The Constraint Satisfaction Problem: Complexity and Approximability

Dagstuhl Seminar 15301

Edited by
Andrei Krokhin
Stanislav Živný

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Editors

Andrei Krokhin
School of Engineering and Computing Sciences
University of Durham, UK
andrei.krokhin@durham.ac.uk

Stanislav Živný
Department of Computer Science
University of Oxford, UK
standa.zivny@cs.ox.ac.uk

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Preface

This volume is based on the Dagstuhl Seminar 15301 "The Constraint Satisfaction Problem: Complexity and Approximability" held in July 2015 and organised by Andrei A. Bulatov (Simon Fraser University), Venkatesan Guruswami (Carnegie Mellon University), Andrei Krokhin (Durham University), and Dániel Marx (Hungarian Academy of Sciences).

Overview of the Seminar

The constraint satisfaction problem, or CSP in short, provides a unifying framework in which it is possible to express, in a natural way, a wide variety of computational problems dealing with mappings and assignments, including satisfiability, graph colourability, and systems of equations. The CSP framework originated 30-35 years ago independently in artificial intelligence, database theory, and graph theory, under three different guises, and it was realised only in the late 1990s that these are in fact different faces of the same fundamental problem. Nowadays, the CSP is extensively used in theoretical computer science, being a mathematical object with very rich structure that provides an excellent laboratory both for classification methods and for algorithmic techniques, while in AI and more applied areas of computer science this framework is widely regarded as a versatile and efficient way of modelling and solving a variety of real-world problems, such as planning and scheduling, software verification and natural language comprehension, to name just a few. An instance of CSP consists of a set of variables, a set of values for the variables, and a set of constraints that restrict the combinations of values that certain subsets of variables may take. Given such an instance, the possible questions include (a) deciding whether there is an assignment of values to the variables so that every constraint is satisfied, or optimising such assignments in various ways, (b) counting satisfying assignments, exactly or approximately, or (c) finding an assignment satisfying as many constraints as possible. There are many important modifications and extensions of this basic framework, e.g. those that deal with valued or global constraints.

Constraint satisfaction has always played a central role in computational complexity theory; appropriate versions of CSPs are classical complete problems for most standard complexity classes. CSPs constitute a very rich and yet sufficiently manageable class of problems to give a good perspective on general computational phenomena. For instance, they help to understand which mathematical properties make a computational problem tractable (in a wide sense, e.g. polynomial-time solvable or non-trivially approximable, fixed-parameter tractable or definable in a weak logic). It is only natural that CSPs play a role in many high-profile conjectures in complexity theory, exemplified by the Dichotomy Conjecture of Feder and Vardi and the Unique Games Conjecture of Khot.

The recent flurry of activity on the topic of the seminar is witnessed by three previous Dagstuhl seminars, titled "Complexity of constraints" (06401) and "The CSP: complexity and approximability" (09441, 12541), that were held in 2006, 2009, and 2012 respectively. This seminar was a follow-up to the 2009 and 2012 seminars. Indeed, the exchange of ideas at the 2009 and 2012 seminars has led to new ambitious research projects and to establishing regular communications channels, and there is a clear potential of a further systematic interaction that will keep on cross-fertilizing the areas and opening new research directions. The 2015 seminar brought together forty three researchers from different highly advanced areas of constraint satisfaction and involved many specialists who use universal-algebraic,

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combinatorial, geometric and probabilistic techniques to study CSP-related algorithmic problems. The participants presented, in 28 talks, their recent results on a number of important questions concerning the topic of the seminar. One particular feature of this seminar was a significant increase in the number of talks involving multiple subareas and approaches within its research direction – a definite sign of the growing synergy, which is one of the main goals of this series of seminars.

The seminar was well received as witnessed by the high rate of accepted invitations and the great degree of involvement by the participants. Because of the multitude of impressive results reported during the seminar and the active discussions between researchers with different expertise areas, the organisers regard this seminar as a great success. With steadily increasing interactions between such researchers, we foresee a new seminar focussing on the interplay between different approaches to studying the complexity and approximability of the CSP.

Follow-Up

For some of the topics presented at the Dagstuhl Seminar 15301 there are excellent surveys. Some other topics are still too nascent to justify survey articles at this point. For several topics for which no surveys presently exist or the current ones are already outdated due to the recent progress, we felt that the time is ripe to produce such surveys as a follow-up to the Dagstuhl Seminar 15301.

Overview of the Volume

The first chapter in this volume is introductory and provides detailed explanations of the so-called algebraic approach to decision CSPs over finite domains. The algebraic approach has been behind several breakthroughs in the last decade. The remaining chapters are more advanced and specialised.

The second chapter gives an overview of absorption, a powerful algebraic technique used in the proof of the "bounded width theorem" and more generally in the study of decision CSPs. The third chapter is about CSPs with infinite numerical domains and constraints defined by using arithmetical operations. The fourth chapter explores so-called hybrid CSPs, which are classes of CSPs that are neither language- nor structure-based. The fifth chapter provides an overview of research on CSPs and backdoors, which is a concept that allows for a formalisation of being "close to a tractable class". The sixth chapter deals with Holant problems, which are special types of CPSs in which every variable appears in exactly two constraints. The seventh chapter studies CSPs from the parametrised perspective. The eight chapter gives an overview of the counting variants of CSPs. The ninth chapter is concerned with valued CSPs, which are generalisations of CSPs to optimisation problems. The tenth chapter investigates CSPs specialised to digraphs. The eleventh chapter gives a good account of the available approximation algorithms for CSPs. Finally, the twelfth chapter provides an overview of quantified CSPs.

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January 2017 *Andrei Krokhin and Stanislav Živný*

List of Authors

Libor Barto
Charles University in Prague, Czech republic
libor.barto@gmail.com

Manuel Bodirsky
TU Dresden, Germany
manuel.bodirsky@tu-dresden.de

Martin C. Cooper
University of Toulouse III, France
cooper@irit.fr

Serge Gaspers
UNSW Australia and Data61, CSIRO
sergeg@cse.unsw.edu.au

Heng Guo
Queen Mary, University of London, UK
h.guo@qmul.ac.uk

Gregory Gutin
Royal Holloway, University of London, UK
gutin@cs.rhul.ac.uk

Mark Jerrum
Queen Mary, University of London, UK
m.jerrum@qmul.ac.uk

Marcin Kozik
Jagiellonian University, Poland
marcin.kozik@uj.edu.pl

Andrei Krokhin
University of Durham, UK
andrei.krokhin@durham.ac.uk

Benoît Larose
Université du Québec à Montréal, Canada
blarose@lacim.ca

Pinyan Lu
Shanghai University of Finance and Economics, China
lu.pinyan@mail.shufe.edu.cn

Konstantin Makarychev
Microsoft Research, USA
komakary@microsoft.com

Yury Makarychev
Toyota Technological Institute at Chicago, USA
yury@ttic.edu

Marcello Mamino
TU Dresden, Germany
marcello.mamino@tu-dresden.de

Barnaby Martin
University of Durham, UK
barnaby.d.martin@durham.ac.uk

Sebastian Ordyniak
TU Wien, Austria
ordyniak@ac.tuwien.ac.at

Stefan Szeider
TU Wien, Austria
szeider@ac.tuwien.ac.at

Ross Willard
University of Waterloo, Canada
ross.willard@uwaterloo.ca

Anders Yeo
Singapore University of Technology and Design, Singapore and
University of Johannesburg, South Africa
anders.yeo.work@gmail.com

Stanislav Živný
University of Oxford, UK
standa.zivny@cs.ox.ac.uk

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