Representation of Interdependencies Between Urban Networks by a Multi-Layer Graph

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Abstract

The RGC4 (Urban resilience and Crisis Management in a Context of Slow Flood to Slow Kinetics) project aims to develop tools to help manage critical technical networks as part of the management process of crisis in a context of slow kinetic flooding in Paris. This project focuses on cascading models to identify a number of inter-dependencies between networks and to define tools capable of coordinating the actions of managers before and during the crisis. This paper revisits the conceptual and methodological bases of networks approach to study the inter-dependencies between networks. Research that studies the return to service of infrastructure networks often angle it from the perspective of operational research. The article proposes a graph theory perspective based on a multi-layer network approach and shows how to characterize the inter-dependencies between networks at three process levels (macro, meso, micro)

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1 Introduction

The ever-increasing city services are based on the growing complexity of urban technical networks (electricity, water supply, transport, telecommunications, etc.). However, these networks, generally interdependent, are highly vulnerable to hazards and extreme weather events. The localized failure of a network component can impact several services over large areas, sometimes well beyond the areas directly subject to the trigger hazard. Recent disasters, such as Hurricane Katrina in New Orleans, have contributed to the development of the concept of urban resilience. Urban resilience is a key focus of current approaches to flood management. The notion of resilience encompasses pre-disaster planning and warning systems, emergency handling procedures and post-disaster reconstruction. The concept of resilience leads in particular to an interest in the post-disaster period and consequently in the phenomenon of "reconstruction" and "return to normal" [9]. Research that studies the return to service of urban networks often responds to this from the perspective of operational

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research. The objective is to optimize the return to service of these networks under the constraints of availability of resources [13], [10].

This research proposes to identify a number of inter-dependencies between networks related to *resilience*. While we may know enough inter-dependencies between networks to be able to simulate the impacts of flood on networks and their cascading failures, it is not clear how to schedule infrastructure return-to-service / troubleshooting, with the additional difficulty of recognizing that managers may have conflicting interests. This analysis of inter-dependencies requires to model cascading effects[7]. These cascades significantly increase the vulnerability of the urban system and makes recovery and reconstruction processes more difficult and slower after a disruption. We described our networks as multi-layer-graphs upon which we modeled this "inverted domino effect" by topological operations.

In this paper we chose to describe in depth the topological structure of our model, but not go into details of the involved algorithms.

2 Modeling urban services with a multi-layer graph

We use graphs to model networks and our resilience issue because graph algorithms and metrology on large graphs highlight possible structural and functional properties related to interactions. To avoid a terminological confusion, further on we will employ "Network" when we mean the real world organization and "Graph" when we mean our model of the Network.

Choice of modeling by graph

To understand the resilience of networks during flood periods, it is important to model their failure dynamics. This modeling, through a graph, requires identifying the entities (vertices) and the relationships (edges/arcs) that connect them either in space or through a more abstract dependency link [1]. Graph modeling represents the information either by a global vision or by a representation at lower scales (structural properties of networks) [14]. The objective is to use the structure and the semantics of the graph obtained to answer the problem of inverted domino effects and to produce indicators to characterize the inter-dependencies between networks in a given territory.

A model we found of particular interest was the multiplex graph of[11]. A multiplex graph is a graph composed of a set of vertices of the same type, linked by different types of relationships. A multiplex is therefore a multi-relational graph that is often represented by a multi-layer graph. Multi-layer graphs explicitly incorporate multiple channels of connectivity and constitute the natural environment to describe systems interconnected [2]. Layers can be interdependent and they contain information which would be lost if we only considered the corresponding aggregated network. It has also been shown recently that different types of dynamics that are run on top of multi-layer systems also provide new insights into the problems being modeled [4], [5]. So, a multi-layer systems consists of several distinct classical layers, each one encoding a specific type of information about the system. Many complex systems can be represented as networks consisting of distinct types of interactions, which can be categorized as links belonging to different layers [6]. The question is then how many layers are indeed necessary to accurately represent our structure of a multilayered complex system to model urban service systems.

Relation between graphs and layers

In this work we model urban services and associated technical networks with a multi-layer graph whose layers represent three levels of study: macro, meso, micro (relationships between the same components (micro level), relations between different infrastructures (meso level) and relations between different urban systems (macro level)). Assuming that urban services are defined by their infrastructure and components and are interconnected, urban services should not be defined as objects but rather through the networks that create them.

We model urban services (ex: electricity, water, railway, buses and metros, etc.) at one scale with vertices which can be detailed by connecting them to graphs of a different layer, representing a different scale. The aggregation approach by urban service and infrastructure leads us to consider not only urban technical networks (macro level) but also the infrastructures that structure each urban service (meso level) and their components (micro level). We thus build a particular type of multi-layer graph with the idea of a multiplex graph, that is to say a sequence of interconnected graphs.

To give an example (Figure 2, C and D), Paris RATP urban service (macro level) consists in a subset of infrastructures S such as metro stations, railway, etc. (meso level). The metro station concept belongs to the meso level while each individual metro station belongs to the micro level. The infrastructures of the micro level are spatialized. The metro stations taken one by one (e.g. Auber, Bercy, Créteil, etc. respectively $S1$, $S2$, $S3$) are geo-referenced and form a new layer(micro level). At each level, relationships and inter-dependencies exist between respectively urban services, infrastructures and spatialized components. Because of the peculiar interconnected structure, it is possible to move from one layer to another one.

3 Our model

The objective is to study the return to service strategies for different urban networks based on multi-layer graphs. Knowing that, we have chosen to focus on Paris' own urban technical networks: the RATP rail network, the ENEDIS electricity network and the road network as well as their infrastructures and components. In this section we present in details our model.

Methodology

We model the disruption of a network by the suppression of one or more arcs or vertices [12]. This modelling leads to the study of failure scenarios. In our funding project, flood are the failure causing event we are meant to investigate. To create these flood scenarios, two steps are necessary. First of all, we would like to simulate the impact of the crisis to obtain a graph representing the disturbed network. This first step is an application of deconstruction rules. Then, the objective is to reconstruct the graph in order to return to the state of the initial network. The idea in this second step is to propose schedules for the return to service of the installations. To this end, we would like to add semantic information on vertices and arcs of graphs, depending on the information and data available. This information will allow us to take into account the network disruption in the graph.

The multi-layer graph

The model is represented by a multi-layer graph $\mathbf{M}$. The structure is defined by : $\mathbf{M} = (G, C)$ where G is a directed graph such as:

$G = \{G^\alpha \mid \alpha \in [\![1..3]\!]\}$ with $G^\alpha = (V^\alpha, E^\alpha, \mu^\alpha, \varepsilon^\alpha)$

V^α is the set of vertices, E^α the set of arcs and

μ^α the set of weights/attributes on the arcs and

ε^α the set of weights/attributes on the vertices

C is the set of interconnections between vertices of different layers (G^α)

We write G^1 for the macro level, G^2 for the meso level and G^3 for the micro level. These G^α are called layers. We will call *semantic sub-graphs* sub-graphs of G^α connected through C to one vertex of $G^{\alpha-1}$ (Figure 1).

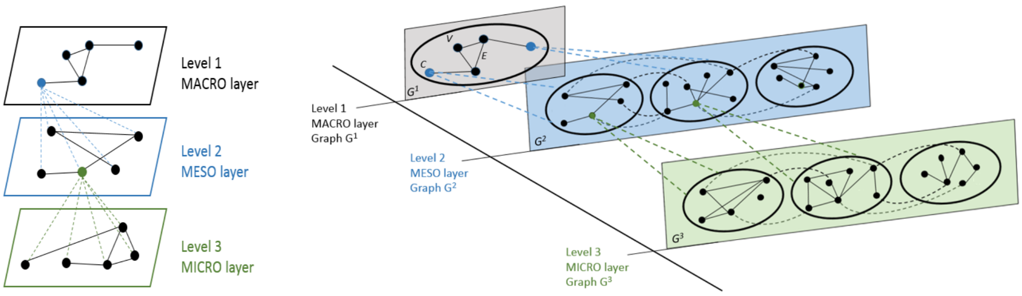

Figure 1 Illustration of a multi-layer graph at different levels representing three analysis layers: macro, meso, micro.

4 Semantic in the graph

Since networks consist of a large number of infrastructures and components, it is necessary to define the most important and primary elements (i.e.critical) which are essential for civil society. The aim is to determine whether the impacts and repercussions of an urban network, infrastructure or component are really disastrous or if they generate only low incidents during a flood. This is what we will call *criticality*. The concepts of vulnerability, resilience, and criticality are interrelated. Resilience is defined as "the capacity of a system to absorb disturbance and re-organize while undergoing change so as to still retain essentially the same function, structure, identity and feedbacks" [16]. We identify specific vulnerability to flooding of a particular critical facility by looking at factors such as its use, past flooding issues, location of critical systems like primary and back-up power to better understand its criticality. In addition, knowing the vulnerability and criticality of an element provides recommendations and/or resources to critical facility managers for short or long-term changes that could be made to reduce their facilities risk to flooding. If we take the example of the ENEDIS (electrical network) and its electrical installations (micro level), they have a variable vulnerability to flooding [8]. Lines, buried or overhead, are considered as not very vulnerable. On the other hand, some equipment clearly appears vulnerable, such as pylons in case of high flows [8]. The vulnerability of transformer stations (meso level) depends on water level (and turbidity too). The electrical network (macro level) is generally meshed to a high level of detail (almost everywhere up to distribution to individuals (micro level)). This mesh allows a station to temporarily or permanently take over from a failed station. However, in the case of floods, this mesh provides little protection. Indeed, floods often affect very large areas, so all stations of the same mesh can be affected and cuts occur directly at customers' premises when they are flooded. If the power grid is very large aggressive towards other networks, it seems little dependent on other networks. Nevertheless, telecommunication networks and road networks are necessary for on-site repairs and communication in times of crisis. The criticality of the elements of the multi-layer network is modeled by vertices and arcs attributes. The idea is to evaluate the criticality of the actors of different networks according to their environment.

Criticality expression

At each level of detail, networks, infrastructure and components are vulnerable to flooding. They will have repercussions and impacts on their own networks. The criticality and vulnerability assessment process establishes priority between urban services, infrastructures and components.

Criticality on arcs is evaluated in order to know their vulnerability and to prioritize the actions to be taken (Figure 2, A and B) . This comparative assessment is based on certain criteria, in particular the dependence of the networks on each other, both in normal times and during floods. The notion of criticality is closely linked to the damage to the network, infrastructure and components (depending of the level of detail) after a flood [15]. Values of criticality may change during events depending on the water level. Over the three studied levels, a criticality scale is established and each edge $e \in E$ in the graph is valuated by this measure (*Criticalitylevel(e)*). The criticality levels are:

- Level 1: no special vigilance. The impact of the flood will be very low or even zero
- Level 2: pay attention. Impacts low but may become higher if the flood persists
- Level 3: very critical. Impacts heavy and will lead to a dysfunction in the resilience capacity of components, infrastructures and networks.
- Level 4: absolute criticality. Impacts exceptional and lead to a secession of activities.

For the macro and meso level, two other values (in percentage) value arcs. The first value (*networkDependencyNorm(e)*) corresponds to the level of dependence between the two vertices linked under normal circumstances, for users. The impact of flooding on vertex dependency defines the second value (*networkDependencyFlood(e)*). These values are intended to assess the fragility of the vertex and the importance of the vertex in the graph (Figure 2, A).

For example, RATP network (macro level) is very dependent on ENEDIS network. RATP network used by nearly 10 million passengers per day depends on ENEDIS' resources. RATP needs the electricital network to operate its transportation network. A simple network interruption on few lines can cause losses and repercussions for travellers to their workplace. On the other hand, in the event of a flood, ENEDIS' impact on RATP network will be low (provided that the network is not damaged, too) since in the event of flooding, RATP closes the exits to the transport routes with cofferdams. Above all, RATP wishes to resist damage caused by ice jam shocks and the pressures of other urban components. Thus, to give an example, the arc between RATP and ENEDIS network is valued as such: 4; 80; 30.

At the meso level, in the ENEDIS graph, the transformer used to transmit and distribute electricity (adapt the voltage) is highly dependent on the source stations. In the event of a flood, the flooding of the transformer (leakage problems) will lead to a deterioration of the source substations and will lead to a disruption or even interruption of the power supply. The transformer to the source stations therefore has a high criticality (for users), as well as a dependency and strong impacts during flood periods when normal weather conditions prevail. Thus, to give an example, the arc between transformer and source stations is valued as such: 3; 80; 80.

Criticality on vertices is taken into account according to their neighbourhood. Each vertex $v \in V$ is valuated with $d(v)$, $d^-(v)$, $d^+(v)$ (degree, input degree and output degree of a vertex v). Applied to this context, the notion of neighbourhood implies that a link with a poorly connected vertex is less critical than a link with a highly connected vertex (Figure 2, A, B).

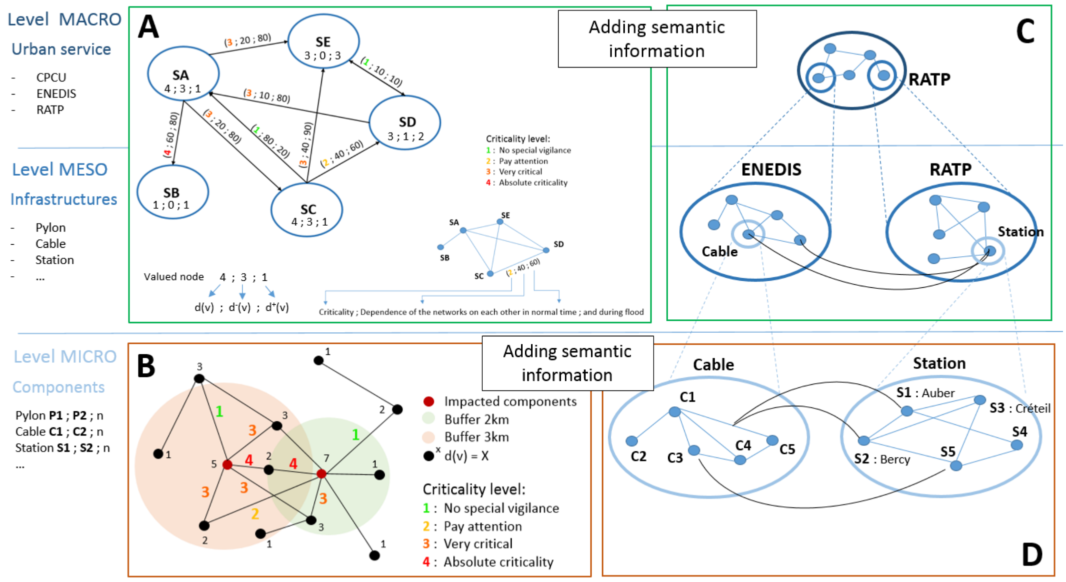

Figure 2 Formalization of inter-dependencies at three levels of study with the addition of semantics on our different graphs.

5 Environmental modelling

5.1 Formalisation

As illustrated in Figure 1, vertices from a G^α layer are connected to vertices of a $G^{\alpha+1}$ layer.

Let $hol^{\alpha+1}$ be:

$$hol^{\alpha+1} : \begin{array}{ccc} \mathbf{V}^{\alpha+1} & \longrightarrow & \mathbf{V}^{\alpha} \\ v^{\alpha+1} & \longmapsto & v^{\alpha} \end{array}$$

For example if v^3 is the power station at the angle of street A and Avenue B, $v^2 = hol^3(v^3)$ is the infrastructure "Power station"."hol" is short for "holon" which describes the same notion in other formalisms.

Let RC^α be:

$$RC^{\alpha} : \begin{array}{ccc} \mathbf{V}^{\alpha} & \longrightarrow & \mathscr{P}(\mathbf{V}^{\alpha+1}) \\ v^{\alpha} & \longmapsto & (hol^{\alpha+1})^{-1}(v^{\alpha}) \end{array}$$

"RC" is short for "returnComponent" which describes the same notion in other formalisms. Finally we call $C^\alpha = \{(v_l^\alpha, RC(v_l^\alpha)), l \in [1.. \mid V^\alpha \mid]\}$ and $C = \bigcup_{\alpha=1}^{3} C^\alpha$ (section 3)

Macro level / Meso level

The graph $G^1 = (V^1, E^1, \mu^1, \varepsilon^1)$, models the macro level. It is defined by its set of vertices V^1 (urban service) and its set of arcs E^1 where

$V^1 = \{v_i^1 \mid i \in [1..nbServices]\}$ and
$E^1 = \{(v_k^1, v_j^1) \mid \exists$ an interdependence link between v_k^1 and v_j^1, $v_k^1, v_j^1 \in V^1, j \neq k\}$ with
$\mu^1 : V^1 \longrightarrow \mathbb{N} \times \mathbb{N} \times \mathbb{N}$
$v \longmapsto (d(v), d^-(v), d^+(v))$
$\varepsilon^1 : E^1 \longrightarrow \mathbb{N} \times \mathbb{R} \times \mathbb{R}$
$e \longmapsto (Criticalitylevel(e), networkDependencyNorm(e), networkDependencyFlood(e))$

The formalisation of the meso level is quite similar to the macro level except that it deals with infrastructures.

Micro level

The graph $G^3 = (V^3, E^3, \mu^3, \varepsilon^3)$, models the micro level. It is defined by its set of V^3 vertices (components) and its set of E^3 arcs where
$V^3 = \{v_i^3 \mid i \in [\mid nbComponent \mid]\}$ and
$E^3 = \{(v_k^3, v_j^3) \mid \exists$ a link of functional or spatial interdependence between v_k^3 and v_j^3, $v_k^3, v_j^3 \in V^3, j \neq k\}$

For the micro level, since the dependencies are not necessarily functional but geographical, the neighbourhood is based on the closest neighbours, by defining a buffer area. Each component have its own buffer with its own distance. The distance then becomes the radius of a circle since the vertex (representing our component) is a point, the induced surface of the circle, the buffer area (e.g Figure 2 B). According to this definition μ^3 and ε^3 formal definition is quite similar to macro and meso level without $networkDependencyNorm(e)$ and $networkDependencyFlood(e)$
$\varepsilon^3 : E^3 \longrightarrow \mathbb{N}$
$e \longmapsto Criticalitylevel(e)$

5.2 Consideration of scheduling algorithms

This formalization gives the definition of the graph structure modeling the interdependency of technical networks. It models the relationships of the urban networks, at different levels of study and with their semantics. However, in order to identify strategies for rebuilding the network after a flood, different constraints must be taken into account, particularly in terms of resources, time and materials [3]. The objective is to prioritize these needs. The multi-layer graph models several networks. Our scheduling problem is therefore to give an order on operating tasks for the reconstruction of the activity of these urban networks while respecting the constraints. This structure, combined with scheduling algorithms, will make it possible to: identify and characterize “critical vertices” and their links, decide on the allocation of a network's need and arbitrate between the managers needs. After the flood, it can take days or even months for affected networks to return to normal operation. A poor consideration of this risk of impacts on the networks may lead to a significant additional delay before the territory is restored to normal.

6 Conclusion

This paper describes a methodology to model the vulnerability and resiliency of interdependant urban services networks (water, electricity, transportation etc.) and introduces a multi-layer approach to provide a sound support for resilience issues ans crisis management. We define a multi-layer graph to model each network, their infrastructural elements (duct, filter station etc.; pylon, transformer etc.; railway, station etc.) and their individual, spatialized components (the filter station at the angle of A street and B street; transformer number 1657 etc.). As it is, the graph model suggested is mainly dynamic how interactions and cascading affects can be modelled.

On this graph, short range reliance between two connected elements (two close pylons, a railway joining two stations) can be provided by the operators of the network. When rebuilding after a disaster, this operational knowledge is put to use to order the necessary operations. Nonetheless, long range dependency is much more difficult to assess for operators, especially if it requires knowledge external to the network they operate, and can therefore lead to clearly sub-optimal decisions. Topological operations on the proposed structure can compute this long range dependency, thus aiding rebuilding ordering decision making, and improving the resilience of the urban networks in the process.

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