Brief Announcement: Sinkless Orientation Is Hard Also in the Supported LOCAL Model

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Abstract

We show that any algorithm that solves the sinkless orientation problem in the supported LOCAL model requires $\Omega(\log n)$ rounds, and this is tight. The supported LOCAL is at least as strong as the usual LOCAL model, and as a corollary this also gives a new, short and elementary proof that shows that the round complexity of the sinkless orientation problem in the deterministic LOCAL model is $\Omega(\log n)$.

2012 ACM Subject Classification Theory of computation → Distributed computing models

Keywords and phrases Supported LOCAL model, sinkless orientation, round elimination

Digital Object Identifier 10.4230/LIPIcs.DISC.2021.58

Related Version *Full Version*: http://arxiv.org/abs/2108.02655

Funding *Janne H. Korhonen*: Project has received funding from the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme (grant agreement No 805223 ScaleML).
Ami Paz: We acknowledge the Austrian Science Fund (FWF) and netIDEE SCIENCE project P 33775-N.
Stefan Schmid: Research supported by the Austrian Science Fund (FWF) project ADVISE, I 4800-N, 2020-2023.

1 Introduction

Sinkless orientations. In the *sinkless orientation problem*, the task is to orient the edges of a graph such that all nodes of degree at least 3 have out-degree at least 1. The problem is always solvable, and easy to solve in the centralized setting, but a lot more challenging to solve efficiently in distributed or parallel settings.

The sinkless orientation problem is the canonical example of a problem that has round complexity $\Theta(\log \log n)$ rounds in the randomized LOCAL model but $\Theta(\log n)$ rounds in the deterministic LOCAL model. It is a rare example of a *locally checkable problem* in which randomness helps exponentially, and also an example of a locally checkable problem with an *intermediate* complexity – not solvable in $O(\log^* n)$ rounds but solvable in sub-diameter time.

Supported LOCAL model. While the complexity landscape in the usual LOCAL model is nowadays well understood [1, 3–5, 7, 8, 10, 11], and also some *weaker* models of distributed computing have been already explored [2, 9, 14], it has been wide open how the landscape changes when we switch to *stronger* models of computing. In this work we focus on the *supported* LOCAL model, which is strictly stronger than the LOCAL model.

In the supported LOCAL model [12, 15], the communication network $G = (V, E)$ and the unique identifiers are all known to all nodes, and the input is a subgraph H of G. That is, each node v receives as input *the entire structure* of the communication network G, including all the unique identifiers, and a list of its incident edges in H; we refer to the latter edges as *input edges*. Otherwise, the computation proceeds as in the standard LOCAL model, using all edges of G for communication. In our case, we would like to find a sinkless orientation in the input graph H.

The availability of the underlying globally-known communication graph G (a.k.a. the *support*) helps a lot with many problems. For example, all locally checkable problems with complexity $O(\log^* n)$ admit constant-time algorithms in the supported model – in essence, the support can be used to *break symmetry for free* [12]. Also if we had the promise that the support G is a *tree*, then the sinkless orientation problem would become trivial: we can orient all edges of G towards a leaf, and this orientation is also a valid orientation for any subgraph H. However, in this work we show that this trick only works in trees – we show that if, for example, G is a 5-regular graph, then the support is essentially useless.

The supported LOCAL model was introduced in the context of *software-defined networks* (SDNs). The underlying idea is that the communication graph G represents the unchanging physical network, and the input graph represents the logical state of the network to which the control plane (here, distributed algorithm) needs to respond to; see reference [15] for more details. However, supported LOCAL has proven useful as a theoretical model for lower bounds (this work, Foerster et al. [12], and the very recent work of Haeupler et al. [13]).

Our contributions. We prove that the round complexity of the sinkless orientation problem in the deterministic supported LOCAL model is $\Omega(\log n)$ rounds. By prior work, we also know that this is tight: the problem is solvable in $O(\log n)$ rounds (with or without support). Furthermore, the same problem can be solved in the randomized supported LOCAL model in $O(\log \log n)$ rounds.

In particular, we learn that in the supported LOCAL model there are locally checkable problems in which randomness helps exponentially. As a corollary, we cannot use the support to efficiently derandomize algorithms.

Relation to prior work. As a by-product, our work gives a new, short and elementary proof that shows that the round complexity of the sinkless orientation problem in the deterministic LOCAL model is $\Omega(\log n)$.

The standard proof is somewhat long and complicated. It builds on the *round elimination* technique [1, 7, 8], but round elimination has so far been unable to handle unique identifiers. Hence in prior work one has always taken a detour: first prove an $\Omega(\log \log n)$ lower bound in the randomized model without unique identifiers [8], and then apply the *deterministic gap result* of Chang et al. [10] to derive a deterministic $\Omega(\log n)$ lower bound.

By switching to the supported LOCAL model, we can give a direct proof without any detours through randomness and gap results. We directly show with elementary arguments that the complexity of sinkless orientation is $\Omega(\log n)$, both in the usual LOCAL model and also in the supported LOCAL model.

The underlying idea is, in essence, the same as the *ID graph technique* from the very recent work by Brandt et al. [9]. The ID graph in their work plays a role similar to the support in our work. However, Brandt et al. aimed at proving a lower bound for randomized local computation algorithms, while our aim is at proving a lower bound for deterministic supported LOCAL algorithms. The key technical difference is that ID graphs [9] need to have a large chromatic number, while our proof goes through even if the support is a bipartite graph. On the other hand, we need to do more work in the base case when we argue that 0-round algorithms do not exist.

2 Sinkless orientation lower bound

For technical convenience, we prove the result in a stronger *bipartite* version of the model. In bipartite supported LOCAL, we are given a promise that the support graph G is bipartite, and a 2-coloring is given to the nodes as an input; we refer to the two colors as black and white. We consider either the black or white nodes to be *active*, and the other color to be *passive*. All nodes of the graph run an algorithm as per supported LOCAL model; upon termination of the algorithm, the active nodes produce an output, and the passive nodes output nothing.

The outputs of the active nodes must form a globally valid solution; in particular, in sinkless orientation, the outputs of the active nodes already orients all edges, and both active and passive nodes of degree at least 3 must not be sinks. Note that any (supported) LOCAL algorithm can be turned into a bipartite algorithm with no round overhead, by running the algorithm as is and discarding the outputs of passive nodes.

We summarise the key lemmas of the proof next. For the full technical details and proofs, we refer the reader to the full version of the paper.

▶ **Lemma 1.** *Let G be a fixed 5-regular bipartite graph with girth g, and fixed unique identifiers and 2-coloring of the nodes. Let $0 < T < g/2$, and assume there is a T-round bipartite supported* LOCAL *algorithm $\mathcal{A}_T$ that solves sinkless orientation on G. Then there is a $(T-1)$-round bipartite supported* LOCAL *algorithm $\mathcal{A}_{T-1}$ that solves sinkless orientation on G.*

▶ **Lemma 2.** *Let G be a fixed 5-regular bipartite graph with girth g, and assume unique identifiers and 2-coloring on G are fixed. There is no algorithm solving sinkless orientation in bipartite supported* LOCAL *in 0 rounds on G.*

▶ **Theorem 3.** *Any deterministic algorithm solving sinkless orientation in the supported* LOCAL *model requires $\Omega(\log n)$ rounds.*

Proof. Let G be a bipartite 5-regular graph with girth $g = \Omega(\log n)$. Observe that we can obtain one e.g. by taking the bipartite double cover of any 5-regular graph of girth $\Omega(\log n)$, which are known to exist (see e.g., [6, Ch. 3]).

Assume that there is a supported LOCAL algorithm $\mathcal{A}_T$ that solves sinkless orientation in $T < g/2$ rounds on communication graph G. This implies that there is a bipartite supported LOCAL algorithm for sinkless orientation on G running in time T. By repeated application of Lemma 1, there is a sequence of bipartite supported LOCAL algorithms $\mathcal{A}_T, \mathcal{A}_{T-1}, \dots, \mathcal{A}_1, \mathcal{A}_0$, where algorithm $\mathcal{A}_i$ solves sinkless orientation in i rounds. In particular, $\mathcal{A}_0$ solves sinkless orientation in 0 rounds. By Lemma 2, this is impossible, so algorithm $\mathcal{A}_T$ cannot exist. ◀

▶ **Corollary 4.** *Any deterministic algorithm solving sinkless orientation in the* LOCAL *model requires $\Omega(\log n)$ rounds.*

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