Why Classificatory Information of Geographic Regions Is Quantum Information

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Abstract

This paper gives an information - theoretic argument in support of the claim that there is geographic quantum information. Quantum information is information in the sense of Shannon's information theory, that, in addition, satisfies two characteristic postulates. The paper aims to show that if the density of information (bits per unit of space) that is possible for classificatory geographic qualities is limited, then it follows that the two characteristic postulates of quantum information are satisfied for information about those geographic qualities.

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1 Introduction

The aim of this paper is it to give an information - theoretic argument in support of the claim that there is geographic quantum information. For this purpose it is necessary to specify what geographic quantum information is, and how it can be distinguished from classical geographic information. In this context it is assumed that the notion of geographic quantum information arises from the notion of quantum information which in turn arises from the notion of information in Shannon's information theory [13].

At it's core, Shannon's information theory abstracts from what information is about by (a) understanding processes that yield information, i.e., measurements/observations, as interactions that establish correlations between observed and observing systems; and (b) by quantifying the amount of information that one system has of another system as the number of the elements of a set of alternatives out of which the correlation arises [13].

Within this abstract framework the distinction between classical and quantum information arises because classical information satisfies Postulate 1:

▶ **Postulate 1** (Unlimited amount of classical information). *There is an unlimited amount of information that an observing system can obtain from an observed system.*

By contrast, quantum information satisfies Postulates 2 and 3 [7]:

▶ **Postulate 2** (Limited information [11]). *There is a maximum amount of (relevant[1]) information that an observing system can obtain from an observed system.*

▶ **Postulate 3** (Unlimited information [11]). *It is always possible for an observing system to acquire new information about an observed system.*

[1] There is some discussion on what "relevant information" is. For details see [11, 7].

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Since Postulates 1 and 2 cannot be jointly true, information can be analyzed either within the framework of classical information or the framework of quantum information but not both frameworks simultaneously. Although Postulates 2 and 3 "look" somewhat contradictory, they are known to be consistent [7].

What classical or quantum information is and how the former can be distinguished from the latter can be described in an abstract, information - theoretic framework in the context of the above postulates. Once this is understood, one can inquire why the information that an observing system can have of an observed system is classical or quantum in nature. It is the hypothesis of this paper that (at least in some domains) this question can be answered by analyzing the interrelations of given observing and observed systems and by determining whether or not certain features of the two systems and their interaction *entail* either (a) that the information that the observing system can obtain of the observed system satisfies Postulate 1 and thereby renders the systems and their information-theoretic interrelation classical; or (b) that the information that the observing system can obtain of the observed system satisfies Postulates 2 and 3 and thereby renders the systems and their interrelation non-classical.

For those who accept this analytical and information - theoretic methodology, to argue that information that is obtained by geographic classification and delineation is quantum information, is to argue why it follows from the nature of geographic regions and their qualities that information obtained by geographic classification and delineation satisfies Postulates 2 and 3.

2 Amounts of information

Information is a *discrete* quantity, i.e., there is a minimum amount of information exchangeable: a single bit, or the information that distinguishes between two alternatives. Therefore, the process of acquisition of information (a measurement/ an observation) can be framed as a question that an observing system asks an observed system [18]. Since information is discrete, any process of acquisition of information can be decomposed into acquisitions of elementary bits of information by elementary (i.e., yes/no) questions.

▶ **Proposition 1.** *Only yes/no questions that in a given domain can at least in principle be answered in both ways are in a position to provide information about that domain.*

Proof. Consider the formula $\phi = P \vee \neg P$. A yes answer to the yes/no question Q="Is the formula ϕ true?" does not yield information in any domain in which the law of the excluded middle holds. This is because in those domains there cannot be a no answer to Q. Therefore a yes answer to Q does not exclude any alternatives/possibilities and is void of information. Similarly for questions that cannot have a yes answer. ◀

Any observed system S is characterized by the elementary yes/no questions that can be asked to it. The answers to a sequence of elementary yes/no questions $(Q_1, Q_2, Q_3, \ldots)$ to S, can be represented as a binary string $(e_1, e_2, e_3, \ldots)$, where each e_i is either 0 or 1 (no or yes) and represents the response of the system S to the elementary question Q_i [18]. In what follows, the focus is on binary strings of length L that represent finite sequences of answers to L elementary yes/no questions. Combinatorially, there are 2^L binary strings of length L. One can identify 2^L *complete* questions $Q^1, \ldots, Q^{2^N}$ that consist of sequences of L elementary yes/no questions such that the complete question Q^i corresponds to the bit string s^i if and only if there is a yes answer to Q^i iff the yes/no answer to the elementary question Q_j is recorded in the bit s^i_j for $1 \leq j \leq L$. This is illustrated in Table the left part of Table 1 for the specific case of two yes/no questions Q_1 and Q_2 which give rise to the set Q_c of $2^2 = 4$ combinatorially possible *complete* questions of length 2 [2, 8, 11].

Table 1 Left: The set $Q_c = \{Q_c^{(i)} \mid 0 \leq i < 4\}$ of 2^4 combinatorially possible complete questions $Q_c^{(i)}$ formed by two yes/no questions Q_1, Q_2; Right: Two sets of complete questions Q_S and Q_R for 2 bits of information.

i	$s_c^{(i)}$ Q_1	Q_2	$Q_c^{(i)}$
0	0	0	$\neg Q_1 \wedge \neg Q_2$
1	0	1	$\neg Q_1 \wedge Q_2$
2	1	0	$Q_1 \wedge \neg Q_2$
3	1	1	$Q_1 \wedge Q_2$

$Q_S^{(i)}$	$s_S^{(i)}$ Q_1	Q_2	$\bigwedge_i Q_i$	$s_R^{(i)}$ R_1	R_2	$\bigwedge_i R_i$	$Q_R^{(i)}$
Q_S^1	1	0	$Q_1 \wedge \neg Q_2$	1	0	$R_1 \wedge \neg R_2$	Q_R^1
Q_S^1	1	0	$Q_1 \wedge \neg Q_2$	0	1	$\neg R_1 \wedge R_2$	Q_R^2
Q_S^2	0	1	$\neg Q_1 \wedge Q_2$	1	0	$R_1 \wedge \neg R_2$	Q_R^1
Q_S^2	0	1	$\neg Q_1 \wedge Q_2$	0	1	$\neg R_1 \wedge R_2$	Q_R^2

The fact that there are 2^2 combinatorially possible pattern of answers to two yes/no questions does not guaranty that all the combinatorial possibilities are also kinematically possible, i.e., possible when domain-specific constraints are in place. This is illustrated in the table in the right of Table 1.

Suppose that a string of L bits of information of a system S has been obtained by an observer O. According to Postulate 1 for classical information the fact that only L bits were obtained does not mean that $L+1$ bits of information could not be obtained by posing complete questions that arise from $L+1$ yes/no questions. That is, in domains in which Postulate 1 holds, it is always possible to acquire more information by asking longer and longer complete questions.

3 The Wheeler-Sinton paradigm

The systematic investigation of the nature of geographic information and geographic information processing from an information-theoretic perspective was pioneered by Sinton [15]. In the language of Wheeler's [18] information-theoretic view of measurement/observation processes, Sinton's conception of the nature of geographic information can be expressed as follows:

▶ **Postulate 4** (Wheeler-Sinton paradigm)**.** *For a string* s *of* L *bits of information to count as geographic information of a system* S *for the observer* O*,* s *must have the following properties:*

(WS1) s *is constituted by bits that result from a yes answer to a* complete *question that is formed by* elementary *yes/no questions that fall in three broad classes: (q) elementary yes/no questions about measurable/observable (geographic) qualities of* S*; (s) elementary yes/no questions about the location of* S *in geographic space; and (t) elementary yes/no questions about the temporal location of* S*.*

(WS2) The complete question that gives rise to s *is such that:*

(1) *One type of* elementary *yes/no questions (q), (s), (t) is* fixed*. That is, only one elementary yes/no question of this type has a yes answer and thereby picks out what is fixed.*

(2) *One type of* elementary *yes/no questions (q), (s), (t) is* controlled*. That is, there is a fixed number of yes/no questions of this type –* control questions *– that must have a yes answer. Yes answers to control questions pick out the cells of a* fiat *subdivision (Def 2).*

(3) *One type of* elementary *yes/no questions (q), (s), (t) is* measured*. That is, every yes answer to a control question is complemented by a yes answer to at least one of the yes/no questions of this type – elementary yes/no questions in Wheeler's standard understanding.*

To control some domain via a fiat subdivision means that this domain is partitioned by a set of cells which boundaries are fiat in nature:

▶ **Definition 2** (Fiat subdivision). *Let x be a (region of) some domain (e.g., a region of geographic space, a range of temperatures, etc.). A set of regions $X = \{x_1, \ldots x_n\}$ is a* subdivision *or a* partition *of x iff (1) jointly, the $x_1, \ldots, x_n \in X$ mereologically [14] sum up to x and (2) the members $x_i, x_j \in X$ are pair-wise disjoint [14]. In a* fiat *subdivision X the boundaries of the $x_i \in X$ are not aligned with physical discontinuities of the domain x that is subdivided [16, 17].*

In what follows the Wheeler-Sinton paradigm is employed in the context of fixing time, controlling space via fiat spatial subdivisions, and measuring/observing qualities in cells of fiat subdivisions.

▶ **Example 3** (adapted from [4]). Consider Fig. 1 and suppose that (a) the information that is obtained by an observer O via measurement/observation of some portion of the surface of the Earth (the observed system S) is information about the quality of elevation across S; (b) the information about spatial location is controlled by projecting a fiat [16, 17] raster-shaped partition onto S as indicated in the top left part of the figure; and (c) information about temporal location is fixed by allowing for a single time stamp.

In the context of the Wheeler-Sinton paradigm (a–c) mean: (1) the yes answer to one of the yes/no questions $Q_t^1, \ldots, Q_t^{10}$ picks out a particular time stamp; (2) the yes answers to the yes/no questions $Q_c^1, \ldots, Q_c^{36}$ pick out particular cells in the grid structure projected onto S; (3) for every control region picked out by a control question Q_c^i there is a yes answer to at least one of the yes/no questions $Q_m^1, \ldots, Q_m^{110}$. This is displayed in the table in the top right of Fig. 1. Jointly, the yes/no questions of (1-3) give rise to 2^L possibilities for strings of L bits of information and the associated complete questions. This is partly displayed in the table in the middle of Fig. 1. This table "implements" Sinton's methodology of measure/control/fix as laid out in (a-c) in Wheeler's information-theoretic framework of modeling information obtained by measurement/information using answers to yes/no questions. The constraints imposed by Sinton's paradigm of measure/control/fix allows for a more efficient encoding of L bits of information. The constraints reduce the combinatorial possibilities of 2^L strings of length L to the members of set S_S.

Consider the complete question $Q_S^{img} \in Q_S$ as depicted in the bottom of Fig. 1. A yes answer to Q_S^{img} yields L bits of information. This information is encoded in the string $s_S^{img} \in \mathsf{S}_\mathsf{S}$. The same information is encoded (more implicitly) in the image in the top left of Fig. 1. ◀

In the context of fixing time via the yes answer to the question $Q_f = \{Q_t^i\}$, controlling space via yes answers to the questions $Q_c = \{Q_l^1 \ldots Q_l^j\}$ and yes/no answers to classificatory questions $Q_m = \{Q_m^1 \ldots Q_m^k\}$ a family of complete questions (Sec. 2) arises and is denoted by $\mathcal{Q}(Q_f, Q_c, Q_m)$. A yes answer to a complete question in $\mathcal{Q}(Q_f, Q_c, Q_m)$ yields L bits of information. In the context of Example 3 one has: $Q_S = \mathcal{Q}(Q_f, Q_c, Q_m)$ and $Q_S^{img} \in Q_S$.

Example 3 satisfies Postulate 1: There does not seem to be a limit to the amount of information about elevation across S that can be had by an observer O. More information can be obtained by refining the partition cells and asking yes/no questions about the elevation in these refined cells. Similarly, more information can be obtained by allowing for more precise elevation measurements, i.e., by enlarging the set Q_m. The elevation information of Example 3 is a prototypical example of *classical* geographic information.

645	650	654	658	653	648
664	666	670	672	668	659
678	682	684	693	689	680
703	708	714	721	719	716
728	732	738	744	745	732
730	739	744	749	748	735

time	Q_t^1	Time is 11/20/2018?
	$\dots$	$\dots$
	Q_t^{10}	Time is 11/29/2018?
location	Q_l^1	Location is cell 1?
	$\dots$	$\dots$
	Q_l^{36}	Location is cell 36?
quality	Q_q^1	Elevation is 640?
	$\dots$	$\dots$
	Q_q^{110}	Elevation is 750?

$s_S \backslash Q_S$	Q_t^1	$\dots$	Q_t^{10}	Q_l^1	$\dots$	Q_l^{36}	$(Q_q)_1^1$	$\dots$	$(Q_q)_1^{110}$	$(Q_q)_2^1$	$\dots$	$(Q_q)_2^{110}$	$\dots$	$(Q_q)_{36}^{110}$
s_S^1	1	$\dots$	1	1	$\dots$	1	1	$\dots$	1	1	$\dots$	1	$\dots$	1
s_S^2	1	$\dots$	1	1	$\dots$	1	1	$\dots$	1	1	$\dots$	1	$\dots$	0
$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$
s_S^L	0	$\dots$	0	0	$\dots$	0	0	$\dots$	0	0	$\dots$	0	$\dots$	0

where $L = 1 + 36 + (36 * 110)$

$$\mathsf{S}_\mathsf{S} = \left\{ s_S^i \in s_S \;\middle|\; \begin{array}{ll} (\Sigma_{j=1}^{10} s_S^i[j]) = 1 = s_S^i[1] \; \& & \text{(fix time)} \\ (\Sigma_{j=11}^{46+1} \; s_S^i[j]) = 36 \; \& & \text{(36 yes-bits of control information)} \\ (\bigwedge_{j=0}^{36-1} \; (\Sigma_{k=k_1}^{k_m} \; s_S^i[k] = 1)) \geq 1 & \text{(at least one of 110 measurement bits is 1} \\ & \text{for each control location)} \\ k_1 = 10 + 36 + (j * 110) + 1 & \\ k_m = 10 + 36 + (j * 110) + 110 & \end{array} \right\}$$

$$Q_S^{img} = \mathrm{Y} \quad \text{iff} \quad Q_t^1 = \mathrm{Y} \wedge \neg(Q_t^2 = \mathrm{Y}) \wedge \ldots \wedge \neg(Q_t^{10} = \mathrm{Y}) \wedge Q_c^1 = \mathrm{Y} \wedge \ldots \wedge Q_c^{36} = \mathrm{Y} \wedge \neg((Q_q)_1^1 = \mathrm{Y}) \wedge \ldots \wedge ((Q_q)_1^6 = \mathrm{Y}) \wedge \ldots \wedge \neg((Q_q)_1^{110} = \mathrm{Y}) \wedge \neg((Q_q)_2^1 = \mathrm{Y}) \wedge \ldots \wedge ((Q_q)_2^{11} = \mathrm{Y}) \wedge \ldots \wedge \neg((Q_q)_{36}^{110} = \mathrm{Y})?$$

Figure 1 The Wheeler-Sinton paradigm of geographic information: temporal location is fixed (11/20/2018), spatial location is controlled by fiat, and a geographic quality is measured. Q_S^{img} is a complete yes/no question the yes answer to which yields L bits of information. The same information is encoded in the image in the top left. [4] (The image in the top left is from [5].)

4 Geographic regions and their qualities

The purpose of this and the next two sections is it to argue why it follows from the nature of geographic regions and their qualities that information about geographic classification and delineation satisfies Postulates 2 and 3 and thereby entails the quantum nature of classification and delineation information. The focus of the discussion is on Bailey's classification and delineation of ecoregions [1]. In this discussion the distinction between quality determinables and quality determinates [12] is taken for granted. Quality determinables are qualities such as energy, temperature, climate type, climate regime, land-surface quality, etc. Those quality determinables subsume the quality determinates such as 100 Joule, 70 degrees Fahrenheit, Irregular planes, etc.

▶ **Definition 4** (Classificatory quality determinables). *The determinable ϕ is a classificatory quality determinable if and only if ϕ has the following properties: (i) the quality determinates of ϕ are the leafs of a finite isA tree; (ii) ϕ is the immediate parent of its determinates in the isA tree; (iii) no instance of ϕ can fail to instantiate one of ϕ's determinates; and (iv) distinct quality determinates cannot be instantiated in partially overlapping entities/regions.*

Examples of classificatory quality determinables include land-surface qualities such as land-surface form, and climate qualities such as climate types and climate regimes (Fig. 2).

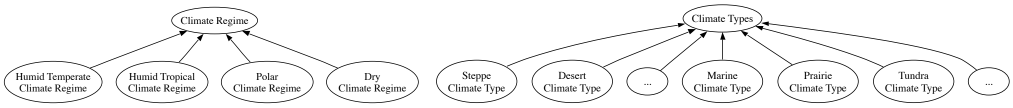

Figure 2 Climate regime qualities (left) and Climate type qualities (right).

▶ Remark 5. Condition (iv) of Def. 4 is consistent with the classical intuition that distinct classificatory determinates cannot be co-instantiated in the same region. It is also consistent with the possibility of the superposition of multiple qualities in the same region. This is because no entity/region can only partially overlap itself. (If x partially overlaps x then there is a part of x that is not a part of x.)

4.1 Land-surface and climate qualities

There are at least four land-surface determinables [10, Table 1]: *Land-surface form*, potential natural vegetation (*Climax Vegetation*), *Land use*, and *Soil type*. Determinates such as *Irregular Plains* inhere in specific regions such as the Central Great Plains (CGP). In the body of Table 2 a number of quality determinates that inhere in CGP are listed.

Table 2 Land-surface qualities of geographic regions – Central Great Plains as an example [10, Table 1].

Geographic region	*Land-surface form* (*LF*)	*Climax vegetation* (*CV*)	*Land use* (*LU*)	*Soil type* (*S*)
...	...	...	...	...
Central Great Plains	Irregular plains	Bluestem / grama prairie, bluesteam prairie, buffalo grass	Cropland, cropland with grazing land, some irrigated agriculture	Dry Mollisols
...	...	...	...	...

Climate qualities fall into at least two major groups [1]: climate types and climate regimes [9, 1]. Relevant climate types are listed in the right of Fig. 2. Relevant climate regimes are listed in the left of Fig. 2.

4.2 Taxonomic structure

In Bailey's system geographic regions that are characterized by a given climate regime determinate are called *domains*. The geographic regions that count as domains and that overlap the territory of the contiguous US are displayed in the top left map of Fig. 3. Geographic regions that are characterized by climate types (in the context of a given climate regime) are called *divisions*. The geographic regions that count as divisions and that overlap the territory of the contiguous US are displayed in the bottom left map of Fig. 3. Finally, geographic regions that are characterized by Land-surface types (in the context of a given climate type) are called *provinces*. In Fig. 4 the solid arrows represent the taxonomic hierarchy for ecoregions. A dotted line connecting a quality determinable to a kind of ecoregion indicates that a quality determinate of this determinable is instantiated at the respective ecoregion and thereby determines the kind of this ecoregion. For details, see [3].

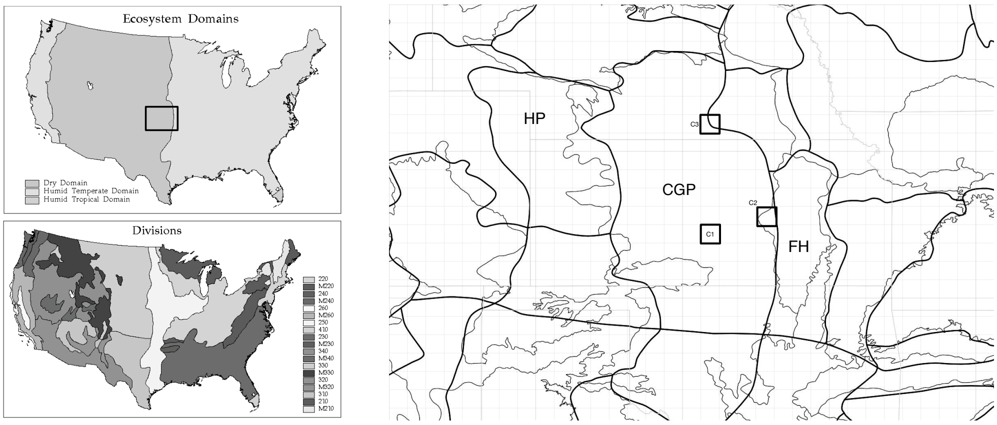

Figure 3 Ecoregion domains of the US (top left); Ecoregion Devisions of the US (bottom left); right: Ecoregions of regional scale in Baliey's framework (bold black boundaries) and in the framework of the EPA (gray boundaries).

4.3 Partonomic structure and scale

In Bailey's system land-surface qualities and climate qualities also give rise to a partonomic nesting of the geographic regions in which they inhere – the partonomic hierarchy of ecoregions is depicted in Fig. 4 using dashed arrows. The partonomic structure ensures that regions in which climate type qualities inhere (divisions) are (proper) parts of regions in which climate regime qualities inhere (domains) and that regions in which land-surface qualities inhere (provinces) are (proper) parts of regions in which climate type qualities inhere (divisions).

In conjunction with the partonomic structure Bailey also identifies scales of regions in which certain kinds of qualities inhere: climate regime qualities mainly inhere in geographic regions of global scale, climate type qualities mainly inhere in geographic regions of continental scale, and land-surface form qualities mainly inhere in geographic regions of regional scale (Fig. 4).

▶ **Postulate 5** (Qualities and scale). *The relation between classificatory quality determinables such as* Land-surface form*,* Climate regime*,* Climate regime *and regions of the surface of the Earth is* scale dependent*: Every quality determinable has an associated scale which constrains the size of the regions in which its determinates can be instantiated.*

Consequently, classificatory qualities such as *Land-surface form*, *Climate regime*, *Climate regime* are non-dissective [6].

5 Classificatory geographic information

In Bailey's ecoregion framework geographic regions are classified according to their climate regimes, climate types and land-surface qualities (classificatory quality determinables). In the Wheeler-Sinton paradigm classificatory information that can be obtained by measurement/observation at given controlled locations/regions is information about instantiated quality determinates that fall under those classificatory quality determinables (Fig. 2, Tab. 2). In this context the choice of controlled locations/regions is *constrained* by the scale of the regions in which the respective quality determinates can inhere.

▶ **Example 6.** Suppose that information is sought about the instantiation of the quality determinates (e.g. prairie climate type, Q^i_m) of a given classificatory quality determinable (e.g., climate type, Q_m). In the Wheeler-Sinton framework this information is obtained via a

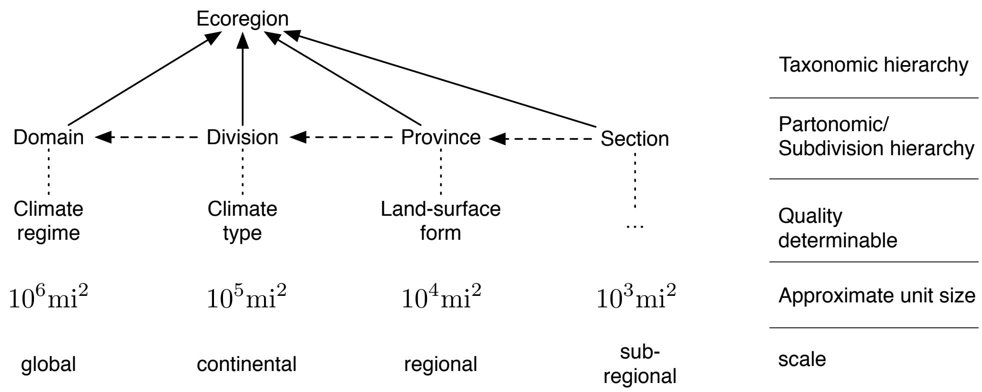

Figure 4 Taxonomic and partonomic hierarchy in Bailey's [1] classification and delineation system for ecoregions (adapted from [3]).

yes answer to a yes/no question Q^j_m that is posed to the control region identified by a yes answer to a control question Q^i_l. (As above, the same symbol, Q^j_m, is used for the yes/no question and the quality determinate of which information about instantiation is sought. Similarly the same symbol is used for the control questions and the control regions they pick out.) A yes answer to a question Q^j_m posed to a control region Q^i_l is possible only if Q^i_l is of a scale (continental scale, order of 10^5 mi^2) that permits the instantiation of a quality determinate Q^j_m of which information is to be acquired. ◀

This example motivates the following granularity postulate:

▶ **Postulate 6** (Granularity [4]). *If the Wheeler-Sinton scheme is applied in contexts in which (a) information about the instantiation of the quality determinates $Q^1_m \ldots Q^h_m$ of the classificatory quality determinable Q_m is to be obtained by answers to yes/no questions $Q^1_m \ldots Q^h_m$; (b) time is fixed; and (c) space is controlled via control cells referenced by yes answers to control questions of the form $Q^1_l \ldots Q^n_l$, then there is a minimal size of control cells – a finest level of resolution/granularity – for which elementary yes/no questions of the form:*

$$\text{“Does the control cell } Q^i_l \text{ instantiate the quality determinate } Q^j_m\text{?”}$$

still can have a yes answer.

As pointed out in Proposition 1, only yes/no questions that *can* have a yes answer as well as a no answer when posed to a system S are in a position to yield information about S. This is because only if a question can be answered in either way, then a specific yes or no answer rules out possibilities and thereby provides information in Shanon's sense. From this information-theoretic perspective in conjunction with Postulate 6 it follows:

▷ Claim 7. There is a minimal size of control cells – a finest level of resolution/granularity – below which no information about the instantiation of classificatory quality determinates that are associated with this level of resolution can be obtained.

Support for this claim is provided in what follows in the specific context of climate and land surface form qualities in Bailey's ecoregion framework.

5.1 Information about climate regime qualities

Consider information that can be obtained by the measurement/observation of climate regime qualities in the context of the Wheeler-Sinton paradigm applied to Bailey's ecoregions of global scale. As displayed in the left of Fig. 2 there are four climate regime determinates $Q^1_{\text{CRQ}} \dots Q^4_{\text{CRQ}}$ which inhere, according to Bailey, in regions of global scale at the order of 10^6 mi^2 (Fig. 4). A lower bound to regions in which climate regime qualities can inhere arises from the order of the size of regions at which climate types are instantiated in conjunction with the partonomic nesting of ecoregions (Fig. 4). This lower bound on the size of regions in which the climate regime qualities $Q^1_{\text{CRQ}} \dots Q^4_{\text{CRQ}}$ can inhere is somewhere between the order of 10^5 mi^2 and 10^6 mi^2.

Consider the map in the right of Fig. 2. The area covered by this map as a whole corresponds to the black rectangle in the top left map of the figure. This region, CGP-Map, – which is larger than the state of Kansas – may still be too small for a climate regime determinate to be instantiated in it. If the region CGP-Map is indeed too small for climate regimes to inhere in this region, then the question

$Q^{\text{CGP-Map}}_{\text{CRQ}}$ = "Does the region CGP-Map referenced by a yes answer to the question $Q^{\text{CGP-Map}}_{l}$ instantiate the quality determinate Q^{i}_{CRQ}?"

cannot have a yes answer for any of the four climate regime determinates (Q^i_{CRQ} with $1 \leq i \leq 4$). Since a yes answer is impossible, a no answer does not provide information.

▶ Remark 8. The lack of information that can be obtained from a no answer to the yes/no question $Q^{\text{CGP-Map}}_{\text{CRQ}}$ does NOT mean that similar information cannot be *inferred* from yes/no questions that collect information of qualities that can inhere in CGP-Map. The partonomic structure of ecoregions (Fig. 4) in conjunction with information about qualities that are instantiated at a part of a region may make it possible to *infer* information about qualities of that larger embedding region [3]. See also Remark 9.

5.2 Information about climate type qualities

Now consider information that can be obtained by the measurement/observation of climate type qualities in the context of the Wheeler-Sinton paradigm applied to geographic regions of continental scale. As displayed in the left of Fig. 2, in Bailey's framework there are more than five climate type determinates $Q^1_{\text{CTQ}} \dots Q^{\#\text{CTQ}}_{\text{CTQ}}$ ($\#_{\text{CTQ}} > 5$) which inhere in regions of continental scale at the order of 10^5 mi^2 (Fig. 4). Again, a lower bound arises from the order of the size of regions at which land-surface qualities are instantiated in conjunction with the partonomic nesting of ecoregions (Fig. 4). This means that information of the classificatory qualities $Q^1_{\text{CTQ}} \dots Q^{\#\text{CTQ}}_{\text{CTQ}}$ can be obtained by measurement/observation of those qualities in (control) cells of subdivisions in which the cell size is between the order of 10^4 mi^2 and 10^5 mi^2. In this case the region CGP-Map is of the right scale and the question

$Q^{\text{CGP-Map}}_{\text{CTQ}}$ = "Does the region CGP-Map referenced by a yes answer to the question $Q^{\text{CGP-Map}}_{l}$ instantiate the quality determinate Q^{i}_{CTQ}?"

can have a yes answer for all of the $1 \leq i \leq \#_{\text{CTQ}}$ climate type determinates. In particular, there is a yes answer to the question

$Q^{\text{CGP-Map}}_{\text{Steppe}}$ = "Does the region CGP-Map referenced by a yes answer to the question $Q^{\text{CGP-Map}}_{l}$ have the quality $Q^{\text{Steppe}}_{\text{CTQ}}$?"

The yes answer to $Q^{\mathsf{CGP}\text{-Map}}_{\mathrm{Steppe}}$ does yield information to the effect that the climate type *Steppe climate* (as opposed to the other possible climate type qualities) is instantiated in the region CGP-Map.

▶ Remark 9. In Bailey's system climate types are defined in the context of given climate regimes. From a yes answer to the question $Q^{\mathsf{CGP}\text{-Map}}_{\mathrm{Steppe}}$ it can be *inferred* in Bailey's system that CGP-Map is a (proper) part of a region of continental scale in which the quality determinate *temperate climate regime* inheres. As pointed out in Remark 8, this kind of inference is based on structural properties within Bailey's framework and not on the information-theoretic principles of Wheeler and Sinton.

5.3 Information about land surface qualities

Finally, consider information that can be obtained by the measurement/observation of land-surface qualities in the context of the Wheeler-Sinton paradigm applied to geographic regions of regional scale. Suppose that there are $\#_{\mathrm{LSQ}}$ land-surface determinates $Q^1_{\mathrm{LSQ}} \dots Q^{\#_{\mathrm{LSQ}}}_{\mathrm{LSQ}}$ which can inhere in regions of regional scale at the order of 10^4 mi^2 (Fig. 4). Again, a lower bound arises from the order of size of regions at which sub-regional qualities are instantiated in conjunction with the partonomic structure that is entailed in Bailey's system (Fig. 4). This means that the classificatory qualities $Q^1_{\mathrm{LSQ}} \dots Q^{\#_{\mathrm{LSQ}}}_{\mathrm{LSQ}}$ are to be measured/observed in control cells which size is between the order of 10^3 mi^2 and 10^4 mi^2. In this case the region $cell_1$ – the cell with bold boundaries labeled c_1 that is part of the region CGP in the right of Fig. 3 – is of a scale that is compatible with the measurement/observation of land-surface qualities. Therefore, the question

$Q^{cell_1}_{\mathrm{LSQ}}$ = "Does the region $cell_1$ referenced by a yes answer to the question $Q^{cell_1}_{l}$ have the quality (pattern) Q^i_{LSQ}?"

can have a yes (as well as a no) answer for all Q^i_{LSQ} with $1 \leq i \leq \#_{\mathrm{LSQ}}$. In fact, there is a yes answer to the question:

$Q^{cell_1}_{\mathrm{LSQ}}$ = "Does the region $cell_1$ referenced by a yes answer to the question $Q^{cell_1}_{l}$ have the land surface form irregular plains and (some of) the climax vegetation listed in row three of Tab. 2, (some of) the land uses listed in row three of Tab. 2, and (some of) the Soil type listed in row three of Tab. 2?"

The yes answer to $Q^{cell_1}_{\mathrm{LSQ}}$ encodes the information about the respective land surface qualities in the region $cell_1$.

The region $cell_1$ is a cell in a fiat subdivision of the area that is depicted in the right of Fig. 3. This fiat subdivision arises from the control of space that is imposed in the context of the Wheeler-Sinton paradigm. The control by fiat ensures that all the cells are of the same size and therefore all the control questions that are associated with each of the control cells have an answer that yields information. The fiat subdivision in the figure gives rise to $32 * 22 = 704$ control questions. In what follows the symbol $\#_{\mathrm{LSC}}$ is used for the number of control cells which size is between the order of 10^3 mi^2 and 10^4 mi^2. Similar but coarser subdivisions arise in the context of the application of the Wheeler-Sinton paradigm to climate regime and climate type qualities.

The sets $\mathcal{Q}_{\mathrm{LSQ}}$, $\mathcal{Q}_{\mathrm{CTQ}}$, and $\mathcal{Q}_{\mathrm{CRQ}}$ of complete questions about the measurement/observation of land-surface qualities as well as climate regime and climate type qualities arise in the same way as discussed in Example 3. Yes answers to those complete questions give rise respectively to the set S_{LSQ} of bit strings of information about land-surface qualities, the set S_{CTQ} of bit strings of information about climate type qualities, and the set S_{CRQ} of bit strings of information about climate regime qualities.

6 Quantum information

In contrast to the unlimited amount of classical information that is possible (Postulate 1, Example 3), the amount of quantum information that an observing system O can obtain about an observed system S is limited as specified in Postulate 2. Any quantum system (S, O) has a maximal information capacity L, where L is an amount of information in bits. L bits of information exhaust the amount of information an observing system O can have about the observed system S.

6.1 Multiple families of complete questions

Given the limited information capacity that characterizes a quantum system (S, O), the question arises how Postulate 3 can possibly be true of (S, O). In the context of the example illustrated in Table 1 the set $Q_S = \{Q_S^1, Q_S^2\}$ of kinematically possible complete questions for two bits of information was introduced. Limiting the amount of information to two bits does not entail that $Q_S = \{Q_S^1, Q_S^2\}$ is the only kinematically possible set of complete questions. That is, there may be a second set of two yes/no questions $\{R_1, R_2\}$ which give rise to two bits of information via the set $Q_R = \{Q_R^1, Q_R^2\}$ of kinematically possible complete questions. The logical structure of the questions in Q_R mirrors the structure of the questions in Q_S as illustrated in Table 1.

Postulate 3 captures what happens if, after having obtained $L = 2$ bits of information by asking Q_S^1, O asks another question, say Q_R^1, as permitted by Postulate 3. Jointly, Postulates 2 and 3 can be understood as follows [11]: Since the amount of information that O can have about S is limited by Postulate 2, it follows that, if O has a maximal amount of information about S, then, if new information about S is acquired by O, O must loose information. In particular, if a yes answer to the question Q_S^1 is followed by a yes answer to the question Q_R^1, then the information obtained by O via a yes answer to Q_R^1 overwrites the information obtained by O via a yes answer to Q_S^1. In virtue of the information that O obtains by a yes answer to Q_R^1, O looses all of its two bits of information that was obtained by a yes answer to Q_S^1. If the yes answer to the question Q_R^1 is in turn followed by a yes answer to the question Q_S^1, then genuinely new information about S is obtained by O. And so on.

6.2 Complete questions, control and resolution

Consider the information of a system of geographic regions (the observed system S) where, in the framework of the Wheeler-Sinton paradigm, the observer O fixes time, controls space (by partitioning S into raster cells), and measures/observes classificatory geographic qualities that can inhere in S. Let $\mathcal{Q}(Q_t, Q_l, Q_m)$ be a set of complete question that O can pose to S. Q_m is the set of yes/no questions about measurable/observable quality determinates of the classificatory quality determinable Q_m. Here and in what follows, the symbol Q_m is used for the quality determinable, as well as for the set of quality determinates that fall under the determinable Q_m, as well as for the set of yes/no questions that gather information about the instantiation of the determinates that fall under Q_m. The context will disambiguate.

In the framework of the Wheeler-Sinton paradigm and its families of complete questions that obtain information about the instantiation of classificatory quality determinates, families of *maximally* complete questions are defined as follows:

▶ **Definition 10** (Maximally complete questions). *Let $\mathcal{Q}(Q_f, Q_l, Q_m)$ be a family of complete questions that is associated with the classificatory quality determinable Q_m. The subdivision that is picked out by the control questions in $Q_l = \{Q_1, \ldots, Q_{\#C}\}$ is of* maximal *resolution if and only if (1) Q_l is the set of control questions with associated control cells of (roughly) equal size in the range of order in which all the complete questions in $\mathcal{Q}(Q_f, Q_l, Q_m)$ can have a yes answer; (2) there is a set of control questions $Q'_l = \{Q'_1, \ldots, Q'_{2*\#C}\}$ that arises when refining the subdivision picked out by $Q_l = \{Q_1, \ldots, Q_{\#C}\}$ by replacing every cell of Q_l by two cells of (roughly) equal size; and (3) the complete questions in $\mathcal{Q}(Q_f, Q'_l, Q_m)$ with the measurable/observable classificatory quality determinates in Q_m and the control questions $Q'_1, \ldots, Q'_{2*\#C}$ fail to yield information because the Q'_i are too small to instantiate the determinates in Q_m.*

A family $\mathcal{Q}(Q_f, Q_l, Q_m)$ of complete questions that is associated with the classificatory quality determinable Q_m is maximal if and only if the resolution of the subdivision that is picked out by Q_l is of maximal resolution.

One can prove the following proposition:

▶ **Proposition 11** (adapted from [4]). *Let $\mathcal{Q}(Q_f, Q_l, Q_m)$ be a set of complete questions that is associated with the classificatory quality determinable Q_m. The members of $\mathcal{Q}(Q_f, Q_l, Q_m)$ have a maximal information capacity (and therefore satisfy Postulate 2), only if the control questions in Q_l pick out cells of* maximal *resolution in the sense of Def. 10.*

Proof. Since every question in $\mathcal{Q}(Q_f, Q_l, Q_m)$ is complete and adheres to the Wheeler-Sinton scheme, a yes answer to any of the complete questions in $\mathcal{Q}(Q_f, Q_l, Q_m)$ yields the amount of L bits of information about the instantiation of quality determinates in Q_m in the cells that are picked out by the control questions in Q_l. Since Q_m is a classificatory quality determinate, it has a finite and fixed number of quality determinates (Def. 4), more information about the instantiation of quality determinates in Q_m in (parts of) the observed system S can be had only by further subdividing the control cells in Q_l. But such a refinement would render the questions that are associated with the cells of the refined subdivision void of information because those questions cannot have a yes answer. This is because, by assumption, the cells that are associated with the control questions in Q_l are already of maximal resolution in the sense of Def. 10. Thus, the amount of information of complete questions in $\mathcal{Q}(Q_f, Q_l, Q_m)$ associated with control questions in Q_l that acquire information about instantiation at cells of maximal resolution is maximal. Hence Postulate 2 is satisfied. ◀

6.3 Multiple families of maximally complete questions

Proposition 11 demonstrated that there is a limit to the amount of information about the instantiation of certain kinds of classificatory quality determinates that is possible. The question now arises whether Postulate 3 is true for such systems with limited information capacity.

Let $\mathcal{Q}(Q_t, Q_l, Q_m)$ be a set of maximally complete question that O can pose to S and suppose that (1) the subdivision that is picked out by Q_l is raster-shaped, and (2) a yes answer to a question in $\mathcal{Q}(Q_t, Q_l, Q_m)$ delivers L bits of information to O about S. Limiting the amount of information that O can have of S to L bits does not entail that $\mathcal{Q}(Q_t, Q_l, Q_m)$ is the only set of maximally complete questions that O can pose to S in the context of the Wheeler-Sinton paradigm. There may be a second set $\mathcal{Q}'(Q_t, Q'_l, Q_m)$ of maximally complete questions which give rise to L bits of information when posed by O to S. Let Q'_l be the set of yes/no questions that pick out the cells of a subdivision of S that arises when the (raster-shaped) subdivision that is associated with Q_l is translated by half of a cell size either along the rows or along the columns of the partition. From the construction of $\mathcal{Q}(Q_t, Q_l, Q_m)$ and $\mathcal{Q}'(Q_t, Q_l', Q_m)$ it follows that: (a) the family of complete questions $\mathcal{Q}(Q_t, Q_l, Q_m)$ has an information capacity of L bits if and only if $\mathcal{Q}'(Q_t, Q_l', Q_m)$ has an information capacity of L bits; and (b) $\mathcal{Q}(Q_t, Q_l, Q_m)$ is a family of maximally complete questions if and only if $\mathcal{Q}'(Q_t, Q_l', Q_m)$ is.

▶ **Proposition 12.** *Let $\mathcal{Q}(Q_t, Q_l, Q_m)$ and $\mathcal{Q}'(Q_t, Q_l', Q_m)$ be as above. For every maximally complete question $Q \in \mathcal{Q}(Q_t, Q_l, Q_m)$ and for every maximally complete question $Q' \in \mathcal{Q}'(Q_t, Q_l', Q_m)$: A yes answer to the question Q posed by O to S that is (immediately) followed by a yes answer to the question Q' leaves O with a total of L bits of information about S.*

Proof. Assume that O has zero bits of information of S before asking Q. A yes answer to Q then gives O exactly L bits of information about S. Now suppose that a yes answer to Q' after a yes answer to Q leaves O with $L + 1$ bits of information about S. From the construction of the questions Q and Q' and the underlying raster-shaped subdivisions, it follows, that there must be two control cells $c_1, c_2 \in Q_l$ and one control cell $c' \in Q_l'$ such that c_1 and c_2 both partially overlap c' and jointly contain c' in their mereological sum. The following cases are relevant: (a) $q_1 = q_2 = q'$ where q_1, q_2, q' are the qualities that are instantiated respectively at c_1, c_2 , c' according to the information provided by S to O. In this case Q' cannot yield any new information. Thus, a yes answer to Q' following a yes answer to Q leaves O with exactly L bits of information. This contradicts the assumption that $L+1$ bits of information were obtained; (b) The second case is $q_1 \neq q'$ or $q_2 \neq q'$. Focus on $q_1 \neq q'$ and the regions $c_1 \cap c'$ and $c_1 \setminus c'$. It follows that if $L + 1$ bits of information can be obtained by O from a yes answer to Q' following a yes answer to Q, then this information originates from the instantiation of q' in the region $c_1 \cap c'$ and q_1 in $c_1 \setminus c'$. This is because, by Def. 4, distinct determinates of a classificatory quality determinable cannot inhere in partially overlapping regions. Thus, via the additional bit of information, jointly Q and Q' provide information about instantiation at regions that are of roughly half the size of cells of maximal resolution. This contradicts the assumption that the questions Q and Q' are maximally complete questions. Thus, a yes answer to Q' after a yes answer to Q does not leave O with $L + 1$ bits of information about S. Similarly for the sub-case $q_2 \neq q'$ of (b).

Since the thesis that a yes answer to Q' after a yes answer to Q leaves O with $L + n$ bits of information about S can be ruled out for $n = 1$, the thesis can also be ruled out for $n > 1$. ◀

From Proposition 12 it follows that Postulate 3 is satisfied for families of maximally complete questions of classificatory quality determinates:

▶ **Corollary 13.** *It is always possible to obtain new classificatory information.*

Proof. Consider the two families of maximally complete questions $Q \in \mathcal{Q}(Q_t, Q_l, Q_m)$ and $Q' \in \mathcal{Q}'(Q_t, Q_l', Q_m)$ that O can ask S. Suppose that O obtains L bits of information via a yes answer to the question Q' after having obtained L bits of information via a yes answer to the question Q. On those assumptions the L bits obtained via a yes answer to Q' must overwrite/erase/destroy the L bits of information previously obtained via a yes answer to Q. This is because the information capacity of L bits cannot be exceeded (Proposition 12). Therefore a yes answer to the question Q following a yes answer to the question Q' yields genuinely new information: information that was not available (anymore) to O after obtaining L bits of information via the yes answer to Q'. ◀

The questions $Q \in \mathcal{Q}(Q_t, Q_l, Q_m)$ and $Q' \in \mathcal{Q}'(Q_t, Q'_l, Q_m)$ are geographic examples of what in quantum information theory are called *complimentary* questions.

7 Information about geographic delineation

Now suppose that there is a third set, $\mathcal{Q}(Q_t, Q_l, Q^{i/b})$, of complete questions which enable the observing system O to obtain information about discontinuities in the distribution of classificatory qualities across the observed system S. Those questions obtain information about the location of bona fide boundaries [17] that separate regional parts of S in which distinct classificatory determinates are instantiated. In what follows the members of $\mathcal{Q}(Q_t, Q_l, Q^{i/b})$ are called *delineatory* questions while the members of $\mathcal{Q}(Q_t, Q_l, Q_m)$ are called *classificatory* questions.

The set $\mathcal{Q}(Q_t, Q_l, Q_m)$ of complete classificatory questions and the set $\mathcal{Q}(Q'_t, Q'_l, Q^{i/b})$ of complete delineatory questions are *compatible* if and only if they share the same yes/no questions for fix/control, i.e., $Q_t = Q'_t$ and $Q_l = Q'_l$. While in $\mathcal{Q}(Q_t, Q_l, Q_m)$ the fix/control questions are complemented by classificatory yes/no questions, in $\mathcal{Q}(Q'_t, Q'_l, Q^{i/b})$ the fix/control questions are complemented by delineatory yes/no questions of the form:

- $(Q_{i/b})^1_j$: “Is the cell associated with a yes answer to the control question Q^j_l an interior part of a region in which a classificatory quality determinate of Q_m inheres?”
- $(Q_{i/b})^2_j$: “Does the cell associated with a yes answer to the control question Q^j_l contain a boundary which separates regions in which distinct classificatory quality determinates of Q_m inhere?”

According to Fig. 3, the answer to the question $(Q^{i/b}_{\text{LSQ}})^1_{cell_1}$ is yes and the answer to question $(Q^{i/b}_{\text{LSQ}})^2_{cell_1}$ is no. By contrast, the answer to question $(Q^{i/b}_{\text{LSQ}})^1_{cell_3}$ is no and the answer to question $(Q^{i/b}_{\text{LSQ}})^2_{cell_3}$ is yes.

The family $\mathcal{Q}(Q_t, Q_l, Q^{i/b})$ of maximally complete delineatory questions and the family $\mathcal{Q}(Q_t, Q_l, Q_m)$ of maximally complete classificatory questions are complementary. This is because if a yes answer to $Q_b \in \mathcal{Q}(Q_t, Q_l, Q^{i/b})$ following a yes answer to $Q_c \in \mathcal{Q}(Q_t, Q_l, Q_m)$ could yield more than L bits of information then so could a complementary classificatory question $Q'_c \in \mathcal{Q}'(Q_t, Q'_l, Q_m)$. But this would contradict Propositions 12 and 13.

8 Conclusion

The arguments of the previous sections about the quantum nature of maximally complete classificatory (and delineatory) questions depended critically on the assumption that the underlying control questions refer to cells at a maximal level of resolution. Linking a maximal amount of information to a minimal unit of space, as it is evident in Postulate 6 as well as in Propositions 11 and 12, makes explicit that, in the context of the processing of classificatory and delineatory information about geographic regions, there is a maximal *information density* associated with every classificatory quality determinable Q_m. The notion of maximal information density then opens the possibility that larger amounts of information can be had at coarser levels of granularity. At those coarser levels the density of information would be lower and quantum information would behave very much like classical information.

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