On the Fundamental Limits of Broadcasting in Wireless Mobile Networks

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Abstract—In this paper, we investigate the fundamental properties of broadcasting in *mobile* wireless networks. In particular, we characterize broadcast capacity and latency of a mobile network, subject to the condition that the stationary node spatial distribution generated by the mobility model is uniform. We first study the intrinsic properties of broadcasting, and present a broadcasting scheme, called RIPPLECAST, that simultaneously achieves asymptotically optimal broadcast capacity and latency, subject to a weak upper bound on the maximum node velocity. This study intendedly ignores the burden related to the selection of broadcast relay nodes within the mobile network, and shows that optimal broadcasting in mobile networks is, in principle, possible. We then investigate the broadcasting problem when the relay selection burden is taken into account, and present a combined distributed leader election and broadcasting scheme achieving a broadcast capacity and latency which is within a $\Theta((\log n)^{1+\frac{2}{\alpha}})$ factor from optimal, where n is the number of mobile nodes and $\alpha > 2$ is the path loss exponent. However, this result holds only under the assumption that the upper bound on node velocity converges to zero (although with a very slow, poly-logarithmic rate) as n grows to infinity.
To the best of our knowledge, our is the first paper investigating the effects of node mobility on the fundamental properties of broadcasting, and showing that, while optimal broadcasting in a mobile network is in principle possible, the coordination efforts related to the selection of broadcast relay nodes lead to sub-optimal broadcasting performance.

Index Terms—wireless networks; mobile networks; broadcast capacity; broadcast latency; SINR interference model.

I. INTRODUCTION

Investigation of fundamental properties of wireless networks has received considerable attention in the research community, starting from the seminal Gupta and Kumar [7] work that characterized the capacity of a wireless multi-hop network for unicast transmissions. Since then, fundamental properties of wireless multi-hop networks have been investigated for a variety of communication patterns including unicast [6], [16], [18], [24], broadcast [8], [20], [25], multicast [12], [23], and convergecast [13], [14]. It has been shown that wireless multi-hop network scaling laws significantly change depending on network parameters such as node deployment (e.g., random vs. arbitrary), traffic pattern, and node mobility. Node mobility in particular has been shown to have considerable effects on wireless network scaling laws: for instance, per-node capacity of unicast transmission has been shown to be asymptotically vanishing with the number n of network nodes independently on the node deployment (see [7]), but to become *constant* (i.e., asymptotically optimal) in case network nodes are mobile [6] (under the assumption that very large delays in packet delivery can be tolerated). The reason of the beneficial effect of node mobility on per-node capacity is that what limits per-node

unicast capacity in a static wireless multi-hop network is the relaying burden, i.e., the fact that the same packet has to be sent several times before it can reach the destination[1]. If nodes are mobile, the relay burden can be avoided (or at least significantly reduced) by exploiting a "wait and deliver" strategy[2]: since nodes move randomly, there is a high probability that the sender and the destination eventually come into each other reach, and the packet can be delivered to the destination with no (or only few) re-transmission(s).

To the best of our knowledge, none of the existing papers has investigated the effect of mobility on *broadcasting* scaling laws. Broadcasting scaling laws have been recently characterized in a series of papers [8], [25], including our work [20], [21] showing that, contrary to what happens for unicast transmission, asymptotically optimal capacity *and* latency can be achieved simultaneously for broadcast communication. However, all these results are based on the assumption that network nodes are static. An implicit consequence of this assumption is that the communication burden induced by the need of selecting broadcast relaying nodes within the network (called the *coordination burden* in the following) is consistently ignored in the analysis. This is acceptable in a static network, since the selection of broadcast relaying nodes can be assumed to be done once and for all at the beginning of the broadcasting session, implying that the coordination burden can be safely ignored in the analysis as long as the duration of the broadcasting session is sufficiently long. However, if relay nodes are mobile, a change in their position might cause an incomplete coverage of the broadcast packets, which must be received by *all* network nodes. Thus, the role of broadcast relay node must be continuously rotated amongst network nodes in a mobile network, in order to ensure broadcast coverage in spite of node mobility. Given this, evaluating the coordination burden cost becomes an integral part of the characterization of broadcasting scaling laws in mobile networks.

In this paper, we make a first step towards gaining a better understanding of the effect of mobility on the broadcasting communication paradigm. We first show that *broadcasting is not inherently capacity nor latency limited by node mobility*: we present a simple cell-based broadcasting scheme, called RIPPLECAST, that simultaneously achieves optimal broadcast capacity and latency under the assumption that: $i)$ nodes move in a bounded region according to a mobility model

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[1]This is true unless the destination is the vicinity of the sender, which occurs with vanishingly probability in a sufficiently large network with randomly selected source/destination pairs.

[2]This strategy has become the fundamental communication paradigm in delay tolerant networks [4].

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whose stationary node spatial distribution is uniform; and $ii)$ maximum node velocity is upper bounded by a (very large) constant. However, when the cost related to the coordination burden is taken into account the picture changes considerably: broadcasting capacity and latency degrades of a factor $\Theta((\log n)^{1+\frac{2}{\alpha}})$ with respect to optimal – n is the number of network nodes and $\alpha > 2$ is the path loss exponent – , and the upper bound on maximum node velocity becomes asymptotically vanishing as $n \to \infty$. We thus formally prove that what limits broadcast performance in a mobile network *are not* the inherent properties of broadcast communication, but the *coordination burden* induced by the need of frequent re-selection of relay nodes within the network.

II. RELATED WORK

The fundamental properties of broadcasting in wireless multi-hop networks have been investigated only very recently. In [25], Zheng investigated the broadcast capacity of random networks with single broadcast source under the generalized physical interference model, and presented a broadcast scheme providing asymptotically optimal capacity. The author also presented a different broadcast scheme, and proved its asymptotically optimal performance with respect to information diffusion rate, which is closely related to latency. The authors of [8] confirmed that optimal broadcast capacity can be achieved in a more general network model, in which arbitrary node positions are allowed, an arbitrary subset of the network nodes is assumed to generate broadcast packets, and accurate SINR-based interference models are used. In [20], we have shown that asymptotically optimal broadcast capacity and latency can be simultaneously achieved in a static network, under the assumption of single broadcast source. This result has been recently extended to the case of an arbitrary number of broadcast sources in [21].

While several papers have proposed broadcasting schemes for mobile networks (see, e.g., [17], [19]), to the best of our knowledge none of them attempted at characterizing the fundamental properties of broadcasting in mobile networks. The work that is closest to our is [2], where the authors present a location-based broadcasting protocol for mobile ad hoc networks, and formally characterize the number of communication steps needed to deliver a broadcast packet to all network nodes. Similarly to our approach, the authors propose selecting broadcast relay nodes based on their position, and present theoretical results that hold under the assumption that node velocity is upper bounded by certain constants. However, the authors in [2] are concerned with delivering a *single* broadcast packet, while in this paper we are interested in characterizing the maximum *rate* at which broadcast packets can be sent by the source. Furthermore, the results of [2] are valid under a simplistic interference model based on the notion of conflict graph, while ours hold under the more realistic, SINR-based physical interference model.

III. NETWORK MODEL AND PRELIMINARIES

We consider a wireless network composed of $n+1$ wireless nodes distributed in a square region R of side $L = L(n)$. One of the nodes is stationary, and is located in the center of the deployment region. This node, denoted s in the following, is the broadcast source. The remaining n nodes are mobile, and move within R according to some mobility model $\mathcal{M}$. Model $\mathcal{M}$ is such that the induced stationary node spatial distribution (which is assumed to exist) is *uniform*. In other words, a snapshot taken at time t of the positions of n nodes moving according to $\mathcal{M}$, for a sufficiently large t, is statistically equivalent to a uniform random distribution of n nodes into R. Examples of mobility models satisfying this assumption are random walks, brownian motion, random direction model with proper border rules, etc (see [11] and references therein).

We assume nodes communicate through a shared wireless channel of a certain, constant capacity W, and that the nodes transmission power is fixed to some value P. Correct message reception at a receiver node is subject to an SINR-based criterion, also known as *physical interference model* [7]. More specifically, a packet sent by node u is correctly received at a node v (with rate W) if and only if

$$\frac{P_v(u)}{N + \sum_{i \in \mathcal{T}} P_v(i)} \geq \beta \ ,$$

where N is the background noise, β is the capture threshold, $\mathcal{T}$ is the set of nodes transmitting concurrently with node u, and $P_v(x)$ is the received power at node v of the signal transmitted by node x.

We also make the standard assumption that radio signal propagation obeys the log-distance path loss model, according to which the received signal strength at distance d from the transmitter (for sufficiently large d, say, $d \geq 1$) equals $P \cdot d^{-\alpha}$, where α is the path loss exponent. In the following, we make the standard assumption that $\alpha > 2$, which is often the case in practice. We then have[3] $P_v(x) = P \cdot d(x,v)^{-\alpha}$, where $d(x,v)$ is the Euclidean distance between nodes v and x, and the SINR value at node v can be rewritten as follows

$$SINR(v) = \frac{d(u,v)^{-\alpha}}{\frac{N}{P} + \sum_{i \in \mathcal{T}} d(i,v)^{-\alpha}} \ .$$

For given values of P, β, α, and N, we define the transmission range r_{max} of a node as the maximum distance up to which a receiver can successfully receive a message *in absence of interference*. From the definition of physical interference model, we have $r_{max} = \sqrt[\alpha]{P/(\beta N)}$.

The *maximal communication graph* is a graph $G = (\mathcal{V}, \mathcal{E})$ representing all possible communication links in the network, i.e., $\mathcal{V}$ is the set of the $n+1$ nodes, and (undirected) edge $(u,v) \in \mathcal{E}$ if and only if $d(u,v) \leq r_{max}$. Given that existence of a link in G depends only on distance between node, graph G is equivalent to a unit disk graph, which has well-known limitations in modeling wireless networks [10]. However, up to straightforward technical details, the results presented in this paper can be extended to the more realistic cost-based radio propagation model of [22], which is shown to closely resemble log-normal shadowing propagation.

We define the *broadcast capacity* of the network as the maximum possible rate $\lambda(n)$ such that all packets generated by source s are received by the remaining n nodes within a certain time T_{max}, with $T_{max} < \infty$. The *broadcast latency*

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[3]To simplify notation, in the following we assume that the product of the transmitter and receiver antenna gain is 1.
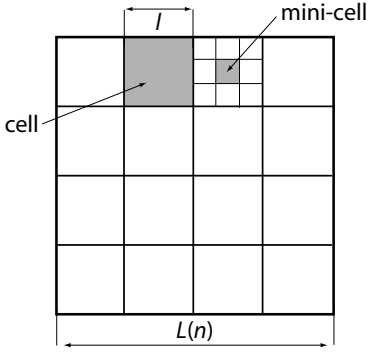
Fig. 1. Cell subdivision of the deployment region.

of the network is the *minimal* time $T(n)$ such that the packet generated by s at time t is received by *all* the n nodes within time $t + T(n)$. It is clear that, in order to have meaningful values of broadcast capacity and latency, the maximal communication graph of a network must be connected. Thus, the assumption of connected maximal communication graph is made throughout this paper. More specifically, we assume that graph G is connected w.h.p. under the assumption that nodes are distributed according to the asymptotic node spatial distribution resulting from mobility model $\mathcal{M}$ which, we recall, is assumed to be uniform[4].

Given the assumption of stationary uniform node spatial distribution of the mobility model $\mathcal{M}$, the *critical transmission range* for connectivity is [3]:

$$ctr(n) = \Theta\left(L(n)\sqrt{\frac{\log n}{n}}\right) .$$

We recall that the critical transmission range for connectivity is the minimal common value of the transmission range such that the resulting maximal communication graph is connected.

Assume the deployment region R is divided into non-overlapping square cells of side l, with $l = \frac{r_{max}}{2h\sqrt{2}}$, for some constant $h > 1$. In turn, each of these cells is partitioned into 9 square mini-cells of side $\frac{l}{3}$ (see Figure 1). The following proposition defines a value of $L(n)$ such that several properties of the resulting node deployment hold, w.h.p.

Proposition 1: Assume $L(n) = \frac{r_{max}}{6h\sqrt{2}}\sqrt{\frac{n}{\log n}}$ for some constant $h > 1$, and assume n nodes are distributed uniformly at random in a square region of side $L(n)$. Then, the following properties hold w.h.p.:
(a) the minimally occupied mini-cell contains at least one node;
(b) the maximally occupied mini-cell contains $\Theta(\log n)$ nodes;
(c) the maximum transmission range r_{max} is asymptotically minimal to ensure network connectivity.

Proof: To prove (a), we observe that when $L(n) = \frac{r_{max}}{6h\sqrt{2}}\sqrt{\frac{n}{\log n}}$, the total number C of mini-cells in the deploy-

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ment region is

$$C = \left(\frac{L(n)}{l/3}\right)^2 = \left(\frac{\frac{r_{max}}{6h\sqrt{2}}\sqrt{\frac{n}{\log n}}}{\frac{r_{max}}{6h\sqrt{2}}}\right)^2 = \frac{n}{\log n} .$$

It follows that the ratio η between the number of nodes and the number of cells is $\log n$. Theorem 5, page 111 of [9] states that, when $\eta = \log n$, the number of nodes in the minimally occupied cell is greater than zero w.h.p., which implies the result when $L(n) = \frac{r_{max}}{6\sqrt{2}}\sqrt{\frac{n}{\log n}}$.

The proof of (b) follows directly from Lemma 1 of [13].

The proof of (c) follows by observing that the critical transmission range for connectivity when n nodes are distributed uniformly at random in a square of side $L(n)$ is [3]

$$\Theta = \left(L(n)\cdot\sqrt{\frac{\log n}{n}}\right) = \left(\frac{r_{max}}{6h\sqrt{2}}\cdot\sqrt{\frac{n\log n}{n\log n}}\right) = \Theta\left(r_{max}\right) .$$

∎

Finally, we introduce the notion of cell distance, which will be extensively used in the following. Given any two cells A and B in the deployment region, the cell distance between A and B, denoted $d(A, B)$, is the minimum number of adjacent cells (horizontal, vertical, and diagonal adjacency) that must be traversed to reach A starting from B (and viceversa).

IV. BOUNDS ON BROADCAST CAPACITY AND LATENCY

The following upper bound on the broadcast capacity trivially follows by observing that the maximum rate at which any receiver can receive broadcast packets is W [8]. The bound holds for an arbitrary network.

Claim 1: In any network with n nodes, we have $\lambda(n) \leq W$.

Define $D(n)$, the *diameter* of the network (relative to the broadcast source), as the maximum Euclidean distance between a network node u and the source s. Given that nodes are mobile, the diameter of the network changes over time. However, Proposition 1 implies an invariant property of network diameter under our deployment assumptions, as stated in the following proposition:

Proposition 2: Let $D(n, t)$ be the network diameter at time t. If t is sufficiently large, $L(n) = \frac{r_{max}}{6h\sqrt{2}}\sqrt{\frac{n}{\log n}}$ for some constant $h > 1$, n nodes move according to a mobility model with stationary uniform node spatial distribution in a square region of side $L(n)$, and the source node is located in the center of the deployment region, then $D(n) \geq \frac{\sqrt{2}}{2}(L(n) - \frac{2}{3}l) = \Omega(L(n))$, w.h.p.

Proof: The proof follows immediately by observing that, by Proposition 1, every mini-cell in the deployment region (and in particular those at the corners) contains at least one node, w.h.p. ∎

We are now ready to prove a lower bound for broadcast latency in mobile networks, subject to an upper bound on node velocity.

Theorem 1: Suppose the same assumptions of Proposition 2 hold, and the maximum node velocity is $\tilde{v} = \frac{r_{max}}{\tau}$, where τ is the (constant) time required to send and correctly receive a packet. Then, the broadcast latency is $\Omega\left(\sqrt{\frac{n}{\log n}}\right)$, w.h.p.

Proof: By Proposition 2, the packet generated by the source at time t has to travel distance at least $\frac{\sqrt{2}}{2}(L(n) - \frac{2}{3}l)$, w.h.p., to reach the nodes that were in the corner mini-cells at time t. Consider one such node u, and consider the segment $\overline{us}$ connecting u to s. Since the progress of the packet generated at time t towards node u is at most r_{max} at each communication step of duration τ, and node u in the best case travels along $\overline{us}$ directed towards s with speed at most $\tilde{v} = \frac{r_{max}}{\tau}$, it is easy to see that at least $\frac{\sqrt{2}}{4}(L(n) - \frac{2}{3}l)$ communication steps (each of duration τ) are required for the packet to reach node u. Observing that τ is a constant, we can conclude that $T(n) = \Omega(L(n)) = \Omega\left(\sqrt{\frac{n}{\log n}}\right)$. ∎

Notice that the upper bound $\tilde{v}$ on node velocity is comparable to the speed of radio signal propagation in the air, i.e., to the speed of light.

V. MATCHING CAPACITY AND LATENCY BOUNDS

In this section we present a broadcasting algorithm achieving asymptotically optimal capacity and latency bounds in mobile networks, under the assumption that broadcast relaying nodes are somewhat magically selected within the network. This assumption, although admittedly not realistic, is made with the purpose of separately studying the fundamental properties of *broadcasting* in mobile networks from those of *electing leaders* (i.e., relay nodes). While using specific relay nodes to forward broadcast packets is indeed the most common approach to broadcasting, strictly speaking leader election is a separate task from broadcasting, which in principle can be achieved also without explicit leader election (e.g., through cooperative communication). Indeed, a major finding of this paper is that, while capacity and latency optimal broadcasting can in principle be achieved in mobile networks (subject to a very reasonable upper bound on node velocity), the burden incurred by leader election causes a poly-logarithmic performance degradation with respect to both capacity and latency, and imposes an asymptotically vanishing upper bound on node velocity.

A. Algorithm overview

While broadcasting in mobile networks is apparently a very complex task due to mobility of individual nodes, this apparent complexity can be tamed by observing that the identity of a specific node within the network is not relevant to a broadcasting scheme, as long as reception of each broadcast packet by each of the (mobile) nodes can be guaranteed. In other words, what is relevant to a broadcasting scheme *it is not the identity of a node*, but *its position within the network*. Thus, instead of selecting specific nodes to relay broadcast packets, a smart broadcasting scheme for mobile networks should focus on invariant properties of the node spatial distribution generated by the mobility model, and use such properties to select relay nodes based on their location within the network.

The broadcasting scheme, which we call RIPPLECAST, is based on the following assumptions:

– a spatial TDMA approach is assumed at the MAC layer: time is divided into transmission slots, and a carefully chosen set of links (transmission set) is activated in each

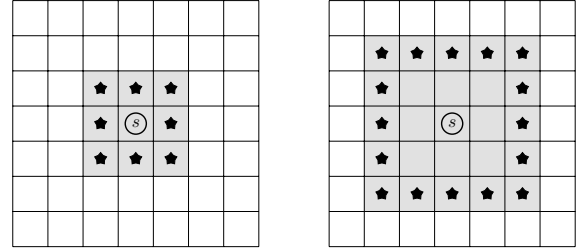Fig. 2. Two-dimensional coloring of parameter $k = 3$.

Fig. 3. The propagation front (ripple) of a broadcast packet. Stars represent cell leaders sending a certain packet p, and shaded cells are those which already received p. Propagation proceeds in a pipelined fashion, and eventually at each step each ripple is propagating a different packet.

slot. The duration of a slot is sufficient to transmit a packet from the sender to the receiver, including propagation time;

– the deployment region is divided into cells and mini-cells, as described in Section III. Cell subdivision is used to virtualize the broadcasting task from a node-related process to a cell-related process. In particular, broadcast relaying nodes (leaders) are chosen within the *central* mini-cell of each cell, and the broadcasting process becomes one of propagating broadcast packets between cells. Without loss of generality, we assume that the source node s is in the central cell.

RIPPLECAST is based on a cell coloring scheme, as in Figure 2, composed of a constant number $\bar{k}^2$ of colors, which is used to spatially separate simultaneously active transmissions. In particular, the coloring scheme ensures that, under the assumption that at most one transmitter is active in each cell with the same color, all transmitted packets are correctly received by *all* the nodes located in the cells adjacent to the transmitter cell. A *round* of transmission is composed of $\bar{k}^2$ transmission slots, one for each color. The color of a cell A is denoted $col(A)$ in the following. Similarly, $col(u)$ denotes the color of the cell to which node u belongs. With RIPPLECAST, propagation of broadcast packets occurs along concentric "waves" (*ripples*, whence the name RIPPLECAST): in the first round, a packet is transmitted to nodes located in cells at cell distance one from s; in the second round, the packet is propagated to nodes located in cells at cell distance two from s, and so on, till the packet is propagated to the furthest cells in the deployment region (see Figure 3). Since a new packet is generated by source s at each round, the propagation proceeds in a pipelined fashion, and eventually at each round each ripple of leaders is propagating a different packet.

Algorithm for source node s:

    Let i be the color of the current time slot; ID is the
current packet ID

  1. **if** $col(s) = i$ **then**

  2.   transmit new packet; ID = ID + 1

Algorithm for a generic node v:

    Let i be the color of the current time slot; if v is a leader node,
let j be the ID of the last packet *received* by node v

  1. listen to the channel

  2. **if** new packet arrive **then**

  3.   receive the packet

  4.   let j' be the ID of the received packet

  5.   **if** $(j' = j + 1)$ **then**

  6.   store packet in transmit buffer

  7. **if** $(col(v) = i)$ **and** $cellLeader(v)$ **then**

  8.   **if** $buffer(v)$ is not empty **then**

  9.   transmit packet and empty transmit buffer

Fig. 4. The RIPPLECAST broadcasting scheme.

B. RIPPLECAST

The RIPPLECAST algorithm is reported in Figure 4. The algorithm for the source node is very simple: when the transmission slot correspondent to $col(s)$ is scheduled, the source node transmits a new packet, and increments the packet ID by one. Any non-source node v acts as follows. Independently of the color of the scheduled slot, node v listens to the channel, and receives new packets. Note that a node in general receive packets with the same ID several times; only *new* packets are received at step 3. of the algorithm. If the ID of the new received packet equals the ID of the most recently received packet increased by one, then the new packet is stored in the transmit buffer. If the color of the current slot equals $col(v)$, v is the cell leader, and the transmit buffer is not empty, the packet is transmitted and the transmit buffer emptied.

Function $cellLeader()$ at step 7. checks whether node v is currently a leader node. Leader selection obeys the following rules. During round t, a node (call it v) currently (more specifically, at the beginning of the round) located within a central mini-cell is selected[5] as leader node for that cell (call it A) for that round. More specifically, during round t node v will be in charge of transmitting the packet received by the cell it belonged to at round $t-1$. As we shall see in the next section, the fact that leader nodes are selected amongst the nodes in the central-mini cell, coupled with an upper bound on node velocity, ensures that node v was in cell A also during the entire round $t-1$, thus guaranteeing a correct propagation of broadcast packets. If node v is still in the central mini-cell of cell A at the beginning of round $t+1$, it keeps the leader role also in the next round, otherwise a new node amongst the ones currently present in the central mini-cell is selected as leader for round $t+1$.

C. Analysis

We start borrowing a result from [20], which shows that cells can be colored using $\bar{k}^2 = \Theta(1)$ colors, in such a way that the packet transmitted by a leader node is correctly received by *all* nodes located in neighboring cells (horizontal, vertical, and diagonal adjacency), under the assumption that at most

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[5]The actual rule used to selected leaders in case more than one nodes are present in a mini-cell is irrelevant.

one node per cell with the same color is transmitting. The coloring scheme depicted in Figure 2 assigns the same color to cells at cell distance $\bar{k}$ along the horizontal and vertical direction (details can be found in [20]). The following result has been proved in [20].

Proposition 3: Given a deployment region divided into square cells of side $l = \frac{r_{max}}{2h\sqrt{2}}$, for some constant $h > 1$, it is possible to devise a coloring scheme with k^2 colors, where $k \geq \bar{k} = \left\lceil 2 + 2^{\frac{3}{2}+\frac{4}{\alpha}} \left(\beta\zeta(\alpha-1)h^\alpha/(h^\alpha-1)\right)^{\frac{1}{\alpha}} \right\rceil$, and ζ is the Riemann's zeta function, such that the packets transmitted by leader nodes with the same color are received by all nodes located in cells adjacent to the cell of a transmitter node (horizontal, vertical, and diagonal adjacency), under the assumption that at most one node per cell with the same color is transmitting.

Note that, being h, α and β constants, the number of colors $\bar{k}^2$, which coincides with the number of transmission slots in a round, is $\Theta(1)$.

The next Lemma, whose straightforward proof is omitted, states that source node s generates new packets at rate $\frac{W}{k^2} = \Omega(W)$, which is asymptotically optimal.

Lemma 1: Assume algorithm RIPPLECAST is used to broadcast packets in the network. The source node s generates packets at rate $\frac{W}{k^2} = \Omega(W)$.

We next show that each packet generated by the source is correctly received *all* network nodes within time $T(n) = O\left(\sqrt{\frac{n}{\log n}}\right)$.

Lemma 2: Assume n nodes move within a square region of side $L(n) = \frac{r_{max}}{6h\sqrt{2}}\sqrt{\frac{n}{\log n}}$ according to a mobility model $\mathcal{M}$ with: i) uniform stationary node spatial distribution, and ii) maximum node velocity equal to $\bar{v} = \frac{l}{3\bar{k}^2\tau}$, where τ is the duration of a transmission slot and l is the side of a cell. Furthermore, assume algorithm RIPPLECAST is used to broadcast packets. Then, a packet generated by the source node at round t is received by all network nodes within round $t + O\left(\sqrt{\frac{n}{\log n}}\right)$, w.h.p.

Proof: Let us call the cells at cell distance i from the cell containing the source node the i-th ripple. We start showing that: a) for each cell A in the i-th ripple, the leader node of cell A transmits during round $t+i$ the packet generated by the source node at round t. The proof is by induction on i. Property a) trivially holds when $i = 0$. Assume now property a) holds for each $j < i$. In order for a) to hold also for i, we need to show that, for any cell A in the i-th ripple, the node selected as leader for A during round $t+i$, which is going to transmit during round $t+i$, has received the packet generated by the source at round t *before* its transmission opportunity during round $t+i$. Given that a) is assumed to hold for $j < i$, we have that the leader node of cell B, where B is any of the cells in the $(i-1)$-th ripple adjacent to A (note that at least one such cell always exists), has transmitted during round $t+i-1$ the packet generated by source node at round t. Given the rules for selecting leader nodes, we have that the leader node at round $t+i-1$ for cell B is selected amongst the nodes located in the central mini-cell of B at the beginning of round $t+i-1$. By Proposition 1, we have that at least one such node
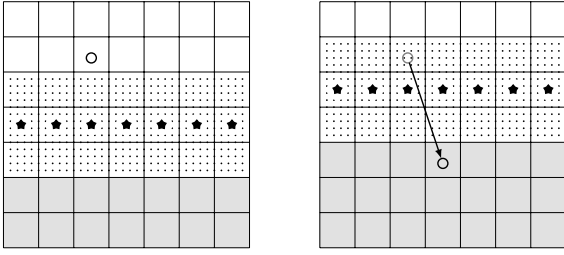
Fig. 5. Assume the source s is somewhere south of the diagrams, and the propagation front of packet p moves northward. Stars represents cell leaders active in a certain round, and the checkered region is the region covered by them. The white area has not yet been covered by packet p, while the gray area represents cells in $Cov(p)$ during a certain round. On the left, a circle represents a node lying in the white area which has not yet received p at a certain round $t+j-1$. To avoid reception of packet p, the node must cut the propagation front and reach the gray area during round $t+j$ (right), where p is no longer transmitted. Thus, the node should travel distance at least $2l$ between the two consecutive rounds.

exists, w.h.p. Furthermore, the upper bound $\bar{v}$ on node velocity guarantees that a node travels at most $\bar{v}\bar{k}^2\tau = \frac{l}{3}$ in the time elapsing between the beginning of round $t+i-1$ and the beginning of round $t+i$. Since the leader node of cell B was within the central mini-cell of cell B at the beginning at round $t+i-1$ and given the above observation about the distance traveled by nodes, we have that the leader node of cell B is still within cell B when it is scheduled for transmission during round $t+i-1$. Hence, by Proposition 3, we have that the packet transmitted by the leader node of cell B during round $t+i-1$, which by induction is the packet generated by the source node at round t, is correctly received by all nodes within cell A at the time of transmission. In particular, the leader node w for cell A at round $t+i$ is within the central mini-cell of A at the beginning of round $t+i$, which given the above observation about maximum traveled distance, ensures that w was within cell A also during the entire round $t+i-1$. Thus, node w can correctly receive the packet sent by the leader node of cell B during round $t+i-1$, and can forward it in the network when scheduled for transmission at round $t+i$, which implies property $a)$.

Let us now define the set of *covered cells* $Cov(p)$ for a certain packet p as the set of cells such that their respective leader nodes have already transmitted packet p. By property $a)$, and assuming packet p is generated by the source at round t, we have that $Cov(p)$ at round $t+i$ is the union of all the cells in ripples $0,\ldots,i$. Given the assumption on the size $L(n)$ of the deployment region, we have that $Cov(p)$ contains *all* the cells in the deployment region at round $t+\frac{L(n)}{2l} = t+O(\sqrt{\frac{n}{\log n}})$. Let us now consider an arbitrary mobile node u, and assume by contradiction that node u has not received packet p by the end of round $t+\frac{L(n)}{2l}$. Since $Cov(p)$ contains all the cells in the deployment region by then, and considering that each of the ripples propagating packet p is a "closed curve"[6], the only possible way for node u to avoid receiving p is to cut through the ripple propagation front during round j, for some $0 < j < \frac{L(n)}{2l}$. However, for this to be possible, node u should travel distance at least $2l$ between two successive rounds (see

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[6]For the sake of simplicity, we use the intuitive notion of "closed curve" when referring to a ripple, although the ripple is not a curve in standard geometric sense.

Figure 5), which is possible only if node velocity is at least $v' = \frac{2l}{2\bar{k}^2\tau} > \bar{v}$. Thus, the assumption about maximum node velocity is contradicted, and the Lemma is proved. ∎

Is the upper bound on node velocity imposed by Lemma 2 restrictive? The answer, for typical values of the network parameters, is *no*, owing to the very high packet propagation speed within the network. For instance, assuming an outdoor propagation environment with path-loss $\alpha = 3$, channel parameters typical of an 802.11a/g network with $54Mbs$ data rate (more specifically, $\beta = 22dB$, $P = 100mW$, and $N = -90dBm$), a packet size of $1KB$, and setting $h = 2$ in the cell partitioning scheme, we have that $\bar{k} = 50$, $r_{max} = 858m$, $l = 151m$, $\tau = 180\mu sec$ (leaving adequate margin for radio signal propagation time), and the upper bound on velocity is $\bar{v} = 111.852m/sec \approx 403km/h$.

We are now ready to prove the main result of this section:

Theorem 2: Assume n nodes move within a square region of side $L(n) = \frac{r_{max}}{6h\sqrt{2}}\sqrt{\frac{n}{\log n}}$ according to a mobility model $\mathcal{M}$ with: $i)$ uniform stationary node spatial distribution, and $ii)$ maximum node velocity equal to $\bar{v} = \frac{l}{3\bar{k}^2\tau}$, where τ is the duration of a transmission slot and l is the side of a cell. Algorithm RIPPLECAST provides asymptotically optimal broadcast capacity and latency.

Proof: The proof is a straightforward consequence of lemmas 1 and 2, and of the observation that the duration of a round (which is composed of $\bar{k}^2$ transmission slots, each of constant duration τ) is $\Theta(1)$. ∎

VI. BROADCASTING WITH LEADER ELECTION

In this section, we revisit the broadcasting problem taking into account also the burden incurred by leader election. Distributed leader election is one of the most investigated problems in the distributed computing literature. Though, the leader election problem we face is non-standard: although each single leader election in a mini-cell corresponds to the classical single-hop leader election problem [15], we have to perform several such elections: one for each of the $\frac{n}{\log n}$ mini-cells in the deployment region. Since sequentially executing these elections would considerably impact both broadcast capacity and latency, we propose running as many simultaneous leader elections as possible, subject to the condition that simultaneously active leader elections do not corrupt each other.

The approach we pursue to tackle the problem at hand is a combination of the ID-based leader election scheme proposed in [1] for network-wide election of a single leader node in a wireless multihop network, and of the carrier sense based technique used in [22] to distributedly build a dominating set in a wireless multihop network. The main idea is to run parallel leader elections in the cells colored with the same color. As we shall see, in order to ensure that mutual interference does not corrupt concurrent leader elections, we have to use a relatively larger (and non-constant) number of colors, which leads to a poly-logarithmic broadcast capacity and latency degradation with respect to optimal. Even worse, using a non-constant number of colors leads to an asymptotically vanishing upper bound on node maximum velocity.

The leader election process in a cell is performed as follows. Each of the n mobile nodes in the network is assigned with

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Algorithm for any non source node u:
Time is divided into $\log n$ phases of constant duration
    1. **if** u is not in a central mini-cell **then** exit //not a leader
    2. **for** $i = 1$ **to** $\log n$ **do** //phase i
    3.   **if** $bit(ID(u), (\log n - i)) = 1$ **then**
    4.   transmit a "1" bit
    5.   **else**
    6.   listen to the channel
    7.   **if** sensed signal $> T_s$ **then** exit //not a leader
    8. set $leader =$ **true** //node u is leader

Fig. 6. The leader election algorithm.

a unique binary ID. Let $ID(u)$ denote the ID of node u. Similarly to [1], the binary representation of node IDs is used to elect leader nodes: at the end of the election process, the leader node for a certain cell A is the node with highest ID among the nodes within the central mini-cell of A at the beginning of the leader election process. The leader election process, reported in Figure 6, is divided into $\log n$ phases of constant duration, where the duration of a phase is sufficient to transmit a single bit of information on the channel. During phase i, node u, if still active, checks whether the i-th most significant bit of its ID is 1, and in that case transmits a "1" bit on the channel. Otherwise, it listens to the channel, and becomes inactive if the signal sensed on the channel exceeds a certain threshold T_s. As we shall see, threshold T_s is set in such a way that the following two properties are satisfied:

   a) if at least one node within the same mini-cell of u is transmitting, then the sensed signal at u is $> T_s$.
   b) if no node within the same mini-cell of u is transmitting, then the sensed signal at u is $< T_s$.

Note that, in order for such threshold T_s to exist, we must be able to upper bound the aggregate power received at u generated by nodes in other mini-cells; furthermore, in order for $a)$ and $b)$ to simultaneously hold, this upper bound must not depend on n. We now show that a threshold T_s satisfying properties $a)$ and $b)$ above can actually be defined if we use $(k^*)^2$ colors, where $k^* = \Theta(\log n)$.

We first prove the following technical lemma, which provides a bound on the amount of power received by a node from nodes in cells with the same color.

Lemma 3: Let us assume a cell coloring with k^2 colors as defined in Figure 2 and that each cell contains at most m nodes. Let us fix an arbitrary node u in an arbitrary cell C. If $k \geq 2$ and all the nodes in all the cells (apart C) with the same color as C transmit simultaneously, then the interference P_I experienced by u satisfies

$$P_I < m \frac{16 P \zeta(\alpha - 1)}{(k-1)^\alpha l^\alpha} , \qquad (1)$$

where ζ is the Riemann's zeta function.

    Proof: Let us consider now the interference experienced by u under the condition in each cell with the same color there are at most m nodes. Assume w.l.o.g. that $cell(u)$ has coordinates $(0,0)$. Given the coloring scheme, interferers lie in the cells with bottom left corner at $(x \cdot k \cdot l, y \cdot k \cdot l)$ with $x, y \in \mathbb{Z}$ and $(x, y) \neq (0, 0)$ (shaded cells in Figure 2).

The distance $d(x, y)$ between u and an interferer located in cell $(x \cdot k \cdot l, y \cdot k \cdot l)$, with $x, y \neq 0$, can be lower bounded as
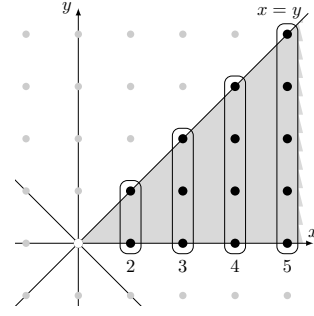

Fig. 7. 8-fold symmetry in the derivation of the upper bound to the total interference.

follows:

$$d(x,y) \geq \sqrt{(|x|kl - l)^2 + (|y|kl - l)^2} , \qquad (2)$$

where the term $-l$ depends on the actual positions of u and I inside their respective cells.

Since $a^2 + b^2 \geq (\max\{a, b\})^2$, from (2) we obtain the following lower bound on $d(x, y)$:

$$\begin{aligned}
d(x,y) &\geq \max\{|x|, |y|\}kl - l = l(k \max\{|x|, |y|\} - 1) \geq \\
&\geq (k-1)l \max\{|x|, |y|\} .
\end{aligned}$$

Note that the last bound is always strictly positive, since we are assuming $k \geq 2$ and $|x|, |y|$ are not both 0.

The interference received by u thus satisfies

$$\begin{aligned}
P_I &< m \sum \frac{P}{((k-1)l \max\{|x|, |y|\})^\alpha} = \\
&= m \frac{P}{(k-1)^\alpha l^\alpha} \sum \frac{1}{\max\{|x|, |y|\}^\alpha} ,
\end{aligned}$$

where the sum is extended over all the pairs $(x, y) \neq (0, 0)$, with $x, y \in \mathbb{Z}$.

Counting twice the contributions along $x = 0$, $y = 0$, and $|x| = |y|$, we have

$$\sum_{(x,y) \neq (0,0)} \frac{1}{\max\{|x|, |y|\}^\alpha} < 8 \sum_{x=1}^{\infty} \sum_{y=0}^{x} \frac{1}{x^\alpha}$$

due to the 8-fold symmetry of the summation shown in Figure 7. Collecting the values for which $\max(x, y) = x$ we obtain

$$8 \sum_{x=1}^{\infty} \sum_{y=0}^{x} \frac{1}{x^\alpha} = 8 \sum_{x=1}^{\infty} \frac{x+1}{x^\alpha} < 16 \sum_{x=1}^{\infty} \frac{1}{x^{\alpha-1}} = 16\zeta(\alpha - 1) ,$$

where $\zeta(\cdot)$ is the Riemann's zeta function and summarizing we obtain formula (1). ∎

Lemma 4: Assume the cell coloring scheme is composed of k^2 colors, with

$$k > k^* = 1 + \frac{2^{4/\alpha} \cdot \sqrt{2} \cdot \zeta(\alpha-1)^{1/\alpha}}{3} \cdot (c \log n)^{1/\alpha} ,$$

for some constant $c > 1$. Then, there exists a (constant) threshold T_s such that properties $a)$ and $b)$ above are satisfied.

    Proof: We first lower bound the intensity P_T of the signal received by a node u within a mini-cell when another node within the same mini-cell is transmitting. Given the assumed radio propagation model, we have

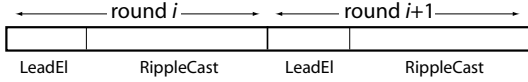$$P_T \geq P \left(\frac{l}{3} \sqrt{2} \right)^{-\alpha} = T_s' ,$$

Fig. 8. The broadcast scheme with leader election.

which implies that we must have $T_s < T_s'$.

We now upper bound the intensity of the signal received at node u generated by nodes belonging to other mini-cells with the same color within the network. Observing that the maximally occupied mini-cell contains at most $c \log n$ nodes, for some constant $c > 1$ (see Proposition 1), and letting $m = c \log n$ and $k = k^*$ in Lemma 3, we obtain that the aggregate power P_I at node u generated by nodes within mini-cells with the same color is upper bounded by:

$$P_I < (c \log n) \frac{16 P \zeta (\alpha - 1)}{(k-1)^\alpha l^\alpha} = T_s'',$$

for some constant $c > 1$, where k is the step of the coloring scheme (i.e., we have k^2 colors in total). Thus, if we set $T_s > T_s''$, we are guaranteed to satisfy property $b)$. The proof of the Lemma follows by observing that, when $k > k^*$, we have $T_s' > T_s''$, and a threshold satisfying both properties $a)$ and $b)$ above can be obtained by choosing any value T_s such that $T_s'' < T_s < T_s'$. ∎

We are now ready to introduce the broadcasting scheme, which is a combination of RIPPLECAST with the leader election scheme presented above. A round of the broadcast scheme is composed of two steps (see Figure 8): in the first step, leader nodes *for each cell* are elected according to the leader election algorithm described above; in the second step, RIPPLECAST is executed using leader nodes elected in the first step to propagate broadcast packets.

We are now ready to characterize the asymptotic properties of this combined broadcasting scheme.

Lemma 5: Assume n nodes move within a square region of side $L(n) = \frac{r_{max}}{6h\sqrt{2}} \sqrt{\frac{n}{\log n}}$ according to a mobility model $\mathcal{M}$ with: $i)$ uniform stationary node spatial distribution, and $ii)$ maximum node velocity equal to $v^* = \frac{l}{3(\tau' \cdot (k^*)^2 \cdot \log n + \bar{k}^2 \tau)}$, where τ' is the duration of a phase of the leader election process, τ is the duration of a transmission slot, and l is the side of a cell. Furthermore, the above described combined leader election and broadcasting scheme is used to broadcast packets. Then, a packet generated by the source node at round t is received by all network nodes within round $t + O\left(\sqrt{\frac{n}{\log n}}\right)$, w.h.p.

Proof: Similarly to Lemma 2, we can show that the packet transmitted by the source during round t is received by each network node within round $t + O\left(\sqrt{\frac{n}{\log n}}\right)$. However, the upper bound on node velocity must take into account the longer duration of a communication round, which comprises also the leader election step. The leader election step lasts for $(k^*)^2 \cdot \log n$ phases overall (leader election processes, each lasting $\log n$ phases, are performed in parallel for each of the $(k^*)^2$ colors). Note that the duration τ' of a phase should be sufficient to send a single bit of information of the channel, i.e., $\tau' \ll \tau$. The total duration of a communication round is then $\tau' \cdot (k^*)^2 \cdot \log n + \bar{k}^2 \tau$. Similarly to Lemma 2, the maximum

node velocity must be set in such a way that the maximal traveled distance within a communication round equals $\frac{l}{3}$, from which we derive $v^* = \frac{l}{3(\tau' \cdot (k^*)^2 \cdot \log n + \bar{k}^2 \tau)}$. ∎

Theorem 3: Assume n nodes move within a square region of side $L(n) = \frac{r_{max}}{6h\sqrt{2}} \sqrt{\frac{n}{\log n}}$ according to a mobility model $\mathcal{M}$ with: $i)$ uniform stationary node spatial distribution, and $ii)$ maximum node velocity equal to $v^* = \frac{l}{3(\tau' \cdot (k^*)^2 \cdot \log n + \bar{k}^2 \tau)}$, where τ' is the duration of a phase of the leader election process, τ is the duration of a transmission slot, and l is the side of a cell. The above described combined leader election and broadcasting scheme provides broadcast capacity and latency within a factor $\Theta((\log n)^{1+\frac{2}{\alpha}})$ from optimal.

Proof: The duration of step 1 (leader election) in each round is $\Theta((\log n)^{1+2/\alpha})$. In fact, leaders must be elected for each cells, which are divided into $(k^*)^2 = \Theta((\log n)^{2/\alpha})$ groups. The leader election process, which lasts $\log n$ time, goes on in parallel for all the cells in a group, implying that the overall duration of step 1 in a round is $\Theta(\log n \cdot (\log n)^{2/\alpha}) = \Theta((\log n)^{1+2/\alpha})$. Even if step 2 of a round has a constant time duration (this is because the required number of colors $\bar{k}^2$ for RIPPLECAST is a constant), the overall duration of a round of communication is $\Theta((\log n)^{1+2/\alpha})$. Since the source transmits a new broadcast packet at each round, we have that the broadcast rate is $\Theta\left(\frac{W}{(\log n)^{1+\frac{2}{\alpha}}}\right)$, which, according to Claim 1, is within a factor $\Theta((\log n)^{1+\frac{2}{\alpha}})$ from optimal. By Lemma 5, the packet generated by the source at round t is received by each network node within round $t + O\left(\sqrt{\frac{n}{\log n}}\right)$. Given that the duration of a round is $\Theta((\log n)^{1+\frac{2}{\alpha}})$ and Claim 1, we have that the broadcast latency achieved by our scheme is also within a factor $\Theta((\log n)^{1+\frac{2}{\alpha}})$ from optimal. ∎

Comparing theorems 2 and 3, we observe a polylogarithmic performance degradation with respect to both capacity and latency when the burden for leader election is taken into account. Most importantly, the burden related to the leader election process considerably strengthen the upper bound on node velocity, *which becomes asymptotically vanishing as n grows to infinity*. Thus, the larger the network, the more stationary the nodes must be in order to achieve near-optimal broadcast capacity and latency. However, owing to the orders of magnitude smaller value of τ' as compared to τ (we recall that τ' is the time necessary to transmit a single bit of information, instead of an entire packet) and logarithmic dependence on n, the actual bound on maximal node velocity is only marginally influenced by the number of network nodes. For instance, the upper bound v^* on node velocity is $v^* = 111.816 m/sec$ when $n = 2^{10} = 1024$ (assuming the same parameters as in Section V-C, and setting $\tau' = \frac{\tau}{1000}$), which should be compared to $\bar{v} = 111.852 m/sec$ when the leader election burden is ignored. When $n = 2^{50}$ (far above the size of any practical network), the upper bound becomes $v^* = 111.415 m/sec$, which is only marginally smaller than $\bar{v}$.

VII. DISCUSSION AND FUTURE WORK

In this paper, we have investigated the fundamental limits of broadcasting in mobile wireless networks, and we have shown

that, while broadcasting is not inherently limited (in terms of both capacity and latency) by node mobility, the coordination burden caused by the need of repeatedly selecting broadcast relay nodes does indeed reduce broadcast performance of a poly-logarithmic factor. Our results hold under a set of assumptions: nodes move within a square region according to a mobility model with stationary uniform node spatial distribution, and node velocity is upper bound by a constant (which becomes an asymptotically vanishing function of n when the coordination burden is taken into account).

We first observe that some generalizations of our results are straightforward: up to tedious technical details, our findings can be extended to deployment regions of different compact shapes, as long as broadcast ripples are still "closed curves". Extension to mobility models whose stationary node spatial distribution is "almost uniform" is also straightforward; by "almost uniform", we mean that the ratio between the larger and smaller value of the two-dimensional probability density function describing stationary node positions within the deployment region is an arbitrary *positive* constant. A more challenging generalization of our results, which we leave as open problem, is considering the case of *mobile* broadcast source. Here, the challenge is that broadcast ripples move along with the source, thus the analysis presented in this paper should be considerably re-formulated.

What are the implications of our findings for the design of practical broadcasting protocols for mobile networks? The main implication is that network designers should focus their design on identifying *invariant* properties of the mobile network (e.g., node spatial distribution), and then build their protocol exploiting these properties. Clearly, location-awareness is likely to be a key feature in designing efficient broadcasting protocols for mobile wireless networks.

It is interesting also to discuss the relative effect of node mobility in case of unicast and broadcast communications: in unicast communication – under the assumption that arbitrarily high delays can be tolerated–, node mobility can be used as a mean to suppress (or considerably reduce) the relaying burden, thus bringing capacity up to the optimal value; on the contrary, in case of broadcast, node mobility introduces the need of frequently re-selecting broadcast relay nodes, thus inducing a coordination burden which causes a poly-logarithmic capacity and latency degradation with respect to optimal. However, it is important to observe that this performance degradation is not inherently due to the broadcast communication pattern, but rather to a "common practice" of performing broadcast communications based on the selection of broadcast relay nodes. Hence, a promising research direction is to investigate whether alternative broadcasting approaches can be used to reach the capacity and latency limits. In particular, we intend to explore cooperative communications, which have already been successfully used to improve capacity limits for unicast communications (see, e.g., [5], [18]).

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