

**06501 Abstracts Collection**  
**Practical Approaches to Multi-Objective  
Optimization**  
— Dagstuhl Seminar —

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**Abstract.** From 10.12.06 to 15.12.06, the Dagstuhl Seminar 06501 “Practical Approaches to Multi-Objective Optimization” was held in the International Conference and Research Center (IBFI), Schloss Dagstuhl. During the seminar, several participants presented their current research, and ongoing work and open problems were discussed. Abstracts of the presentations given during the seminar as well as abstracts of seminar results and ideas are put together in this paper. The first section describes the seminar topics and goals in general. Links to extended abstracts or full papers are provided, if available.

**Keywords.** Multi-criteria optimization, evolutionary and classical methods, interaction

## 06501 Summary – Practical Approaches to Multi-Objective Optimization

One can say that there are two communities dealing with multiobjective optimization problems: MCDM (multiple criteria decision making) and EMO (evolutionary multiobjective optimization) communities and they have remained rather isolated from each other during the years: they have their own conferences, journals, etc. This was the starting point and motivation of the First Dagstuhl Seminar on Practical Approaches to Multi-Objective Optimization which was organized in November 2004 (see <http://www.dagstuhl.de/de/program/calendar/semhp/?semnr=04461> for the seminar and <http://drops.dagstuhl.de/portals/index.php?semnr=04461> for the proceedings). The organizers were Juergen Branke (University of Karlsruhe, Germany), Kalyanmoy Deb (IIT Kanpur, India), Kaisa

Miettinen (Helsinki School of Economics, Finland) and Ralph E. Steuer (University of Georgia, USA).

During the First Dagstuhl Seminar, two aspects clearly emerged and were unanimously agreed by all the participants. Firstly, getting both MCDM and EMO researchers and applicationists together in one seminar for five days and in a Dagstuhl environment was beneficial to both groups in terms of understanding each other's approaches better and fostering collaboration. Secondly, all the participants thought that it was a good starting event, but there was an urgent need for the two groups to arrange more such extended meetings to continue the interactions. For these reasons, the Second Dagstuhl Seminar was organized in December 2006.

In the Second Dagstuhl Seminar on Practical Approaches to Multi-Objective Optimization, about 80 researchers were invited, about 40 from the MCDM and 40 from the EMO community and, in all, about 50 researchers were able to attend the seminar. The organizers of the Second Dagstuhl Seminar were the same as in the First Seminar with the exception that Roman Slowinski (Poznan University, Poland) replaced Ralph E. Steuer.

In connection with the Second Dagstuhl Seminar, we (the organizers) decided to initiate an ambitious project of writing a book covering both MCDM and EMO approaches and their hybridization possibilities.

We believe that this book has the potential to become a key reference and inspiration for the growing community dealing with the challenges of multiobjective optimization. To start with, some of the world's best experts from both communities were invited to write chapters for the book, for example, about different approaches in MCDM and EMO and how their benefits can be put together in order to get new hybrid methods. Special attention was paid to interactive methods and methods utilizing preference information because many EMO approaches have lacked these properties until recently. The contents of the chapters were discussed in the seminar in order not to miss any important topics and, also to avoid overlaps. Besides talks devoted to book chapters, the seminar program consisted of talks on recent research trends. In addition, an important part of the seminar was active work in working groups.

The topics of the working groups were: real-world applications of multiobjective optimization, software, quality of Pareto set approximations, MODM V a learning perspective, parallel approaches for multiobjective optimization, and future challenges. Besides the invited chapters, a book chapter will be prepared based on the work of each working group.

The title of the book was decided to be Multiobjective Optimization:

Interactive and Evolutionary Approaches and it will be published by Springer in the LNCS Series as a LNCS State-of-the-Art Survey. For further information about the Dagstuhl Seminar and participants, see <http://www.dagstuhl.de/program/calendar/semhp/?semnr=06501>.

On behalf of all the organizers, I would like to thank the participants for active discussions and attendance as well as for a very positive attitude towards the book project.

## EMO and a priori preferences

*Jürgen Branke (Universität Karlsruhe, D)*

Evolutionary multi-objective optimization usually attempts to find a good approximation to the complete Pareto optimal front. However, often the user has at least a vague idea about what kind of solutions might be preferred. In this chapter, we argue that by incorporating such knowledge, it is possible to focus the search on the most interesting (from a user's perspective) areas of the Pareto-optimal front, thereby reducing computation time and/or increasing resolution in the most relevant part of the front.

Different attempts to take user preferences into account are surveyed and compared.

## Evolutionary Multi-Objective Optimization and Interactive Approaches

*Kalyanmoy Deb (Indian Inst. of Technology - Kanpur, IND)*

In this talk, a brief introduction to evolutionary multi-objective optimization (EMO) methodologies will be presented. EMO methodologies use a population of solutions in each iteration, thereby allowing such a method to find multiple Pareto-optimal solutions in a single simulation run. This talk will also demonstrate other benefits of finding multiple trade-off optimal solutions, particularly in helping to unveil important insights in a problem and in solving other kinds of optimization problems, such as constrained optimization and goal programming. Finally, a couple of possibilities of using EMO with decision-making aides will be described to show ways to combine both multi-objective optimization and decision-making tasks together in an efficient manner.

*Keywords:* Multi-objective optimization, Evolutionary optimization, Decision-making, Interactive methodologies

## The attainment-function approach to stochastic multiobjective optimiser assessment and comparison

*Carlos Fonseca (University of Algarve, P)*

Since the (random) outcomes of stochastic multiobjective optimisers, such as multiobjective evolutionary algorithms, are sets of non-dominated solutions, analysing their statistical performance is challenging in that it involves studying the distribution of those sets. The attainment function, so named because it indicates the probability of an algorithm attaining arbitrary goals in objective space, has been related to results from random closed-set theory which cast it

as a mean-like, first-order moment measure of the distribution of the optimisation outcomes, and also suggest suitable definitions of higher order attainment functions.

This talk will overview the use of attainment function techniques in the experimental assessment of multiobjective optimisers, from performance characterisation to comparison. In addition, ways in which other approaches to multiobjective optimiser assessment may be related to the attainment function will be suggested. Finally, the attainment function will be shown to support a more general, multiobjective view of optimiser performance, in which runtime plays the role of an additional objective to be minimised.

*Joint work of:* Fonseca, Carlos M.; Grunert da Fonseca, Viviane

*Full Paper:*

<http://www.imada.sdu.dk/marco/EMAA/Papers/EMAA06-fonseca.pdf>

## **Interactive Optimization of Postal Routes**

*Pablo Funes (Icosystem, USA)*

This application optimizes postal routes for both efficiency and for individual postmen's preferences. Each postman's satisfaction is considered a separate objective.

*Keywords:* Multiple decision makers; collective interactive evolution, evolution, multiobjective optimization, postal routes, postman problem

*Joint work of:* Funes, Pablo; Bonabeau, Eric; Buchsbaum, Daphna; Budynek, Julien; Martens, Siegfried

## **Geometric Clustering (and its application to the consolidation of farmland)**

*Peter Gritzmann (TU München, D)*

In geometric clustering points of some finite point set in some Minkowski space have to be grouped together according to some balancing constraints so as to optimize some objective function. The prime example of a real-world clustering problem that motivates and guides our study is that of a lend-lease initiative for the consolidation of farmland.

Of course, the underlying mathematical clustering problem is NP-hard even in the most simple cases. We give and analyze a new norm maximization model for geometric clustering where in effect the centers of gravity of the clusters are pushed apart.

On the theoretical side, we show that this model facilitates appropriate separation. On the algorithmic side we derive a polynomial-time approximation algorithm that can handle the underlying large size convex maximization problem efficiently, yet lending itself to a tight worst case analysis showing how favourably this model compares with other possible formulations of the task.

*Keywords:* Geometric clustering, combinatorial optimization, norm maximization, approximation algorithms

*Joint work of:* Brieden, Andreas; Metzger, Christoph; Gritzmann, Peter

## **Incorporation of Scalarizing Fitness Functions into EMO Algorithms**

*Hisao Ishibuchi (Osaka Prefecture University, J)*

In this talk, we discuss two issues related to the incorporation of scalarizing fitness functions into evolutionary multiobjective optimization (EMO) algorithms. One is the use of EMO to optimize scalarizing fitness functions. We assume that a scalarizing fitness function to be optimized has already been generated from an original multiobjective problem. Our task is to optimize the given scalarizing fitness function. In order to efficiently search for its optimal solution without getting stuck in local optima, we generate a new multiobjective problem to which an EMO algorithm is applied. The point is to specify multiple objectives, which are similar to but different from the given scalarizing fitness function, so that the location of the optimal solution is near the center of the Pareto front of the generated multiobjective problem. The use of EMO algorithms helps escape from local optima. It also helps find a number of alternative solutions around the optimal solution. Difficulties of Pareto ranking-based EMO algorithms in the handling of many objectives are avoided by the use of similar objectives. The other issue is the use of scalarizing fitness functions in EMO algorithms to improve their search ability. We introduce two probabilities to specify how often scalarizing fitness functions are used for parent selection and generation update in EMO algorithms. In a special case with a single scalarizing fitness function, our idea can be viewed as the probabilistic use of an EMO scheme in single-objective evolutionary algorithms. From this point of view, we examine the effectiveness of our idea. In a general case with multiple scalarizing fitness functions, the use of scalarizing fitness functions has a potential ability to improve both the diversity of individuals and their convergence to the Pareto front.

## **Multiobjective Search Algorithm with Subdivision Technique**

*Johannes Jahn (Univ. Erlangen-Nürnberg, D)*

A multiobjective search algorithm with subdivision technique (MOSAST) for the global solution of multiobjective constrained optimization problems with possibly noncontinuous objective or constraint functions is presented. This method is based on a random search method and a new version of the Graef-Younes algorithm and it uses a subdivision technique. Numerical results are given for bicriterial test problems.

## Interactive EMO procedures, A. Jaszkiewicz

*Andrzej Jaszkiewicz (Poznan University of Technology, PL)*

- Introduction
- Classification of MOO methods
- Interactive evolution
- EA/metaheuristics in interactive MOO
- Traditional approach to interactive analysis with the use of single objective metaheuristics
- Main ideas, pros & cons, efficiency, examples
- Semi-a posteriori approach &#8211; interactive selection from a set of solutions generated by a multiobjective metaheuristic
- Main ideas, pros & cons, efficiency, examples
- Interactive multiobjective metaheuristics
- Main ideas, pros & cons, efficiency, examples
- Future trends and research directions

*Keywords:* Interactive procedures, evolutionary multiobjective optimization, computational efficiency

## Metamodeling in Multiobjective Optimization

*Joshua D. Knowles (Univ. of Manchester, GB)*

We survey the use of metamodeling in the iterative search of multiobjective functions. A broad definition of metamodeling is: any process of building an explicit model \*during\* search of a function, where that model is used to inform subsequent search. The reason behind metamodeling is almost always that the function one begins with is, in some sense, expensive to use (and it may be more or less a black box). Two distinct but not altogether orthogonal problems of interest arise as a result. One is essentially the problem of modeling the function itself with some simpler or more wieldy function—this is often called inverse modelling—and the job for the iterative search is how best to sample the original function in order to construct an informative analogue of it. The other is how to optimize the function, given a constraint or objective relating to the financial or time cost associated with conducting the search. The latter is the more difficult task because the need to optimize must be balanced against the need to improve the evolving model. Even in the case of single objective optimization, numerous factors impinge upon the metamodeling endeavour (e.g. noise, uncertainty, constraints); we review the effect of these factors and relate them to existing and potential approaches. In the case of metamodeling of multiobjective functions,

some additional factors come into play and there are extra dimensions to what one may hope to achieve as a result of the process.

In particular, for many-objective problems, there arises the possibility of using perceived correlations between objectives in some way, perhaps even reducing the dimension of the objective space during the search.

*Keywords:* Metamodeling, surrogate models, Kriging, learnable evolution model, multiobjective

## Pareto Frontier Visualization

*Alexander Lotov (Dorodnicyn Computing Center, RUS)*

Ch. 7 of the Dagstuhl book project is outlined. The content of the talk is as follows.

1. Why visualization of the Pareto frontier as a whole is important?
2. Stability of the Pareto frontier: why approximating the Pareto frontier is an ill-posed problem and what to do?
3. Classification of MCO problems related to Pareto frontier visualization.
4. Several words concerning two-criterion visualization.
5. Visualization in the case of a finite number of alternatives:
  - a) tools for a small number (not greater than a dozen) of Pareto-efficient alternatives;
  - b) tools for a medium number (not greater than 1000) of alternatives;
  - c) tools for a large number (greater than 1000) of alternatives.
6. Visualization in the case of infinite number of alternatives:
  - a) tools for the general case based on Pareto frontier approximation by a number of criterion points (both classical and EMO);
  - b) tools for the convex case based on polyhedral approximation:
    - b1) visualization in the case of three criteria; b2) visualization in the case of more than three criteria.
7. Interactive Pareto frontier visualization in interactive procedures (Pareto Race & Interactive Decision Maps, Pareto Step & Interactive Decision Maps).
8. Pareto frontier visualization in Web.

*Keywords:* Pareto frontier visualization

## Chapter 2: Classical Approaches

*Kaisa Miettinen (Helsinki School of Economics, FIN)*

We describe basic terminology and methods of multi-criterion optimization.

*Keywords:* Multiobjective optimization, Pareto optimality, methods

## Chapter 8: Interactive Methods

*Kaisa Miettinen (Helsinki School of Economics, FIN)*

We describe different interactive methods developed for multi-criterion optimization. They are based on trade-off information, reference points or classification.

*Keywords:* Interactive approaches, multiobjective optimization, methods, Pareto optimality

*Joint work of:* Miettinen, Kaisa; Wierzbicki, Andrzej; Ruiz, Francisco

### Reference point based dominance

*Julian Molina Luque (Universidad de Malaga, E)*

One of the main tools for including DMŠs preferences in the MOP literature is the use of reference points ( $g$ ) and achievement scalarizing functions (Wierzbicki, 1980). The main point in these approaches is converting the original multi-objective problem into a single-objective optimization problem through the use of a scalarizing function based on a reference point. As a result, a single efficient point adapted to the DMŠs preferences is obtained. However, a single solution can be less interesting than an approximation of the efficient set around this area. In this paper, we propose a variation of the concept of Pareto dominance, called  $g$ -dominance, based on the information included in a reference point  $g$ . This concept will let us approximate the efficient set around the area of the most preferred point without using any scalarizing function. On the other hand, we will show how it can be easily used with any MO evolutionary method or any MO metaheuristic (just changing the dominance concept) and, as example, we will show some results with some state-of-the-art-methods and some test problems.

*Keywords:* Reference point methods, interactive methods, MOEA, MOMH

*Joint work of:* Molina, Julian; Coello, Carlos; Hernandez-Diaz, Alfredo; Santana, Luis; Caballero, Rafael

### Handling a large number of objectives / Use of Metamodeling

*Hiroataka Nakayama (Konan University - Kobe, J)*

In this talk, I will introduce methods for treating a large number of objective functions along with practical examples in engineering design.

In particular, since trade-off analysis is important in practical situations, several devices for trade-off analysis for a large number of objectives will be discussed.

Another topic is metamodelling (sequential approximate optimization) techniques for multi-objective optimization in engineering design. In many practical engineering design problems, the form of objective functions is not given explicitly in terms of design variables. Given the value of design variables, under this circumstance, the value of objective functions is obtained by real/computational experiments such as structural analysis, fluidmechanic analysis, thermodynamic analysis, and so on. Usually, these experiments are considerably expensive. In order to make the number of these experiments as few as possible, optimization is performed in parallel with predicting the form of objective functions. Response Surface Methods (RSM) are well known along this approach. This paper presents a brief review of machine learning approaches to metamodelling (sequential approximate optimization) for multi-objective optimization such as Radial Basis Function Networks (RBFN) and Support Vector Machines (SVM). One of the most important tasks in this approach is to find effective sample data moderately in order to make the number of experiments as small as possible. Several methods are compared along with numerical examples.

Table of Contents: Handling a Large Number of Objectives:

1. Practical Examples
  - i) lens design
  - ii) construction accuracy control of cable-stayed bridges
2. Aspiration-based Interactive Multi-objective Programming
  - i) formulation and procedure
  - ii) optimization engines
3. Trade-off Analysis
  - i) automatic trade-off
  - ii) exact trade-off
  - iii) non-Lagrangean information
4. Graphical Interface
5. Real Applications

Using Meta-modeling (Sequential Approximate Optimization)

1. Review of Response Surface Method
2. Using Design of Experiment
3. Kriging Method
4. Computational Intelligence
  - i) radial basis function networks
  - ii) support vector machines
5. hybrid methods based on aspiration-based methods and generation of Pareto frontier
6. Examples
  - i) comparison through bench mark problems
  - ii) examples of real applications
    - reinforcement in antiseismic design of cable-stayed bridges
    - operation management in power generating plants

*Keywords:* Metamodelling, sequential approximate optimization, RBFN, SVM, a large number of objectives

## Optimization for Black-box Objective Functions

*Hiroataka Nakayama (Konan University - Kobe, J)*

In many practical engineering design problems, the form of objective functions is not given explicitly in terms of design variables. Given the value of design variables, under this circumstance, the value of objective functions is obtained by real/computational experiments such as structural analysis, fluidmechanic analysis, thermodynamic analysis, and so on. Usually, these experiments are considerably expensive. In order to make the number of these experiments as few as possible, optimization is performed in parallel with predicting the form of objective functions. Response Surface Methods (RSM) are well known along this approach. This paper presents a brief review machine learning approaches to RSM such as Radial Basis Function Networks (RBFN) and Support Vector Machines (SVM). One of the most important tasks in this approach is to find effective sample data moderately in order to make the number of experiments as small as possible. Several methods are compared along with numerical examples.

*Keywords:* Response surface method, RBF networks, Support vector machines, Genetic algorithms, Expected improvement

*Joint work of:* Nakayama, Hiroataka; Arakawa, Masao; Washino, Koji

*See also:* Optimization and Optimal Control, pp. 185-210; P.M. Pardalos, I. Tsevendorj, and R. Enkhbat, Editors; 2003 World Scientific Publishing Co.

## Nucleolar Reference Point Method and Fair Aggregations of Individual Achievements

*Wlodek Ogryczak (Warsaw Univ. of Technology, PL)*

The Reference Point Method (RPM) is based on the so-called augmented max-min aggregation.

Thus, the worst individual achievement is essentially maximized but the optimization process is additionally regularized with the term representing the average achievement.

The regularization by the average achievement is easily implementable but it may disturb the basic max-min model. The only consequent regularization of the max-min aggregation is the lexicographic max-min (nucleolar) solution concept where in addition to the worst achievement, the second worst achievement is also

optimized (provided that the worst remains on the optimal level), the third worst is optimized (provided that the two worst remain optimal), and so on. Such a nucleolar regularization satisfies the addition/deleting principle thus making the corresponding nucleolar RPM not affected by any passive criteria. The nucleolar regularization is more complicated in implementation but the recent progress made in optimization methods for ordered averages allows one to implement the nucleolar RPM quite effectively.

Both the theoretical and implementation issues of the nucleolar RPM are analyzed.

*Keywords:* Multi-criteria optimization, reference point method, lexicographic max-min, fairness

## **Multi-objective optimization and decision making process in engineering design**

*Silvia Poles (ESTECO - Trieste, I)*

This presentation shows a real world application of a multi-disciplinary and multi-objective optimization and decision making process in engineering design. Optimal design of complex engineering systems requires analysis that accounts for interactions amongst the disciplines. In this presentation both the hydraulic performances and the electro-mechanical design of a washing machine motor-pump are optimized. The main aim of this presentation is to demonstrate that a complete optimization software is needed for these kind of problems. A complete optimization software should contain tool for design of experiments, statistical analysis, several different optimization methods, decision making support tools, visualization charts, meta-modeling and even have the possibility to parallelized the entire computation.

*Keywords:* Multi-objective optimization, multi-disciplinary, engineering design

## **Interactive Methods for MCDM**

*Francisco Ruiz (Universidad de Malaga, E)*

We give an overview of interactive methods developed for solving nonlinear multi-objective optimization problems. In interactive methods, a decision maker plays an important part and the idea is to support her/him in decision making. In interactive methods, a solution pattern is formed and repeated and the decision maker progressively provides preference information so that the most preferred solution can be found. We identify three types of specifying preference information in interactive methods and give some examples of methods. The types are methods based on trade-off information, reference points and classification.

*Keywords:* Multiobjective Programming, Interactive Methods, Reference Point Approaches

*Joint work of:* Miettinen, Kaisa; Ruiz, Francisco; Wierzbicki, Andrzej P.

## MO problems in aerospace industry and universities

*Daisuke Sasaki (Cambridge University, GB)*

Industrial design problems often have many design objectives, which may have conflicting requirements. Many designers or engineers are interested in obtaining the trade-offs between objectives to choose the best design or to understand the trade-off space. To identify such trade-offs, the problem has to be treated as Multi-Objective (MO) optimisation. Nowadays, MOEAs (Multi-Objective Evolutionary Algorithms) have gained popularity because of its efficient and effective search for obtaining Pareto solutions. As it is well-known, MOEAs require a large number of evaluations. This could be a major inhibitor in using MOEAs for aerodynamic optimisation using time-consuming high-fidelity CFD (Computational Fluid Dynamics). Efficient algorithms, parallel algorithms, or using metamodel may be a candidate for this problem. In addition to the computational time, there are many other issues to be considered for conducting the optimisation. For example, visualisation and analysis of results are quite important for designers for the further improvements. In the presentation, the application of MOEAs to turbomachinery optimisation problems is presented.

*Keywords:* EMO, aerodynamic optimization, multi-criteria optimization

## Some Exact and Heuristic Algorithms in Multiple Objective Discrete Optimization

*Serpil Sayin (Koc University, TR)*

Vector optimization approaches in Multiple Objective Discrete Optimization (MODO) propose to find the entire set of efficient solutions of the problem to provide a complete picture of trade-off information inherent in the decision making situation. This has proven to be a computationally challenging approach except for cases that possess some special mathematical structure. Recently, a few exact algorithms for the general bicriteria discrete optimization problem, and numerous ones for special structured problems such as the knapsack, assignment, or network problems have been introduced. Some of these algorithms suggest computationally tractable heuristic modifications. In this talk, we will briefly revisit the findings of the traditional heuristic approaches to MODO problems. The issue of quality measures will be elaborated on. We will then discuss how the information derived from the heuristic efficient sets may be incorporated into an evolutionary setting.

*Keywords:* Multiple objective discrete optimization, heuristic efficient set, quality measures

## Multi-Objective Optimization for Robust Airfoil Design Considering Design Errors and Uncertainties

*Koji Shimoyama (Tohoku University, J)*

A new optimization approach for robust design, design for multi-objective six sigma (DFMOSS) has been developed and applied to robust aerodynamic airfoil design for Mars exploratory airplane. The present robust aerodynamic airfoil design optimization using DFMOSS successfully showed the trade-off information between maximization and robustness improvement in aerodynamic performance by one optimization run without careful input parameter tuning. The obtained trade-off information indicated that an airfoil with a smaller maximum camber improves robustness of lift to drag ratio, and that with a larger curvature near the shock wave location improves robustness of pitching moment against the variation of flight Mach number.

*Keywords:* Robust Optimization, Design for Multi-Objective Six Sigma, Airfoil Design

*Joint work of:* Shimoyama, Koji; Oyama, Akira; Fujii, Kozo

## Slides for talk on Real World Applications

*Theodor J. Stewart (Univ. of Cape Town, ZA)*

Much of our experience arises from discrete choice problems involving intensive interaction with decision makers and stakeholders. Optimization methods are sometimes used to generate a short list for deeper evaluation with groups. This approach is illustrated by two examples in which a reference point method is solved using a GA in order to generate an efficient solution. Repeated application with systematically varying reference points interactively generates the required short list for further evaluation.

*Keywords:* MCDM, Reference point, genetic algorithm

## Preference information levels and the robustness of MCDM classification methods

*Rudolf Vetschera (Universität Wien, A)*

Most multi-criteria decision methods rely on subjective information about preferences from the decision maker. Since the direct specification of preference parameters like attribute weights is rather difficult for decision makers, several methods have been proposed to simplify this cognitive task. One class of methods, which are particularly popular in multicriteria classification problems, are

case-based methods, for example [1]. In the case based approach, the decision maker is asked to classify a number of example cases into different categories. From these examples, a set of preference parameters (attribute weights) is estimated, which then can be used to classify additional cases according to the decision maker's preferences.

While the case based approach greatly simplifies the cognitive task of the decision maker in specifying preference parameters, it also leads to two problems: the first problem is the robustness of results. Since the classifications elicited from the decision maker provide only limited information on the decision maker's preferences, the preference parameters and the resulting classifications of additional cases are not necessarily unique. We propose a measure of robustness based on random sampling of possible preference parameters and potential classification changes, which provides an indicator of the quality of the preference information obtained.

The second problem concerns the amount of information to be provided by the decision maker.

Obviously, the more information a decision maker provides (in the form of additional classifications of sample cases), the more robust the solution will become. In a second step of our analysis, we therefore study the relationship between the amount of information provided by the decision maker, and the resulting robustness of results using a computational model. We test several hypotheses about the influence of additional information and problem parameters on robustness. Our results indicate that non-uniqueness of parameters in multi-criteria classification problems is a rather frequent phenomenon. Providing additional information (cases) reduces the problem, but surprisingly we do not find evidence for decreasing returns of information. Various problem parameters like the number of classes or attributes also have a significant effect. [1] Ye Chen, Kevin W. Li, D. Marc Kilgour, Keith W. Hipel: A case-based distance model for multiple criteria ABC analysis. *Computers and Operations Research* 2006 (forthcoming)

*Keywords:* Multicriteria classification, robustness, information

*Joint work of:* Vetschera, Rudolf; Hipel, Keith; Chen, Richard

## Reference Point Approaches and Objective Ranking

*Andrzej Wierzbicki (JAIST - Ishikawa, J)*

The paper presents a reflection on some of the basic assumptions and philosophy of reference point approaches, stressing their unique concentration on the sovereignty of the subjective decision maker. As a new development in reference point approaches also the concept of objective ranking is stressed, defined as dependent only on a given set of data, relevant for the decision situation, and independent from any more detailed specification of personal preferences than that given by defining criteria and the partial order in criterion space. Rational objective ranking can be based on reference point approach, because reference

levels needed in this approach can be established objectively statistically from the given data set. Examples show that such objective ranking can be very useful in many management situations.

*Keywords:* Multiple criteria optimization and decisions; reference point approaches; objectivity and subjectivity in decision support

*Full Paper:* <http://drops.dagstuhl.de/opus/volltexte/2007/1121>

## Design of Set Quality Measures Using the Hypervolume Indicator

*Eckart Zitzler (ETH Zürich, CH)*

The design of quality measures for approximations of the Pareto-optimal set is of high importance not only for the performance assessment, but also for the construction of multiobjective optimizers. Various measures have been proposed in the literature with the intention to capture different preferences of the decision maker. A quality measure that possesses a highly desirable feature is the hypervolume measure: whenever one approximation completely dominates another approximation, the hypervolume of the former will be greater than the hypervolume of the latter. Unfortunately, this measure is as any measure inducing a total order on the search space; is biased, in particular towards convex, inner portions of the objective space. Thus, an open question in this context is whether it can be modified such that other preferences such as a bias towards extreme solutions can be obtained. This talk presents a methodology for quality measure design based on the hypervolume measure and demonstrates its usefulness for three types of preferences.

*Keywords:* Quality indicators, hypervolume indicator, multiobjective optimization, evolutionary algorithms

*Joint work of:* Zitzler, Eckart; Brockhoff, Dimo; Thiele, Lothar