Suballowable sequences of permutations

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Abstract

We introduce a notion of a suballowable sequence of permutations and prove a few combinatorial and geometric results on such sequences.

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1 The notion of a suballowable sequence of permutations

We define a *suballowable sequence of permutations* as an infinite periodic sequence of permutations of the set $\{1, 2, \ldots, n\}$ such that:

(i) Each term in the second half period is the reverse of the corresponding term in the first half period;

(ii) In the course of each half period, each pair of labels switches just once.

Any suballowable sequence of permutations is completely determined by its half-period.

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This notion generalizes that of an *allowable sequence of permutations* which was introduced in 1980 by Goodman and Pollack [1]. Combinatorial properties of allowable sequences were used in solving several problems in Discrete Geometry (on configurations of points, arrangements of lines, k-sets, $(\leq k)$-sets etc.).

2 Results

We prove the following results:

- Geometric permutations of a planar family of pairwise disjoint convex sets, ordered according to directions of corresponding transversal lines, form half a period of a suballowable sequence (a *geometric permutation* is the order in which a transversal line of such a family meets its members, described by a permutation of their labels).
- *Characterization of allowable sequences in the class of suballowable sequences:* Let $\mathcal{L}$ be a suballowable sequence. It is allowable if and only if it does not contain a move of the form $(\ldots a \ldots b \ldots c \ldots) \rightarrow (\ldots b \ldots c \ldots a \ldots)$ or of of the form $(\ldots a \ldots b \ldots c \ldots) \rightarrow (\ldots c \ldots a \ldots b \ldots)$.
- Each suballowable sequence may be completed to an allowable sequence.
- *A Helly-Type result on sets of permutations forming suballowable sequences:* Let $\mathcal{P}$ be a set of undirected permutations (an *undirected permutation* is a pair formed by a permutation and its reverse). It is possible to choose a representative of each member of $\mathcal{P}$ and to order them into half a period of suballowable sequence if and only if the same is true for any restriction of $\mathcal{P}$ to six labels.
- A suballowable sequence defined by half-period $(123456) \rightarrow (412563) \rightarrow (541632)$ is non-realizable by a configuration of points or by an arrangement of straight lines. It is the only (up to relabeling) suballowable non-realizable sequence on six labels with three terms in half-period.

References

[1] J. E. Goodman and R. Pollack. On the combinatorial classification of nondegenerate configurations in the plane, *J. of Combinatorial Theory, Ser. A* **29** (1980), 220-235.