_{j=1}^{n} w_j\left(\begin{array}{c}\dfrac{\min\limits_{1\le i\le m}\min\limits_{1\le j\le n} d\left(PLA_{ij},PLPIS_j\right)+\rho\max\limits_{1\le i\le m}\max\limits_{1\le j\le n} d\left(PLA_{ij},PLPIS_j\right)}{d\left(PLA_{ij},PLPIS_j\right)+\rho\max\limits_{1\le i\le m}\max\limits_{1\le j\le n} d\left(PLA_{ij},PLPIS_j\right)}-\\ \dfrac{\min\limits_{1\le i\le m}\min\limits_{1\le j\le n} d\left(PLA_{ij},PLNIS_j\right)+\rho\max\limits_{1\le i\le m}\max\limits_{1\le j\le n} d\left(PLA_{ij},PLNIS_j\right)}{d\left(PLA_{ij},PLNIS_j\right)+\rho\max\limits_{1\le i\le m}\max\limits_{1\le j\le n} d\left(PLA_{ij},PLNIS_j\right)}\end{array}\right)\\ subject\ to:\ w\in H\end{cases}$$

By solving the model (SOOM-1), we get the optimal solution $w=(w_1,w_2,\cdots,w_n)$, which can be regarded as the weight vector of attributes. Then, we can get $PLPIS\left(\xi_i\right)$ and $PLNIS\left(\xi_i\right)$ by Eqs. (15) and (16).

Step 7. Derive the probabilistic linguistic relative relational degree (PLRRD) of all possible alternatives from PLPIS.

$$PLRRD\left(\xi_i\right)=\frac{PLPIS\left(\xi_i\right)}{PLPIS\left(\xi_i\right)+PLNIS\left(\xi_i\right)}, i=1,2,\cdots,m \tag{17}$$

Step 8. According to $PLRRD\left(\xi_i\right)$, the sorting order of all possible alternatives can be obtained. If any alternative has the largest $PLRRD\left(\xi_i\right)$, then, it is optimal choice.

4. NUMERICAL CASE AND COMPARATIVE ANALYSIS

4.1. Numerical Case

Along with the acceleration of urbanization and the rapid growth of urban population, the output of municipal solid waste also increases rapidly. More and more city or region will create more and more waste incineration plants in accordance with "The Twelfth Five-Year Plan." However, the waste incinerator is a NIMBY facility, and if the site selection is not scientific and reasonable, it is likely to cause NIMBY conflicts. Therefore, how to reasonably carry on the waste incineration plant scientific location is particularly important. Waste incineration plants location problem should be regarded as the corresponding MAGDM [45–51]. Thus, in this chapter we developed a case study concerning waste incineration plants location problem to demonstrate the approach presented in this essay. There are five potential waste incineration plants sites $A_i\,(i = 1, 2, 3, 4, 5)$ proposed to select. The experts select the following four beneficial attributes to assess the five potential waste incineration plants sites: ① G_1 is waste incineration plants area; ② G_2 is transportation cost of waste; ③ G_3 is site far from the residential area; ④ G_4 is seepage control rainwater and sewage diversion. The transportation cost (G_2) is not beneficial attribute, others are beneficial attributes. The five potential waste incineration plants sites $A_i\,(i = 1, 2, 3, 4, 5)$ are to be assessed by utilizing the linguistic term set

$$L = \{l_{-3} = extremely\,poor\,(EP)\,, l_{-2} = very\,poor\,(VP)\,,\\ l_{-1} = poor\,(P)\,, l_0 = medium\,(M)\,,\ l_1 = good\,(G)\,,\\ l_2 = very\,good\,(VG)\,, l_3 = extremely\,good\,(EG)\}$$

by the five DMs within the above four attributes, as listed in the Tables 1–5. In the light of the three experts' judgment, the attribute weights are partly known in the following:

$$H = \{0.15 \le \omega_1 \le 0.21, 0.19 \le \omega_2 \le 0.29,\\ 0.22 \le \omega_3 \le 0.35, \omega_3 - \omega_4 \ge 0.11\}$$

Following that, the PL-GRA method is utilized to select the optimal waste incineration plants sites.

Step 1. Shift cost attribute G_2 into beneficial attribute. If the cost attribute value is l_τ, then the corresponding beneficial attribute value is $l_{-\tau}\,(\tau = -3, -2, -1, 0, 1, 2, 3)$ (See Tables 6–10).

Step 2. Transform the linguistic variables into PLTSs (Table 11).

Table 1 Linguistic decision matrix by the first DM.

Alternatives	G_1	G_2	G_3	G_4
A_1	G	VG	M	VG
A_2	G	P	VG	P
A_3	VG	VG	P	VG
A_4	VG	VP	VP	P
A_5	EG	VP	G	EG

DM, decision maker.

Table 2 Linguistic decision matrix by the second DM.

Alternatives	G_1	G_2	G_3	G_4
A_1	EG	VP	VG	VG
A_2	G	EP	EG	M
A_3	VG	M	G	EG
A_4	VG	VP	P	P
A_5	VG	P	G	VG

DM, decision maker.

Step 3. Calculate the decision matrix with normalized PLTSs (Table 12).

Step 4. Defining the PLPIS and PLNIS by Eqs. (7–9) (Table 13):

Step 5. Computing the corresponding GRC of each alternative from PLPIS and PLNIS (Tables 14 and 15):

Step 6. The model (SOOM-1) is utilized to set up the single-objective programming model:

$$\begin{cases} \min d\,(w) = -0.1315w_1 - 0.0215w_2 + 0.0137w_3 - 0.1197w_4 \\ subject\,:\ w \in H \end{cases}$$

Table 3 Linguistic decision matrix by the third DM.

Alternatives	G_1	G_2	G_3	G_4
A_1	EG	G	VG	VG
A_2	G	EP	EG	G
A_3	G	M	G	VG
A_4	G	EP	P	P
A_5	VG	VP	M	G

DM, decision maker.

Table 4 Linguistic decision matrix by the fourth DM.

Alternatives	G_1	G_2	G_3	G_4
A_1	VG	G	VG	VG
A_2	G	P	VG	M
A_3	G	EG	G	VG
A_4	G	EP	VP	EP
A_5	EG	P	VG	VG

DM, decision maker.

Table 5 Linguistic decision matrix by the fifth DM.

Alternatives	G_1	G_2	G_3	G_4
A_1	EG	VP	M	G
A_2	P	EP	VG	M
A_3	VG	VG	G	VG
A_4	VP	VP	P	P
A_5	EG	VP	VG	VG

DM, decision maker.

Table 6 Linguistic decision matrix by the first DM.

Alternatives	G_1	G_2	G_3	G_4
A_1	G	VP	M	VG
A_2	G	G	VG	P
A_3	VG	VP	P	VG
A_4	VG	VG	VP	P
A_5	EG	VG	G	EG

DM, decision maker.

Table 7 Linguistic decision matrix by the second DM.

Alternatives	G_1	G_2	G_3	G_4
A_1	EG	VG	VG	VG
A_2	G	EG	EG	M
A_3	VG	M	G	EG
A_4	VG	VG	P	P
A_5	VG	G	G	VG

DM, decision maker.

Table 8 Linguistic decision matrix by the third DM.

Alternatives	G_1	G_2	G_3	G_4
A_1	EG	P	VG	VG
A_2	G	EG	EG	G
A_3	G	M	G	VG
A_4	G	EG	P	P
A_5	VG	VG	M	G

DM, decision maker.

Table 9 Linguistic decision matrix by the fourth DM.

Alternatives	G_1	G_2	G_3	G_4
A_1	VG	P	VG	VG
A_2	G	G	VG	M
A_3	G	EP	G	VG
A_4	G	EG	VP	EP
A_5	EG	G	VG	VG

DM, decision maker.

Table 10 Linguistic decision matrix by the fifth DM.

Alternatives	G_1	G_2	G_3	G_4
A_1	EG	VG	M	G
A_2	P	EG	VG	M
A_3	VG	VP	G	VG
A_4	VP	VG	P	P
A_5	EG	VG	VG	VG

DM, decision maker.

Table 11 Decision matrix with PLTSs.

Alternatives	G_1	G_2
A_1	$\{l_1(0.2), l_2(0.2), l_3(0.6)\}$	$\{l_{-2}(0.2), l_{-1}(0.4), l_2(0.4)\}$
A_2	$\{l_{-1}(0.2), l_1(0.8)\}$	$\{l_1(0.4), l_3(0.6)\}$
A_3	$\{l_1(0.4), l_2(0.6)\}$	$\{l_{-3}(0.2), l_{-2}(0.4), l_0(0.4)\}$
A_4	$\{l_{-2}(0.2), l_1(0.4), l_2(0.4)\}$	$\{l_2(0.6), l_3(0.4)\}$
A_5	$\{l_2(0.4), l_3(0.6)\}$	$\{l_1(0.4), l_2(0.6)\}$
Alternatives	G_3	G_4
A_1	$\{l_0(0.4), l_2(0.6)\}$	$\{l_1(0.2), l_2(0.8)\}$
A_2	$\{l_2(0.6), l_3(0.4)\}$	$\{l_{-1}(0.2), l_0(0.6), l_1(0.2)\}$
A_3	$\{l_{-1}(0.2), l_1(0.8)\}$	$\{l_2(0.8), l_3(0.2)\}$
A_4	$\{l_{-2}(0.4), l_{-1}(0.6)\}$	$\{l_{-3}(0.2), l_{-1}(0.8)\}$
A_5	$\{l_0(0.2), l_1(0.4), l_2(0.4)\}$	$\{l_1(0.2), l_2(0.6), l_3(0.2)\}$

PLTS, probabilistic linguistic term set.

Solve this model, the weight vector of attributes can be got: $w = (0.2100, 0.2000, 0.3500, 0.2400)^T$.

Step 7. Calculating the degree of GRC of all possible alternatives from PLPIS and PLNIS, respectively (Table 16):

Step 8. Calculating the $PLRRD\left(\xi_i\right)$ of each alternative from PLPIS by Eq. (14) (Table 17).

Step 9. According to the $PLRRD\left(\xi_i\right)$ $(i = 1, 2, 3, 4, 5)$, all the waste incineration plants sites can be ranked. Evidently, the order is $A_5 > A_2 > A_3 > A_1 > A_4$ and the most desirable waste incineration plants site among five alternatives is A_5.

Table 12 Decision matrix with Nnormalized PLTSs.

Alternatives	G_1	G_2
A_1	$\{l_1(0.2), l_2(0.2), l_3(0.6)\}$	$\{l_{-2}(0.2), l_{-1}(0.4), l_2(0.4)\}$
A_2	$\{l_{-1}(0), l_{-1}(0.2), l_1(0.8)\}$	$\{l_1(0), l_1(0.4), l_3(0.6)\}$
A_3	$\{l_1(0), l_1(0.4), l_2(0.6)\}$	$\{l_{-3}(0.2), l_{-2}(0.4), l_0(0.4)\}$
A_4	$\{l_{-2}(0.2), l_1(0.4), l_2(0.4)\}$	$\{l_2(0), l_2(0.6), l_3(0.4)\}$
A_5	$\{l_2(0), l_2(0.4), l_3(0.6)\}$	$\{l_1(0), l_1(0.4), l_2(0.6)\}$
Alternatives	G_3	G_4
A_1	$\{l_0(0), l_0(0.4), l_2(0.6)\}$	$\{l_1(0), l_1(0.2), l_2(0.8)\}$
A_2	$\{l_2(0), l_2(0.6), l_3(0.4)\}$	$\{l_{-1}(0.2), l_0(0.6), l_1(0.2)\}$
A_3	$\{l_{-1}(0), l_{-1}(0.2), l_1(0.8)\}$	$\{l_2(0), l_2(0.8), l_3(0.2)\}$
A_4	$\{l_{-2}(0), l_{-2}(0.4), l_{-1}(0.6)\}$	$\{l_{-3}(0), l_{-3}(0.2), l_{-1}(0.8)\}$
A_5	$\{l_0(0.2), l_1(0.4), l_2(0.4)\}$	$\{l_1(0.2), l_2(0.6), l_3(0.2)\}$

PLTS, probabilistic linguistic term set.

Table 13 PLPIS and PLNIS.

	G_1	G_2
PLPIS	$\{l_2(0), l_2(0.4), l_3(0.6)\}$	$\{l_2(0), l_2(0.6), l_3(0.4)\}$
PLNIS	$\{l_{-1}(0), l_{-1}(0.2), l_1(0.8)\}$	$\{l_{-3}(0.2), l_{-2}(0.4), l_0(0.4)\}$
	G_3	G_4
PLPIS	$\{l_2(0), l_2(0.6), l_3(0.4)\}$	$\{l_2(0), l_2(0.8), l_3(0.2)\}$
PLNIS	$\{l_{-2}(0), l_{-2}(0.4), l_{-1}(0.6)\}$	$\{l_{-3}(0), l_{-3}(0.2), l_{-1}(0.8)\}$

PLPIS, probabilistic linguistic positive ideal solution; PLNIS, probabilistic linguistic negative ideal solution.

Table 14 GRC of each alternative from PLPIS.

Alternatives	G_1	G_2	G_3	G_4
A_1	0.6241	0.4864	0.5284	0.3333
A_2	0.5631	0.5355	1.0000	0.4835
A_3	0.7467	0.4261	0.4387	1.0000
A_4	0.5613	1.0000	0.4261	0.3459
A_5	1.0000	0.5826	0.5745	0.6241

GRC, grey relational coefficient; PLPIS, probabilistic linguistic positive ideal solution.

Table 15 GRC of each alternative from PLNIS.

Alternatives	G_1	G_2	G_3	G_4
A_1	0.6511	0.6868	0.5051	0.4427
A_2	1.0000	0.4283	0.4124	0.5000
A_3	0.6230	1.0000	0.5012	0.3333
A_4	0.5405	0.4124	1.0000	1.0000
A_5	0.5493	0.4816	0.5627	0.3910

GRC, grey relational coefficient; PLNIS, probabilistic linguistic negative ideal solution.

4.2. Comparative Analysis

Then, our proposed method is compared with probabilistic linguistic weighted average (PLWA) operator [25] and PL-TOPSIS method [25] as in Table 18.

In terms of the above analysis, it can be found that these above-mentioned methods have the same best waste incineration plants site A_5 and the bad waste incineration plants site A_4, and there are

Table 16 | $PLPIS(\xi_i)$ and $PLNIS(\xi_i)$ of each alternative.

Alternatives	$PLPIS(\xi_i)$	$PLNIS(\xi_i)$
A_1	0.4933	0.5571
A_2	0.6914	0.5600
A_3	0.6356	0.5863
A_4	0.5500	0.7860
A_5	0.6774	0.5025

PLPIS, probabilistic linguistic positive ideal solution; PLNIS, probabilistic linguistic negative ideal solution.

Table 17 | PLRRD of each alternative from PLPIS.

Alternatives	A_1	A_2	A_3	A_4	A_5
$PLRRD(\xi_i)$	0.4696	0.5525	0.5202	0.4117	0.5741

PLRRD, probabilistic linguistic relative relational degree; PLPIS, probabilistic linguistic positive ideal solution.

Table 18 | Ordering of the waste incineration plants sites by using diverse methods.

Methods	Computing Results	Ordering
PL-TOPSIS [25]	$d_1 = -1.2035, d_2 = -0.3420, d_3 = -1.4208,$ $d_4 = -2.1670, d_5 = 0.0000.$	$A_5 \succ A_2 \succ A_1 \succ A_3 \succ A_4$
PLWA operator [25]	$E(Z_1(w)) = s_{0.4520}, E(Z_2(w)) = s_{0.4687},$ $E(Z_3(w)) = s_{0.2647}, E(Z_4(w)) = s_{-0.0593},$ $E(Z_5(w)) = s_{0.5887}.$	$A_5 \succ A_2 \succ A_1 \succ A_3 \succ A_4$
PL-GRA method	$PLRRD(\xi_1) = 0.4696, PLRRD(\xi_2) = 0.5525$ $PLRRD(\xi_3) = 0.5202, PLRRD(\xi_4) = 0.4117$ $PLRRD(\xi_5) = 0.5741$	$A_5 > A_2 > A_3 > A_1 > A_4$

PLWA, probabilistic linguistic weighted average; PL_GRA, probabilistic linguistic grey relational analysis.

slightly different in three methods' ranking results, which can confirm the presented method is feasible and effective in this essay. All these methods have their good advantages: (1) PL-TOPSIS method emphasis the distance similarity degree from the positive and negative ideal solution; (2) PLWA operator emphasis group influences; (3) our proposed PL-GRA method emphasis emphasizes he shape similarity degree from PIS and NIS simultaneously with incomplete weight information.

5. CONCLUSION

In this essay, the GRA method is expanded to the PL-MAGDM with incomplete weight information. First and foremost, the definition, comparative method and distance of PLTs are simply reviewed. Additionally, the extended GRA method is employed to tackle PL-MAGDM issues with incomplete weight information. We construct the multiple objective optimization models on the basis of the conventional GRA method. Besides, the multiple objective optimization models can be converted into a single-objective programming model by making use of the linear equal weighted method. By calculating the single-objective programming model, the weight information can be acquired. In the light of the conventional GRA, the optimal choice is derived by obtaining "largest degree of GRC" from PLPIS and "smallest degree of GRC" from PLNIS. Finally, a practical case study concerning waste incineration plants location problem is designed to validate the proposed algorithms and some comparative studies are also designed to verify the applicability. In our future research, the proposed methods and algorithm will be needful and meaningful for other real decision making problems [52–60] and the developed approaches can also be extended to other fuzzy [48,61,62] and uncertain information [63–73].

CONFLICT OF INTEREST

The authors declare that they have no conflict of interest.

AUTHORS' CONTRIBUTIONS

Fan Lei, Guiwu Wei, Jianping Lu, Jiang Wu and Cun Wei conceived and worked together to achieve this work, **Fan Lei** compiled the computing program by Excel and analyzed the data, **Fan Lei and Guiwu Wei** wrote the paper. Finally, all the authors have read and approved the final manuscript.

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REFERENCES

[1] J.L. Deng, Introduction to grey system, J. Grey Syst. 1 (1989), 1–24.

[2] M.F. Yeh, H.C. Lu, Evaluating weapon systems based on grey relational analysis and fuzzy arithmetic operations, J. Chin. Inst. Eng. 23 (2000), 211–221.

[3] X.P. Xiao, X.L. Chong, Grey relational analysis and application of hybrid index sequences, Dyn. Cont. Discrete Impulsive Syst. Ser. B Appl. Algorithms. 13 (2006), 915–919.

[4] Y.M. Xie, H.P. Yu, J. Chen, X.Y. Ruan, Application of grey relational analysis in sheet metal forming for multi-response quality characteristics, J. Zhejiang Univ. Sci. A. 8 (2007), 805–811.

[5] C.Y. Kung, K.L. Wen, Applying grey relational analysis and grey decision-making to evaluate the relationship between company attributes and its financial performance - a case study of venture capital enterprises in Taiwan, Decis. Support Syst. 43 (2007), 842–852.

[6] T.M. Yan, C.Y. Kung, Business performance assessment of insurance company via grey relational analysis, J. Grey Syst. 23 (2011), 83–90.

[7] W.S. Wu, Grey relational analysis method for group decision making in credit risk analysis, Eurasia J. Math. Sci. Technol. Educ. 13 (2017), 7913–7920.

[8] Y.S. Tan, H. Chen, S. Wu, Evaluation and implementation of environmentally conscious product design by using AHP and grey relational analysis approaches, Ekoloji. 28 (2019), 857–864.

[9] Z.P. Chiang, Extended grey relational analysis for dependent criteria decision making problems, J. Grey Syst. 23 (2011), 273–280.

[10] A. Malek, S. Ebrahimnejad, R. Tavakkoli-Moghaddam, An improved hybrid grey relational analysis approach for green resilient supply chain network assessment, Sustainability. 9 (2017), 1433.

[11] O. Alptekin, N. Alptekin, B. Sarac, Evaluation of low carbon development of European Union Countries and Turkey using grey relational analysis, Tehnicki Vjesnik-Technical Gazette. 25 (2018), 1497–1505.

[12] B.Z. Zhu, L.L. Yuan, S.X. Ye, Examining the multi-timescales of European carbon market with grey relational analysis and empirical mode decomposition, Phys. a-Stat. Mech. Appl. 517 (2019), 392–399.

[13] M. Yazdani, C. Kahraman, P. Zarate, S.C. Onar, A fuzzy multi attribute decision framework with integration of QFD and grey relational analysis, Expert Syst. Appl. 115 (2019), 474–485.

[14] C.H. Chen, A new multi-criteria assessment model combining GRA techniques with intuitionistic fuzzy entropy-based TOPSIS method for sustainable building materials supplier selection, Sustainability. 11 (2019), 2265.

[15] G.W. Wei, Grey relational analysis model for dynamic hybrid multiple attribute decision making, Knowl. Based Syst. 24 (2011), 672–679.

[16] S. Abdullah, N.U. Amin, Analysis of S-box image encryption based on generalized fuzzy soft expert set, Nonlin. Dyn. 79 (2015), 1679–1692.

[17] J. Wang, H. Gao, G.W. Wei, Some 2-tuple linguistic neutrosophic number Muirhead mean operators and their applications to multiple attribute decision making, J. Exp. Theor. Artif. Intell. 31 (2019), 409–439.

[18] S.Q. Zhang, H. Gao, G.W. Wei, Y. Wei, C. Wei, Evaluation based on distance from average solution method for multiple criteria group decision making under picture 2-tuple linguistic environment, Mathematics. 7 (2019), 243.

[19] L.A. Zadeh, The concept of a linguistic variable and its application to approximate reasoning, Inf. Sci. 8 (1975), 301–357.

[20] R.M. Rodriguez, L. Martinez, F. Herrera, Hesitant fuzzy linguistic term sets for decision making, IEEE Trans. Fuzzy Syst. 20 (2012), 109–119.

[21] X.J. Gou, Z.S. Xu, H.C. Liao, Hesitant fuzzy linguistic entropy and cross-entropy measures and alternative queuing method for multiple criteria decision making, Inf. Sci. 388 (2017), 225–246.

[22] G.W. Wei, The generalized dice similarity measures for multiple attribute decision making with hesitant fuzzy linguistic information, Econ. Res. Ekonomska Istrazivanja. 32 (2019), 1498–1520.

[23] H.C. Liao, Z.S. Xu, X.J. Zeng, Hesitant fuzzy linguistic VIKOR method and its application in qualitative multiple criteria decision making, IEEE Trans. Fuzzy Syst. 23 (2015), 1343–1355.

[24] M.W. Lin, Z.S. Xu, Y.L. Zhai, Z.Q. Yao, Multi-attribute group decision-making under probabilistic uncertain linguistic environment, J. Oper. Res. Soc. 69 (2018), 157–170.

[25] Q. Pang, H. Wang, Z.S. Xu, Probabilistic linguistic linguistic term sets in multi-attribute group decision making, Inf. Sci. 369 (2016), 128–143.

[26] M.W. Lin, Z.Y. Chen, H.C. Liao, Z.S. Xu, ELECTRE II method to deal with probabilistic linguistic term sets and its application to edge computing, Nonlin. Dyn. 96 (2019), 2125–2143.

[27] X.Q. Feng, Q. Liu, C.P. Wei, Probabilistic linguistic QUALIFLEX approach with possibility degree comparison, J. Intell. Fuzzy Syst. 36 (2019), 719–730.

[28] C.Z. Bai, R. Zhang, L.X. Qian, Y.N. Wu, Comparisons of probabilistic linguistic term sets for multi-criteria decision making, Knowl. Based Syst. 119 (2017), 284–291.

[29] S.X. Chen, J.Q. Wang, T.L. Wang, Cloud-based ERP system selection based on extended probabilistic linguistic MULTIMOORA method and Choquet integral operator, Comput. Appl. Math. 38 (2019), 88.

[30] P.D. Liu, F. Teng, Some Muirhead mean operators for probabilistic linguistic term sets and their applications to multiple attribute decision-making, Appl. Soft Comput. 68 (2018), 396–431.

[31] P.D. Liu, F. Teng, Probabilistic linguistic TODIM method for selecting products through online product reviews, Inf. Sci. 485 (2019), 441–455.

[32] P.D. Liu, Y. Li, Multi-attribute decision making method based on generalized maclaurin symmetric mean aggregation operators for probabilistic linguistic information, Comput. Ind. Eng. 131 (2019), 282–294.

[33] D.C. Liang, A. Kobina, W. Quan, Grey relational analysis method for probabilistic linguistic multi-criteria group decision-making based on geometric Bonferroni mean, Int. J. Fuzzy Syst. 20 (2018), 2234–2244.

[34] J. Wang, G.W. Wei, Y. Wei, Models for green supplier selection with some 2-tuple linguistic neutrosophic number Bonferroni mean operators, Symmetry-Basel. 10 (2018), 131.

[35] G.W. Wei, R. Wang, J. Wang, C. Wei, Y. Zhang, Methods for evaluating the technological innovation capability for the high-tech enterprises with generalized interval neutrosophic number Bonferroni mean operators, IEEE Access. 7 (2019), 86473–86492.

[36] B. Zhu, Z.S. Xu, Hesitant fuzzy Bonferroni means for multi-criteria decision making, J. Oper. Res. Soc. 64 (2013), 1831–1840.

[37] B. Dutta, D. Guha, Partitioned Bonferroni mean based on linguistic 2-tuple for dealing with multi-attribute group decision making, Appl. Soft Comput. 37 (2015), 166–179.

[38] J.P. Lu, C. Wei, J. Wu, G.W. Wei, TOPSIS method for probabilistic linguistic MAGDM with entropy weight and its application to supplier selection of new agricultural machinery products, Entropy. 21 (2019), 953.

[39] N. Zhao, Z.S. Xu, Z.L. Ren, Some approaches to constructing distance measures for hesitant fuzzy linguistic term sets with applications in decision-making, Int. J. Inf. Technol. Decis. Making. 17 (2018), 103–132.

[40] C.P. Wei, N. Zhao, X.J. Tang, Operators and comparisons of hesitant fuzzy linguistic term sets, IEEE Trans. Fuzzy Syst. 22 (2014), 575–585.

[41] X.B. Mao, M. Wu, J.Y. Dong, S.P. Wan, Z. Jin, A new method for probabilistic linguistic multi-attribute group decision making: application to the selection of financial technologies, Appl. Soft Comput. 77 (2019), 155–175.

[42] K.S. Park, S.H. Kim, Tools for interactive multiattribute decision-making with incompletely identified information, Eur. J. Oper. Res. 98 (1997), 111–123.

[43] S.H. Kim, H.C. Sang, J.K. Kim, An interactive procedure for multiple attribute group decision making with incomplete information: range-based approach, Eur. J. Oper. Res. 118 (1999), 139–152.

[44] S.H. Kim, B.S. Ahn, Interactive group decision making procedure under incomplete information, Eur. J. Oper. Res. 116 (1999), 498–507.

[45] J.P. Lu, X.Y. Tang, G.W. Wei, C. Wei, Y. Wei, Bidirectional project method for dual hesitant Pythagorean fuzzy multiple attribute decision-making and their application to performance assessment of new rural construction, Int. J. Intell. Syst. 34 (2019), 1920–1934.

[46] P. Wang, J. Wang, G.W. Wei, C. Wei, Similarity measures of q-rung orthopair fuzzy sets based on cosine function and their applications, Mathematics. 7 (2019), 340.

[47] L.P. Wu, J. Wang, H. Gao, Models for competiveness evaluation of tourist destination with some interval-valued intuitionistic fuzzy Hamy mean operators, J. Intell. Fuzzy Syst. 36 (2019), 5693–5709.

[48] G.W. Wei, J. Wu, C. Wei, J. Wang, J.P. Lu, Models for MADM with 2-tuple linguistic neutrosophic Dombi Bonferroni mean operators, IEEE Access. 7 (2019), 108878–108905.

[49] B. Matic, S. Jovanovic, D.K. Das, E.K. Zavadskas, Z. Stevic, S. Sremac, M. Marinkovic, A new hybrid MCDM model: sustainable supplier selection in a construction company, Symmetry-Basel. 11 (2019), 353.

[50] J. Roy, H.K. Sharma, S. Kar, E.K. Zavadskas, J. Saparauskas, An extended COPRAS model for multi-criteria decision-making problems and its application in web-based hotel evaluation and selection, Econ. Res. Ekonomska Istrazivanja. 32 (2019), 219–253.

[51] E.K. Zavadskas, J. Antucheviciene, P. Chatterjee, Multiple-Criteria Decision-Making (MCDM) techniques for business processes information management, Information. 10 (2019), 4.

[52] A. Valipour, N. Yahaya, N. Md Noor, I. Valipour, J. Tamosaitiene, A SWARA-COPRAS approach to the allocation of risk in water and sewerage public-private partnership projects in Malaysia, Int. J. Strateg. Property Manage. 23 (2019), 269–283.

[53] X.Y. Tang, G.W. Wei, Dual hesitant Pythagorean fuzzy Bonferroni mean operators in multi-attribute decision making, Arch. Control Sci. 29 (2019), 339–386.

[54] X.Y. Tang, G.W. Wei, H. Gao, Models for multiple attribute decision making with interval-valued pythagorean fuzzy muirhead mean operators and their application to green suppliers selection, Informatica. 30 (2019), 153–186.

[55] M. Pishdar, F. Ghasemzadeh, J. Antucheviciene, J. Saparauskas, Internet of things and its challenges in supply chain management: a rough strength-relation analysis method, E & M Ekonomie Manage. 21 (2018), 208–222.

[56] J. Wang, H. Gao, G.W. Wei, Y. Wei, Methods for multiple-attribute group decision making with q-rung interval-valued orthopair fuzzy information and their applications to the selection of green suppliers, Symmetry-Basel. 11 (2019), 56.

[57] D.L. Zheng, L.J. Yu, L.Z. Wang, J.G. Tao, Integrating willingness analysis into investment prediction model for large scale building energy saving retrofit: using fuzzy multiple attribute decision making method with Monte Carlo simulation, Sustain. Cities Soc. 44 (2019), 291–309.

[58] P.D. Liu, S.F. Cheng, Y.M. Zhang, An extended multi-criteria group decision-making Promethee method based on probability multi-valued neutrosophic sets, Int. J. Fuzzy Syst. 21 (2019), 388–406.

[59] S.Q. Zhang, G.W. Wei, H. Gao, C. Wei, Y. Wei, EDAS method for multiple criteria group decision making with picture fuzzy information and its application to green suppliers selections, Technol. Econ. Dev. Econ. 26 (2019), 1123–1138.

[60] G.F. Yu, W. Fei, D.F. Li, A compromise-typed variable weight decision method for hybrid multiattribute decision making, IEEE Trans. Fuzzy Syst. 27 (2019), 861–872.

[61] G.W. Wei, S.Q. Zhang, J.P. Lu, J. Wu, C. Wei, An extended bidirectional projection method for picture fuzzy MAGDM and its application to safety assessment of construction project, IEEE Access. 7 (2019), 166138–166147.

[62] G.W. Wei, J. Wang, M. Lu, J. Wu, C. Wei, Similarity measures of spherical fuzzy sets based on cosine function and their applications, IEEE Access. 7 (2019), 159069–159080.

[63] X.M. Deng, J. Wang, G.W. Wei, Some 2-tuple linguistic Pythagorean Heronian mean operators and their application to multiple attribute decision-making, J. Exp. Theor. Artif. Intell. 31 (2019), 555–574.

[64] G.W. Wei, 2-tuple intuitionistic fuzzy linguistic aggregation operators in multiple attribute decision making, Iran. J. Fuzzy Syst. 16 (2019), 159–174.

[65] R. Yager, Generalized orthopair fuzzy sets, IEEE Trans. Fuzzy Syst. 25 (2017), 1222–1230.

[66] F.B. Yeni, G. Ozcelik, Interval-valued atanassov intuitionistic fuzzy CODAS method for multi criteria group decision making problems, Group Decis. Negot. 28 (2019), 433–452.

[67] D. Stanujkic, D. Karabasevic, E.K. Zavadskas, F. Smarandache, W.K.M. Brauers, A bipolar fuzzy extension of the MULTIMOORA method, Informatica. 30 (2019), 135–152.

[68] S.H. Zolfani, M. Yazdani, E.K. Zavadskas, An extended Stepwise Weight Assessment Ratio Analysis (SWARA) method for improving criteria prioritization process, Soft Comput. 22 (2018), 7399–7405.

[69] J.Y. Dong, Y. Chen, S.P. Wan, A cosine similarity based QUALIFLEX approach with hesitant fuzzy linguistic term sets for financial performance evaluation, Appl. Soft Comput. 69 (2018), 316–329.
[70] J.Y. Dong, F.F. Yuan, S.P. Wan, Extended VIKOR method for multiple criteria decision-making with linguistic hesitant fuzzy information, Comput. Ind. Eng. 112 (2017), 305–319.
[71] S.P. Wan, G.L. Xu, J.Y. Dong, Supplier selection using ANP and ELECTRE II in interval 2-tuple linguistic environment, Inf. Sci. 385 (2017), 19–38.
[72] H. Gao, J. Wu, C. Wei, G.W. Wei, MADM method with interval-valued bipolar uncertain linguistic information for evaluating the computer network security, IEEE Access. 7 (2019), 151506–151524.
[73] D.F. Li, G.H. Chen, Z.G. Huang, Linear programming method for multiattribute group decision making using IF sets, Inf. Sci. 180 (2010), 1591–1609.