Graphs Shortcuts: New Bounds and Algorithms

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Abstract

For an n-vertex digraph $G = (V, E)$, a *shortcut set* is a (small) subset of edges H taken from the transitive closure of G that, when added to G guarantees that the diameter of $G \cup H$ is small. Shortcut sets, introduced by Thorup in 1993, have a wide range of applications in algorithm design, especially in the context of parallel, distributed and dynamic computation on directed graphs. A folklore result in this context shows that every n-vertex digraph admits a shortcut set of linear size (i.e., of $O(n)$ edges) that reduces the diameter to $\widetilde{O}(\sqrt{n})$. Despite extensive research over the years, the question of whether one can reduce the diameter to $o(\sqrt{n})$ with $\widetilde{O}(n)$ shortcut edges has been left open.

In this talk, I will present the first improved diameter-sparsity tradeoff for this problem, breaking the $\sqrt{n}$ diameter barrier. Specifically, we show an $O(n^{\omega})$-time randomized algorithm for computing a linear shortcut set that reduces the diameter of the digraph to $\widetilde{O}(n^{1/3})$. I also present time efficient algorithms for computing these shortcuts and explain the limitations of the current approaches. Finally, I will draw some connections between shortcuts and several forms of graph sparsification (e.g., reachability preservers, spanners). Based on a joint work with Shimon Kogan (SODA 2022, ICALP 2022, FOCS 2022, SODA 2023).

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