Structured Markov Chains Arising from Homogeneous Finite-Source Retrial Queues with Orbital Search*

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Extended Abstract

We consider retrial queueing systems with a finite number of homogeneous sources of service requests, a single reliable server, and the search for orbiting customers by the server after service completion. A graphical representation of this model is given in Figure 1.

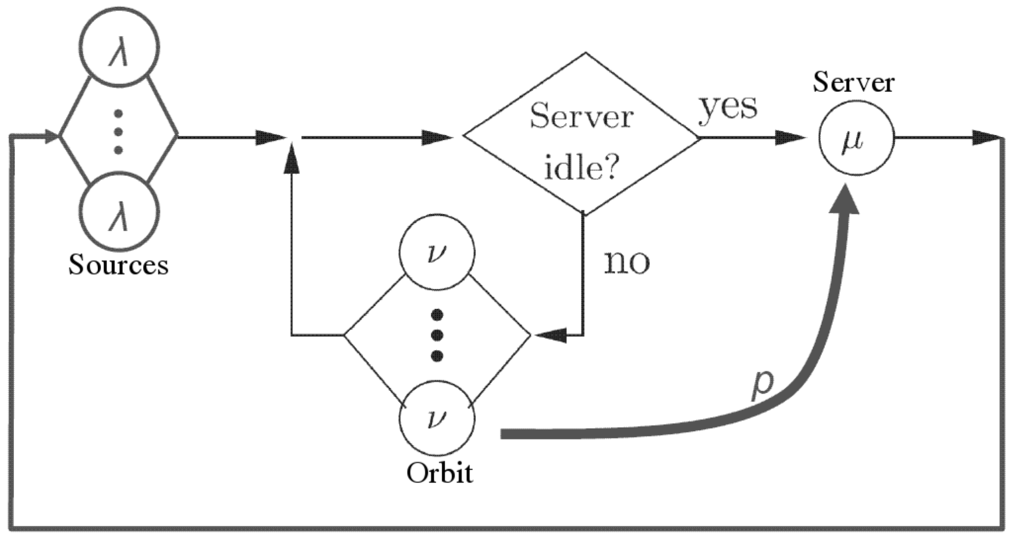

Fig. 1. Finite-source retrial queue with orbital search.

There are K heterogeneous sources. Each active source, i.e., each source that does not currently wait for a response, generates a new request with rate[1] λ. Each request tries to get service by the single server. The server's service rate is given by μ. If an arriving request finds the server busy, it joins the orbit. All requests in the orbit will retry to get

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[1] The request-generation, service, and retrial times are assumed to be exponentially distributed.

service with retrial rate ν. After service completion, the server will actively search for a request in the orbit with probability p. Then, unless the orbit is empty, the server will drag one orbiting request to the service station and immediately start to give service to this request. Consequently, if p is set to 1, the model is equivalent to a M/M/1/K/K queue (in Kendall's notation), and, if p is set to 0, the model is equivalent to the classical finite-source retrial queue (see, e.g., [3]).

For more details on the model and for an application example, please see our recent publications [6] and [7], respectively.

The behavior of the system under consideration can be described by a continuous-time Markov chain (CTMC) that has a finite number of states. The state space $\mathbf{S}$ can be given by

$$\mathbf{S} = \{S_{2i+j} = (i,j)\,|0 \leq i \leq K-1, j \in \{0,1\}\}, \tag{1}$$

where i refers to the number of requests in the orbit, and $j = 0$ if the server is idle and $j = 1$ if it is busy.

Figure 2 shows the graphical representation of the CTMC for $K = 4$ sources.

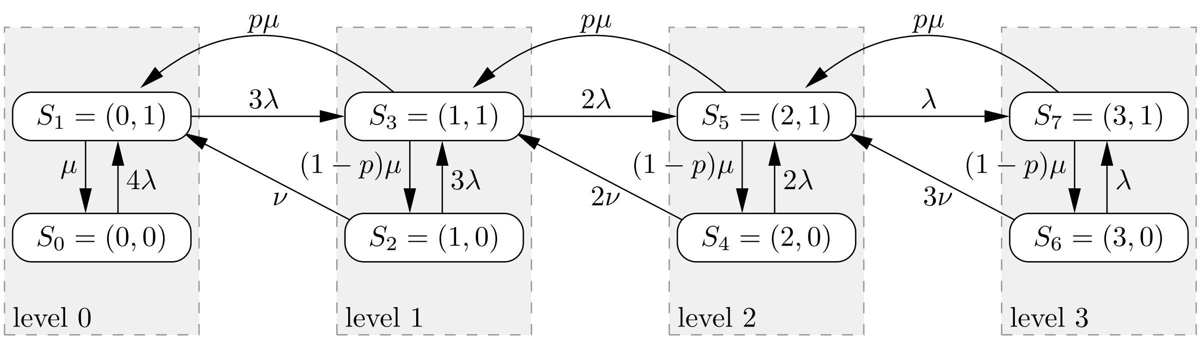

Fig. 2. Continuous-time Markov chain of finite-source retrial queue with orbital search.

Thus, the underlying CTMC takes a level-dependent QBD-like form, where the levels are determined by the number of jobs in the orbit (specified by i), and the phases within the levels by the state of the server (specified by j).

The corresponding infinitesimal generator $\mathbf{Q}$ of the underlying CTMC can be given by:

$$\mathbf{Q} = \begin{pmatrix} -4\lambda & 4\lambda & 0 & 0 & 0 & 0 & 0 & 0 \\ \mu & -\mu-3\lambda & 3\lambda & 0 & 0 & 0 & 0 & 0 \\ 0 & p\mu & -\mu-2\lambda & (1-p)\mu & 2\lambda & 0 & 0 & 0 \\ 0 & \nu & 3\lambda & -\nu-3\lambda & 0 & 0 & 0 & 0 \\ 0 & 0 & p\mu & 0 & -\mu-\lambda & (1-p)\mu & \lambda & 0 \\ 0 & 0 & 2\nu & 0 & 2\lambda & -2\nu-2\lambda & 0 & 0 \\ 0 & 0 & 0 & 0 & p\mu & 0 & -\mu & (1-p)\mu \\ 0 & 0 & 0 & 0 & 3\nu & 0 & \lambda & -3\nu-\lambda \end{pmatrix}. \tag{2}$$

After solving for the steady state probabilities using numerical analysis by applying the MOSEL-2 tool (see [5]), the results show a surprising maximum of the mean response time (see Fig. 3). This maximum was already discovered by other researchers dealing with finite-source retrial queues (e.g.,[1–3]). However, to our best knowledge, no thorough investigation was done yet why this maximum exists and in which way it depends on the system parameters.

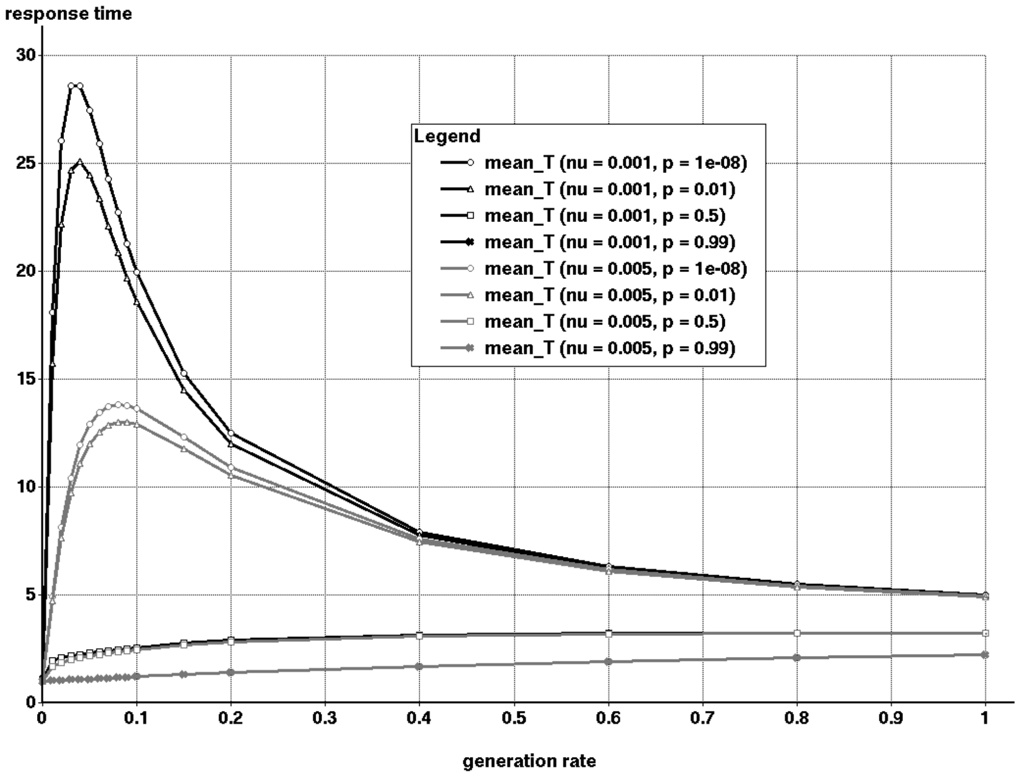

Fig. 3. MOSEL-2 results: Mean response time of finite-source retrial queue with orbital search.

Thus, the question arises, how this maximum can be discussed in more detail. To this end, helpful directions were given and fruitful discussions took place during the Dagstuhl seminar. We will use these hints to elaborate our next steps.

The first of these steps is to get an explicit closed-form solution of the steady state probabilities and of the mean response time. The algorithm proposed in [4] can be used to develop such a closed-form solution. The thorough discussion of the closed-form solution should then help to deepen the insights into the behavior of finite-source retrial queues with and also without orbital search.

Furthermore, in future work, in addition to the mean response time, the upper bounds of the response time should be discussed, as well as the mean number of retrials before service.

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