09471 Abstracts Collection
Computer-assisted proofs - tools, methods and applications
— Dagstuhl Seminar —

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Abstract. From 15.11. to 20.11.2009, the Dagstuhl Seminar 09471 "Computer-assisted proofs - tools, methods and applications " was held in Schloss Dagstuhl – Leibniz Center for Informatics. During the seminar, several participants presented their current research, and ongoing work and open problems were discussed. Abstracts of the presentations given during the seminar as well as abstracts of seminar results and ideas are put together in this paper. The first section describes the seminar topics and goals in general. Links to extended abstracts or full papers are provided, if available.

Keywords. Verification methods, computer algebra, computer-assisted proofs

09471 Executive Summary – Computer-assisted proofs - tools, methods and applications

From November 15-20, 2009, the Dagstuhl seminar on "Computer-assisted proofs - tools, methods and applications" continued a series of previous successful seminars. Participants from 10 different countries presented recent results in verification methods, computer algebra, and other computer-assisted-proof related areas.

We had lively talks and discussions, during the regular times for talks, during meals and afterwards. In the following links to abstracts and/or the presentation are given were applicable.

Keywords: Verification methods, computer algebra, computer-assisted proofs

Joint work of: Brown, Malcolm B.; Kaltofen, Erich; Oishi, Shin'ichi; Rump, Siegfried M.

Extended Abstract: http://drops.dagstuhl.de/opus/volltexte/2010/2531

A computer-assisted proof for stable/unstable behaviour of periodic solutions for the forced damped pendulum

Balazs Banhelyi (University of Szeged, HU)

We consider a simple mechanical system, the forced damped pendulum. According to earlier publications [3] this system has chaotic behaviour.

Among other things, the chaotic behavior is implied by the existence of an unstable periodic solution.

First we describe an automatic technique that is able to enclose periodic solutions. To find such regions that satisfy the respective conditions, we applied an interval inclusion and a recursive subdivision technique. In the considered system we find two periodic solutions. One of them is unstable and the other one is stable. The unstable 2π-periodic solution is bifurcated from the top equilibrium position, and the other, asymptotically stable 2π-periodic solution is bifurcated from the bottom equilibrium position of the damped unforced pendulum.

To find out whether the periodic solution is stable, we can calculate the Jacobian matrix for the related variables and analyze their eigenvalues. Unfortunately, the Jacobian matrix of this pendulum cannot be given in closed form. One of the possibilities is to apply the characteristic multiplicator technique. To this end we can determine the Poincaré map, and while we follow the trajectory [2], we calculate the derivatives as well. Rounding and other errors were considered, both in the trajectory following and in the calculation of eigenvalues. With this method we can provide the anticipated result for the mentioned 2π-periodic solutions [1].

Finally, we discuss some stabilization techniques. We analyze how to stabilize the unstable periodic solution of the considered pendulum. One of these methods uses the current values of the state variables (the angle and the speed of the pendulum), thus this is a feedback control. The other method does not use this kind of information, so it is not a feedback control. We also analyze whether these methods really stabilize the unstable solution. We are able to prove necessary conditions for the stabilization of the unstable solution.

[1] Bánhelyi, B., T. Csendes, B. Garay, and L. Hatvani: A computer–assisted proof of Σ_3-chaos in the forced damped pendulum equation. SIAM J. Applied Dynamical Systems, accepted for publication.

[2] Nedialkov, N.: VNODE, A Validated Solver for Initial Value Problems for Ordinary Differential Equations.
http://www.cas.mcmaster.ca/~nedialk/Software/VNODE/VNODE.shtml

[3] Hubbard, J.H.: The Forced Damped Pendulum: Chaos, Complication and Control. American Mathematical Monthly, 8(1999) 741-758.

Keywords: Forced damped pendulum, chaos, control

Joint work of: Bánhelyi, Balázs; Hatvani, László

Verified Enclosures for Sloshing Frequencies

Henning Behnke (TU Clausthal, DE)

The calculation of the frequencies ω for small oscillations of an ideal liquid in a container results in a Steklov eigenvalue problem.

A procedure for calculating lower and upper bounds to the eigenvalues is presented. The calculation of upper bounds is done by means of the well known Rayleigh-Ritz procedure.

For the lower bound computation Goerich's generalization of Lehmann's method is applied, trial functions are constructed with finte elements. It is shown that Lehmann's method can not be applied in this context, whereas a specification of Goerisch's method is possible. Rounding errors in the computation are controlled with interval arithmetic. Numerical results for different cross sections are given.

Keywords: Eigenvalue problem, variational methods, Rayleigh-Ritz, Lehmann-Goerisch, interval arithmetic

The Role of Computer-Assisted Numerical Proofs in Efficient Hardware Design

George A. Constantinides (Imperial College London, GB)

This talk will discuss the use of computer assisted proofs in high performance embedded computing using Field-Programmable Gate Arrays (FPGAs). I will discuss the importance of static verification in this context, the role of parallelism, the need for numerical optimization, and the role that precision plays in determining performance. I will then discuss the vision of my group for an automated algebraic methodology for answering some of these questions, and present some preliminary results.

Keywords: FPGA, optimization, numerical algorithm, proof

GasDay: Forecasting Customer Demand for Natural Gas

George F. Corliss (Marquette University - Milwaukee, US)

Natural gas for heating takes 2 - 3 days to get to Milwaukee from Oklahoma or Louisiana. If our local utility does not order enough, they buy it on the spot market at premium prices. If they order too much, the pipeline companies charge them penalties.

The GasDay lab at Marquette University licenses to local natural gas utilities software to forecast the gas demands of their customers hours, days, and months in the future.

We license to 21 utilities across the US. Each day, software written by our students helps forecast 20% of the gas used by residential, commercial, and industrial customers in the US. Our customers tell us our forecasts save their customers tens of millions of dollars each winter. We also offer analysis consulting services including peak day studies and data cleaning.

We will discuss some of the mathematical modeling challenges, as well as the entrepreneurial challenges of running a $500K business with students within the university.

This talk has **no** interval content. I will not be offended if the organizers choose not to schedule it.

Keywords: Applied mathematics, modeling, forecast natural gas demand

The role of optimization in computer assisted proofs

Tibor Csendes (University of Szeged, HU)

The talk will present an effective way of using unreliable, approximative but quick optimization methods to prove theoretical mathematical statements with rigour. We use chaos detection as sample problems, and highlight the important points of the composed fully automatic computational procedure. Some new results, obtained with the introduced technique are listed and shortly introduced.

1. B. Bánhelyi, T. Csendes, B.M. Garay, and L. Hatvani: Computer assisted proof of chaotic behaviour of the forced damped pendulum. Folia FSN Universitatis Masarykianae 16(2006) 9-20.
2. B. Bánhelyi, T. Csendes, and B.M. Garay: Optimization and the Miranda approach in detecting horseshoe-type chaos by computer. Int. J. Bifurcation and Chaos 17(2007) 735-748
3. T. Csendes, B. Bánhelyi, and L. Hatvani: Towards a computer-assisted proof for chaos in a forced damped pendulum equation. J. Computational and Applied Mathematics 199(2007) 378-383
4. T. Csendes, B.M. Garay, and B. Bánhelyi: A verified optimization technique to locate chaotic regions of Hénon systems. J. of Global Optimization 35(2006) 145-160

Keywords: Computer assisted proof, optimization

GloptLab - a rigorous, configurable framework for solving algebraic constraint satisfaction problems

Ferenc Domes (Universität Wien, AT)

GloptLab is an easy-to-use testing and development platform for solving algebraic constraint satisfaction problems, written in Matlab.

Various new and state-of-the-art algorithms implemented in GloptLab are used to reduce the search space: scaling, constraint propagation, linear relaxations, strictly convex enclosures, conic methods, probing and branch and bound. By using interval arithemic and directed rounding it is guarateed that all methods in GloptLab are rigorous.

From the method repertoire one can build custom made strategies with a user-friendly graphical interface. During the solution process each method may produce a certificate, containing all information to reproduce the theoretical proof of the method. The certificates can be used to generate a human readable proof.

Keywords: Constraint satisfaction problems, verified computing

Joint work of: Domes, Ferenc; Neumaier, Arnold

Full Paper:
http://www.mat.univie.ac.at/~dferi/publications.html

Computing enclosures for the matrix square root

Andreas Frommer (Universität Wuppertal, DE)

We present a slight generalization of a well-established theorem on the Krawczyk operator. This theorem will be useful to develop an $\mathcal{O}(n^3)$ method to compute enclosures for a primary square root of a matrix from a given accurate approximation.

Keywords: Matrix functions, square root, Krawczyk's method, complexity

Joint work of: Frommer, Andreas; Behnam, Hashemi

Computation of dot products in finite fields with floating-point arithmetic

Stef Graillat (UPMC - Paris 6, FR)

Finite fields are widely used in numerous fields like computer algebra, cryptography or error code correcting. It is then important to be able to deal with them efficiently.

The approach we will present here is to use floating-point arithmetic with which computations can be done efficiently. The main concern is then to deal with rounding errors that can appear during the computation. To solve this problem, we use error-free transformations (EFT). Using theses EFT on recent processors (with an FMA), we show that it is possible to deal with high finite fields. We will present two different methods for the computation of dot product.

This is a joint work with Jérémy Jean.

Verified Numerical Computations in Sums-Of-Squares Optimization

Viktor Haerter (TU Hamburg-Harburg, DE)

In the last decade sum of squares optimization problems have been popularized and found many applications in applied mathematics. Often the user is interested only on an approximation, but sometimes rigorous SOS certificates are required. A well known approach to compute rigorous SOS certificates is based on rationalization of numerical solutions.

We present a verification method for SOS problems based on verified semidefinite programming. In many cases this method seems to be an alternative to rational sums of squares providing comparable results in less time.

Keywords: Sums of squares, semidefinite programming, global optimization

Dispersion relations for Split-Ring Resonator media

Vu Hoang (Universität Karlsruhe, DE)

We report on ongoing work and introduce a simple mathematical model for arrays made of Split-Ring Resonators in a 2D setting.

These are given as thin metallic rings having a gap. These structures show a strong magnetic response and are considered as building blocks for so-called "metamaterials" with unusual optical properties. We are not studying the homogenization of such structures, but rather look for propagating modes in photonic crystals made out of such elements. Enclosure methods for spectral problems, applied to these structures, could be useful for understanding their electromagnetic properties.

Joint work of: Hoang, Vu; Radosz, Maria

Reliable numerical evaluation of eigenvalues involved in polynomial systems solving

Fabienne Jezequel (UPMC - Paris, FR)

In order to improve the performances of symbolic algorithms for solving large polynomial systems, certified numerical computations may be performed.

We present here how to control the validity of the numerical evaluation of eigenvalues involved in polynomial systems solving. The accuracy of the computed eigenvalues can be estimated from the eigenvalue condition number. Round-off errors due to the use of floating-point arithmetic for eigenvalues computation can be estimated using Discrete Stochastic Arithmetic implemented in the CADNA library. Guaranteed inclusions of the eigenvalues can be computed using the INTLAB toolbox. These numerical validation tools have been compared, taking into account the exact eigenvalues computed using symbolic algorithms. In order to improve the numerical quality of the results, experiments have also been carried out using extended precision arithmetic and the MPFR multiple precision library.

Keywords: Conditioning, Discrete Stochastic Arithmetic, extended precision, interval arithmetic, eigenvalues, numerical validation, QR algorithm

On convergence of optimizers of relaxations to the set of optimizers

R. Baker Kearfott (Univ. of Louisiana - Lafayette, US)

A common practice in leading software and ad-hoc codes for non-convex global optimization is to combine branch and bound processes with linear relaxations, convex relaxations, or non-convex relaxations that are easy to solve. The objective value at a solution of a relaxation gives a lower bound on the global optimum of the original problem, something that is valuable in making the branch and bound process efficient. In a seminal 2004 work, Neumaier and Shcherbina showed how the duality gap can be used to obtain, with little extra computation, a mathematically rigorous lower bound on the optimum of the original problem, giving an approximate optimizing point of a linear relaxation, thus making approximate solution of linear relaxations available for use in mathematically rigorous non-convex optimization.

If the relaxation approximates the original problem sufficiently well, one would expect not only the optimum, but also optimal parameters of the relaxation to be close to optimal parameters of the original problem.

However, to our knowledge, a careful analysis of this heretofore had not been done, nor has a rigorous quantification of the distance from an optimizer. Such information could be of use both in mathematically rigorous and non-rigorous codes.

After defining what it means for a relaxation to be close to a problem, we present a theorem that states that solution parameters of any sequence of ever-closer relaxations converges to a solution of the original problem. In particular cases, elements of the proof of the theorem might form a computational method for ascertaining the distance between the two parameter sets. We also show, through a counterexample, that there are, in general, solutions to the original problem that are not limit points to solutions of any particular sequence of relaxations.

Keywords: Global optimization, bounds on solution set

Interval Computations, Rigor and Non-Rigor in Deterministic Continuous Global Optimization

R. Baker Kearfott (Univ. of Louisiana - Lafayette, US)

Deterministic branch and bound methods for the solution of general nonlinear programs have become increasingly popular during the last decade or two, with increasing computer speed, algorithmic improvements, and multiprocessors. There are presently several commercial packages. Although such packages are based on exhaustive search, not all of them rigorously take account of round-off error, singularities, and other possibilities. Popular non-rigorous packages have much in common with mathematically rigorous packages, including overall structure and basic techniques. Nonetheless, it is not trivial to make non-rigorous packages rigorous. We

1. Define different kinds of answers that global optimization software can claim to provide.
2. Explain where rigor might be needed and where it is not practical.
3. Briefly review salient techniques common to deterministic branch and bound methods.
4. Illustrate pitfalls in non-rigorous branch and bound methods.
5. Outline some of the techniques to make non-rigorous software rigorous, and provide guidance for research into and implementation of these techniques.
6. Provide some theoretical backing, with examples, for convergence of common relaxation techniques

Keywords: Deterministic global optimization, mathematical rigor, relaxations, branch and bound, interval computations

Full Paper:
http://interval.louisiana.edu/preprints/2009-verified-vs-non-verified.pdf

See also: This paper has been accepted for publication in the journal "Optimization Methods and Software"

Robust centralized and distributed bounded-error parameter estimation

Michel Kieffer (CWRS - Supélec - Université Paris-Sud, FR)

This contribution proposes guaranteed robust bounded-error distributed estimation algorithm. It may be employed to perform parameter estimation from data collected in a network of wireless sensors. The algorithm is robust to an arbitrary number of outliers. Using interval analysis, one is able, provided that the network is connected, to evaluate at each sensor, an outer approximation of the set of all parameter values which are consistent with a given number of measurements, and with noise bounds. An application to a robust distributed source localization problem is considered.

Keywords: Bounded errors, distributed estimation, outliers, parameter estimation, robust estimation

An a priori constant which appears in bi-harmonic problems

Kenta Kobayashi (Kanazawa University - Ishikawa-ken, JP)

To apply numerical verification method (Nakao's method) to the solution of the 2-D driven cavity problem:

$$\begin{cases} \Delta^2 u + \Delta^2 \phi + RJ\Big(u+\phi, \Delta(u+\phi)\Big) = 0 \quad \text{in} \ \ \Omega, \\ \hfill u = \dfrac{\partial u}{\partial n} = 0 \quad \text{on} \ \ \partial\Omega, \end{cases}$$

where Ω is a rectangle domain, $\phi = 16x^2(1-x)^2y^2(1-y)$, $u+\phi$ denotes stream function, R denotets Reynolds number and $J(u,v) = u_x v_y - u_y v_x$, we have to establish an a priori error estimation for finite element solution of the following bi-harmonic equation:

$$\begin{cases} \qquad \Delta^2 u = f \quad \text{in} \ \ \Omega, \\ u = \dfrac{\partial u}{\partial n} = 0 \quad \text{on} \ \ \partial\Omega. \end{cases}$$

It is known that if f belongs to $L^2(\Omega)$, the solution u belongs to $H^4(\Omega) \cap H_0^2(\Omega)$.

To verify this solution, we have to bound $|u - \vartheta_\Omega u|_{H_0^2(\Omega)}$ by $\|\Delta^2 u\|$ where ϑ denots cubic spline inetrpolation.

It is easy to prove that

$$|u - \vartheta_\Omega u|_{H_0^2(\Omega)} \le \frac{2h^2}{\pi^2} |u|_{H^4(\Omega)},$$

where

$$|u|_{H^4(\Omega)} = \|u_{xxxx}\|^2_{L^2(\Omega)} + 4\|u_{xxxy}\|^2_{L^2(\Omega)} + 6\|amp; u_{xxyy}\|^2_{L^2(\Omega)} \\ amp; +4\|u_{xyyy}\|^2_{L^2(\Omega)} + \|u_{yyyy}\|^2_{L^2(\Omega)}.$$

Therefore, our remained task is to fined the constant which satisfy

$$|u|_{H^4(\Omega)} \le C\|\Delta^2 u\|, \qquad u \in H^4(\Omega) \cap H_0^2(\Omega).$$

Although the existance of such constant is known, the upper bound have not been known.

However, we obtained the upper bound of C by an analytical way.

Keywords: A priori constant, bi-harmonic problems, driven cavity problem

On a Root Bound Asserted by Lagrange

Werner Krandick (Drexel University - Philadelphia, US)

In 1798, Lagrange gave a bound for the real roots of a polynomial without supplying a proof. I will present a proof that was found by G. E. Collins and myself in 2009. The theorem gives rise to a bound for the absolute values of the complex roots. Conversely, a complex root bound found by Westerfield in 1931 gives rise to the Lagrange bound. The proof of either bound is nontrivial. I will describe a general duality between real and complex root bounds by combining results of Cauchy (1829) and Kioustelidis (1986). I will then use the Lagrange bound to improve Hong's bound (1998). Hong's bound is relevant for polynomial real root computation because, due to a result of Sharma (2007), the use of Hong's bound in the continued fractions method for polynomial real root isolation guarantees that the computing time function is dominated by a polynomial in the degree and the maximal length of the coefficients. An algorithm by Mehlhorn and Ray (2009) for the computation of Hong's bound can be modified so that it computes the improved bound instead.

Keywords: Root bounds, real root isolation

See also: TBD

Eigenvalue enclosures and exclosures for non-self-adjoint problems in hydrodynamics

Matthias Langer (The University of Strathclyde - Glasgow, GB)

In this talk an algorithm is presented which is based on interval arithmetic and yields provably correct enclosures for eigenvalues of non-self-adjoint boundary value problems. Combined with analytical information about the localisation of the spectrum, this algorithm can also be used to prove that certain areas of the complex plane remain free of eigenvalues.

The algorithm is applied to boundary value problems on bounded and unbounded intervals, in particular, the Orr–Sommerfeld equation, other problems from hydrodynamics and resonances of Schroedinger operators.

Processor simulation: a new way for the performance analysis of numerical algorithms

Philippe Langlois (Université de Perpignan, FR)

Several accurate and validated numerical algorithms using error-free transformations have been introduced these last years: summation and dot product (Ogita, Oishi, Rump), polynomial evaluation with the Horner schema and triangular linear system resolution (Graillat, Langlois, Louvet).

These algorithms appear to be fast in terms of *measured computing time*: faster than extended precision computations (double-double, quad-double) and even surprisingly faster than what the classic floating-point operation count predicts. This interesting practical observation has already been explained with a "hand driven" analysis of the instruction level parallelism (ILP) present in these algorithms (Langlois, Louvet). The ILP of a program measures its performance potential, i.e., how fast the program would run within an ideal processor (a processor with no physical resource constraint)

In this talk, we illustrate how to automatize and experiment such a performance analysis thanks to *processor simulation*. We describe two simulators: one derives from the Unisim environment (http://unisim.org), the other from the Pin tool from Intel (http://pintool.org). We experiment these tools for some previously cited algorithms. We exhibit that some of them tend to compute twice more accurate results for free (no extra computing time over-cost) when run on “tomorrow processors”.

Keywords: Error-free transformation, compensated algorithm, measured computing time, performance analysis

Joint work of: Langlois, Philippe; Goossens, Bernard; Parello, David

A Symbolic-Numeric Approach toward Polynomial Algebra

Wen-Shin Lee (University of Antwerp, BE)

Recent advances in Pade approximation theory reveal unforeseen links to polynomial algebra. The connection between the detection of poles of univariate rational approximants and the sparse interpolation of multivariate polynomials leads to a deterministic algorithm for sparse polynomial interpolation in floating point arithmetic. This numerical sparse interpolation requires neither the number of terms nor the partial degrees of the target polynomial to be supplied as input and is extended to the sparse interpolation of multivariate rational functions. By further exploring the correspondence between the poles and the sparsity of a polynomial, algorithms for computing approximate sparse representations are being developed. These algorithms seek the approximation by a polynomial or rational function that is sparse when represented in a basis after a shift or linear transform.

A detailed study of a multivariate algorithm for the detection of pole curves and surfaces provides us with a new tool to extract algebraic structure from numerical partial data. Among the examples are numerical factorization, GCD, and irreducibility.

Beside to report our current research under this framework, we plan to comment on a latest research direction that is connected to resultant theory and motivated by the observation of a curious analogy among the formally orthogonal Hadamard polynomials, Jacobi's method for approximating dominant roots, and the generating polynomial of a linear recurrence sequence.

Keywords: Symbolic-numeric computation, computer algebra

TellHiM&S: Interval Based Methods For Adaptive Hierarchical Models In Modeling And Simulation Systems

Wolfram Luther (Universität Duisburg-Essen, DE)

In this talk we present recent work on interval based methods for adaptive hierarchical structures and verified dynamic modeling in multibody simulation software.

Keywords: Reliable geometric modeling, verified hierarchical structures, comparison of verified solvers for initial value problems, applications in biomechanics

Joint work of: Luther, Wolfram; Auer, Ekaterina; Dyllong, Eva; Kiel, Stefan

Verification and Validation for Femur Prosthesis Surgery

Wolfram Luther (Universität Duisburg-Essen, DE)

In this paper, we describe how verified methods we are developing in the course of the project TellHim&S (Interval Based Methods For Adaptive Hierarchical Models In Modeling And Simulation Systems) can be applied in the context of the biomechanical project PROREOP (Development of a new prognosis system to optimize patient-specific pre- operative surgical planning for the human skeletal system). On the one hand, it includes the use of verified hierarchical structures for reliable geometric modeling, object decomposition, distance computation and path planning. On the other hand, we cover such tasks as verification and validation assessment and propagation of differently described uncertainties through system models in engineering or mechanics.

Keywords: Graphical interface construction, superquadrics, 3D modeling, biomedical engineering

Joint work of: Auer, Ekaterina; Cuypers, Roger; Dyllong, Eva; Kiel, Stefan; Luther, Wolfram

Full Paper: http://drops.dagstuhl.de/opus/volltexte/2010/2513

Orbital stability investigations for travelling waves in a nonlinearly supported beam

Kaori Nagatou (Kyushu University, JP)

We consider the fourth-order problem $\varphi_{tt} + \varphi_{xxxx} + f(\varphi) = 0, (x,t) \in IR \times IR^+$, with a nonlinearity f vanishing at 0.

Solitary waves $\varphi = u(x+ct)$ satisfy the ODE $u'''' + c^2u'' + f(u) = 0$, and for the case $f(u) = e^u - 1$, the existence of at least 36 travelling waves was proved in [1] by computer-assisted means.

We investigate the orbital stability of these solutions by computing the Morse index of them. In order to achieve it we make use of both analytical ([2,3]) and computer-assisted techniques.

[1] B. Breuer, J. Horák, P. J. McKenna, M. Plum, A computer-assisted existence and multiplicity proof for travelling waves in a nonlinearly supported beam, Journal of Differential Equations 224 (2006), pp. 60-97.

[2] M. Grillakis, J. Shatah and W. Strauss, Stability Theory of Solitary Waves in the Presence of Symmetry, I, Journal of Functional Analysis 74 (1987), pp. 160-197.

[3] M. Grillakis, J. Shatah and W. Strauss, Stability Theory of Solitary Waves in the Presence of Symmetry, II, Journal of Functional Analysis 94 (1990), pp. 308-348.

Keywords: Travelling waves, orbital stability

Joint work of: Nagatou, Kaori; McKenna, P. Joseph; Plum, Michael

Some remarks on the a priori L^2 error estimates of a finite element H_0^1-projection on nonconvex domains

Mitsuhiro T. Nakao (Kyushu University, JP)

The Aubin-Nitsche trick to the linear finite element method of the second order Dirichlet boundary value problem is well-known as the technique to obtain a priori L^2 error estimation with $O(h^2)$ using $O(h)$ estimates in H_0^1 sense for the dual problem.

But, we could no longer use such a technique for the problems on the nonconvex and nonsmooth domains because of the less regularity of the solution for dual problems.

Actually, as far as we concern, for example, it was not known at all up to now whether the order of the a priori L^2 error is still one order higher than H_0^1 error for the piecewise linear element, even if the solution of the original equation is sufficiently smooth.

In this talk, we consider an apriori L^2 error estimate of a linear finite element H_0^1-projection on the L-shape domain.

We show that, under some smoothness assumption of the original solution, a priori L^2 error is still $O(h^2)$ by using a computer assisted proof.

This result strongly suggests that the order of convergence should be independent of the smoothness of the solutions of dual problem.

Keywords: Constructive error estimates, finite element method, elliptic problems

FMathL - Formal Mathematical Language

Arnold Neumaier (Universität Wien, AT)

FMathL (= Formal Mathematical Language) is the working title for a modeling and documentation language for mathematics, suited to the habits of mathematicians, to be developed in a project at the University of Vienna.

The project complements efforts for formalizing mathematics from the computer science and automated theorem proving perspective.

In the long run, the FMathL system might turn into a user-friendly automatic mathematical assistant for retrieving, editing, and checking mathematics (but also computer science and theoretical physics) in both informal, partially formalized, and completely formalized mathematical form.

For more details, see http://www.mat.univie.ac.at/ neum/FMathL.html

Robust and accurate matrix factorizations

Takeshi Ogita (Tokyo Woman's Christian University, JP)

In this talk, robust algorithms for accurate factorizations of extremely ill-conditioned matrices are proposed, which include LU, QR and Cholesky factorizations, eigenvalue decomposition and singular value decomposition.

The proposed algorithms are based on standard numerical algorithms using pure floating-point arithmetic and accurate dot product. As applications of the algorithms, methods of computing accurate solutions of linear systems and accurate determinant are also proposed. Numerical results are presented for illustrating the performance of the proposed algorithms. Computing times for the algorithms adaptively change according to difficulty of given problems.

Keywords: Matrix factorization, accurate numerical algorithms, floating-point arithmetic

Numerical Existence Theorem for Boundary Value Problems of Differential Equations

Shinichi Oishi (Waseda University - Tokyo, JP)

In 1994, I have proposed a numerical verification theorem for fixed point type equations. In this talk, I will discuss its applications to boundary value problems of differential equations. I will also present some comments to the relationship among this method, Nakao's method and Plum's method.

Keywords: Boundary value problems, verification theorem

Error-Free Transformation of Matrix Multiplication and its Related Topics

Katsuhisa Ozaki (Waseda University - Tokyo, JP)

This talk is concerned with accurate matrix multiplication.

Recently, topics of error-free transformation of floating-point arithmetic are frequently discussed. By specializing the technique for matrix operations, an algorithm for an error-free transformation of matrix multiplication is developed. The algorithm transforms matrix multiplication into summation of some floating-point matrices. There are several applications of this transformation. For example, accurate matrix multiplication, tight inclusion of matrix multiplication and multi-precision arithmetic for matrix multiplication. Dominant computations in the transformation are usual floating-point matrix products. It is known that 'gemm' in optimized BLAS achieves near-peak performance. Therefore, the proposed algorithm receives much benefit from such fast routines in terms of computational performance. In this talk, the error-free transformation of matrix multiplication is first introduced. Next, some applications are presented with numerical examples.

Keywords: Matrix Computations, Accurate Computations

Exact linear algebra tools for computer assisted proofs

Clement Pernet (Université Joseph Fourier, FR)

Computer algebra plays a major role in computer assisted proofs. Beyond the assistance of symbolic computation in handling and simplifying formula, certified, and exact computations are also mandatory in proving actual results.

In the special case of testing a conjecture, the larger the collection of objects computed, the higher the confidence in the conjecture. This motivates a special care for intensive computations, and consequently the asymptotic behavior of algorithms and their implementations.

Through the study of a few study cases, we will describe how experiments in mathematics make use of intensive exact computations that often boil down to large linear algebra computations over the rationals or finite fields.

We'll present the different algorithmic and implementation techniques used in this area, that help dealing with larger instances.

Keywords: Computer algebra, Exact Linear Algebra

A computer-assisted proof for photonic band gaps

Michael Plum (KIT - Karlsruhe Institute of Technology, DE)

We investigate photonic crystals, modeled by a spectral problem for Maxwell's equations with periodic electric permittivity. Here, we specialize to a two-dimensional situation and to polarized waves. By Floquet-Bloch theory, the spectrum has band-gap structure, and the bands are characterized by families of eigenvalue problems on a periodicity cell, depending on a parameter k varying in the Brillouin zone K. We propose a computer-assisted method for proving the presence of band gaps: For k in a finite grid in K, we obtain eigenvalue enclosures by variational methods supported by finite element computations, and then capture all $k \in K$ by a perturbation argument.

Keywords: Photonic crystal, Maxwell's equations, band gap, computer-assisted proof

Joint work of: Hoang, Vu; Plum, Michael; Wieners, Christian

Progress of IEEE Working Group P1788: A Standard for Interval Arithmetic

John D. Pryce (Wiltshire, GB)

In June 2008 the IEEE set up Working Group P1788 for the standardisation of interval arithmetic. Four members of the executive committee are at this Dagstuhl seminar: Revol (chair), Kearfott (vice-chair), Pryce (technical editor), Corliss (vote tabulator). This talk reports on progress to date. In particular for three issues:

- the underlying number system and the definition of "interval";
- managing complexity by a layered design of the standard;
- what exceptional conditions are appropriate to interval work and how they may be handled;

I summarise the difficulties, the alternatives and some conflicting views, and some of the decisions made. If interval arithmetic concerns you, participate!

Keywords: Interval arithmetic, standard, verification

Toward 1000000000000000 Automatic Proofs

Stefan Ratschan (Academy of Science - Prague, CZ)

Mathematicians only produce a tiny fraction of all proofs done everyday worldwide. Not only that: I claim that computers, already nowadays, routinely do many more proofs per day than all living mathematicians together. Yet, the subject of almost all computer-generated proofs today are computer systems themselves. However, since computers are becoming increasingly intermingled into their environment, it is increasingly important to integrate algorithms for automatic reasoning about physical systems into algorithm that do the same for computer systems. In the talk, we will discuss some attempts at such an integration.

Interval Approaches to Reliable Control of Dynamical Systems

Andreas Rauh (Universität Rostock, DE)

Recently, we presented an implementation of interval-based algorithms which can be applied in real-time to control dynamical processes and to estimate internal states and disturbances. The approach is based on verified methods for sets of algebraic equations, ordinary differential equations as well as differential-algebraic equations. Due to this fact, the same program code can be used for two different tasks. On the one hand, we can use it online to estimate non-measurable internal system states which are necessary for nonlinear model-based control strategies. On the other hand, we can verify the admissibility and feasibility of these control strategies offline.

Although we use the same code for the online and offline tasks, there is an important difference between them. While the computing time is of minor importance in offline applications, we have to guarantee that the necessary online computations are completed successfully in a predefined time interval. For that reason, the role of verification is slightly different depending on the task. In offline applications, our goal is to compute tightest possible bounds for the sets of all solutions to the control problem under consideration. In contrast to that, we restrict the online mode to a search for a single solution that matches all demands on feasibility of control inputs and admissibility of the trajectories of the state variables in a reliable way.

To highlight the practical applicability of the underlying computational routines, we present the following cases for the use of verified solvers in real-time [1-3].

Case 1: Direct computation of feedforward control strategies with the help of differential-algebraic equation solvers. In this application, both verified and non-verified solvers can be used to determine open-loop control strategies for a dynamical system such that its output coincides with a predefined time response within given tolerances. This procedure corresponds to a numerical inversion of the dynamics of the system to be controlled. In this case, verified solvers are used to prove the existence of a control law within given physical bounds for the admissible range of the system inputs.

Case 2: If measured data and their time derivatives are available, the same procedures as in case 1 can be used to estimate non-measured state variables as well as non-measurable disturbances. Since the verified algorithms used in this context are capable of propagating bounded measurement uncertainties, the quality of the state and disturbance estimates can be expressed in terms of the resulting interval widths. Moreover, assumptions about the parameters and the structure of the underlying model can be verified.

Case 3: Routines for verified sensitivity analysis provide further information on the influence of variations of control inputs on the trajectories of the state variables. We present novel procedures implementing a sensitivity-based framework for model-predictive control. These procedures can be integrated directly in a feedback control structure.

Sometimes it is necessary to combine verified and non-verified algorithms to solve a given control problem. In this case, it is important to certify the results of the algorithm appropriately. Based on the four-tier hierarchy presented in earlier works [4], we develop a measure for characterizing such mixed approaches.

The presentation is concluded with simulation and experimental results for the example of temperature control of a distributed heating system.

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Keywords: Robust control, Ordinary differential equations, Differential-algebraic equations

Joint work of: Rauh, Andreas; Auer, Ekaterina

Full Paper: http://drops.dagstuhl.de/opus/volltexte/2010/2512

An interval iterative refinement method to certify the solution of a linear system

Nathalie Revol (ENS - Lyon, FR)

We address the question to certify the solution of a linear system, computed using floating-point arithmetic. Our method is inspired from the "verifylss" algorithm of Rump, available in IntLab. The goal is to compute an enclosure of the solution, starting from the computed floating-point solution. We use iterative refinement to enclose the error bound and to refine this enclosure. To get an efficient method, we enlarge the computed quantities to work only with quantities which are symmetrical with respect to 0: this enables us to use fast BLAS routines to perform interval computations.

We will present our technique as well as some experimental results.

Keywords: Interval arithmetic, linear system solving, certified result, performance, implementation

Joint work of: Revol, Nathalie; Nguyen, Hong Diep

A computer-assisted uniqueness proof for a semilinear elliptic boundary value problem

Dagmar Roth (Universität Karlsruhe, DE)

In our talk we use computer-assistance to prove existence and even global uniqueness of solutions to a semilinear elliptic problem. We consider positive solutions of $-\Delta u - \lambda u - u^2 = 0$ in Ω, $u = 0$ on $\partial\Omega$ and λ ranging between 0 and the first Dirichlet eigenvalue $\lambda_1(\Omega)$ of $-\Delta$ on Ω. In case Ω is a ball or $\lambda = 0$ uniqueness of solutions to this problem has been proven, but only little is known, when Ω is different from a ball and $\lambda \neq 0$.

We consider the case $\Omega = (0,1)^2$ and show, that the positive solution is unique for all λ in the considered range. The proof is split in two parts: For λ close to $\lambda_1(\Omega)$ we use analytic arguments to show non-degeneracy of all solutions. For the remaining values of λ we compute a branch of approximate solutions and use a fixed point argument to enclose the true solutions. Moreover we prove non-degeneracy of all these solutions, which finally gives uniqueness, due to the known bifurcation structure of the problem. To verify the assumptions needed for the fixed point theorem we make heavy use of the computer.

Joint work of: McKenna, P. Joseph; Pacella, Filomena; Plum, Michael; Roth, Dagmar

Variant Quantifier Elimination

Mohab Safey El Din (UPMC - Paris 6, FR)

We study a variant of the real quantifier elimination problem (QE). The variant problem requires the input to satisfy a certain extra condition, and allows the ouput to be almost equivalent to the input. In a sense, we are strengthening the pre-condition and weakening the post-condition of the standard QE problem.

The motivation/rationale for studying such a variant QE problem is that many quantified formulas arising in applications (such as Initial Boundary Value Problems or the stability analysis of numerical schemes for solving PDEs) do satisfy the extra conditions. Furthermore, in most applications, it is sufficient that the ouput formula is almost equivalent to the input formula. Thus, we propose to solve a variant of the initial quantifier elimination problem.

We present an algorithm (VQE), that exploits the strengthened pre-condition and the weakened post-condition. The main idea underlying the algorithm is to substitute the repeated projection step of CAD by a single projection without carrying out a parametric existential decision over the reals.

We find that the algorithm VQE can tackle important and challenging problems, such as numerical stability analysis of the widely-used MacCormack's scheme. The problem has been practically out of reach for standard QE algorithms in spite of many attempts to tackle it. However the current implementation of VQE can solve it in about 1 day.

Joint work of: Hong, Hoon; Safey El Din, Mohab

The COCONUT Environment for global optimization

Hermann Schichl (Universität Wien, AT)

The COCONUT Environment is an open source software platform for global optimization originally developed during an EU IST project between 2000 and 2004 and now maintained and enhanced at the University of Vienna.

The platform provides a modular system designed for developing complete algorithms for solving non-linear mixed-integer global optimization problems and constraint satisfaction problems. This presentation will give an overview of the newest version and will provide information on new modules, like a projective constraint propagator designed for unbounded regions, the automatic computation of sparse second order information like Hessians, interval Hessians, and second order slopes.

For more details on the COCONUT environment see
http://www.mat.univie.ac.at/coconut-environment

Keywords: Global optimization, constraint propagation, open source software

Comparison of Zero Separation Bounds for Division-Free Arithmetic Expressions Involving Radicals

Stefan Schirra (Universität Magdeburg, DE)

Separation bounds are used for detecting zeros in exact decision computations based on expression dags. After briefly reviewing the concept of separation bounds and its application in the exact geometric computation paradigm, we turn to the comparison of known bounds for the special, but relevant case of division-free expressions, more precisely, arithmetic expressions with integer operands over the operations $+$, $-$, $\cdot$, and $\sqrt[k]{\ }$. In particular, we show that the so-called BFMS bound is never worse than a more recent bound proposed by Sekigawa which is never worse than the so-called degree-measure bound.

Keywords: Zero separation bounds, exact geometric computation

Estimating Regions of Attraction for Dynamical Systems

Bernd Tibken (Universität Wuppertal, DE)

In order to analyse the asympototic stability of nonlinear systems the main method is Lyapunovs second method. In this presentation an overview is given for the special case of polynomial systems and polynomial Lyapunov functions. It is shown how results from real algebraic geometry can be used to derive semidefinite relaxations to the corresponding optimization problems. The results are illiustrated with the help of benchmark examples from literature and a comparison is made with other existing and recently developed methods. In a second part of the talk ist is shown how methods from interval arithmetic can be used to extend the analysis to nonpolynomial systems. For this case also benchmark examples from literature will serve to illustrate the results.

The presentation will conclude with an summary and an outlook.

Keywords: Asymptotic Stability, Lyapunov Second Method, Semidefinite Programming, Interval Approach

Iterative scheme for a free boundary problem defined with the Hadamard variations

Takuya Tsuchiya (Ehime University - Matsuyama, JP)

In this talk we deal with the dam problem, a typical free boundary problem which seeks the free boundary of static flow inside a dam. To construct a numerical iterative scheme, we have developed a theory of variations with respect to perturbations of the boundary.

Such a variation is called the Hadamard variation. We present an iterative scheme and discuss on an its properties and convergence.

Keywords: Free boundary problem, Hadamard variation, the traction method
Joint work of: Tsuchiya, Takuya; Suzuki, Takashi

An abstract framework for verified constrained minimization

Christian Wieners (Universität Karlsruhe, DE)

In the context of computer-assisted existence proofs for the solution of partial differential equations we consider problem classes, where the solution can be characterized as a minimizer of a suitable functional. If (1) a sufficiently accurate approximate solution is available, (2) spectral estimates of a linearization at this approximation can be computed, and (3) explicit estimates for derivatives of the minimization functional can be derived, a variant of the Newton-Kantorovich theorem yields the existence of a minimizer together with explicit error bounds for the approximation.

We discuss the application of this concept to semi-linear equations.

Finally, we consider minimization problems with additional (nonlinear) equality constraints. For this purpose the Newton-Kantorovich theorem is extended to manifolds, where the Newton linearization is computed in the tangent space and the Newton update is realized along geodetic curves.

Boundary triples - an abstract setting for forward and inverse problems

Ian Wood (University of Kent, GB)

This talk will introduce boundary triples, an abstract setting which can be useful for studying both forward and inverse problems.

An important tool in studying these problems for ODEs is given by the Weyl-Titchmarsh m-function. In PDE problems, a similar role is played by the Dirichlet-to-Neumann map. Both of these can be determined solely from the boundary behaviour of solutions.

Boundary triples are a framework in which these ideas and methods can be extended to a larger class of operators, giving rise to operator M-functions. We will discuss properties of these M- functions and their relation to the resolvent and the spectrum of the associated operator.

A Verified Automatic Contour Integration Algorithm

Naoya Yamanaka (Waseda University - Tokyo, JP)

A verified automatic integration algorithm is proposed for calculating contour integration over complex field using numerical computations. The proposed algorithm is based on trapezoidal rule for angle.

The error analysis of the method have been presented by several authors, however, these investigations are done basically for examining the rates of convergence, and several constants in these error formula were left unevaluated. In order to construct verified numerical integrator using the algorithm, the error formula is presented. To construct efficient verified numerical integrator, an efficient a priori method of evaluating function calculation errors is adopted. Combining these, a verified automatic integration algorithm is proposed. Numerical examples are presented for illustrating effectiveness of the proposed algorithm.

Keywords: Numerical integration, verification, contour integraion

Exact certification in global polynomial optimization via sums-of-squares of rational functions with rational coefficients

Zhengfeng Yang (East China Normal University - Shanghai, CN)

We present a hybrid symbolic-numeric algorithm for certifying a polynomial or rational function with rational coefficients to be non-negative for all real values of the variables by computing a representation for it as a fraction of two polynomial sum-of-squares (SOS) with rational coefficients. We have proved the Monotone Column Permanent Conjecture for dimension four and solved some examples from the SOS literture and other polynomial inequalities. This is joint work with Erich Kaltofen, Bin Li and Lihong Zhi.

Keywords: Semidefinite programming, sum-of-squares, hybrid method

Joint work of: Kaltofen, Erich; Li,Bin; Yang, Zhengfeng; Zhi,Lihong

Compute Certified Global Optimum of Rational Functions in Maple

Lihong Zhi (MMRC - Beijing, CN)

We provide algorithms in Maple to compute and certify the global optimum of a rational function by solving semidefinite programs (SDP) and converting the numerical polynomial sum-of-squares (SOS) into an exact rational identity. We can either perform high precision Newton iterations on the numerical SOS computed by SDP solvers in Matlab [2] or use the high precision SDP solver in Maple [1] to get the SOS with necessary precision, then we can convert the numerical SOS into an exact rational SOS by orthogonal projection or rational coefficient vector recovery. We demonstrate our algorithms on certifying lower bounds for Rump's model problem and various exceptional SOS problems in the literature.

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Keywords: Semidefinite programming, sum-of-squares, certified global optimium

Joint work of: Guo, Feng; Kaltofen, Erich Leo; Yang, Zhengfeng; Zhi, Lihong