Unfolding Urban Structures: Towards Route Prediction and Automated City Modeling

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Abstract

This paper extends previous work concerning intersection classification by including a new set of statistics that enable to describe the structure of a city at a higher level of detail. Namely, we suggest to analyze sequences of intersections of different types. We start with sequences of length two and present a probabilistic model to derive statistics for longer sequences. We validate the results by comparing them with real frequencies. Finally, we discuss how this work can contribute to the generation of virtual cities as well as to spatial configuration search.

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1 Introduction

Modeling the structure of a city is a long-term goal in the GIScience community, as well as in other communities such as Urban Planning, Transportation Planning, Civil Engineering, and Spatial Cognition. Indeed, developing a formal model for describing the structure of a city can be beneficial for a variety of scenarios. For example, to look for structurally similar areas in different cities or different areas of the same city, to generate virtual look-alike cities (i.e., virtual environments exposing a similar structure to a reference city), or to (re)design a street network to minimize the probability of traffic congestion.

The structure of a city can be regarded as consisting of topological and metrical information. In this work we introduce an approach for capturing a topological aspect of the structure that will be complemented in future work with distance and directional information to obtain a complete structural representation.

In [8] a novel approach was introduced that approaches the problem by analyzing the intersections making up the street network of a city. The paper presents a classification for

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intersections and introduces and formally defines so-called regular intersections that are used as a baseline for comparing real intersections. The introduced model is utilized to derive statistics about and compare four cities as well different districts of the same city.

We extend the previous work by introducing a novel metric for the representation of urban structures. Building on top on the intersection data provided in [8] we draft a model to predict sequences of consecutive intersections. We start by counting the occurrences of sequences of two intersections and we present a probabilistic model to infer the frequencies of longer sequences. We validate the model by comparing the inferred frequencies with real data for sequences of 3 and 4 intersections.

Finally, we envision how this model can be used in future work to automatically generate virtual look-alike environments that expose a similar structure with respect to a reference city and to find structurally similar areas in different cities or in different parts of the same city.

2 Related Work

While our work takes on different disciplines such as spatial cognition, network analysis, graph theory, and space syntax, at the best of our knowledge this is a novel approach to the problem of understanding the structure of a city.

Probably the most famous work about the analysis of urban spaces is the work from Lynch [12]. In this work, Lynch analyzes properties of cities that affect the perceptual and cognitive aspects. He argues that the environmental image of a city consists of three main components: identity, structure, and meaning. The structure of a city is described as the spatial relations occurring among the city objects as well as between those and the observer.

Spatial networks (see [1] for a detailed survey) are spatially embedded graphs representing spatial features and connections among them. A typical example of spatial networks are street networks where intersections are reported as nodes of the graph and street segments as its edges. In [14] an open source toolbox for ArcGIS is introduced that allows for computing five types of network centrality measures on spatial networks: reach, gravity index, betweenness, closeness, and straightness (see [14] for details about these metrics).

Graph theory provides the mathematical foundation to topological analysis (see [4] for an extensive discussion on the topic). Network analysis and other spatial studies typically model the domain of investigation by means of a graph or a hypergraph and employ typical graph properties (e.g., node degree, and reachability) and operations (e.g. shortest path, connected components) to perform the necessary analyses.

Finally, space syntax [11, 10] is a set of theories aiming at identifying how urban structure affects social structure. Theories of space syntax typically represent the urban space as a graph, using different abstractions for nodes and edges. In simple terms, space syntax approaches model spatial environments with a dual graph where nodes represent empty space (e.g., streets in a street network) and edges represents some sort of connection among them (e.g., intersections). One of the earliest approaches [11] to space syntax is based on the concepts of axial line and convex space. However, it has been argued [2] that the lack of formality in the original definition of these concepts does not allow for an automatic generation of a so-called axial map. Another popular approach to space syntax resorts to the concept of isovist: the set of all points visible from a given vantage point in space and with respect to an environment [3].

2.1 Types of Intersections

In [8] an intersection is classified according to two main metrics. First, the number n of street segments stemming out of an intersections (i.e., its branches). An intersection with n branches is called an n-way intersection.

Second, the angular arrangement of the branches of an n-way intersection I^n. This is described as the angular distance $\Delta(I^n, R^n)$ between I^n and the corresponding *regular* n-way intersection R^n: an intersection whose branches split a revolution (2π) into n equal angles, each of width $\frac{2\pi}{n}$. $\Delta(I^n, R^n)$ is the minimum sum of angles that we have to rotate the branches of I^n to perfectly match R^n, while preserving the circular order of I^n's branches.

Finally, intersections are classified according to the type of transportation mode they allow. *Path*-intersections allow only for pedestrian transit; *road*-intersections are passable by both pedestrians and cars; *car*-intersections only allow cars.

3 Predicting Route Sequences

3.1 Modeling

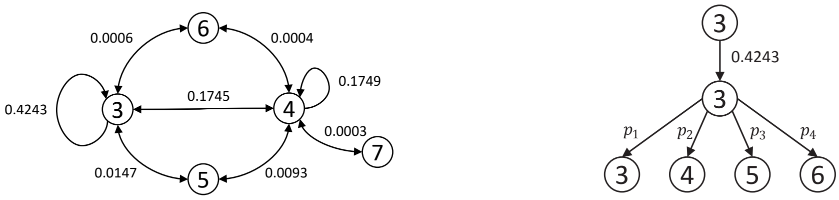
In [8] statistics about the intersections of a city have been derived: the type of intersections (3-way, 4-way, ...) and their angular distance to the corresponding regular intersection. Assume to represent the street network of a city as a graph $G = (V, E)$ whose nodes V represent intersections and whose edges E represent street segments among them. Then, the type of an intersection denotes the degree of a node and the corresponding statistics provide a first approximate description of the graph and, thus, of the city structure.

In order to fully characterize the city structure we shall compute more information. In this paper we focus only on topological information. More specifically, we focus on the prediction of intersection sequences as we route through the street network.

Say $\pi =< t_1, ..., t_n >$ is the shortest path between two nodes in the graph, where t_i denotes the type of node that is traversed – i.e., its degree or branching factor – and (t_i, t_{i+1}) is an edge of the graph – i.e., a street segment connecting two consecutive intersections. So, for example $\pi =< 3, 4, 5 >$ denotes a path starting at a 3-way node, passing through another 4-way node and terminating in a 5-way node. We say that π is a sequence of three consecutive intersections. In short we denote this as a 3-sequence of type [3,4,5]. Paths can overlap but cannot be identical – i.e., two paths π_i and π_j can share proper sub-paths. So, the number of paths starting at a node is equal to its degree. In this work we consider undirected graphs – i.e., we do not account for traffic direction – and want to efficiently compute the number of occurrences of n-sequences of any possible type $[x_1, x_2, ..., x_n]$.

Assume that we know the number of occurrences of 2-sequences of all possible types. See Figure 1a for the transitional probabilities of the city of Vienna obtained by counting. Then we can derive the statistics for all n-sequences with $n > 2$ by probabilistic reasoning. Assume that $P([x_1, x_2])$ is the probability that a 2-sequence of type $[x_1, x_2]$ occurs. Such a 2-sequence can only be followed by another 2-sequence starting at a node with degree x_2 – this is illustrated in Figure 1b. Then the probability $P([x_1, x_2, x_3])$ that a 3-sequence of type $[x_1, x_2, x_3]$ occurs is equal to the probability that a 2-sequence of type $[x_1, x_2]$ occurs times the probability that a 2-sequence of type $[x_2, x_3]$ occurs rescaled over the possible 2-sequences that start on a node of type x_2. This can be generalized as follows:

$$P([x_1, x_2, \dots, x_n]) = P([x_1, x_2]) \cdot \prod_{i \in C} \left[P(S_i) + P(S_i) \cdot \left(1 - \sum_{j \in A_i} P(S_j) \right) \right] \quad (1)$$



(a) Transition probabilities. **(b)** Composition of 3-sequences.

Figure 1 The left graph illustrates the transition probabilities (computed for the city of Vienna, Austria) from one type of intersection to another. We omitted the transitions with zero probability. The right figure illustrates an example where the probabilities have to be reassigned after the first transition is known since the options to continue are constrained.

Table 1 Distribution of 2-sequences (a) and 3-sequences (b, c) of intersections.

(a) distribution of 2-sequences.

Type	Count	Percentage
[3,3]	3778	0.4243
[3,4]	1554	0.1745
[3,5]	131	0.0147
[3,6]	6	0.0006
[4,4]	1558	0.1749
[4,5]	83	0.0093
[4,6]	4	0.0004
[4,7]	3	0.0003
[5,5]	6	0.0006
Total count: 8904		

(b) distribution of 3-sequences.

Type	Count	Percentage	Predicted
[3,3,3]	3952	0.2525	0.2494
[3,3,4]	1404	0.0897	0.1026
[3,3,5]	121	0.0077	0.0086
[3,3,6]	4	0.0002	0.0003
[3,4,3]	1998	0.1276	0.0499
[3,4,4]	1550	0.0990	0.0501
[3,4,5]	98	0.0062	0.0026
[3,4,6]	2	0.0001	0.0001
[3,4,7]	2	0.0001	0.0001
[3,5,3]	234	0.0149	0.0004
[3,5,4]	146	0.0093	0.0002
[3,5,5]	13	0.0008	0.0002
[3,6,3]	12	0.0007	0.000001
[3,6,4]	11	0.0007	0.000001
Total count: 15650			

(c) distribution of 3-sequences.

Type	Count	Percentage	Predicted
[4,3,4]	693	0.0442	0.0422
[4,3,5]	58	0.0037	0.0035
[4,3,6]	2	0.0001	0.0001
[4,4,4]	1709	0.1092	0.0502
[4,4,5]	57	0.0036	0.0026
[4,4,6]	5	0.0003	0.0001
[4,4,7]	2	0.0001	0.0001
[4,5,4]	75	0.0047	0.0001
[4,5,5]	2	0.0001	0.00001
[4,6,4]	3	0.0001	0.0000004
[4,7,4]	5	0.0003	0.0000002
[5,3,5]	6	0.0003	0.0002
[5,4,5]	6	0.0003	0.0001
[6,4,6]	2	0.0001	0.0000003
Total count: 15650			

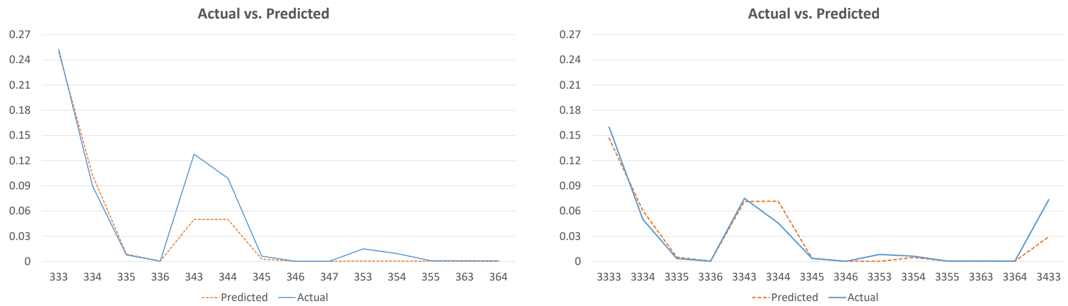
where S_i denotes a generic 2-sequence, C is the set of 2-sequences that have to be concatenated to $[x_1, x_2]$ to obtain the n-sequence $[x_1, x_2, \ldots, x_n]$ and S^B is the set of admissible 2-sequences that can follow the generic i-sequence $[x_1, x_2, \ldots, x_i]$. We show in Section 3.2 that probabilistic inference through the formula given in Equation 1 is reliable.

Since we can infer the probability of any n-sequence with $n > 2$, we only have to compute the probabilities for 2-sequences. This can be done straightforwardly by checking all the edges E in the graph. Since, in the worst case the number of edges is quadratic with the number of vertices, this can be done in $O(|V|^2)$. In practice, since we are dealing with graphs representing street networks we expect the number of edges to be much lower than that.

3.2 Validation

To validate the model presented in Section 3.1 we analyzed[1] a subset of the OpenStreetMap[2] dataset of the city of Vienna: districts 1, 3, 4, 5, 6, 7, 8, and 9 that, together, form a connected region. More specifically, we used the intersection dataset computed in [8]. We only considered intersections of type *Road* and we focused on pedestrian navigation (i.e., we

[1] The analysis has been performed on a PostGIS database.
[2] `http://www.openstreetmap.org/`



(a) Distribution of 3-sequences.

(b) Distribution of 4-sequences.

Figure 2 Validation of predicted distributions computed with Equation 1 for intersection sequences of length 3 (a) and 4 (b).

assumed that each street segment to be traversable in both directions).

We counted the occurrences of n-sequences of intersections, with $n \in \{2, 3, 4\}$. The result of this operation for $n = 2$ (reported in Tables 1a) has been used in equation 1 to generate predictions about the distribution of n-sequences of length $n = 3$ and $n = 4$. Note that, since we assumed that each street segment is traversable in both directions, the results for pairs $[x, y]$ and $[y, x]$ are the same and we only report them once. Tables 1b and 1c show the prediction and the actual count for each type of 3-sequence. A graphical representation is reported in Figure 2a. The results for the case of 4-sequences is only reported in graphical form in Figure 2b.

3.3 Discussion and Outlook

The results produced by the introduced model look very promising (see Figure 2) and can be already utilized for a variety of applications.

The data we derived can be put together in a graph representation. This would allow for looking for structurally similar areas in different regions by applying graph matching algorithm – e.g, the algorithms presented in [15] or in [6]. The first [15] is one of the first algorithms conceived for subgraph isomorphism and is still today one of the most used techniques. It enumerates all (sub)graph matchings employing a tree search with backtracking and forward checking. It basically creates the matching incrementally; at each step it tries to match a new node. If the matching fails it backtracks to the last matched subgraph. The forward checking is used to prune the search space by looking at node adjacency. The more recent algorithm presented in [6] is based on a depth-first search strategy, also employing a set of forward-checking rules to prune the search space. For a survey on graph-matching techniques, please refer [5].

We plan to extend the work presented in this paper by also including spatial relations and semantics. The former include other quantitative measurements such as the angles formed by consecutive intersections (as already computed in [8]) or the distance between two intersections. Similarly, one can also include qualitative spatial relations such as relative direction, orientation, and visibility as done, for example, in [9].

Semantics can be included in different ways. For example, one may extend the model by considering not only intersections but also point of interests of a given types (e.g., recreational and sightseeing features). Extending the representation in such a way would allow for semantic similarity analysis and search among different regions.

We argue that the statistics derived in this paper, extended with more information as described in the paragraphs above, provide the base for the generation of virtual look-alike environments. The idea is that of incrementally generating a (mostly[3]) planar graph that fits to the different statistics that we generated: intersection type (3-way, 4-way,...) and shape (angular distance to regular intersections), length of intersection sequences (2-sequences, 3-sequences,...) and type of intersection sequences (3-3, 3-4, ...). A simple solution would be to resort to a brute-force procedure that deploys in the plane a number of intersections of type 3-way, 4-way, and so on according to the given statistics and then tries all possible combination of connecting those. Clearly, this is computationally very expensive and may become unfeasible already for small graphs. More feasible approaches would resort to the adaptation of random graph generation techniques [7, 13]. These techniques are capable of generating a graph uniformly at random, so they have to be adapted to fit the statistical distributions derived with our model.

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[3] In first approximation we assume a street network to be a planar graph: so we exclude special situations such as tunnels that may break planarity.