Solving Classical String Problems on Compressed Texts

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Abstract

Here we study the complexity of string problems as a function of *the size of a program that generates input*. We consider *straight-line programs* (SLP), since all algorithms on SLP-generated strings could be applied to processing LZ-compressed texts.

The main result is a new algorithm for pattern matching when both a text T and a pattern P are presented by SLPs (so-called fully compressed pattern matching problem). We show how to find a first occurrence, count all occurrences, check whether any given position is an occurrence or not in time $O(n^2m)$. Here m, n are the sizes of straight-line programs generating correspondingly P and T.

Then we present polynomial algorithms for computing fingerprint table and compressed representation of all covers (for the first time) and for finding periods of a given compressed string (our algorithm is faster than previously known). On the other hand, we show that computing the Hamming distance between two SLP-generated strings is NP- and coNP-hard.

I. Introduction

Background. How to solve text problems, if instead of an input string we get only program generating it? Is it possible to find a faster solution than just "generate text + apply classical algorithm"? In this paper we consider strings generated by *straight-line programs* (SLP). These are programs using only assignment operator. The exact definition and discussion on this notion are given in Section II.

We come to this question studying algorithms on compressed texts. Many algorithms for direct search in compressed texts without unpacking were proposed in the last decade. Finally (see Rytter's work [22]), it turns out that only decompression stage really matters. More precisely, we can consider a pair (decompression algorithm, archive file) as a *generating description* for the original text. Rytter presents an effective translation from any LZ-compressed text to the straight-line program generating the same text.

Here we study complexity of solving problems as a function of *the size of a program that generates input*. Since the ratio between the size of SLP and the length of original string might be exponential, even any polynomial algorithm is a matter of interest. Main purpose of such algorithms is to speed up naive approach and to save memory (since we never generate the original text). Moreover, even if we have just original text data, but containing a lot of repetitions, we can find generating description first and then apply one of our algorithms for SLP-generated texts. A similar idea (for OBDD-described functions) is used in symbolic model checking.

There are already promising applications for work with compressed objects: solving word equations in polynomial space [20], and pattern matching in message sequence charts [9]. Potential applications include all fields with highly compressible data like bio-informatics (genomes contain a lot of repetitions), pattern matching in statistical data (like internet log files) and any kinds of automatically generated texts.

Problem. For any text problem on SLP-generated strings we ask two questions: (1) does any polynomial algorithm exist? (2) If yes, what is exact complexity of the problem? We can think about negative answer

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to the first question (say, NP-hardness) as an evidence that naive "generate-and-solve" is the best way for that particular problem.

First of all we consider complexity of pattern matching problem on SLP-generated strings (sometimes called fully compressed pattern matching or FCPM). That is, given a SLPs generating pattern P and text T to answer whether P is a substring of T and provide a succinct description of all occurrences. An important special case is *equivalence problem* for SLP-generated texts. Then we expand the obtained algorithm/technique for several other string problems: finding shortest period/cover and constructing fingerprint table. Finally we consider the Hamming distance problem.

Results. The key result of the paper is a new $O(n^2m)$ algorithm for pattern matching on SLP-generated texts, where m and n are sizes of SLPs generating P and T, correspondingly. This is an improvement of the $O((n+m)^5 \log^3 |T|)$ algorithm by Gąsieniec et. al [8] and $O(n^2m^2)$ algorithm by Miyazaki et al. [16]. For one quite special class of SLP the FCPM problem was solved in time $O(mn)$ [11].

After presenting the main algorithm in Section III we sketch (more details will follow in a full version of paper) the polynomial algorithms for finding covers and periods, and for computing a fingerprint table for SLP-generated texts in Section IV. On the other hand, we show that surprisingly, computing the Hamming distance is NP-hard. Here we have a closely related problems (Hamming distance and equivalence) from different sides of the border between efficiently solvable problems on SLP-generated texts and intractable ones.

Proofs. The main ingredients of the algorithm are dynamic programming method, operations with arithmetical progressions and special tricks in case of dense sequence of pattern occurrences. The following way to work with SLP-generated strings turns out to be the most productive: (1) invent some auxiliary property of strings (2) compute it for all intermediate texts (3) derive the answer from the computed array. In our pattern matching algorithm we consecutively compute elements of a special $n \times m$ table. This is exactly the same idea as in [16]. However, the routine for computing a new element is completely different: we use only $O(n)$ time while in the paper [16] $O(mn)$ time is used. One of the key tricks comes from checking occurrence of false mismatches for Rabin-Karp algorithm as in [17]. However we just apply the same intuition but not directly the same procedure.

Immediate consequence of our result is an $O(n^2m)$ algorithm for pattern matching in LZ-compressed texts. Our algorithm uses only linear time for one step of dynamic programming. Hence, in order to get faster method, we now need a radically new approach.

A. Related results

The whole field started from papers by Amir, Benson and Farach [2] and by Farach and Thorup [6] presenting algorithms for compressed pattern matching with working time depending on *size of compressed text*. Compression models vary from run-length encoding, different members of Lempel-Ziv family [24], straight-line programs (SLP) and collage systems [13]. The last two are good theoretical models that generalize all previously studied compression algorithms. Namely, Rytter [22] proved that any LZ-encoding could be efficiently translated to SLP of approximately the same size without unpacking.

In addition to pattern matching, also algorithms for window subsequence (i.e. scattered substring) search [5], membership in regular language [18], and approximate pattern matching [12] were presented. From the other side, it was shown that fully compressed subsequence problem and the longest common subsequence are Θ_2-hard (this is a kind of closure of NP and coNP), compressed and fully compressed two-dimensional pattern matching are NP-complete and Σ_2-complete, respectively [4], while membership in a context-free language is even PSPACE-complete [15].

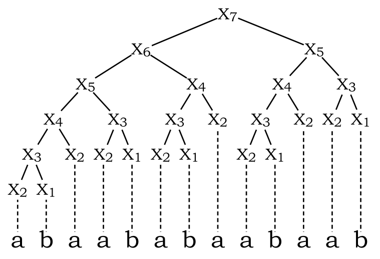
Encouraging experimental results are reported in [19]. The papers [21], [23] survey the field of processing compressed texts, while the book [10] provides a general overview of classical pattern matching algorithms.

II. Compressed Strings Are Straight-Line Programs

A *Straight-line program* is a context-free grammar generating exactly one string. Moreover, we allow only two types of productions: $X_i \to a$ and $X_i \to X_p X_q$ with $i > p, q$. The string presented by a given SLP is a unique text corresponding to the last nonterminal X_m. Although in previous papers X_i denotes only a nonterminal symbol while the corresponding text was denoted by $val(X_i)$ or $eval(X_i)$ we will identify this notions and use X_i both as a nonterminal symbol and as the corresponding text. We say that the size of SLP is equal to the number of productions.

Example. Consider string *abaababaabaab*. It could be generated by the following SLP:

$X_7 \to X_6X_5, \quad X_6 \to X_5X_4,$
$X_5 \to X_4X_3, \quad X_4 \to X_3X_2,$
$X_3 \to X_2X_1, \quad X_2 \to a,$
$X_1 \to b$



In fact, the notion of SLP describes only decompression operation. We do not care how such an SLP was obtained. Surprisingly, while the compression methods vary in many practical algorithms of Lempel-Ziv family and run-length encoding, the decompression goes in almost the same way. In 2003 Rytter [22] showed that given a LZ-encoding of string T we could efficiently get an SLP encoding for the same string which is at most $O(\log |T|)$ times longer than the original LZ-encoding. This translation allows us to construct algorithms only in the simplest SLP model. If we get a different encoding, we just translate it to SLP before applying our algorithm. Moreover, if we apply Rytter's translation to LZ77-encoding of a string T, then we get an $O(log|T|)$-approximation of the *minimal* SLP generating T.

The straight-line programs allow the exponential ratio between the size of SLP and the length of original text. Consider, for example

$$X_n \to X_{n-1}X_{n-1}, \qquad \dots \qquad X_2 \to X_1X_1, \qquad X_1 \to a.$$

In complexity analysis we use both $\log |T|$ and n (number of rules in SLP). For example, we prefer $O(n \log |T|)$ bounds to $O(n^2)$, since in practice the ratio between the size of SLP and the length of the text might be much smaller than exponential.

III. A New Algorithm for Fully Compressed Pattern Matching

Decision version of the fully compressed pattern matching problem (FCPM) is as follows:

INPUT: Two straight-line programs generating P and T
OUTPUT: Yes/No (whether P is a substring in T?)

Other variations are: to find the first occurrence, to count all occurrences, to check whether there is an occurrence from the given position and to compute a "compressed" representation of all occurrences. Our plan is to solve the last one, that is, to compute auxiliary data structure that will contain all necessary information for effective answering on the other questions.

We need some notation and terminology. We call a *position* in the text a point between two consequent letters. Hence, the text $a_1 \dots a_n$ has positions $0, \dots, n$ where first is in front of the first letter and the last one after the last letter. We say that some substring *touches* a given position if this position is either inside or on the border of this substring. We use the term *occurrence* both as a corresponding substring and as its starting position. Hopefully, the right meaning is always clear from the context.

Let $P_1, \dots, P_m$ and $T_1, \dots, T_n$ be the nonterminal symbols of SLPs generating P and T. For each of these texts we define a special *cut* position. It is a starting position for one-letter texts and merging position for $X_i = X_r X_s$. In the example above, the cut position for the intermediate text X_6 is between 5th and 6th letters: $abaab|aba$, since X_6 was obtained by concatenating $X_5 = abaab$ and $X_4 = aba$.

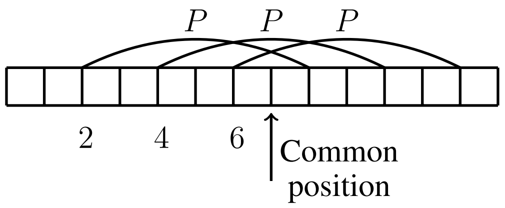
We use a computational assumption which was used in all previous algorithms but not stated explicitly. In analysis of our algorithm we count arithmetical operations on text positions as unit operations. In fact, text positions are integers with at most $\log |T|$ bits in binary form. Hence, the bit operation complexity is larger than our $O(n^2 m)$ up to some $\log |T|$-dependent factor.

Explanation of the algorithm goes in three steps. We introduce a special data structure (*AP-table*) and show how to solve pattern matching problem using this table in Subsection III-A. Then we show how to compute AP-table using *Local PM* procedure in Subsection III-B. Finally, we present an algorithm for Local PM in Subsection III-C.

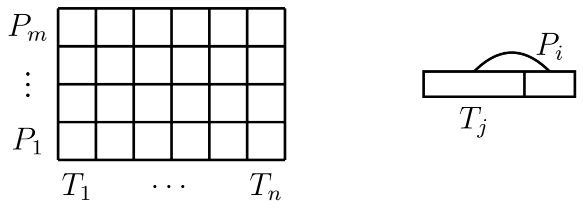
A. Idea of the algorithm

Our algorithm is based on the following theoretical fact (it was already used in [16]):

Lemma 1 (Basic lemma): All occurrences of P in T touching any given position form a single arithmetical progression (ar.pr.)



The *AP-table* is defined as follows. For every $1 \le i \le m, 1 \le j \le n$ the value $AP[i, j]$ is a code of ar.pr. of occurrences of P_i in T_j that touch the cut of T_j. Note that any ar.pr. could be encoded by three integers: first position, difference, number of elements. If $|T_j| < |P_i|$ we define $AP[i, j] = 0$, and if text is large enough but there are no occurrences we define $AP[i, j] = \varnothing$



Claim 1: Using AP-table (actually, only top row is necessary) we can solve decision, count and checking versions of FCPM in time $O(n)$.

Claim 2: We can count the whole AP-table by dynamic programming method in time $O(n^2 m)$.

Proof of Claim 1. We get the answer for decision FCPM by the following rule: P occurs in T iff there is j such that $AP[m, j]$ is nonempty. Checking and counting are slightly more tricky. Recursive algorithm for checking: test whether the candidate occurrence touches the cut in the current text. If yes, use AP-table and check the membership in the corresponding ar.pr., otherwise call recursively this procedure either for the left or for the right part. We will inductively count the number of P-occurrences in all $T_1, \ldots, T_n$. To start, we just get the cardinality of the corresponding ar.pr. from AP-table. Inductive step: add results for the left part, for the right part and cardinality of central ar.pr. without "just-touching" occurrences.

B. Computing AP-table

Sketch of the algorithm for computing AP-table.

1) Precomputation: compute lengths and cut positions for all intermediate texts;
2) Compute the first row and the first column of AP-table;
3) From j=2 to n do From i=2 to m do Compute AP[i,j]
    a) Compute occurrences of the larger part of P_i in T_j around the cut of T_j;
    b) Compute occurrences of the smaller part of P_i in T_j that starts at ending positions of the larger part occurrences;
    c) Intersect occurrences of smaller and larger part of P_i and merge all results to a single ar.pr.

At the very beginning we compute tables for lengths, cut positions, first letter, last letter of the texts $P_1, \ldots, P_m, T_1, \ldots, T_n$ in time $O(n+m)$.

We compute elements of AP-table one-by-one. At the beginning we compute the first row and the first column, then in the following order:

From j=2 to n do From i=2 to m do Compute AP[i,j]

Case of $|P_i| = 1$: compare with T_j if $|T_j| = 1$ or compare with the last letter of the left part and the first one of the right part. The resulting ar.pr. will be one or two neighbor positions. Hence, just O(1) time used for computing every cell in the table.

Case of $|T_j| = 1$: if $|P_i| > 1$ return 0, else compare letters. Also $O(1)$ time is enough for every element.

Induction step: let P_i and T_j be both of length greater than one. We are going to compute a new element in time $O(n)$ using already computed values of AP-table. For this purpose, we design a special auxiliary procedure that extracts useful information from already computed part of AP-table.

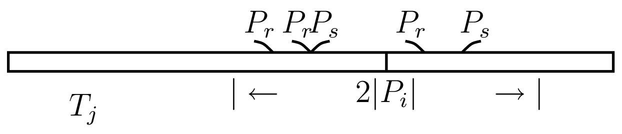
This is a border between ideas of Miyazaki et al. and the new approach presented in the paper. They compute the same table, but new element routines are completely different.

Procedure $LocalPM(i, j, [\alpha, \beta])$ returns occurrences of P_i in T_j inside the interval $[\alpha, \beta]$.

Important properties:

- Local PM uses values AP[i,k] for $1 \leq k \leq j$,
- It is defined only when $|\beta - \alpha| \leq 3|P_i|$,
- It works in time $O(j)$,
- The output of Local PM is a pair of ar.pr., all occurrences inside each ar.pr. have a common position, and all elements of the second are to the right of all elements of the first.

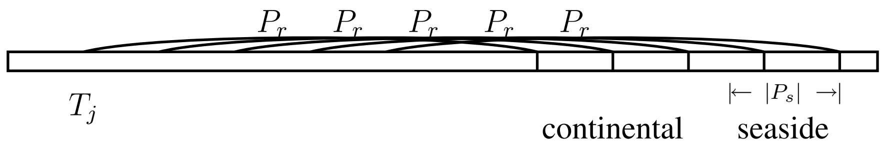
We now show how to compute a new element using 5 Local PM calls. Let $P_i = P_r P_s$, the cut position in T_j be γ and, without loss of generality, let $|P_r| \geq |P_s|$. The intuitive way is (1) to compute all occurrences of P_r "around" cut of T_j, (2) to compute all occurrences of P_s "around" cut of T_j, and (3) shift the latter by $|P_r|$ and intersect.



Any occurrence of P_i that touches the cut γ of T_j consists of occurrence of P_r and occurrence of P_s inside $|P_i|$-neighborhood of cut where ending position of P_r equals starting position of P_s.

We apply Local PM for finding all occurrences of P_r in the interval $[\gamma - |P_i|, \gamma + |P_r|]$. Its length is $|P_i| + |P_r| \leq 3|P_r|$. As an answer we get two ar.pr. of all potential starts of P_i occurrences that touch the cut. Unfortunately, we are not able to do the same for P_s, since the length of interesting interval is not necessarily constant in terms of $|P_s|$. So we are going to find only occurrences of P_s that are starting from two arithmetical progressions of endings of P_r occurrences.

We will process each ar.pr. separately. We call an ending *continental* if it is at least $|P_s|$ far from the last ending in progression, otherwise we call it *seaside*.


P_r P_r P_r P_r P_r
T_j
$|\leftarrow |P_s| \rightarrow|$
continental
seaside

Since we have an ar.pr. of P_r occurrences that have common position (property 4 of Local PM), all substrings of length $|P_s|$ starting from continental endings are identical. Hence, we need to check all seaside endings and *only one* continental position. For checking the seaside region we just apply Local PM for $|P_s|$-neighborhood of last endpoint and intersect the answer with ar.pr. of seaside ending positions. Intersecting two ar.pr. could be done in time $O(\log |T|)$ by technique similar to extended Euclid algorithm. For checking continental region we apply Local PM for $|P_s|$ substring starting from the first continental ending.

As the answer we obtain all continental endings/or none of them, plus some sub-progression of seaside endings, plus something similar for the second ar.pr. Since all of these four parts are ar.pr. going one after another, we could simplify the answer to one ar.pr. (it must be one ar.pr. by Basic Lemma) in time $O(1)$.

Let us estimate the complexity of the procedure for computing a new element. We use one Local PM call for P_r, four Local PM calls for P_s, and twice compute the intersection of arithmetical progressions. Hence, we have 7 steps of $O(n)$ complexity.

C. Realization of Local PM

By the first step (crawling procedure) we put all occurrences of P_i in $[\alpha, \beta]$ interval of T_j to the list. On the second step (merging procedure) we merge all these occurrences to just two ar.pr.

In crawling procedure on $(i, j, [\alpha, \beta])$ we take the ar.pr. of occurrences of P_i in T_j that touch the cut, leave only occurrences within the interval, and output this truncated ar.pr. to the list. After that, we check whether the intersection of the interval $[\alpha, \beta]$ with left/right part of T_j is at least $|P_i|$ long. If so, we recursively call crawling procedure with the same i, the index of left/right part of T_j, and with this intersection interval.

Consider the set of all intervals we work with during the crawling procedure. Note that, by construction, any pair of them are either disjoint or embedded. Moreover, since the initial interval was at most $3|P_i|$, there are no four pairwise disjoint intervals in this set. If we consider a sequence of embedded intervals, then all intervals correspond to their own intermediate text from $T_1, \dots, T_j$. Therefore, there were at most $3j$ recursive calls in crawling procedure and it works in time $O(j)$.

We assume that crawling procedure always maintain the pointer to the corresponding place in the list. This means that at the end we get a sorted list of at most $3n$ arithmetical progressions. By "sorted" we mean that the last element of k-th progression is less than or equal to the first one of $k+1$-th progression. It follows from construction of crawling procedure, that output progressions could have only first/last elements in common.

Now in the merging procedure we go through the resulting list of progressions. Namely, we compare the distance between the last element of current progression and the first element of the next progression with the differences of these two progressions. If all three numbers are equal we merge the next progression with the current one. Otherwise we just move to the next progression.

If we apply the Basic Lemma to $\delta_1 = \frac{2\alpha+\beta}{3}$ and $\delta_2 = \frac{\alpha+2\beta}{3}$ positions, we will see that all occurrences of P_i in $[\alpha, \beta]$ interval form two (one after another) arithmetical progressions. Namely, those who touch δ_1 and those who don't touch but touch δ_2. Here we use that $\beta - \alpha \le 3|P_i|$, and therefore any occurrence of P_i touches either δ_1 or δ_2. Hence, our merging procedure will start a new progression at most once.

D. Discussion on the Algorithm

Here we point out two possible improvements of the algorithm. Consider in details the "new element routine". Note that Local PM uses only $O(h)$ time, where h is the height of the SLP generating T, while intersection of arithmetical progressions uses even $O(\log |T|)$. Hence, if it is possible to "balance" any SLP up to $O(\log |T|)$ height, then the bound for working time of our algorithm becomes $O(nm \log |T|)$.

It is interesting to consider more rules for generating texts, since collage systems [13] and LZ77 [24] use concatenations and *truncations*. Indeed, as Rytter [22] showed, we could leave only concatenations expanding the archive just by factor $O(\log |T|)$. But there is a hope that the presented technique will work directly for the system of truncation/concatenation rules. More details will follow in a full version.

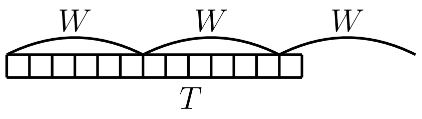
We also claim that AP-table might be translated to a polynomial-sized SLP generating all occurrences of P in T.

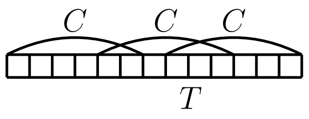
IV. New Algorithms for Related Problems

In this section we apply dynamic programming and our AP-table routine for solving more classical problems: periods, covers and fingerprinting. For highly compressible texts these algorithms might be faster than "unpack-and-solve" approach. These results show that pattern matching is not the only problem that we can speed up. Hence, we get a more general understanding of how processing on compressed texts really works. In this extended abstract we present only the basic theoretical facts and short sketches of the algorithms.

A. Covers and Periods

A *period* of string T is a string W (and also an integer $|W|$) such that T is a prefix of W^k for some integer k. A *cover* (originated from [3]) of a string T is a string C such that any character in T is covered by some occurrence of C in T. Note that every cover/period is uniquely determined by its length since by the definition they all are prefixes of T. We use notation $t = |T|$ in this section.

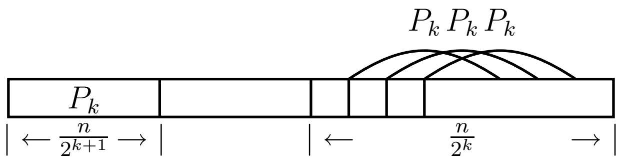




Problem of *compressed periods/covers:* given a compressed string T, to find a length of minimal period/cover and to compute a "compressed" representation of all periods/covers.

Theoretical facts serving as a basis for our algorithms:

1) Given an SLP of size n and for a given substring we can construct a linear-sized SLP (generating this substring) in linear time.
2) Lengths of periods of the t-string between $t - \frac{t}{2^k}$ and $t - \frac{t}{2^{k+1}}$ form a single ar.pr.
3) Any cover is a *border* (prefix and suffix of a given text). Any given text has a border of length u iff it has a period of length $t - u$.
4) If some border in the interval $[\frac{t}{2^{k+1}}, \frac{t}{2^k}]$ is a cover, then all smaller borders in that interval are also covers.



P_k P_k P_k
P_k
$\leftarrow \frac{n}{2^{k+1}} \rightarrow$
$\leftarrow \frac{n}{2^k} \rightarrow$

Sketch of algorithm for compressed periods. Using Fact 3 we will search for borders instead of periods. We will separately find all periods in each of $[\frac{t}{2^{k+1}}, \frac{t}{2^k}]$ intervals. Let us fix some k. Step 1: we take a $\frac{t}{2^{k+1}}$-prefix of T (let us denote it by P_k) and find all occurrences of this prefix in $\frac{t}{2^k}$-suffix of T.

By the Basic Lemma this is a single ar.pr. The distances between starting positions of occurrences of P_k and the end of T are *candidate* borders.

Step 2: we take a $\frac{t}{2^{k+1}}$-suffix of T (let us denote it by S_k) and find all occurrences of this suffix in $\frac{t}{2^k}$-prefix of T. By the Basic Lemma this is a single ar.pr. The distances between the start of T and the ending positions of occurrences of S_k and the end of T are *second candidate* borders.

Fact: to obtain the set of lengths of all borders in our interval we should simply intersect the sets of candidate and second candidate borders. Let us estimate the complexity of processing one interval. We apply FCPM algorithm to patterns that are substrings of initial text T. By Fact 1 we can efficiently construct their SLP representation. Moreover, the sizes of SLPs generating these patterns are larger than SLP generating T by at most constant factor. Step 1 requires a single FCPM call, step 2 also requires a single call, intersection of two ar.pr. could be done in linear time. Hence, we need $O(n^3)$ time for one interval and $O(n^3 \log |T|)$ time for the whole compressed periods problem.

The compressed periods problem was introduced in 1996 in the extended abstract [8]. However, the full version of their algorithm (it works in $O(n^5 \log^3 |T|)$ time) was never published.

Sketch of algorithm for compressed covers. We will separately solve problem for cover size between $\frac{t}{2^{k+1}}$ and $\frac{t}{2^k}$ for every k. As it was shown in the previous algorithm, we can find all borders with lengths in one such interval in time $O(n^3)$. Then using Fact 4 we apply binary search with cover-check procedure.

Cover check procedure (for given compressed strings C and T, to check whether C is a cover of T):

1) Compute AP-table for C and T.
2) For every intermediate text T_j check by Local PM procedure whether all $|C|$-neighborhood of cut in T_j, excepting only characters that are less than $|C|$-close to the ends of T_j, are covered by C-occurrences.
3) Check by Local PM procedure whether $|C|$-prefix and $|C|$-suffix of T are completely covered by C-occurrences.

We can prove by induction that C is a cover iff we get only "yes" answers on Step 2 and Step 3.

Complexity analysis. Finding borders in all intervals requires $O(n^3 \log |T|)$ time. Cover-check procedure requires $O(n^3)$ time, since Step 2 and Step 3 use only $O(n^2)$ time. We call cover-check procedure at most $\log |T|$ times in each of $\log |T|$ intervals. Hence, the total complexity is $O(n^3 \log^2 |T|)$.

B. Fingerprint table

Let Σ be an alphabet. A *fingerprint* is a set of used characters of any substring in a text T. A *fingerprint table* is a set of all fingerprints for a given text. The algorithm for computing fingerprint table for usual (uncompressed) strings is presented in [1]. Important fact: for any string there might be at most $|\Sigma| + 1$ (including) different fingerprints for all its suffixes (prefixes).

Compressed fingerprint table: given a compressed string T, to compute a fingerprint table.

Sketch of the algorithm. For every j compute all prefix-fingerprint, all suffix-fingerprint and add to the table all fingerprints of "cut-containing" substrings of T_j by induction. At the end we clean the table from the repeated fingerprints. Hence, we have $O(n|\Sigma|^2 \log n)$ complexity and $O(n|\Sigma|^2)$ output size.

C. Hardness result

Compressed Hamming distance (inequality version): given integer h and SLPs generating T_1 and T_2, to check whether the Hamming distance (the number of characters which differ) between them is less than h.

Theorem 1: Compressed Hamming distance is NP- and coNP-hard (even inequality version).

Proof: Recall a well-known NP-complete SUBSET SUM problem [7]: Given integers $w_1, \ldots, w_n, t$ in binary form, to determine whether there exist $x_1, \ldots, x_n \in \{0, 1\}$ such that $\sum_{i=1}^{n} x_i \cdot w_i \quad = \quad t$.

Let us fix some input values for SUBSET SUM. We now (efficiently) construct P, Q and h such that $HD(P, Q) < h$ iff SUBSET SUM has a positive answer.

Let $s = w_1 + \dots w_n$. Then we take

$$P = (0^t 1 0^{s-t})^{2^n}, \quad T = \prod_{x=0}^{2^n-1} (0^{\bar{x}\cdot\bar{w}} 1 0^{s-\bar{x}\cdot\bar{w}}).$$

Here $\bar{x}\cdot\bar{w} = \sum x_i w_i$, and $\prod$ denotes concatenation. The string T (we call it *Lohrey string*) was presented for the first time in Lohrey's work [15] and then also used in [14]. It was proved in [15] that given input data of SUBSET SUM we can construct polynomial-size SLP generating P and T in polynomial time. But the inequality $HD(P,T) < 2^{n+1}$ holds iff the SUBSET SUM answer is "yes". Hence, the compressed Hamming distance is NP-hard. Inverting all bits in P and taking $h = |P| - 2^{n+1} + 1$ we can prove coNP-hardness. ■

V. Open Problems and Directions for Further Research

Algorithms.

- To speed up the presented algorithm for fully compressed pattern matching. *Conjecture:* $O(nm \log |T|)$ time is enough. More precisely, to show that computing new element of AP-table could be done in time $O(\log |T|)$.
- To construct a faster algorithm and/or with less memory requirements for compressed window subsequence problem. This problem was solved in paper [5] in $O(nk^2)$ time and space. *Conjecture:* $O(nk^2)$ time, but $O(nk)$ space.
- To construct $O(nm)$ algorithms for (weighted) edit distance, where n is original length of T_1 and m is the size of SLP generating T_2. This will lead to immediate speed up from any "super-logarithmic" compression, since only $O(\frac{n^2}{\log n})$ classical algorithm is known for this problem.
- To speed up the presented algorithm for compressed fingerprint table or find an n-SLP-generated text with $\Omega(n|\Sigma|^2)$ fingerprint table.

Complexity.

- The membership of compressed string in a language described by extended regular language is NP-hard. On the other hand it is in PSPACE. To find exact complexity of the problem (repeated from [21]).
- The fully compressed subsequence problem is Θ_2-hard [14]. On the other hand it is in PSPACE. To find exact complexity of the problem.
- The compressed Hamming distance is NP- and coNP-hard. On the other hand it is in PSPACE. To find exact complexity of the problem.
- Is compressed weighted edit distance NP-hard for any system of weights? What is exact complexity of the problem?

Other directions for further research include considering more classical string problems, more powerful models of string generation and experimental study of new algorithms on compressed texts. Also it is extremely interesting to find effective techniques besides dynamic programming.

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