Towards a Qualitative Reasoning on Shape Change and Object Division

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Abstract

We propose a qualitative representation for handling shape change and object division. We model the shape of a smooth curve in a two-dimensional plane together with its temporal change, using curvature extrema. The representation is based on Process-Grammar, which gives a causal account for each shape change. We introduce several rewriting rules to handle object division, that consist of making a tangent point, reconstruction, and separation. On the treatment of the division process, the expression can clarify the relative locations of multiple objects. We show formalization and application to represent a sequence of shape changes frequently observed in an organogenesis process.

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1 Introduction

There are many examples of shape changes in dynamic systems. Usually simulation is applied using quantitative data to show the process of what happened or what will happen. An alternative way to represent shape change is to use algebraic formulas, for example, differential equations. However, it is difficult to imagine the shapes solely with differential equations and impossible to perform logical reasoning directly via algebraic formulation. We sometimes would like to establish the reasons that something happened to facilitate future predictions. For example, maybe one is interested in why a certain shape has been made or what will happen if a pair of objects become attached. These types of problems can be addressed using logical reasoning based on qualitative data to provide a symbolic description.

In biology and life sciences, division of an object and shape change in a single object are frequently observed. When two smooth curves (probably portions of the same closed circuit) contact each other at a point, we call the contact point *a tangent point*. In the process of division, the shape of an object gradually changes in such a way that a concave part is generated, a tangent point of the border is made, and then separation occurs at that point. Therefore, if we want to analyze such dynamic systems, we must first establish a better understanding of the underlying change mechanism.

There has been almost no symbolic treatment of shape change in the dynamic systems found in the life sciences. Tosue et al. proposed a symbolic expression to represent a

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shape that enables reasoning about its temporal change for an organogenesis process. They approximated the shape using straight edges ignoring lengths and regarded the object as a polygon; transformation rules for the expression were described [12]. More specifically, they represented the border of a polygon using a sequence of rotation angles made by the subsequent edge. They defined shape changes as a set of rewriting rules on this expression, and presented an algorithm for drawing a figure corresponding to the expression [13].

However, it is difficult to perceive transformations using a coarse approximation such as straight edges, because the borders of objects are usually curved. If we wish to apply a more intuitive model using this method, we must use a more refined approximation, which complicates the rewriting rules and introduces high computational complexity.

In this study, we adopt a method that allows the curve to be represented qualitatively without using straight edges. The method is based on Process Grammar proposed by Leyton [7, 8], in which curvatures and extrema are used to represent the shape. Here, a curvature extremum is a part of the curve, where the curvature is at a maximum or minimum when tracing the boundary in a designated direction. Leyton considered that an extremum of a closed curve was formed gradually from a simple convex shape, and he aimed to infer the history of the construction of the shape. For example, the outline of an object in Figure 1(a) is changed to that in (b) by adding the force in the direction shown by the arrow; then to (c) if the force continues; and to (d) if the force diverges into two directions. Leyton formalized this transition as Process Grammar, which is a rewriting rule for symbols.

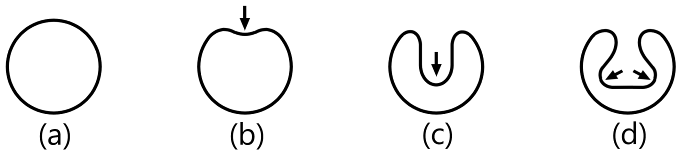


Figure 1 History of changing a shape.

Leyton's Process Grammar treats only a smooth curve that does not cross itself and has no cusps. Moreover, the division of an object was outside of his focus.

Here, we extend Leyton's representation to handle the division of an object. To this end, we define an expression of a shape that can discriminate (1) shapes, i.e., whether the curve has concavity and/or a tangent point, and (2) relative locations of objects, more specifically, whether an object is in the inner or outer part of another object. The second point is an essential factor in the treatment of multiple objects in two-dimensional planes (e.g., [15]).

The division process proceeds as follows. First, a border of a single curve extends to make a tangent point on itself. The connection is then reconstructed so that two closed curves are connected at the tangent point. Finally, the two closed curves are separated. A tangent point is made by connecting two points of a single curve. Therefore, the rewriting rules with respect to the division are defined over the entire expression, while the original Process Grammar is defined as rewriting a symbol locally.

The crucial point in the process of division is reconstruction. For example, in an organogenesis process, the borders of each object consist of a sequence of cells, and a certain force on the cells causes changes in the reconstruction. Here, we introduce a reconstruction rule to reflect such a phenomenon.

The remainder of this paper is organized as follows. In Section 2, we describe Leyton's Process Grammar. In Section 3, we introduce the description language for representing a shape. In Section 4, we define transformation rules for a shape change, and in Section 5, we apply it to the transformations of objects in the organogenesis process. In Section 6, we discuss the extension of the proposed method and also compare our method to related works. Finally, in Section 7, we present our conclusions and future work.

2 Process-Grammar

Process-Grammar is a means of recovering the process history of a smooth shape from its curvature extrema, and expressing that evolution in terms of transitions at these extrema [7]. Here, a smooth curve never intersects itself and has no tangent point nor cusp. The target is the boundary of an object between the solid and the empty. A smooth line is represented by a sequence of curvature extrema, traveling along the curve so that the solid lies on the left side of the curve. Leyton showed that in a two-dimensional plane the evolution of any smooth shape of a smooth curve can be expressed in terms of six process transitions; he named this a "Process-Grammar." In Process-Grammar, a process is understood as creating the curvature extrema. It shows how the shapes form over time, and a direction of change of a curve is shown by an arrow to the curve in the figure. Here, we refer to the cause for the shape change as a "force."

There are four types of extrema curvatures: two maximum extrema M^+ and M^- and two minimum extrema m^+ and m^-. Each one shows how the shapes form over time: M^+ indicates a protrusion that is sharpening outwards, m^- indicates an indentation that is sharpening inwards, m^+ indicates a squashing that is flattening inwards, and M^- indicates an internal resistance that is flattening outwards. The polarity represents the convexity: "+" indicates a convex shape, while "−" is a concavity.

extremum type	explanation	force type	force direction	convexity
M^+	protrusion	sharpening	outwards	convex
m^-	indentation	sharpening	inwards	concave
m^+	squashing	flattening	inwards	convex
M^-	internal resistance	flattening	outwards	concave

A smooth curve in a two-dimensional plane is expressed as a sequence of these symbols. Figure 2 shows an example.

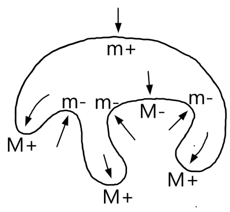


Figure 2 A figure and the corresponding expression.

A Process-Grammar is the transition rule over these sequences to represent changes in the shapes. There are two kinds of rules: continuation (the names of the rules begin with "C") and bifurcation (the names of the rules begin with "B") of the force at each extremum. Below we show the rules associated with the description of changes in the shape[1].

[1] In [7], the symbol "0" was used to represent an inflection point whose curvature is zero; we do not use it, because it can be deduced.

[Rules]	continuation and bifurcation	
Cm^+:	$m^+ \to m^-$	(squashing continues until it indents)
CM^-:	$M^- \to M^+$	(resistance continues until it protrudes)
BM^+:	$M^+ \to M^+m^+M^+$	(shield formation)
Bm^-:	$m^- \to m^-M^-m^-$	(bay formation)
Bm^+:	$m^+ \to m^+M^+m^+$	(breaking through of a protrusion)
BM^-:	$M^- \to M^-m^-M^-$	(breaking through of an indentation)

For example, if protrusion (M^+) continues in the same direction, the shape of the extremum will become steeper, but its shape type does not change; if the force branches forward, then the extremum will move both to the left and right sides, and the original position will be flattened, which is formalized as BM^+ rule. The correspondence between these rules and shape changes are shown in Figure 3. In each figure, the arrow towards a curve indicates a force; the bold black arrow indicates the added force and the white arrow is a newly emerged force.

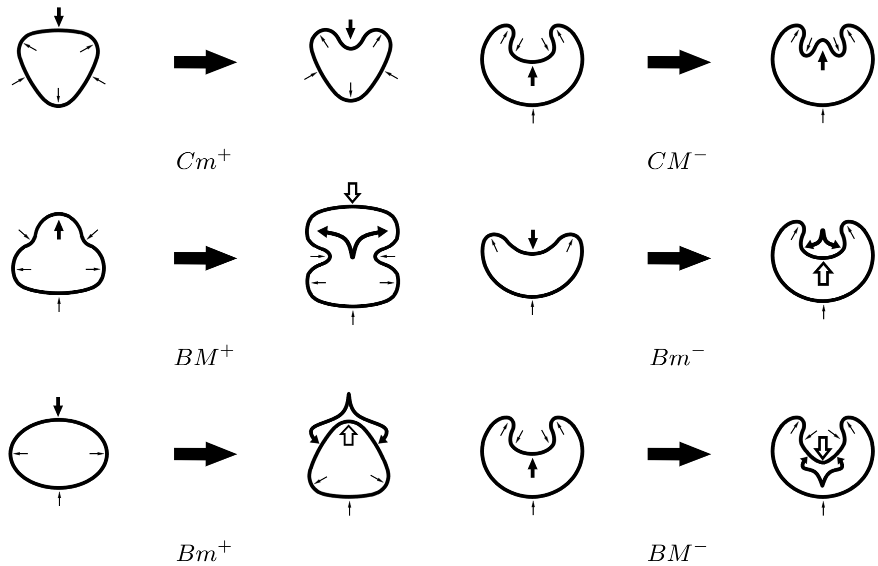


Figure 3 Process-Grammar defined by Leyton.

3 Description Language

We extend the Process Grammar formalism to describe the process of division. Our target figure is a set of smooth closed curves without an intersection. To simplify the problem, we first restrict the case in which there are at most two closed curves with at most one tangent point.

We introduce a description language $\mathcal{L}$ based on the Process Grammar. The language consists of two types of symbols: one with a dot and one without a dot

$\mathcal{L} = \{M^+, m^-, m^+, M^-, \dot{M}^+, \dot{m}^-, \dot{m}^+, \dot{M}^-\}$.

$\dot{M}^+, \dot{m}^-, \dot{m}^+$ and $\dot{M}^-$ denote that there exists a tangent point on the extrema M^+, m^-, m^+ and M^-, respectively. We call the symbols $\dot{M}^+, \dot{m}^-, \dot{m}^+$, and $\dot{M}^-$, *dotted elements*. We also call $M^+, M^-, \dot{M}^+$ and $\dot{M}^-$, *M-elements*, and $m^+, m^-, \dot{m}^+$ and $\dot{m}^-$, *m-elements*, respectively.

An expression is a finite sequence of elements in $\mathcal{L}$. For example, the expression for the figure in Figure 4 is $M^+\dot{m}^-M^+m^+M^+\dot{m}^-M^+m^+$

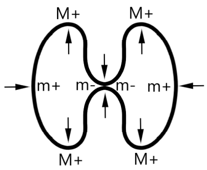
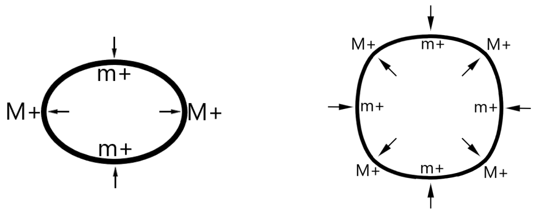


Figure 4 A single curve with a tangent point.

An expression for a single closed curve is cyclic, that is, expressions $e_ie_{i+1}\dots e_ne_1\dots e_{i-1}$, for all i $(1 \leq i \leq n)$ show the same shape. For example, an expression for a simple oval in Figure 5(a) is represented either as $M^+m^+M^+m^+$ or $m^+M^+m^+M^+$. If we use more elements to represent a closed curve, then we can express the shape in a more refined manner (Figure 5(a)(b)).



(a) (b)

Figure 5 Simple ovals.

Let $\mathcal{E}$ be a set of expressions
$\mathcal{E} = \{e_1e_2\dots e_n \ \mid e_i \in \mathcal{L}\ (1 \leq i \leq n)\}$.
We define an inverse function on $\mathcal{E}$ as follows:

$$
\begin{aligned}
&inv(M^+) = m^-, \quad inv(\dot{M}^+) = \dot{m}^-,\\
&inv(m^+) = M^-, \quad inv(\dot{m}^+) = \dot{M}^-,\\
&inv(M^-) = m^+, \quad inv(\dot{M}^-) = \dot{m}^+,\\
&inv(m^-) = M^+, \quad inv(\dot{m}^-) = \dot{M}^+,\\
&inv(e_1e_2\cdots e_n) = inv(e_n)\cdots inv(e_2)inv(e_1) \text{ for } e_1e_2\cdots e_n \in \mathcal{E}\ (1 \leq i \leq n).
\end{aligned}
$$

Let $\dot{s}$ be an expression that includes exactly one dotted element. Then, s is the expression obtained by replacing the dotted element in $\dot{s}$ by the corresponding non-dotted element. That is, for $\dot{s} = e_1\cdots e_n$, there $\exists 1 i$ $(1 \leq i \leq n)$ such that $e_i = \dot{e}$ where $e = M^+, m^-, m^+, M^-$, s denotes $e_1\cdots e_{i-1}ee_{i+1}\cdots e_n$.

An expression for a smooth closed curve satisfies the following conditions (C1) and (C2).

(C1) For $e_1\dots e_n \in \mathcal{E}$, n is more than three.
(C2) For $e_1\dots e_n \in \mathcal{E}$, if e_i is an M/m-element, then e_{i+1} is an m/M-element for all i $(1 \leq i \leq n,\ e_{n+1} = e_1)$.

The first condition requires at least four extrema to form a closed curve in a two-dimensional plane, according to the four-vertex theorem in differential geometry (e.g., [4]). The second condition requires that both M-element and m-element appear in turn, which is critical for smooth curve formation. Specifically, it indicates that there are no cusps between tangent points and guarantees the balance of inward and outward forces.

If there are two closed curves, then we can combine the expressions for each curve. The *combined expression* is either in the form σ, $\sigma \,||\, \tau$ or $\sigma[\tau]$, where σ and τ are expressions that satisfy the above conditions (C1) and (C2). In case it is in the form of σ, then it includes either a no-dotted element or two dotted elements that are not next to each other. In case it is in the form of $\sigma \,||\, \tau$ or $\sigma[\tau]$, both σ and τ have exactly one dotted element. $\sigma \,||\, \tau$ shows that τ is located in the external part of σ, and $\sigma[\tau]$ shows that the closed curve τ is located in the inner part of σ. In the latter case, σ has a hole τ in its inner part. $\sigma \,||\, \tau$ and $\tau \,||\, \sigma$ show the same figure.

This representation can be used to discriminate between the location of closed curves and also the existence of tangent points. We show several simple combined expressions for the shapes shown in Figure 6.

(a) $M^+m^+M^+m^+ \,||\, M^+m^+M^+m^+$
(b) $M^+m^+M^+m^+[M^-m^-M^-m^-]$
(c) $M^+m^+\dot{M}^+m^+ \,||\, M^+m^+\dot{M}^+m^+$
(d) $M^+m^+M^+\dot{m}^+[M^-\dot{m}^-M^-m^-]$

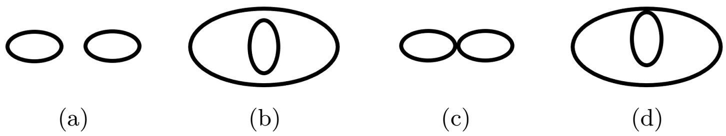


Figure 6 Combined expressions.

There are two closed curves. In cases (a) and (b), they are disconnected; in case (c), they are externally connected; and in case (d), they are internally connected. The tangent point is represented by the dotted expressions. Moreover, in cases (b) and (d), one is inside of the other.

4 Transition System

In addition to continuation and bifurcation rules, we introduce several rewriting rules over the description language to formalize object division: making a tangent point, reconstructing closed curves, and separation.

4.1 Making a tangent point

A tangent point is made by connecting a pair of extrema, which have grown by receiving a force. For example, an extremum m^- grows to reach another extremum m^+, then a tangent point is made both at m^- and m^+. The type of connection is either internal or external depending on the direction of the added force. Only four pairs have the possibility to make a tangent point.

1. internal connection
   A pair of extrema has received forces inward and at least one of them is concave. The pair satisfying this condition is either a pair of m^- and m^-, or a pair of m^- and m^+.
2. external connection
   A pair of extrema has received forces outward and at least one of them is convex. The pair satisfying this condition is either a pair of M^+ and M^+, or a pair of M^+ and M^-.

   The transition rule for each pair is as follows.

[Rules] making a tangent point

$TH : sm^-tm^- \to s\dot{m}^-t\dot{m}^-$
$TU : sm^+tm^- \to s\dot{m}^+t\dot{m}^-$
$TO : sM^+tM^+ \to s\dot{M}^+t\dot{M}^+$
$TP : sM^+tM^- \to s\dot{M}^+t\dot{M}^-$

where $s, t \in \mathcal{E}$ that satisfy (C2), and t contains at least one M^+ and m^- in the rules TH and TO, respectively.

4.2 Reconstruction

Reconstruction is a crucial part of the division process.

When we deal with an alveolus whose boundary is a sequence of cells, a pair of the sequences reconnect with each other in the reconstruction process. Actually, this occurs within a thick boundary at the tangent point. Here, we make a model in which the structure of the boundary is reconstructed.

In reconstruction, the sequences of the extrema located around a tangent point are decomposed and connected differently with new pairs of extrema.

On tracing a boundary which has a tangent point, we find two smooth curves encountered at the tangent point. Considering the directions of these curves on passing the tangent point, there are only two possibilities shown in Figure 7, since there is a constraint that a boundary never crosses. The reconstruction is the process of changing from (a) to (b) or from (b) to (a) in Figure 7. Therefore, we get four types of reconstructions, each of which corresponds to a type of tangent point; type P is divided into P_l and P_r, which are symmetric.

(a) (b)

Figure 7 Directions of curves on tracing a boundary.

[Rules] reconstruction

$RH : s\dot{m}^-t\dot{m}^- \to s\dot{m}^+ \,||\, t\dot{m}^+$ (Type H)
$RU : s\dot{m}^+t\dot{m}^- \to sm^+\dot{M}^+m^+ \,||\, tm^+\dot{M}^+m^+$ (Type U)
$RO : s\dot{M}^+t\dot{M}^+ \to s\dot{M}^-[t\dot{M}^-]$ (Type O)
$RP_l : s\dot{M}^-t\dot{M}^+ \to sM^-\dot{m}^-M^-[tM^-\dot{m}^-M^-]$ (Type P_l)
$RP_r : s\dot{M}^+t\dot{M}^- \to sM^-\dot{m}^-M^-[tM^-\dot{m}^-M^-]$ (Type P_r)

where $s, t \in \mathcal{E}$ that satisfy (C2), $|s|, |t| \geq 3$, and s is the expression for the outer curve in the rules RO, RP_l, and RP_r.

The symbols "H," "U," "O," and "P" used in the names of the rules are based on the entire shape of an object when a tangent point is made.

The constraint on the length of the expressions s and t is applied to obtain a combined expression that satisfies the conditions (C1) and (C2) after reconstruction. If this constraint is not satisfied, then the transformation process should transit to an intermediate state by applying bifurcation rules (BM^+, Bm^-, Bm^+ and BM^-) before applying the reconstruction rule. The other constraint is for distinguishing the locations of curves.

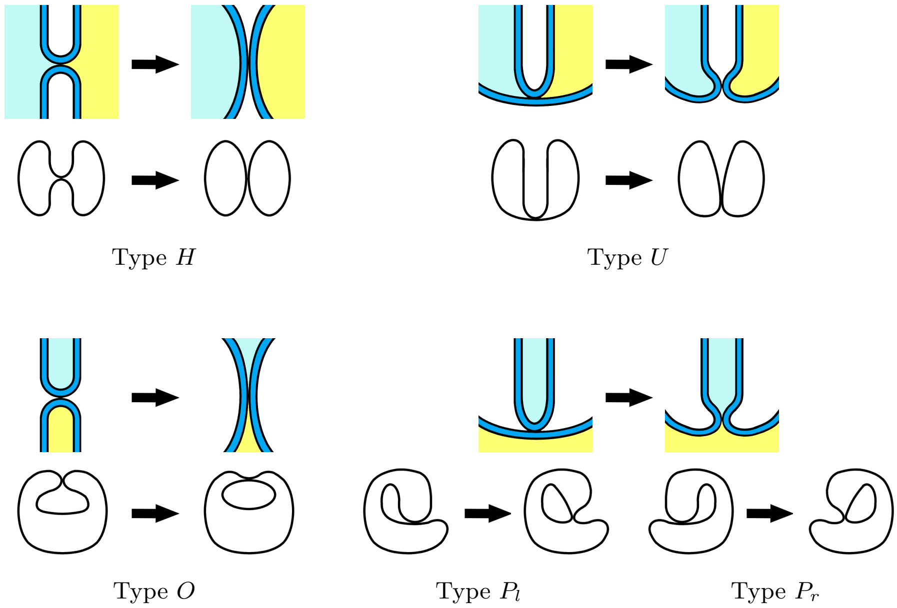


Figure 8 Reconstruction for each type.

Figure 8 illustrates the reconstruction for each type. The details of the neighbor to the tangent point are shown at the top, and an example of the shape of an object is shown at the bottom for each type. The shaded area indicates a solid part, that is, the inside of the object.

The rules of types H and U are the changes in which a part(s) of the border is extended inward to connect itself from the inside; as a result, two externally connected, closed curves are obtained. The rules of type O and P are the changes in which a part(s) of the border is extended to connect itself from the outside; as a result, a hole is made inside.

For example, in type H, two extrema m^- approach to make a tangent point (left side of the figure); then, the directions of the forces are changed to m^+ (right side).

4.3 Separation

The rules for separation are simple. We separate the two closed curves by removing the tangent point.

[Rules] separation

$SE : \dot{s_1} \mid\mid \dot{s_2} \rightarrow s_1 \mid\mid s_2$
$SI : \dot{s_1}[\dot{s_2}] \rightarrow s_1[s_2]$
where $s_1, s_2 \in \mathcal{E}$ that satisfy (C1) and (C2).

5 Application of rules for division processes

We show an application of four types of rules for the division process that frequently appear in an organogenesis process [14].

We start with a convex shape, whose expression is $M^+m^+M^+m^+M^+m^+M^+m^+$. We set this shape as S_0[2].

[2] We follow the Leyton viewpoint that a pure circle cannot be represented as a process. We take a simple

5.1 Type H

In this case, starting from S_0, two protrusions are made, come near, make an internal tangent point, and then the object is separated into two pieces (Figure 9).

$S_0 : M^+m^+M^+m^+M^+m^+M^+m^+$
$\Downarrow \quad (Cm^+)$
$S_1 : M^+m^-M^+m^+M^+m^-M^+m^+$
$\Downarrow \quad (TH)$
$S_2 : M^+\dot{m}^-M^+m^+M^+\dot{m}^-M^+m^+$
$\Downarrow \quad (RH)$
$S_3 : M^+\dot{m}^+M^+m^+ \mid\mid M^+m^+M^+\dot{m}^+$
$\Downarrow \quad (SE)$
$S_4 : M^+m^+M^+m^+ \mid\mid M^+m^+M^+m^+$

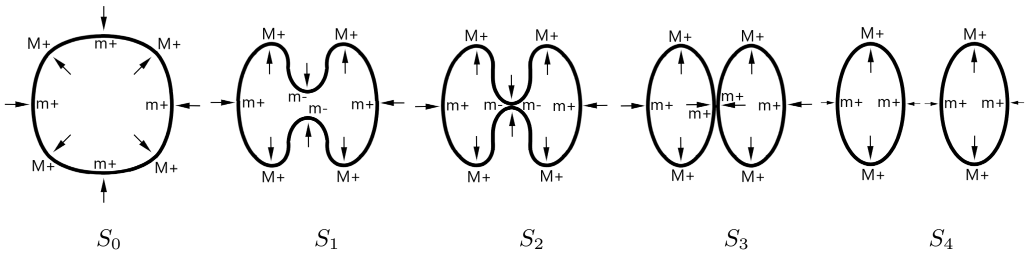


Figure 9 Division process for Type H.

5.2 Type U

In this case, starting from S_0, one indentation arises and reaches another part of the border, makes an internal tangent point, and then the object is separated into two pieces (Figure 10).

$S_0 : M^+m^+M^+m^+M^+m^+M^+m^+$
$\Downarrow \quad (Cm^+)$
$S_5 : M^+m^+M^+m^+M^+m^-M^+m^+$
$\Downarrow \quad (TU)$
$S_6 : M^+\dot{m}^+M^+m^+M^+\dot{m}^-M^+m^+$
$\Downarrow \quad (RU)$
$S_7 : M^+m^+\dot{M}^+m^+M^+m^+ \mid\mid \dot{M}^+m^+M^+m^+M^+m^+$
$\Downarrow \quad (SE)$
$S_8 : M^+m^+M^+m^+M^+m^+ \mid\mid M^+m^+M^+m^+M^+m^+$

5.3 Type O

In this case, the same change as that of type U occurs before S_5. After S_5, the protrusion branches and extends to connect together, and makes an external tangent point, and then the curve is separated into two closed curves, one of which is enclosed by the other (Figure 11).

$S_0 : M^+m^+M^+m^+M^+m^+M^+m^+$
$\Downarrow \quad (Cm^+)$
$S_5 : M^+m^+M^+m^+M^+m^-M^+m^+$

oval as an initial state and apply Bm^+ to get S_0.

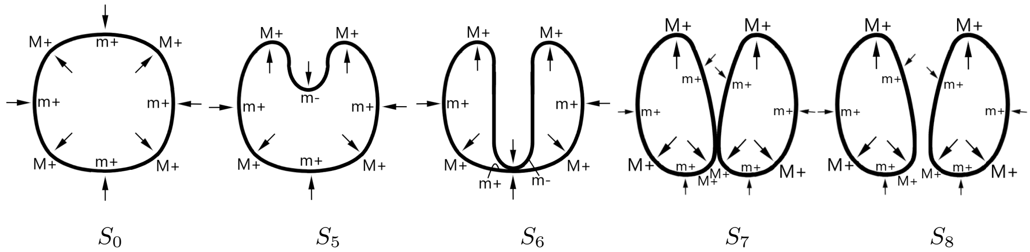


Figure 10 Division process for Type U.

$\Downarrow \quad (Bm^-)$
$S_9 : M^+m^+M^+m^+M^+m^-M^-m^-M^+m^+$
$\Downarrow \quad (TO)$
$S_{10} : M^+m^+M^+m^+\dot{M}^+m^-M^-m^-\dot{M}^+m^+$
$\Downarrow \quad (RO)$
$S_{11} : M^+m^+M^+m^+M^+m^+\dot{M}^-m^+M^+m^+[M^-m^-\dot{M}^-m^-]$
$\Downarrow \quad (SI)$
$S_{12} : M^+m^+M^+m^+M^+m^+M^-m^+M^+m^+[M^-m^-M^-m^-]$

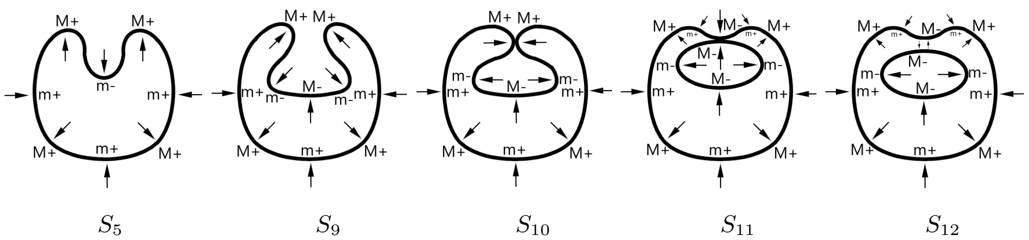
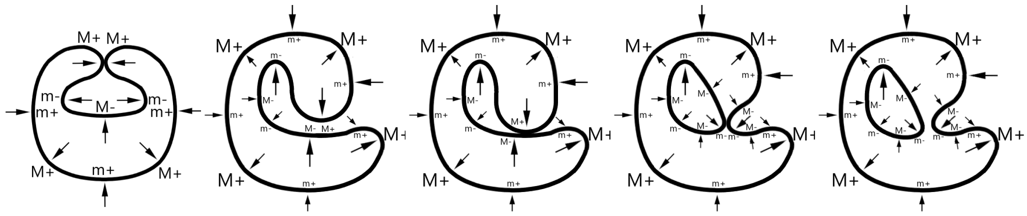


Figure 11 Division process for Type O.

5.4 Type P_l

In this case, the same change as that of type O occurs before S_9. After S_9, only one protrusion extends and bends to reach another part of the border; this makes an external tangent point, and then the curve is separated into two closed curves, one of which is enclosed of by the other (Figure 12). The process is similarly described for type P_r.

$S_0 : M^+m^+M^+m^+M^+m^+M^+m^+$
$\Downarrow \quad (Cm^+)$
$S_5 : M^+m^+M^+m^+M^+m^-M^+m^+$
$\Downarrow \quad (Bm^-)$
$S_9 : M^+m^+M^+m^+M^+m^-M^-m^-M^+m^+$
$\Downarrow \quad (BM^+)$
$S_{13} : M^+m^+M^+m^+M^-m^-M^-m^-M^+m^+M^+m^+M^+m^+$
$\Downarrow \quad (TP)$
$S_{14} : M^+m^+M^+m^+\dot{M}^-m^-M^-m^-\dot{M}^+m^+M^+m^+M^+m^+$
$\Downarrow \quad (RP_l)$
$S_{15} : m^+M^+m^+M^+m^+M^+m^+M^+m^+M^-\dot{m}^-M^-[m^-M^-m^-M^-\dot{m}^-M^-]$
$\Downarrow \quad (SI)$
$S_{16} : m^+M^+m^+M^+m^+M^+m^+M^+m^+M^-m^-M^-[m^-M^-m^-M^-m^-M^-]$



S_9 S_{13} S_{14} S_{15} S_{16}

Figure 12 Division process for Type P_l.

Note that after separation in cases of type O and type P_l, one closed curve becomes a hole, that is, the inner side of the hole is an outside of the original solid object. Therefore, in case of type O, the expression of the inner closed curve is $M^-m^-M^-m^-$, which is equivalent to $inv(m^+M^+m^+M^+)$.

6 Discussion

6.1 Generalization

In the previous sections, we restricted the target figure to one with at most two closed curves and at most one tangent point. We can drop these two restrictions easily.

We can represent multiple closed curves in any location by defining a combined expression recursively: let $\mathcal{E}_\mathcal{C}$ be a set of combined expressions; the combined expression is defined either in the form of σ, $\sigma \,||\, \tau$ or $\sigma[\tau]$ where $\sigma, \tau \in \mathcal{E}_\mathcal{C}$ that satisfy the conditions (C1) and (C2), respectively. We can represent multiple tangent points by adding elements to a description language: "numbered-dotted element" is used instead of dotted element to discriminate each tangent point because tangent points do not affect one another. For example, Figure 13 shows one possible division process in the case of two tangent points, and the following is the corresponding reconstruction rule, in which two tangent points are represented using a single dot and a double dot, respectively.

[Rules] reconstruction for double tangent points

$RU_2 : s\dot{m}^+t\ddot{m}^+[u\ddot{m}^-v\dot{m}^-] \rightarrow sm^+\dot{M}^+m^+um^+\ddot{M}^+m^+ \,||\, tm^+\ddot{M}^+m^+vm^+\dot{M}^+m^+$
where $s, t, u, v \in \mathcal{E}_c$ that satisfy (C2), $|s|, |t|, |u|, |v| \geq 3$.

In this figure, the inner circuit is expanded to reach the border of the outer circuit at two distinct points, and two tangent points are generated (Figure 13(b)). Next, the reconstruction occurs at these tangent points, respectively and as a result, two new curves are generated that are externally connected (Figure 13(c)). The object is then separated into two pieces (Figure 13(d)).

6.2 Extension

So far, we have discussed shape change starting from a simple convex form in the direction in which concave parts are created. Moreover, we have not considered shape change after separation. Then, the following question arises: if an object has a concave part after separation, how does this affect the shape change? The shape may change similarly with the process before the separation; however, it may change to recover the convex form.

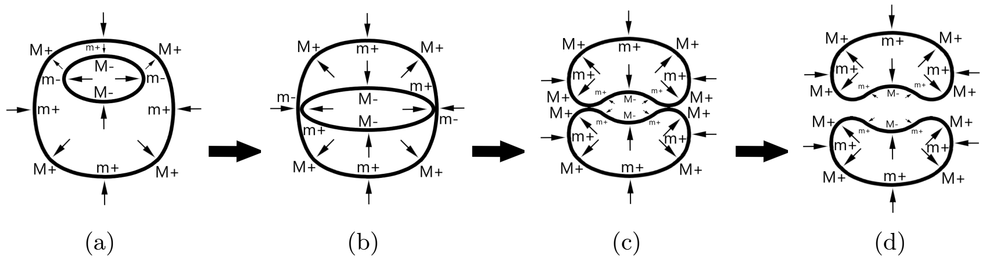


Figure 13 Division process in case of double tangent points.

Figure 14 Stabilization rules.

To address this issue, first, we introduce the concept of a stable state. When an expression consists of only M^+ and m^+, we call it *stable*. It can be considered that a stable curve changes by receiving some force, and an unstable curve likes to change to become stable. To treat this possibility, we need to allow an application of the rules introduced so far in the opposite direction. As such, the following rules are required.

[Rules] stabilization

$$Stm^+ : m^+M^-m^+ \to m^+$$
$$StM^- : M^-m^+M^- \to M^-$$

The first rule shows that if the concave part of the curve is pressed continuously from the inside then this part vanishes. The second rule shows that if the concave part of the curve is pressed continuously from the outside, then the protrusion vanishes (Figure 14). In each figure, the bold black arrow indicates the changing force, and the white arrow is the force that vanishes.

Using all of these rules together, we generate typical shape changes that appear during the organogenesis process (Figure 15).

6.3 Related works

Generally, it is difficult to represent the shape of an object qualitatively compared to other spatial features such as the relative positional relationships and relative directions. The most popular approach is to divide the boundary of an object into segments and represent its shape as a sequence of attributes such as length, direction, curvature, and so on that are attached to each segment. In these methods, the more attributes each segment has, the more accurately a figure can be drawn. This also requires more reasoning rules to interpret more complicated data.

Museros et al. introduced a qualitative shape descriptor (QSD) of each boundary using length, angle, curvature and so on [2, 5]. They also extended this scheme to a juxtaposition of objects in point-point, point-line, or line-point connecting types. They defined this

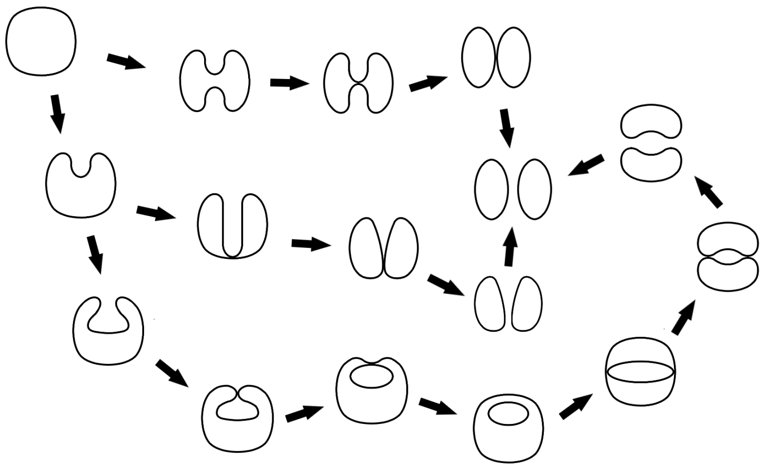


Figure 15 Application for explaining an organogenesis process.

juxtaposition as a shape composition that derives a new shape by this operation [1, 9] and treats rigid objects with non-deformable boundaries. Their focus was to provide a qualitative description of an object and formalize their composition, whereas we describe the change in the shape of an object with deformable boundaries using rewriting rules.

Galton et al. proposed a grammar scheme to describe changes in shapes, including a cusp [6]. Unlike QSD, they addressed deformable boundaries. They did not use extrema but rather local shape patterns to represent a closed curve; additionally, they created a number of transition rules by enumerating possible local changes. However, they did not describe the reason for the change. In contrast, here, we consider the forces involved in deforming the boundaries. Moreover, they did not address tangent points nor the division of an object, whereas we address these aspects.

Cohn used a mereotopological approach to formalize the shape of an object. He proposed a qualitative representation of a concave region using predicates [3]. Various shapes can be distinguished by representing relative position, size, and the direction of concave parts in a refined manner. He also discussed the continuous shape change. Because the number of possible shape descriptions is generally unbounded, he showed an example of a possible continuous transformation under some restricted forms. In contrast to an approach using rewriting rules, it is difficult to define the continuous transformation in the logical framework, and no formal explanation was given regarding this transformation.

In some research activities, shape is represented as a sequence of symbols, and its change is formalized as a set of rewriting rules. Shape grammar is a set of rules applied to an initial shape to generate designs [11]. It is mainly applied to show the structure of architectures. As the rules are defined to transform the initial shapes, the user may decide which rule can be used to achieve the desired outcome. Leyton's Process Grammar uses a set of rewriting rules. It can be considered as an abstract rewriting system [16, 10]. The main reason for our choice of Process Grammar is that it is suitable for resolving dynamic changes in a curve, as the history of shape change can be explained in terms of forces applied to the curve.

The biggest difference between our work and previous works is that our method can address the division of an object. We have defined a language and transition rules to handle the reconstruction of closed curves and the locations of multiple closed curves, which are the main issues involved in the treatment of a division frequently observed in an organogenesis process.

7 Conclusion

We have proposed a system to handle qualitative shape change, using the curvature and extrema of the curve. The proposed system enables the representation of a transformation qualitatively, including the division of an object, and gives a causal account for each transformation.

Our method has the following main features:

- direct representation of a smooth curve, as opposed to using an approximation such as a polygon,
- the ability to accommodate a tangent point and a division process, and
- the ability to describe the relative positional relationships of multiple closed curves.

Our approach can be applied to shape changes in various fields such as an alveolar division in a life science, analysis of a tumor in immunology, change in terrain shape in geomorphology, and so on.

As a future work, we would like to prove the completeness of this transition system, that is, the set of expressions that cover all possible transformations. It may be suitable to make several distinct models, including a conceptual, a theoretical, and a realistic model. We are currently looking into the rules necessary for describing these possible transitions more precisely, depending on the model.

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