07441 Abstracts Collection
Algorithmic-Logical Theory of Infinite Structures
— Dagstuhl Seminar —

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Abstract. From 28.10. to 02.11.2007, the Dagstuhl Seminar 07441 "Algorithmic-Logical Theory of Infinite Structures" was held in the International Conference and Research Center (IBFI), Schloss Dagstuhl. During the seminar, several participants presented their current research, and ongoing work and open problems were discussed. Abstracts of the presentations given during the seminar as well as abstracts of seminar results and ideas are put together in this paper. The first section describes the seminar topics and goals in general. Links to extended abstracts or full papers are provided, if available.

Keywords. Theories of infinite structures , computable model theory and automatic structures , model checking infinite systems

07441 Summary – Algorithmic-Logical Theory of Infinite Structures

One of the important research fields of theoretical and applied computer science and mathematics is the study of algorithmic, logical and model theoretic properties of structures and their interactions. By a structure we mean typical objects that arise in computer science and mathematics such as data structures, programs, transition systems, graphs, large databases, XML documents, algebraic systems including groups, integers, fields, Boolean algebras and so on.

Keywords: Theories of infinite structures , computable model theory and automatic structures , model checking infinite systems

Joint work of: Downey, Rod; Khoussainov, Bakhadyr; Kuske, Dietrich; Lohrey, Markus; Vardi, Moshe Y.

Extended Abstract: http://drops.dagstuhl.de/opus/volltexte/2008/1411

Simple Monadic Theories

Achim Blumensath (TU Darmstadt, D)

We give an overview over recent developments concerning the model theory of monadic second-order logic. On the one hand, there are tree-like structures whose monadic theory is simple enough to develop a structure theory.

On the other hand, there are structures with definable pairing functions where monadic second-order logic is as expressive as full second-order logic.

According to a conjecture of Seese these two cases form a dichotomy: either a structure is 'tree-like' or it has a definable pairing functions.

For graphs (or structures with relations of arity at most 2) a variant of this conjecture has recently been proved by Courcelle and Oum.

In this talk we will present first partial results concerning the general case.

We consider structures without pairing function and we prove that the partition width of such structures is bounded by $2^{2^{\aleph_0}}$.

MSO + cardinality quantifiers on countable trees and linear orders

Vince Bárány (RWTH Aachen, D)

We consider the extension of MSO with quantifiers "there are infinitely/uncountably many sets X such that ..." .

We provide uniform and effective reductions of this logic to MSO on countable linear orders and on finitely branching trees.

On infinitely branching trees our reduction is into the extension of MSO with the second-order predicate "X is a finite set", which can not be eliminated.

These results generalise the work of Niwinski '91, concerning parameter free formulas on regular trees, and of Kuske and Lohrey '06 on the linear ordering ω.

This is joint work with Lukasz Kaiser and Alex Rabinovich. The cooperation was made possible by AutoMathA. Also thanks to Sasha Rubin for discussions.

I will also advertise another result obtained with Lukasz Kaiser and Sasha Rubin on counting subsets of ω modulo an MSO-definable equivalence relation. This is another extension of a result of Kuske and Lohrey '06 and is connected to some problems concerning injective ω-automatic (alias Buchi-automatic) presentations. Cf. talk by Andre Nies.

Joint work of: Bárány, Vince; Kaiser, Lukasz; Rabinovich, Alex

Tree Automata Make Ordinal Theory Easy

Thierry Cachat (LIAFA - Université Paris VII, F)

We give a new simple proof of the decidability of the First Order Theory of $(\omega^{\omega^i}, +)$ and the Monadic Second Order Theory of $(\omega^i, <)$, improving the complexity in both cases. Our algorithm is based on tree automata and a new representation of (sets of) ordinals by (infinite) trees.

Keywords: Ordinals, First Order theory, Monadic Second Order Theory, tree automata

Full Paper: http://drops.dagstuhl.de/opus/volltexte/2008/1408

Full Paper:
http://hal.archives-ouvertes.fr/hal-00110485

See also: T.Cachat, Tree Automata Make Ordinal Theory Easy, proceedings of FSTTCS 2006

Deterministic graph grammars

Didier Caucal (CNRS, F)

Deterministic graph grammars were originally introduced to finitely describe the structure of pushdown automata transition graphs.

Even though these devices have existed for about twenty years, it is quite surprising to see that they are not really used.

In this talk, we present deterministic graph grammars and the graphs they generate (called regular graphs) and describe two recent applications.

First, we extend well-known transformations over finite automata to the context of deterministic graph grammars, namely the synchronized product, determinization and complementation constructions, as well as concatenation and its Kleene closure.

For each deterministic regular graph, we obtain an effective boolean algebra of deterministic context-free languages containing the regular languages, and an effective boolean algebra of deterministic context-free languages closed by concatenation and its Kleene closure.

A second application is to use deterministic graph grammars to extend standard algorithms from finite graph theory to the class of regular graphs, like for instance shortest path algorithms.

A combinatorial theorem for trees, application to logics and infinite structures

Thomas Colcombet (LIAFA - Université Paris VII, F)

Following the idea developed by I. Simon in his theorem of Ramseyan factorisation forests, we develop a result of 'deterministic factorisations'.

This extra determinism property makes it usable on trees (finite or infinite).

We apply our result for proving that, *over trees*, every monadic (second-order) interpretation is equivalent to the composition of a first-order interpretation (with access to the ancestor relation) and a monadic marking. Equivalently, every monadic query is equivalent to a first-order query with access to monadicaly definable unary predicates.

Using this remark, we give new characterisations for prefix-recognisable structures and for the Caucal hierarchy.

Keywords: Semigroups, Ramseyan factorisation, Monadic second-order logic, Trees, Infinite structures

Non-Archimedean Words over Groups

Volker Diekert (Universität Stuttgart, D)

This the handout of my lecture at Dagstuhl seminar 07441. The material is based on joint work with Alexei Miasnikov.

The idea is to use non-archimedean words as a tool to define non-standard Cayley graphs and to apply standard combinatorics on words. A concrete goal is to prove decidability of elementary theries in, say, fully residually free groups.

In my lecture I present some underlying general concepts. No particular knowledge in group theory is required.

Keywords: Algorithmic group theory

Weighted automata and weighted logic

Manfred Droste (Universität Leipzig, D)

In automata theory, a classical result of Büchi states that the recognizable languages are precisely the ones definable by sentences of monadic second order logic. We will present a generalization of this result to the context of weighted automata.

A weighted automaton is a classical nondeterministic automaton in which each transition carries a weight describing e.g. the ressources used for its execution, the length of time needed, or its reliability. The behaviour (language) of such a weighted automaton is a function associating to each word the weight of its execution. We develop syntax and semantics of a quantitative logics; the semantics counts 'how often' a formula is true.

Our main result shows that if the weights are taken in an arbitrary commutative semiring, then the behaviors of weighted automata are precisely the functions definable by sentences of our quantitative logic.

M. Droste, P. Gastin: Weighted automata and weighted logics, Theoret. Comp. Science 380 (2007), 69-86; extended abstract in: 32nd ICALP, LNCS vol. 3580, Springer, 2005, pp. 513-525.

Keywords: Weighted automata, weighted logic, BÜchi theorem, formal power series

Joint work of: Droste, Manfred; Gastin, Paul

See also: M. Droste, P. Gastin: Weighted automata and weighted logics, Theoret. Comp. Science 380 (2007), 69-86; extended abstract in: 32nd ICALP, LNCS vol. 3580, Springer, 2005, pp. 513-525.

PDL with Intersection and Converse is 2EXP-complete

Stefan Göller (Universität Leipzig, D)

The logic ICPDL is the expressive extension of Propositional Dynamic Logic (PDL), which admits intersection and converse as program operators.

The result of this paper is containment of ICPDL-satisfiability in 2EXP, which improves the previously known non-elementary upper bound and implies 2EXP-completeness due to an existing lower bound for PDL with intersection (IPDL). The proof proceeds showing that every satisfiable ICPDL formula has model of tree width at most two. Next, we reduce satisfiability in ICPDL to ω-regular tree satisfiability in ICPDL. In the latter problem the set of possible models is restricted to trees of an ω-regular tree language. In the final step,ω-regular tree satisfiability is reduced the emptiness problem for alternating two-way automata on infinite trees. In this way, a more elegant proof is obtained for Danecki's difficult result that satisfiability in IPDL is in $2EXP$.

Keywords: Satisfiability, Propositional Dynamic Logic, Computational Complexity

Joint work of: Göller, Stefan; Lohrey, Markus; Lutz, Carsten

Extended Abstract: http://drops.dagstuhl.de/opus/volltexte/2008/1409

Full Paper:
http://www.springerlink.com/content/a5mk0x0413q1565p/

See also: S.Göller, M. Lohrey, and C. Lutz. PDL with Intersection and Converse is 2EXP-complete. In Proceedings of the 10th International Conference on Foundations of Software Science and Computational Structures (FoSSaCS 2007), Braga (Portu- gal), number 4423 in Lecture Notes in Computer Science, pages 198–212. Springer, 2007.

Model Checking and Games with Hierarchical Information

Lukasz Kaiser (RWTH Aachen, D)

We introduce games played on graphs by two coalitions of players with imperfect information exchanged with respect to a hierarchical constraint.

These games can be used for model checking first-order logic on omega-automatic structures and winning in such games on finite arenas can be expressed in first-order logic extended with game quantifier. We discuss possible variations in the definition of such hierarchical games and utility of these games for model checking.

Keywords: Games, automatic structures

Open questions in the theory of automatic structures

Bakhadyr Khoussainov (University of Auckland, NZ)

This talk is an attempt to collect several open questions in the theory of automatic structures and present some background to these questions. We classify our questions into three categories. The first category of questions we call *algebraic questions.*

They ask about descriptions of the isomorphism invariants of automatic structures.

The second category of questions we call *computability-theoretic questions.*

These questions address computability-theoretic complexity of algebraic and model theoretic properties of automatic structures. Here complexity is meant to refer to either a degree theoretic complexity or time (space) complexity. Finally, the third category of questions we call *model-theoretic questions.* We aim these questions to develop model theory of automatic structures. We also propose some possible directions towards *generalization* as well as *specialization* of automatic structures.

Keywords: Automata, Automatic structures

On the Automata Size for Automatic Structures

Felix Klaedtke (ETH Zürich, CH)

Various logical theories can be decided by automata-theoretic methods.

Prominent examples are Presburger arithmetic FO(N,+) and real addition FO(R,+,<). An automata-based decision procedure for such a logical theory can be outlined as follows: for a formula, one constructs by recursion over the formula structure an automaton that accepts the word representatives of the satisfying elements. The satisfiability problem is thus reduced to the emptiness problem for automata. The logical theories that admit such automata-based decision procedures are often called automatic.

A crude complexity analysis of such an automata-based decision procedure leads only to a non-elementary upper bound on the size of the constructed automaton. In this talk, we will present a new approach—based on model-theoretic methods—to establish better worst-case upper bounds on the automata size.

We will illustrate this approach by establishing an exponential-quadratic upper bound on the automata size for FO(N,<). Moreover, we report on upper bounds that we obtained by applying the presented approach to other automatic structures, e.g. real addition.

Stochastic Games with PCTL Objectives

Antonin Kucera (Masaryk University, CZ)

We survey recent results about stochastic games and Markov decision processes where the winning objectives are specified by formulae of branching-time probabilistic logics such as PCTL or PCTL*.

Keywords: Stochastic games, Markov decision processes, probabilistic logics

Full Paper:
http://www.fi.muni.cz/usr/kucera/publications.html

Compatibility of Shelah and Stupp's and Muchnik's iteration with fragments of monadic second order logic

Dietrich Kuske (Universität Leipzig, D)

We investigate the relation between the theory of the iterations in the sense of Shelah-Stupp and of Muchnik, resp., and the theory of the base structure for several logics. These logics are obtained from the restriction of set quantification in monadic second order logic to certain subsets like, e.g., finite sets, chains, and finite unions of chains. We show that these theories of the Shelah-Stupp iteration can be reduced to corresponding theories of the base structure, a result that fails for Muchnik's iteration.

Full Paper: http://drops.dagstuhl.de/opus/volltexte/2008/1407

Ideals in computable rings and subspaces of computable vector spaces

Steffen Lempp (University of Wisconsin - Madison, USA)

We show that the existence of a nontrivial proper ideal in a commutative ring with identity which is not a field is equivalent to WKL_0 over RCA_0, and that the existence of a nontrivial proper finitely generated ideal in a commutative ring with identity which is not a field is equivalent to ACA_0 over RCA_0. We also prove that there are computable commutative rings with identity where the nilradical is $Sigma^0_1$-complete, and the Jacobson radical is Π^0_2-complete, respectively. (Joint work of Downey, Lempp and Mileti.)

In a follow-up project (by all coauthors), we show that the existence of a nontrivial proper subspace of a vector space of dimension greater than one (over an infinite field) is equivalent to WKL_0 over RCA_0, and that the existence of a finite-dimensional nontrivial proper subspace of such a vector space is equivalent to ACA_0 over RCA_0.

Keywords: Computable algebra, reverse mathematics, computable ring, computable vector space, ideal, subspace

Joint work of: Downey, Rodney G.; Hirschfeldt, Denis R.; Kach, Asher M.; Lempp, Steffen (presenter); Mileti, Joseph R.; and Montalbán, Antonio

Full Paper:
http://www.math.wisc.edu/∼lempp/papers/ideals.pdf

Full Paper:
http://www.math.wisc.edu/∼lempp/papers/subspace.pdf

See also: Ideals in computable rings, by Rodney G. Downey, Steffen Lempp, and Joseph R. Mileti, J. Algebra 314 (2007) 872-887; and Subspaces of computable vector spaces, by Rodney G. Downey, Denis R. Hirschfeldt, Asher M. Kach, Steffen Lempp, Joseph R. Mileti, and Antonio Montalbán, J. Algebra 314 (2007) 888-894

Application of verification techniques to inverse monoids

Markus Lohrey (Universität Leipzig, D)

The word problem for inverse monoids generated by a set Γ subject to relations of the form $e = f$, where e and f are both idempotents in the free inverse monoid generated by Γ, is investigated. It is shown that for every fixed monoid of this form the word problem can be solved in polynomial time which solves an open problem of Margolis and Meakin. For the uniform word problem, where the presentation is part of the input, EXPTIME-completeness is shown.

For the Cayley-graphs of these monoids, it is shown that the first-order theory with regular path predicates is decidable. Regular path predicates allow to state that there is a path from a node x to a node y that is labeled with a word from some regular language. As a corollary, the decidability of the generalized word problem is deduced. Finally, some results on free partially commutative inverse monoids are presented.

Keywords: Inverse monoids, word problems, Cayley-graphs, complexity

Extended Abstract: http://drops.dagstuhl.de/opus/volltexte/2008/1410

See also: Inverse monoids: decidability and complexity of algebraic questions (mit Nicole Ondrusch), Information and Computation 205(8), S. 1212-1234, 2007

MSO on the infinite binary tree: Choice and order

Christof Löding (RWTH Aachen, D)

We give a new proof showing that it is not possible to define in monadic second-order logic (MSO) a choice function on the infinite binary tree. This result was first obtained by Gurevich and Shelah using set theoretical arguments. Our proof is much simpler and only uses basic tools from automata theory. We discuss some applications of the result concerning unambiguous tree automata and definability of winning strategies in infinite games. In a second part we strengthen the result of the non-existence of an MSO-definable well-founded order on the infinite binary tree by showing that every infinite binary tree with a well-founded order has an undecidable MSO-theory.

Full Paper:
http://automata.rwth-aachen.de/publications/pub-Loeding.html

See also: In Proceedings of the 16th Annual Conference of the European Association for Computer Science Logic, CSL 2007, volume 4646 of Lecture Notes in Computer Science, pages 161-176. Springer, 2007.

Analyzing Markov Reward Models in Infinite Systems with Finite Attractors

Richard Mayr (North Carolina State University, USA)

Probabilistic program models can be used to describe systems that exhibit uncertainty, such as communication protocols over unreliable channels, randomized algorithms in distributed systems, or fault-tolerant systems.

Their semantics is defined in terms of Markov chains.

Their usage of resources (time, memory, bandwidth, etc.) can be modeled by assigning a reward (or cost) to individual transitions, or, more generally, to whole computation paths. The resulting Markov reward model can be analyzed to compute, e.g., (1) the conditional expected reward until reaching a given target state (or a set of target states), or (2) the limit-average expected reward (i.e., the expected reward per step in the long run).

While these problems are well studied for finite-state systems, more recent works consider infinite-state Markov reward models derived from program-like structures (modeling, e.g., communication, counters, concurrency).

Here we give an overview over recent work and open problems in this area.

In particular, we discuss the infinite-state Markov reward models induced by probabilistic lossy channel systems and probabilistic vector addition systems and show that they have a particular structure which can be used for their algorithmic analysis. We show under which circumstances path exploration schemes can be used to compute arbitrarily close approximate solutions to the problems mentioned above.

Keywords: Markov Chains, Markov Reward Models

Model theoretic complexity of automatic structures

Mia Minnes (Cornell University, USA)

We use notions of rank from model theory and logic to study the complexity of automatic structures. In particular, we look at the ordinal height of well-founded relations, the Scott rank of arbitrary structures, and the Cantor-Bendixson rank of trees. We show that, in general, ranks of automatic structures can be as high as possible. But, if we restrict to classes (such as partial orders), then there is a low bound on the ranks.

This is joint work with Bakhadyr Khoussainov.

Countable structures sigma-definable over R,C, and H

Andrey Morozov (Sobolev Institute of Mathematics - Novosibirsk, RUS)

We characterize countable algebraic structures sigma-definable in hereditarily finite superstructures over reals, complex numbers and quaternions.

Keywords: Admissible set, hereditarily finite superstructure, sigma-definable structure, reals, complex numbers, quaternions, computable model

Joint work of: Morozov, Andrey; Korovina, Margarita

Borel Structures

Andre Nies (University of Auckland, NZ)

Traditionally, effectivity is studied for countable structures. Borel structures in contrast allow us to develop a theory of effectivity for the equally natural uncountable structures, such as the field of real numbers. After some initial work by Friedman (1979), the forthcoming paper entitled ŞFrom automatic structures to Borel structuresŠŠ by Khoussainov, Hjorth, Montalban and myself has revived the subject by applying Borel structures to solve a well-known question on Buechi presentable structures; see Section 5 of Nies' Bull. Symb. Logic paper ŞDescribing GroupsŠŠ, Sept 2007. We show that there is a Buechi presentable structure without an injective Buechi representation. Further, there exists a Rabin presentable structure that is not Borel.

Keywords: Borel Structures, Buechi automata, Rabin automata

Joint work of: Hjorth, Greg; Khoussainov, Bakh; Montalban, Antonio; Nies, Andre

Global model-checking of infinite trees

Chih-Hao Luke Ong (Oxford University, GB)

Given a state-transition system/graph and a logic, *global model checking* is the problem of computing a finite representation of the set of states satisfying a given formula. We consider the global model checking problem for the class of (possibly infinite) ranked trees generated by *higher-order recursion schemes*. This is a general class which forms an infinite hierarchy: it includes the regular, algebraic and hyperalgebraic trees (corresponding to orders $0, 1$ and 2 respectively), subsuming the Caucal Tree Hierarchy.

Given an order-n recursion scheme and given an alternating parity tree automaton, we show that the set of pairs (α, q), such that there is an accepting run of the automaton starting from the state q reading node α, is recognizable by an order-n *collapsible pushdown automaton* (CPDA). It follows that the set of nodes of an order-n tree satisfying a given monadic second-order formula is computable by an order-n CPDA.

Keywords: Global model checking, monadic second-order logic, infinite trees, higher-order recursion schemes, (collapsible) pushdown automata

An overview of Automatic Structures

Sasha Rubin (University of Auckland, NZ)

After introducing the relevant background (S1S, S2S), I will focus on recent advances on the quotient problem and extensions of FO decidability. I will point out the advantages of considering specific automatic presentations (automatic groups, automatic words), and mention a natural generalisation of automaticity (corresponding to S2S with decidable unary predicate).

The regular Post embedding problem and lossy channel systems.

Philippe Schnoebelen (ENS - Cachan, F)

Post's embedding problem (PEP) is a variant of Post's correspondence problem (PCP) where one compares strings with the subword relation. PEP_reg is a further variant where solutions are constrained and must belong to a given regular language (or omega-regular).

We describe PEP_reg and discuss related variants. The main result is the equivalence between PEP_reg and verification problems on lossy channel systems.

Algorithmics of Alternating Pushdown Automata

Stefan Schwoon (TU München, D)

Pushdown automata (PDA) are a natural model for programs with procedures. For this reason, algorithms for the analysis of PDA (and probabilistic PDA) have been intensively studied in recent years. In the talk, which builds upon recent joint work with Ahmed Bouajjani, Javier Esparza, and Dejvuth Suwimonteerabuth, I will show how how some of the algorithmic ideas can be transferred to the analysis of alternating PDA. In particular, we identify a subclass of alternating PDA where the analysis problems can be solved efficiently. Based on these ideas, we propose a system for computing a measure of academic reputation.

Playing over extensions of pushdown automata

Olivier Serre (LIAFA - Université Paris VII, F)

In this talk, we propose to discuss several extensions of pushdown automata: higher-order pushdown automata, stack data pushdown automata (i.e. pushdown automata with infinite stack alphabet without any restriction), and collapsible pushdown automata (aka panic automata). For those three models we infer that the spirit of the decidability proof of parity games for pushdown games by Walukiewicz [1996] can be generalized. In particular, it leads optimal decidabiliy bound for games over higher-order pushdown graphs and collapsible pushdown graphs.

We will discuss the ideas behind those results and also consequences, in particular:

- precise description of effective winning strategies in those games;
- finite representation of the winning region (eq. global model-checking for mu-calculus)

Keywords: Parity games, pushdown automata and extension

The Atomic Model Theorem and Type Omitting

Richard Shore (Cornell University, USA)

We continue the analysis of the complexity of constructing atomic models begun in Denis Hirschfeldt's talk. We also deal with other type omitting theorems.

The analysis is given both in terms of Turing degrees and reverse mathematics.

In particular, we provide two type ommitting theorems that make no mention of recursion theoretic notions but are nonetheless equivalent to the existence of hyperimmune and nonrecursive degrees, respectively.

This is joint work with Denis Hirschfeldt and Theodore Slaman.

Keywords: Atomic models, type omitting, reverse mathematics, hyperimmune, nonrecursive

Joint work of: Hirschfeldt, Denis; Shore, Richard; Slaman, Theordore

On Translations between Timed-Arc Petri Nets and Timed Automata

Jiri Srba (Aalborg University, DK)

A timed extension of Petri nets where every token has an associated age from the domain of real numbers is a well-studied formalism for modelling of real-time systems. Unfortunately, there is no tool support for the verification of such nets yet. In this talk I will describe mutual translations between the timed extension of Petri nets and the model of timed automata as used e.g. in the verification tool UPPAAL. A prototype tool with GUI translating Petri net verification questions to UPPAAL-ready timed automata will be presented, including some preliminary experimental results.

Keywords: Timed-arc Petri net, timed extensions, timed automata, UPPAAL, verification

Classifying FA-presentable structures

Rick Thomas (University of Leicester, GB)

In this talk we will discuss the problem of classifying classes of FA-presentable structures. We will concentrate on some classes of groups, semigroups and rings; the intention is to give a survey of some recent results. In the case of finitely generated groups and semigroups one can compare the classes of FA-presentable groups and semigroups on the one hand and the classes of automatic groups and semigroups on the other; we will make some comments about the relationships between these classes.

MSO-Properties over Infinite Graphs

Wolfgang Thomas (RWTH Aachen, D)

This talk is a survey on decidability of theories of finitely presented infinite graphs.

Keywords: Automatic graphs, pushdown graphs, tree rewriting graphs