Center Point of Simple Area Feature Based on Triangulation Skeleton Graph

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Abstract

In the area of cartography and geographic information science, the center points of area features are related to many fields. The centroid is a conventional choice of center point of area feature. However, it is not suitable for features with a complex shape for the center point may be outside the area or not fit the visual center so well. This paper proposes a novel method to calculate the center point of area feature based on triangulation skeleton graph. This paper defines two kinds of centrality of vertices in skeleton graph according to the centrality theory in graph and network analysis. Through the measurement of vertices centrality, the center points of polygon area features are defined as the vertices with maximum centrality.

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1 Introduction

In geographic information science (GIS), skeleton and center point are two important abstract descriptors of area feature which are extensively used in spatial data compression, cartographic generalization, map annotation configuration, multiscale map matching, spatial relation calculation, etc. Skeleton is a dimension reduction representation of area feature which maintains the geometric and topological characteristics of the area feature. Generally, the skeleton of area feature is a graph structure. The branches reflect the topological relation between different part of an area feature. The extension, length, and width of each part indicate the geometric characteristics of an area feature[7][3]. As for the calculation of center point of an area feature, the most popular used method is the centroid of boundary polygon of an area feature[11]. The pole of inaccessibility evaluation is also used to calculate the center point of area feature[9]. Chen presented a method for calculating the shape center through the triangulation skeleton of area feature[7]. As Chen indicated this method heavily relies on the parameter selection, and it will not guarantee the center point within the area feature. Inspired by Chen's work, this paper provides a new center point extraction method based on skeleton graph of a simple polygon.

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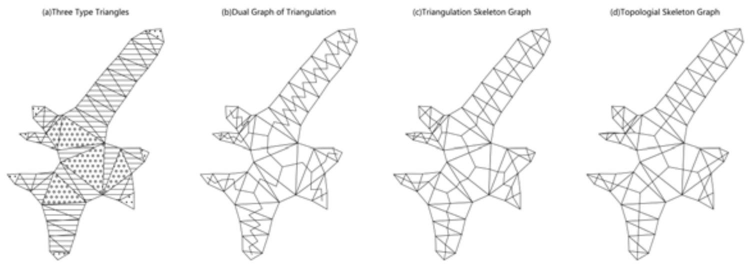

Figure 1 Triangulation of Polygon and Related Structures.

This paper presents a method to define the centrality of polygon area feature based on its triangulation skeleton. On the skeleton graph structure of polygon, we define the betweenness and closeness centrality of skeleton graph vertex which is similar to centrality in graph theory[6]. By the centrality definitions, we can extract different kinds of center points of area features. At last, we discusses the algorithm complexity of the methods presented.

2 Triangulation Skeleton Graph of Simple Area Feature

Skeleton or medial axis is a concept firstly used in biology as the descriptor of biological Shape[5]. In computational geometry, this structure has been studied extensively[8][4]. And there are several different definitions for this structure. Skeleton is wildly studied and used in areas such as image recognition, medicine analysis, geospatial science, etc. In cartography and geographical information science. A kind of skeleton based on triangulation of polygon is generally used for spatial relation calculation, map annotation, and map generalization[2][1]. This section will give some brief formal definition of this kind skeleton structure and some basic concepts for the definition of centrality of polygon.

2.1 Triangulation of Simple Polygon

This paper studies the GIS area feature which is formed by simply polygon P in a two-dimensional plane of Euclidean space. A decomposition of P into triangles by a maximal set of non-intersecting diagonals is called a triangulation of P, noted as T_P[4]. This decomposition is not unique for every simple polygon. The number of different triangulation is a Catalan number related to the number of vertices[8]. In paper[2], the authors studied the influence of different triangulation on the form of the skeleton of a polygon. In GIS science area, constrained Delaunay triangulation is used prevalently in engineering projects and scholar researches. According to the definition of T_P, there are three kinds of triangles classified by edge type(Figure 1.a). The one which contains one diagonal is noted as type I triangle, or ear triangle; the one which contains 2 diagonals as type II triangle, or link triangle; the one which contains three diagonals as type III triangle, or branch triangle. The dual graph of triangulation[4](Figure 1.b) represents the topological link relations between sub-areas of a polygon which shows the topological characteristics of different visual feature parts of a polygon.

2.2 Basic Definitons

The triangulation skeleton graph of P, G_P, is defined by a construction process presented in [2]. The vertices of graph G_P can be the vertices of P (**end vertex**), and middle point of diagonals (**link vertex**) of P, and mass centers of triangles in T_P (**branch vertex**). The structure is shown in Figure 1.c.

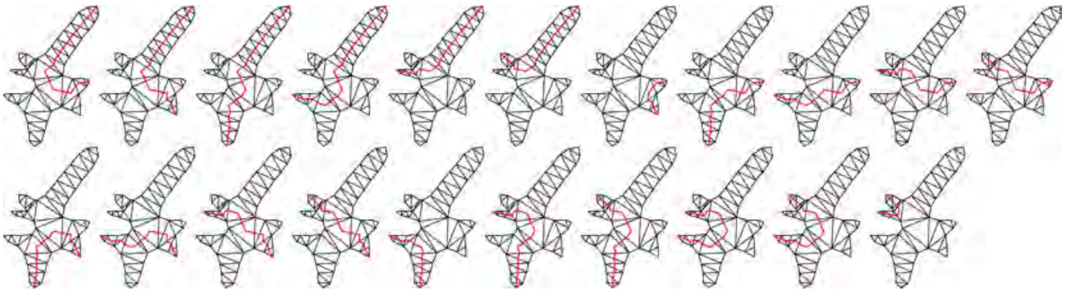

Figure 2 All Skeleton Paths of a Polygon Triangulation.

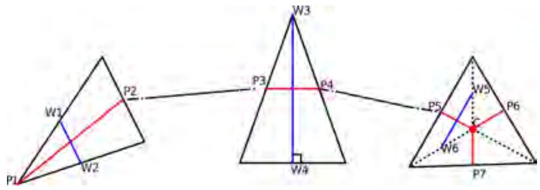

Figure 3 Geometric Definition of Cover Width of Three Type of Triangles.

The shortest path between every two vertices in G_P is defined as a **skeleton branch**. If the path between two vertices of G_P doesn't contain any branch vertex, we call the two vertices directly adjacent. The skeleton branch of two end vertices s, t is called **skeleton path**, noted as $P_{s,t}$. As shown in Figure 2, the red skeleton branches are all the skeleton paths of a polygon. If all the link vertices are removed, and the directly adjacent end vertices and branch vertices are connected, we have a topological skeleton graph of P, as shown in Figure 1.d.

We define the cover length, cover width and cover area of the edge of G_P. The cover area of type I and II edge is the area of the corresponding triangle, and cover area of type III edge is $\frac{1}{3}$ of the area of the corresponding triangle. The cover length of each edge is the geometric length of the edge. The geometric definitions of cover width of three type triangles are in the following description shown in Figure 3. The red line segments are the edges, and the blue line segments are the geometric definition of cover width of each edge. For type II triangle, width is the length of the height of triangle on non-diagonal edge, shown as W_3W_4. For type I triangle, we find a line segment W_1W_2 on triangle parallel to the diagonal edge, and the product of the length of W_1W_2 and the length of edge P_1P_2 will equal to the area of the triangle. For type III triangle, the three sub-area of it can be regarded as type I triangle, the definition of widthW_5W_6 for each sub-area is the same as type I triangle.

3 Centrality of Area Feature

Center point of shape is an important attribute of a geographic feature. Generally, the centroid of a polygon will be regarded as the center point. And for special shapes, the center points would not be within the polygon and they will not suitable for some applications, such as annotation of area features. Chen proposed a method based on the main skeleton of a polygon to calculate the center point. There are parameters to be specified when adopting Chen's method which is subjective and sensitive to different data and it will not guarantee the center point always be within the polygon. This study uses the skeleton to define the center point of a polygon from the perspective of graph centrality. In skeleton graph, the end vertices represent the visual feature points of an area feature. The skeleton paths show the connected characteristics of each pair visual feature points of area feature. The branch vertices are the topological link points of each visual parts. This is the base of our centrality definition of a polygon area.

3.1 Skeleton Graph Vertex Centrality

In graph theory, betweenness centrality is a central measurement of graph vertex based on the shortest path between vertices, which is defined as the number of shortest path through a vertex. In this study, we consider the visual coherence between visual feature parts of a polygon which can be indicated by the shortest path between visual feature vertex of a polygon. We define the betweenness centrality of skeleton graph vertex as the number of skeleton path through a vertex. Through the definition we can conclude that the maximum betweenness centrality vertex is a branch vertex if there are branches in the skeleton of a polygon. Thus, the calculation of betweenness centrality can be applied to the topological skeleton graph which can reduce the calculation complexity for fewer vertices.

▶ **Definition 1** (Betweenness Centrality of Skeleton Vertex)**.** Betweenness centrality of vertex V is the number of skeleton path through V as:

$$C_b(V) = \sum_s \sum_t P_{s,V,t},$$

for $P_{s,V,t}$ is the skeleton path through V.

The closeness centrality of graph vertex measures the balance of all vertices to the specific vertex by the total length of shortest paths through the vertex. In this paper, we consider the balance between each visual feature vertex and the specific vertex. We define the standard deviation of all the weighted length of the specific vertex between each visual feature vertex. We can have three different kinds of closeness centrality when choosing different weight. The cover length indicates the elongation of the visual part shape of a polygon, and the cover width shows the width of the shape of visual parts, and cover area consider this two factors which are similar to Chen's method.

▶ **Definition 2** (Closeness Centrality of Skeleton Vertex)**.** Closeness centrality of vertex V is the inverse of standard deviation of all the weighted lengths of paths from V to each end vertex as:

$$C_c(V) = \frac{1}{std(d_w(V,s))},$$

for $d_w(V,s)$ is the weighted length between V and end vertex s, the weight can be cover area, cover length, and cover width of graph edge.

3.2 Experiments and Results

To calculate the center point of a polygon, we first calculate the centrality of all vertex of the skeleton graph and find the vertex with the maximum value of centrality, which can be used as the center point of a polygon. In Figure 4, there are different center points by our algorithm. Betweenness centrality indicates the topological connections between visual feature parts of a polygon. The betweenness center point shows the center place have greatest topological importance. Closeness center point reflect the geometric nearness to feature

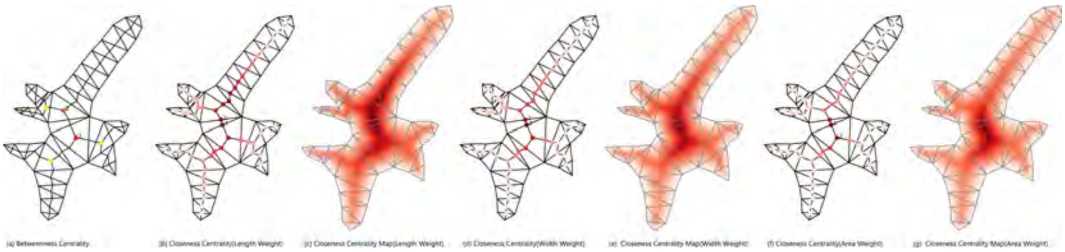

Figure 4 Two kinds center points examples(a,b,d,f) and interpolate map (c,e,g).

points of a polygon. All the two kinds of center points are within the polygon and indicate the different visual center of a polygon.

In Figure 4(a, b, d, f), two kinds of centrality degree of the vertices are illustrated. We also generate closeness centrality pattern map of three different weight (c, e, g) which is calculated by linear interpolation with the centrality degree all the vertices of skeleton graph and the points of the boundary polygon. Our centrality illustrates the geometric visual center of area feature while the method by border number [10] is about urban structure center by road networks blocks.

4 Complexity Analysis and Discussion

4.1 Complexity Analysis

The calculation of centrality related to polygon triangulation and skeleton construction. According to the triangulation theory, we know that each triangulation has $n-2$ triangles, and must have at least 2 type I (ear) triangles. If a triangulation contains e type I triangle, then $n \geq 2$, and the number of type III triangles is $e-2$. By the definition of skeleton graph, each type I triangle and type II triangle form a skeleton edge, and each type III triangle form 3 skeleton edges. Therefore, the number of skeleton edge is $E = n-2+2(e-2)$. At extreme cases, there are only type I and type III triangles, that is $n-2 = e+(e-2)$, thus $2 \leq e \leq n/2$, and we can derive$n-2 \leq E \leq 2n-6$. A skeleton graph can also be regarded as a binary tree structure, therefore, the number of vertex and edge maintains $V = E+1$.

The betweenness centrality needs the calculation of path between two end vertices. And this calculation based on the topological skeleton graph only contains the end vertices and branch vertices. Finding all the paths between end vertices, the complexity is $O(e(2e-2))$. Under extreme cases in which the triangulation only contains type I and type III triangles, the complexity is $O(n^2)$.

For closeness centrality of vertices in skeleton graph, we need to find the all the path between the end vertices and all other vertices. To find the skeleton branch from each vertex to all end vertex needs a traverse of the skeleton graph, thus the complexity is$O(V)$, and $V = E+1 = n+2e-3$. The number of link vertex and branch vertex is $n-e$. According to the range of e discussed above, we have the complexity of closeness centrality is $O((n-e)(n+2e-3)) \sim O(n^2)$.

4.2 Special Cases Discussions

We will consider some special cases. For an "H" shape polygon, the betweenness centrality may have two maximum vertices. In this situation, we can use the closeness center point which can give the difference. For a stripe shape polygon, there is no branch vertex in the skeleton graph. Under this situation, we only consider the closeness center point. For the polygon with a hole, which is not a simple polygon, we can have a skeleton graph which contains ring structures. This kind of polygon will not be considered for the calculation of center points in this study.

5 Conclusion

This paper presents a definition and calculation of center point of area feature formed by simple polygon. The centrality of a polygon is defined based on the triangulation skeleton graph of a polygon. This method takes into account of the topological and geometric characteristics of visual feature points and parts of a polygon. The center point by this method is within the polygon and shows good visual center characteristics of an area feature. The method proposed has several issues need to be considered. One is the calculation complexity is higher than the mass-based center point in theory. Another is the situations when two candidates will occur. In the future study, we consider extending this paper considering formalizing the definition and comparing with other existing methods by cognitive experiments.

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