07461 Abstracts Collection Numerical Methods for Structured Markov Chains

— Dagstuhl Seminar —

Dario Bini 1, Beatrice Meini 2, Vaidyanathan Ramaswami 3, Marie-Ange Remiche 4 and Peter Taylor 5

1 Univ. of Pisa, IT
bini@dm.unipi.it
2 Univ. of Pisa, IT
meini@dm.unipi.it
3 AT&T Florham Park, US
4 Free Univ. of Bruxelles, BE
5 Univ. of Melbourne, AU
p.taylor@ms.unimelb.edu.au

Abstract. From 11.11. to 14.11.07, the Dagstuhl Seminar 07461 "Numerical Methods for Structured Markov Chains" was held in the International Conference and Research Center (IBFI), Schloss Dagstuhl. During the seminar, several participants presented their current research, and ongoing work and open problems were discussed. Abstracts of the presentations given during the seminar as well as abstracts of seminar results and ideas are put together in this paper. The first section describes the seminar topics and goals in general. Links to extended abstracts or full papers are provided, if available.

Keywords. Matrix analytic methods, markov processes, queuing theory, numerical methods, structured matrices, telecommunication modeling, performance evaluation

07461 Executive Summary – Numerical Methods for Structured Markov Chains

This Dagstuhl seminar has brought together leaders and young researchers in the fields of analysis of numerical algorithms, applied stochastic modeling and statistical inference, with the result of stimulating exchange of methodologies and experiences and generating synergetic collaborations.

This has favored a better communication between these worlds where problems from the applications feed the theoretical research and where advanced numerical tools can be utilized in applications with reciprocal advantages.

Keywords: Matrix analytic methods, Markov processes, queuing theory, numerical methods, structured matrices, telecommunication modeling, performance evaluation

Joint work of: Bini, Dario; Meini, Beatrice; Ramaswami, Vaidyanathan; Remiche, Marie-Ange; Taylor, Peter

Extended Abstract: http://drops.dagstuhl.de/opus/volltexte/2008/1400

Algorithmic analysis of the Sparre Andersen model in discrete time

Attahiru Alfa (University of Manitoba, CA)

We set up the Sparre Andersen insurance risk model in discrete time and show that it can be analyzed as a doubly infinite Markov chain of the GI/M/1 type. A computational procedure, based on matrix analytic methods, is then developed for finding the probability distributions associated with fundamental ruin-related quantities, such as the time of ruin, the surplus immediately prior to ruin and deficit at ruin. Numerical examples are then presented. This paper is to be published in ASTIN Bulletin, vol. 37, #2, 2007, 293-317.

Keywords: Sparre Andersen model, discrete time, GI/M/1 Markov chain, matrix analytic methods, time of ruin, deficit at ruin, surplus immediately prior to ruin

Joint work of: Alfa, Attahiru; Drekic, Steve

A use of Phase-Type distributions in Semi-Markov **Decision Processes**

Nigel Bean (University of Adelaide, AU)

We present an investigation of a particular class of Decision Processes with nonexponential lifetimes. We wish to use Phase-Type distributions to model the non-exponential lifetimes and thus return the problem to an ordinary Markov Decision Process. The key question is then: How can we make use of this simplification to efficiently determine the optimal policy?

This is joint work with Jeremy McMahon

Keywords: Markov Decision Processes, PH distributions

First passage times for Markov-additive processes

Lothar Breuer (University of Kent, GB)

We generalise some results for spectrally negative Levy processes to the setting of Markov-additive processes (MAPs).

Among them are Laplace transforms for hitting times, the distribution of maxima, a Wiener–Hopf factorisation, and the stationary distribution of the reflected process. These all depend more or less on one matrix only, which can be regarded as the generalised inverse function of the cumulant matrix. A numerically stable iteration to compute this matrix is given. The theory is first developed for MAPS without positive jumps and then extended to include positive jumps having phase-type distributions, for which the overshoot distribution is given, too.

Keywords: Markov-additive process, hitting time, cumulant matrix, Levy process

Interarrival Times Characterization and Fitting for Markovian Traffic Analysis

Giuliano Casale (College of William and Mary, USA)

We propose a traffic fitting algorithm for Markovian Arrival Processes (MAPs) that can capture statistics of any order of interarrival times. By studying real traffic traces, we show that matching higher order properties, in addition to first and second order descriptors, results in increased queueing prediction accuracy with respect to other algorithms that only match the mean, coefficient of variation, and autocorrelations. The result promotes the idea of modeling traffic traces using the interarrival time process instead of the counting process that is more frequently employed in previous work, but for which higher order moments are difficult to manipulate.

We proceed by first characterizing the general properties of MAPs using a spectral approach. Based on this characterization, we show how different MAP processes can be combined together using Kronecker products to define a larger MAP with predefined properties of interarrival times. We then devise an algorithm that is based on this Kronecker composition and can accurately fit traffic traces. The algorithm employs nonlinear optimization programs that can be customized to fit an arbitrary number of moments and to meet the desired cost-accuracy tradeoff.

Numerical results of the fitting algorithm on real HTTP and TCP traffic data, such as the Bellcore Aug89 trace, indicate that the proposed fitting methods achieve increased prediction accuracy with respect to other state-of-the-art fitting methods.

Keywords: MAP fitting, interarrival time process, higher-order moments

Joint work of: Casale, Giuliano; Zhang, Eddy Z.; Smirni, Evgenia

Some Results Concerning Coxian Distributions of Algebraic Degree Three

Mark Fackrell (University of Melbourne, AU)

It is well known that the algebraic degree of a Coxian distribution is less than or equal to its triangular order. Given a Coxian distribution defined in terms of its Laplace transform, it is a simple matter to calculate its algebraic degree. However, it is unknown, in general, what the triangular order of the distribution is. In this talk we give some results concerning the triangular order of Coxian distributions of algebraic degree three, and explain how they can be extended to Coxian distributions of higher algebraic degree.

Keywords: Coxian distribution, Laplace transform, algebraic degree

Characterizing Coxian Distributions of Algebraic Degree q and Triangular Order p

Mark Fackrell (University of Melbourne, AU)

In this research note we present a procedure to characterize the set of all phasetype distributions of algebraic degree q with real poles, that have Coxian representations of order p where p > q.

Keywords: Phase-type distribution, Coxian distribution, Laplace-Stieltjes transform, algebraic degree, triangular order

Extended Abstract: http://drops.dagstuhl.de/opus/volltexte/2008/1391

QBD processes and matrix values orthogonal polynomials. some new explicit examples

Alberto F. Grünbaum (Univ. California - Berkeley, USA)

In the case of birth-and-death processes there are a few exactly solvable situations where the n-step transition matrix can be written down using the Karlin-McGregor formula. A few of these come from group representation theory.

I plan to show how this can be extended to some instances of QBD processes with an arbitrary finite number of phases. The group involved is the set of all unitary matrices of size N. For a fixed N one gets examples where the number of phases is a free parameter, and there are a few extra parameters to play with. By tunning these parameters one can exhibit examples where states are recurrent or transient.

The rather surprising fact that for these examples one can compute everything explicitly raises the issue of finding a possible network application for this piece of mathematics that involves matrix valued orthogonal polynomials.

I will give an ab-initio discussion of the examples starting with the case of one phase.

 $Keywords: \quad {\rm QBD, \ orthogonal \ polynomials, \ Karlin-McGregor \ formula, \ representation \ theory$

Full Paper: http://drops.dagstuhl.de/opus/volltexte/2008/1392

Matrix Analytic Methods in Branching Processes

Sophie Hautphenne (Université Libre de Bruxelles, B)

A large number of questions in fields like biology and telecommunications may be modeled as continuous-time multi-type branching processes. In order to determine the extinction probability for this kind of processes, it is necessary to find the minimal nonnegative solution of a non-linear matrix fixed point equation. We examine the question of solving that equation, using a mixture of algorithmic and probabilistic arguments. This allows us to analyze the efficiency of various algorithms on the basis of the behavior of the branching process itself. We focus our attention here on a particular class of continuous-time multi-type branching processes, named Markovian binary trees (MBT), but our approach is amenable to more complex systems. In the special case of the MBT, the fixed point equation turns out to be quadratic, which makes its resolution particularly clear.

In the first part of the talk, we analyze two linear algorithms to obtain the extinction probability of an MBT, of which one is new, and, we propose a quadratic algorithm arising from Newton's iteration method for fixed-point equations. We also give them a probabilistic interpretation in terms of the MBT.

In the second part of the talk, we add a catastrophe process to the initial MBT, and we analyze the resulting system. The extinction probability turns out to be much more difficult to compute; we propose a G/M/1-type Markovian process approach to approximate this probability.

Keywords: Matrix analytic methods, branching processes, catastrophe process, extinction probability

Matrix Analytic Methods in Branching processes

Sophie Hautphenne (Université Libre de Bruxelles, B)

We examine the question of solving the extinction probability of a particular class of continuous-time multi-type branching processes, named Markovian binary trees (MBT). The extinction probability is the minimal nonnegative solution of a fixed point equation that turns out to be quadratic, which makes its resolution particularly clear.

We analyze first two linear algorithms to compute the extinction probability of an MBT, of which one is new, and, we propose a quadratic algorithm arising from Newton's iteration method for fixed-point equations.

Finally, we add a catastrophe process to the initial MBT, and we analyze the resulting system. The extinction probability turns out to be much more difficult to compute; we use a G/M/1-type Markovian process approach to approximate this probability.

Keywords: Branching Processes, Matrix Analytic Methods, Extinction Probability, Catastrophe Process

Joint work of: Hautphenne, Sophie; Latouche, Guy; Remiche, Marie-Ange

Extended Abstract: http://drops.dagstuhl.de/opus/volltexte/2008/1393

Moment Characterization of Matrix Exponential and Markovian Arrival

András Horváth (University of Torino, I)

This talk provides a general framework for establishing the relation between various moments of matrix exponential and Markovian processes.

Based on this framework we present an algorithm to compute any finite dimensional moments of these processes based on a set of required (low order) moments. This algorithm does not require the computation of any representation of the given process. We present a series of related results and numerical examples to demonstrate the potential use of the obtained moment relations.

Keywords: Matrix exponential process, Markov arrival process, matrix exponential distribution, phase type distribution

Joint work of: Bodrog, Levente; Horváth, András; Telek, Miklós

On the Properties of Moments of Matrix Exponential Distributions and Matrix Exponential Processes

András Horváth (University of Torino, I)

In this paper we provide properties of moments of matrix exponential distributions and joint moments of matrix exponential processes. Based on the provided properties, an algorithm is presented to compute any finite dimensional moments of these processes based on a set of required (low order) moments. This algorithm does not require the computation of any representation of the given process. We present some related examples to demonstrate the potential use of the properties of moments.

Matrix exponential process, Markov arrival process, Matrix exponential distribution, phase type distribution

Joint work of: Bodrog, Levente; Horváth, András; Telek, Miklós

Full Paper: http://drops.dagstuhl.de/opus/volltexte/2008/1394

Numerical solution of a nonsymmetric algebraic Riccati equation arising in fluid queues

Bruno Iannazzo (Università di Pisa, I)

We study a nonsymmetric algebraic Riccati equation arising in a fluid queues model.

The equation has the form XCX - AX - XD + B = 0 where X is an m-by-n matrix and the coefficients, opportunely arranged, form an M-matrix.

We give a review of the theoretical properties of the equation and we present the most reliable algorithms developed so far. In particular we present some recent techniques which allow the algorithms to works well also in the ill-conditioned cases.

Keywords: Algebraic Riccati equation, matrix iteration, cyclic reduction, logarithmic reduction, fluid queues, shift technique

Joint work of: Bini, Dario; Guo, Chun-Hua; Iannazzo, Bruno; Latouche, Guy; Meini, Beatrice

Nonsymmetric algebraic Riccati equations associated with an M-matrix: recent advances and algorithms

Bruno Iannazzo (Università di Pisa, I)

We survey on theoretical properties and algorithms concerning the problem of solving a nonsymmetric algebraic Riccati equation, and we report on some known methods and new algorithmic advances. In particular, some results on the number of positive solutions are proved and a careful convergence analysis of Newton's iteration is carried out in the cases of interest where some singularity conditions are encountered. From this analysis we determine initial approximations which still guarantee the quadratic convergence.

Keywords: Nonsymmetric algebraic Riccati equations, matrix equation, M-matrices, Newton method, quadratically convergent algorithms, cyclic reduction, doubling algorithm

Joint work of: Bini, Dario A.; Iannazzo, Bruno; Meini, Beatrice; Poloni, Federico

Invariant measures for fluid queues

Guy Latouche (Université Libre de Bruxelles, B)

Positive recurrent fluid queues have a matrix-exponential stationary distribution. We show that a transient fluid queue also has a matrix-exponential invariant measure which is not summable. We give a construction for this measure and we give the probabilistic interpretation for the key matrix which appears in the construction.

Joint work of: Van Lierde, Sarah; da Silva Soares, Ana; Latouche, Guy

On the tail decay of M/G/1-type Markov renewal processes

Beatrice Meini (Università di Pisa, I)

The tail decay of M/G/1-type Markov renewal processes is studied. The Markov renewal process is transformed into a Markov chain so that the problem of tail decay is reformulated in terms of the decay of the coefficients of a suitable power series. The latter problem is reduced to analyze the analyticity domain of the power series.

Keywords: Tail decay, M/G/1-type, Markov renewal process

Joint work of: Meini, Beatrice; Bini, Dario; Ramaswami, Vaidyanathan

Full Paper: http://drops.dagstuhl.de/opus/volltexte/2008/1396

Multivariate matrix-exponential distributions

Bo Friis Nielsen (Technical University of Denmark, DK)

We review what is currently known about one-dimensional distributions on the non-negative reals with rational Laplace transform, also known as matrixexponential distributions. In particular we discuss a flow interpretation which enables one to mimic certain probabilisticly inspired arguments which are known from the theory of phase-type distributions.

We then move on to present ongoing research for higher dimensions. We discuss a characterization result, some closure properties, and a number of examples. Finally we present open problems and future perspectives.

Keywords: Multivariate matrix-exponential distributions, multivariate phasetype distributions, rational Laplace transform

Joint work of: Bladt, Mogens; Nielsen, Bo Friis

Old and new algorithms for algebraic Riccati equations

Federico Poloni (Scuola Normale Superiore - Pisa, I)

We consider the matrix algebraic Riccati equation (ARE)

$$XCX + B - AX - XA^{T} = 0, A, B, C \in \mathbb{R}^{n \times n}$$

with B and C symmetric, and the more general version

$$XCX + B - AX - XD = 0,$$

known as nonsymmetric algebraic Riccati equation.

Their solutions can be computed explicitly in terms of the invariant subspaces of the matrix

$$H = \begin{pmatrix} D - C \\ B - A \end{pmatrix}.$$

Several iterative methods exist in literature for the calculation of the extremal solution X^* .

Among these, we will focus on an approach proposed in [Ramaswami, 1999] that consists in applying a spectral transformation to H, and then reducing the ARE to a unilateral matrix equation (UME) of dimension 2n of the form

$$R + SY + TY^2 = 0,$$

which can then be solved using Logarithmic Reduction [Latouche–Ramaswami, '93] or Cyclic Reduction [Bini–Meini, '96].

We take this approach further, showing that other choices are possible for both the spectral transformation and the reduction step, with lower computational cost. In particular, we are able to show that one of this choices yields the same iteration as the *structured doubling algorithm* (SDA), another algorithm for solving the ARE which had been developed in a completely different setting [X.X. Guo–Lin–Xu, 2006]. This new interpretation leads to better understanding of the convergence properties of the SDA. New variants can be easily introduced, taking the best from each of the two algorithms.

Moreover, we consider another choice for the reduction step of the algorithm, leading to an unilateral equation of size n (instead of 2n), with great computational advantage. Strategies to improve the stability of this method are discussed.

Keywords: Algebraic Riccati Equation, Matrix Equation, Cyclic Reduction, Logarithmic Reduction

Joint work of: Bini, Dario; Iannazzo, Bruno; Meini, Beatrice; Poloni, Federico

From Algebraic Riccati equations to unilateral quadratic matrix equations: old and new algorithms

Federico Poloni (Scuola Normale Superiore - Pisa, I)

The problem of reducing an algebraic Riccati equation XCX-AX-XD+B=0 to a unilateral quadratic matrix equation (UQME) of the kind PX^2+QX+R is analyzed. New reductions are introduced which enable one to prove some theoretical and computational properties.

In particular we show that the structure preserving doubling algorithm of B.D.O. Anderson [Internat. J. Control, 1978] is nothing else but the cyclic reduction algorithm applied to a suitable UQME. A new algorithm obtained by complementing our reductions with the shrink-and-shift tech- nique of Ramaswami is presented. Finally, faster algorithms which require some non-singularity conditions, are designed. The non-singularity re- striction is relaxed by introducing a suitable similarity transformation of the Hamiltonian.

Keywords: Algebraic Riccati Equation, Matrix Equation, Cyclic Reduction, Structured doubling algorithm

Joint work of: Bini, Dario A.; Meini, Beatrice; Poloni, Federico Full Paper: http://drops.dagstuhl.de/opus/volltexte/2008/1398

A feedback fluid queue with two congestion control thresholds

Werner Scheinhardt (University of Twente, NL)

Feedback fluid queues play an important role in modeling congestion control mechanisms for packet networks. In this presentation we analyze a fluid queue with a feedback-based traffic rate adaptation scheme which uses two thresholds. The higher threshold B_1 is used to signal the beginning of congestion while the lower threshold B_2 signals the end of congestion.

These two parameters together allow to make the trade–off between maximizing throughput performance and minimizing delay. The difference between the two thresholds helps to control the amount of feedback signals sent to the traffic source.

In our model the input source can behave like either of two Markov fluid processes. The first applies as long as the upper threshold B_1 has not been hit from below. As soon as that happens, the traffic source adapts and switches to the second process, until B_2 (smaller than B_1) is hit from above.

We present the exact analysis of the stationary distribution of the buffer occupancy and evaluate several other performance measures.

We solve the model by setting up the Kolmogorov forward equations, then solving the corresponding balance equations using a spectral expansion and finally finding sufficient constraints to solve for the unknowns in the solution.

An interesting question is whether such a model could also be solved using matrix-analytic methods.

Keywords: Fluid queue, feedback, spectral expansion

Joint work of: Berg, Hans van den; Malhotra, Richa; Mandjes, Michel; Scheinhardt, Werner

Erlangization for perturbed risk models

David Stanford (University of Western Ontario, CA)

The Erlangization method has been applied in a variety of risk process contexts, starting with Asmussen, Avram and Usabel (2002) for the classical risk model; the extension to the Sparre-Andersen risk process is described in Stanford et al (2005). Recently, Ramswami et al (2008) provided a unified procedure for the analysis of Markov-Modulated fluid flows via Erlangization. In the present work, we develop an algorithmic procedure for the analysis of perturbed risk processes via Erlangization.

This is joint work with Mr. Kaiqi Yu and Prof. Jiandong Ren of the same department.

Joint work of: Stanford, David; Yu, Kaiqi; Ren, Jiandong

Erlangian Approximation to Finite Time Ruin Probabilities in Perturbed Risk Models

David Stanford (University of Western Ontario, CA)

In this work-in-progress, we consider perturbed risk processes that have an underlying Markovian structure, including Markovian risk processes, and Sparre-Andersen risk processes when both inter claim times and claim sizes are phase-type. We apply the Erlangization method to this risk process in order to obtain an accurate approximation of the finite time ruin probability. In addition, we recognize a repeating structure in the probability matrices we work with. This is the key element in developing more efficient algorithms for the computation of the ruin probabilities. Several numerical examples are present to illustrate the model.

Keywords: Perturbed risk processes, finite-time ruin probability, phase-type distribution, fluid flow models, Erlangization

Joint work of: Yu, Kaiqi; Stanford, David A.; Ren, Jiandong

Drift Conditions for Matrix Analytic Models

Peter Taylor (University of Melbourne, AU)

In his seminal work, Marcel Neuts gave drift criteria via which one can determine whether processes of $\mathrm{GI/M/1-or\ M/G/1-type}$ are positive recurrent. Late last century, Ciardo and Smirni used a drift condition to establish positive recurrence, which was not the same as Neuts' conditions and which did not follow directly from them. This condition was, however, plausible and Guy and I were motivated to try to prove it. We subsequently succeeded in doing this and, at the same time, established another two drift conditions. These appeared in a joint publication in 2003. Furthermore, taking advantage of Ramaswami's concept of duality, it is also possible to derive two more drift conditions.

In this talk, I shall tell the story of the development of these drift conditions.

Keywords: Drift conditions, matrix-analytic models

Joint work of: Taylor, Peter

See also: G. Latouche and P. G. Taylor, Drift Conditions for Matrix-Analytic Models, Mathematics of Operations Research, 28 (2003), 346–360.

G. Ciardo and E. Smirni, ETACA: An Efficient Technique for the Analysis of QBD-processes by Aggregation, *Performance Evaluation*, **36–37**, 71–93, 1999.

Current results and open questions on PH and MAP characterization

Miklós Telek (Technical University of Budapest, H)

Stochastic processes with matrix exponential kernels have a wide range of applications due to the availability of efficient matrix analytic methods.

The characterization of these processes is in progress in recent years.

Basic questions like the flexibility, the degree of freedom, the most efficient (canonical) representation of these models are under study. The presentation collects a set of available results and related open questions.

Keywords: PH distribution, ME distribution, MAP, MEP

Joint work of: Bodrog, Levente; Heindl, Armin; Horváth, András; Horváth, Gábor; Telek, Miklós

A markovian-type approach for stream selection in audtiory selective attention

Carlos Trenado (INM - Saarbrücken, D)

Selective attention in humans has been defined as the brain's outstanding ability to select relevant information from a vast amount received at any time. Different selective attention theories have suggested that individuals have a tendency to orient themselves toward, or process information from only one part of the environment with the exclusion of other parts. In relation to this, psychological studies have also supported that selective attention is governed by the humans arousal level, consequently one of the most prevalent questions in current psychophysiological studies of attention is whether the shifts in attention that accompany changes in the arousal level are automatic, or deliberate.

In the auditory modality, the importance of selective attention has been exemplified by the well–known cocktail party effect, in which a listener is able to follow a conversation in a noisy environment most likely assisted by selective attention on the speaker's voice. According to auditory scene analysis (ASA), processing of exogenous information reaching conscious state involve two stages: First, decomposition of perceptual stimuli into discrete sensory elements, and second recombination of such discrete el- ements into perceptual streams. In this paper, we present a preliminary auditory selective attention scheme based on stochastic stream selection. Such model is aimed to gain understanding into the highly complex neural auditory stream selection process.

Keywords: Selective attention, auditory scene analysis, auditory streams

Joint work of: Trenado, Carlos; Strauss, Daniel J.

Structured Markov Chains Arising from Finite-Source Retrial Queues with Orital Search

Patrick Wüchner (Universität Passau, D)

We consider retrial queueing systems with a finite number of homogeneous sources of calls, a single reliable server, and the search for orbiting customers by the server after job completion.

During this investigation, the infinitesimal generator of the underlying (finite) continuous-time Markov chain takes a (level-dependent) QBD-like form.

After solving for the steady state probabilities using the MOSEL-2 tool, the results show a surprising maximum of the mean response time. This maximum was already discovered by other researchers dealing with finite-source retrial queues. However, to our best knowledge, no thorough investigation was done yet why this maximum exists and in which way it depends on the system parameters.

In the talk, after introducing the backgrounds of finite-source retrial queues with orbital search, a generalized stochastic Petri net is used to derive the underlying continuous-time Markov chain and its generator. Finally, using the seminar, we can hopefully bring forward discussions how to make more general statements on the parameter-dependent behavior of the response timeŠs maximum.

Keywords: Structured Markov chain, finite source, retrial queues, orbital search, performance measures, performance tool

Joint work of: Wüchner, Patrick; Sztrik, János; de Meer, Hermann

Full Paper: http://drops.dagstuhl.de/opus/volltexte/2008/1389

Markov Decision Processes skip-free in one direction

Benny van Houdt (University of Antwerp, B)

In this talk we present a new policy iteration algorithm for Markov decision processes (MDPs) skip-free in one direction. This algorithm, which is based on matrix analytic methods, is in the same spirit as the algorithm of White (Stochastic Models, 21:785-797, 2005) which was limited to Markov chains that are skip-free in both directions. Such optimization problems arise in the domain of optical networking, more specifically, when trying to improve loss rates of optical fibre delay line (FDL) buffers.

Keywords: Markov Decision Processes skip-free optical buffering

See also: In Proc. of SMCTools workshop, Nantes (France), Oct 2007.

A policy iteration algorithm for Markov decision processes skip-free in one direction

Benny van Houdt (University of Antwerp, B)

In this paper we present a new algorithm for policy iteration for Markov decision processes (MDP) skip-free in one direction. This algorithm, which is based on matrix analytic methods, is in the same spirit as the algorithm of White (Stochastic Models, 21:785-797, 2005) which was limited to matrices that are skip-free in both directions.

Optimization problems that can be solved using Markov decision processes arise in the domain of optical buffers, when trying to improve loss rates of fibre delay line (FDL) buffers. Based on the analysis of such an FDL buffer we present a comparative study between the different techniques available to solve an MDP. The results illustrate that the exploitation of the structure of the transition matrices places us in a position to deal with larger systems, while reducing the computation times.

 $Keywords\colon$ Markov Decision Process, Policy Evaluation, Skip-Free, Optical buffers, Fibre Delay Lines

Joint work of: Lambert, Joke; van Houdt, Benny; Blondia Chris

Extended Abstract: http://drops.dagstuhl.de/opus/volltexte/2008/1403

 $See\ also:$ J. Lambert, B. Van Houdt, C. Blondia. - A policy iteration algorithm for Markov decision processes skip-free in one direction - In Proc. of SMCTools 07 - Nantes, France, 2007.