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An integral approach to bedrock river profile analysis

1 # An integral approach to bedrock river profile analysis

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9 ## Abstract

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11 Bedrock river profiles are often interpreted with the aid of slope-area analysis, but noisy

12 topographic data make such interpretations challenging. We present an alternative

13 approach based on an integration of the steady-state form of the stream power equation.

14 The main component of this approach is a transformation of the horizontal coordinate

15 that converts a steady-state river profile into a straight line with a slope that is simply

16 related to the ratio of the uplift rate to the erodibility. The transformed profiles, called chi

17 plots, have other useful properties, including co-linearity of steady-state tributaries with

18 their main stem and the ease of identifying transient erosional signals. We illustrate these

19 applications with analyses of river profiles extracted from digital topographic datasets.

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Introduction

Bedrock rivers record information about a landscape's bedrock lithology, tectonic context, and climate history. It has become common practice to use bedrock river profiles to test for steady-state topography, infer deformation history, and calibrate erosion models (see reviews by Whipple, 2004, and Wobus et al., 2006). The most widely used models of bedrock river incision express the erosion rate in terms of channel slope and drainage area, which makes them easy to apply to topographic measurements and incorporate into landscape evolution models. We focus on the stream power equation:

$$\frac{\partial z}{\partial t} = U(x,t) - K(x,t) A(x,t)^m \left| \frac{\partial z}{\partial x} \right|^n \tag{1}$$

where z is elevation, t is time, x is horizontal upstream distance, U is the rate of rock uplift relative to a reference elevation, K is an erodibility coefficient, A is drainage area, and m and n are constants. Although equation (1) is commonly referred to as the stream power equation, it can be derived from the assumption that erosion rate scales with either stream power per unit area of the bed (Seidl and Dietrich, 1992; Howard et al., 1994) or bed shear stress (Howard and Kerby, 1983).

If the stream power equation is used to describe the evolution of a river profile, a common analytical approach is to assume a topographic steady state ($\partial z/\partial t = 0$) with uniform U and K and solve equation (1) for the channel slope:

$$\left| \frac{dz}{dx} \right| = \left(\frac{U}{K} \right)^{\frac{1}{n}} A(x)^{-\frac{m}{n}} \tag{2}$$

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43   Equation (2) predicts a power-law relationship between slope and drainage area. If such a

44   power law is observed for a given profile, it supports the steady state assumption, and the

45   exponent and coefficient of a best-fit power law can be used to infer m/n and $(U/K)^{1/n}$,

46   respectively. Alternatively, deviations from a power law slope-area relationship may be

47   evidence of transient evolution of the river profile, variations in bedrock erodibility, or

48   transitions to other dominant erosion and transport mechanisms (Whipple and Tucker,

49   1999; Tucker and Whipple, 2002; Stock et al., 2005).

50

51   Slope-area analysis has been widely applied to the study of bedrock river profiles (e.g.,

52   Flint, 1974; Tarboton et al., 1989; Wobus et al., 2006), but it suffers from significant

53   limitations. Topographic data are subject to errors and uncertainty and are typically noisy.

54   Estimates of slope obtained by differentiating a noisy elevation surface are even noisier.

55   This typically causes considerable scatter in slope-area plots, which makes it challenging

56   to identify a power-law trend with adequate certainty. Perhaps more concerning is the

57   possibility that the scatter may obscure deviations from a simple power law that could

58   indicate a change in process, a transient signal, or a failure of the stream power model.

59   Another limitation of slope-area analysis is that the slope measured in a coarsely sampled

60   topographic map may differ from the reach slope relevant to flow dynamics.

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62   Strategies have been proposed to cope with some of these problems. In the common case

63   of digital elevation maps (DEMs) that contain stair-step artefacts associated with the

64   original contour source maps, for example, sampling at a regular and carefully selected

65   elevation interval can extract the approximate points where the stream profile crosses the

66 original contours (Wobus et al., 2006). This method requires care, however, and at best it

67 reproduces the slopes that correspond to the original contours, which may be inaccurate.

68 Measuring the slope over elevation intervals that correspond to long horizontal distances

69 can compound the problem of measuring an average slope that differs from the local

70 slope that drives flow. Furthermore, as Wobus et al. (2006) note, the contour sampling

71 approach cannot distinguish between artefacts associated with the DEM generation

72 procedure and real topographic features. Other common techniques for reducing noise

73 and uncertainty in slope-area analyses include smoothing the river profile and logarithmic

74 binning of slope measurements. Some of these approaches have been shown to yield

75 good results (Wobus et al., 2006), but all introduce biases that are difficult to evaluate

76 without field surveys.

77

78 In this paper, we propose a more robust method that alleviates many of these problems by

79 avoiding measurements of channel slope. Our method uses elevation instead of slope as

80 the dependent variable, and a spatial integral of drainage area as the independent variable.

81 This approach has additional advantages that include the simultaneous use of main stem

82 and tributaries to calibrate the stream power law, the ease of comparing profiles with

83 different uplift rates, erosion parameters, or spatial scales, and clearer identification of

84 transient signals. We present examples that demonstrate these advantages.

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Transcription of river profiles

Change of horizontal coordinate

Our procedure is based on a change of the horizontal spatial coordinate of a river
longitudinal profile. Separating variables in equation (2), assuming for generality that U
and K may be spatially variable, and integrating yields

$$\int dz = \int \left(\frac{U(x)}{K(x)A(x)^m} \right)^{\frac{1}{n}} dx \qquad (3)$$

Performing the integration in the upstream direction from a base level x_b to an
observation point x yields an equation for the elevation profile:

$$z(x) = z(x_b) + \int_{x_b}^{x} \left(\frac{U(x)}{K(x)A(x)^m} \right)^{\frac{1}{n}} dx \qquad (4)$$

There is no special significance associated with the choice of x_b; it is merely the
downstream end of the portion of the profile being analysed. The integration can also be
performed in the downstream direction, but it is best to use the upstream direction for
reasons that will become apparent below.

Equation (4) applies to cases in which the profile is in steady state, but is spatially
heterogeneous (if, for example, the profile crosses an active fault or spans different rock
types, or if precipitation rate varies over the drainage basin). In the case of spatially
invariant uplift rate and erodibility, the equation for the profile reduces to a simpler form,

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$$z(x) = z(x_b) + \left(\frac{U}{K}\right)^{\frac{1}{n}} \int_{x_b}^{x} \frac{dx}{A(x)^{\frac{m}{n}}}$$ (5)

107   To create transformed river profiles with units of length on both axes, it is convenient to

108   introduce a reference drainage area, A_0, such that the coefficient and integrand in the

109   trailing term are dimensionless,

$$z(x) = z(x_b) + \left(\frac{U}{KA_0^m}\right)^{\frac{1}{n}} \chi$$ (6a)

110   with

$$\chi = \int_{x_b}^{x} \left(\frac{A_0}{A(x)}\right)^{\frac{m}{n}} dx$$ (6b)

111   Equation (6) has the form of a line in which the dependent variable is z and the

112   independent variable is the integral quantity χ, which has units of distance. The z-

113   intercept of the line is the elevation at x_b, and the dimensionless slope is $(U/K)^{1/n}/A_0^{m/n}$.

114   We refer to a plot of z vs. χ for a river profile as a "chi plot."

115

116   The use of this coordinate transformation to linearize river profiles was originally

117   proposed by Royden et al. (2000), and has subsequently been used to determine stream

118   power parameters (Sorby and England, 2004; Harkins et al., 2007; Whipple et al., 2007).

119   In this paper, we expand on this approach and explore additional applications of chi plots.

120   As we show in the examples below, a chi plot can be useful even if U and K are spatially

121   variable, or if the profile is not in steady state. The coordinate χ in equation (6) is also

122   similar to the dimensionless horizontal coordinate χ in the analysis of Royden and Perron

123   (2012), which can be referred to for a more theoretical treatment of the stream power

124   equation.

125

126   *Measuring χ*

127

128   It is usually not possible to evaluate the integral quantity χ in equation (6) analytically,

129   but given a series of upslope drainage areas measured at discrete values of x along a

130   stream profile, it is straightforward to approximate the value of χ at each point using the

131   trapezoid rule or another suitable approximation. If the points along the profile are spaced

132   at approximately equal intervals, the simplest approach is to calculate the cumulative sum

133   of $[A_0/A(x)]^{m/n}$ along the profile in the upstream direction and multiply by the average

134   distance between adjacent points. (Using the average distance avoids the "quantization"

135   effect introduced by a steepest descent path through gridded data, in which point-to-point

136   distances can only have values of δ or $\delta\sqrt{2}$, where δ is the grid resolution.) If δ varies

137   significantly along the profile, or varies systematically with x, it is preferable to calculate

138   the cumulative sum of $[A_0/A(x)]^{m/n}\delta(x)$. If desired, $\delta(x)$ can be smoothed with a moving

139   average before performing the summation.

140

141   In most cases, the value of m/n required to compute χ will be unknown. In the next

142   section, we illustrate a procedure for finding m/n that improves on conventional slope-

143   area analysis.

144

145   **Examples**

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Identifying steady-state profiles

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The preceding analysis predicts that a steady-state bedrock river profile will have a linear

chi plot. To demonstrate how the coordinate transformation can be used to identify a

steady state river profile, we analysed the longitudinal profile of Cooskie Creek (Fig. 1a),

one of several bedrock rivers in the Mendocino Triple Junction (MTJ) region of northern

California studied previously by Merritts and Vincent (1989) and Snyder et al. (2000,

2003a,b). We determined upstream distance, elevation, and drainage area along the

profile by applying a steepest descent algorithm to a DEM with 10 m grid spacing. For a

range of m/n values ranging from 0 to 1, we calculated χ in equation (6), performed a

linear least-squares regression of elevation against χ, and recorded the R^2 value as a

measure of goodness of fit. A plot of R^2 against m/n (Fig. 1b) has a well-defined

maximum at $m/n = 0.36$, implying that this is the best-fitting value. We then transformed

the longitudinal profile according to equation (6) with $m/n = 0.36$ and $A_0 = 1$ km^2. The

resulting chi plot (Fig. 1c) shows that the transformed profile closely follows a linear

trend, suggesting that the profile is nearly in steady state. The slope of the regression line

is 0.12, which, combined with an uplift rate of 3.5 mm/yr inferred from uplifted marine

terraces (Merritts and Bull, 1989), implies an erodibility $K = 0.0002$ m$^{0.28}$/yr for $n = 1$.

Note that the stair-step features in the longitudinal profile (Fig. 1a), which would produce

considerable scatter in a slope-area plot, do not interfere with the regression analysis, and

introduce only minor deviations from the linear trend in the chi plot of the transformed

profile (Fig. 1c).

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170

171   *Using tributaries to estimate stream power parameters*

172

173   A useful property of the coordinate transformation is that it scales points with similar

174   elevations to similar values of χ, even if those points have different drainage areas. This

175   implies that tributaries that are in steady state and that have the same uplift rate and

176   erosion parameters as the main stem should be co-linear with the main stem in a chi plot.

177   The co-linearity of tributaries and main stem provides a second, independent constraint

178   on m/n: in theory, the correct value of m/n should both linearize all the profiles and

179   collapse the tributaries and main stem to a single line. This highlights one reason for

180   performing the integration in equation (3) in the upstream direction: tributaries have the

181   same elevation as the main stem at their downstream ends, but not at their upstream ends.

182

183   Fig. 2 illustrates this principle with an analysis of Rush Run in the Allegheny Plateau of

184   northern West Virginia. We extracted profiles of the main stem and nine tributaries of

185   Rush Run from a DEM with 3 m grid spacing (Fig. 2a). The tributary longitudinal

186   profiles differ from one another and from the main stem profile (Fig. 2b). Transforming

187   the profiles with three different values of m/n (Fig. 2c-e) demonstrates how the best

188   choice of m/n collapses the tributaries and main stem on a chi plot (Fig. 2d); for other

189   values of m/n, the tributaries have systematically higher (Fig. 2c) or lower (Fig. 2e)

190   elevations than the main stem in transformed coordinates.

191

192   In practice, the value of m/n that best collapses the tributaries and main stem is not

193   always the value that maximizes the linearity of each individual profile. In the case of

194   Rush Run, the value of $m/n = 0.65$ that best collapses the tributaries makes the main stem

195   slightly concave down in transformed coordinates (Fig. 2d). Provided there are no

196   systematic differences in erodibility or precipitation rates between the main stem and

197   tributaries, this minor discrepancy may be an indication that the drainage basin is slightly

198   out of equilibrium. Alternatively, it could be an indication that the mechanics of channel

199   incision are not completely described by the stream power equation. This example

200   illustrates how the comparison of transformed tributary and main stem profiles can

201   provide a perspective on drainage basin evolution that would be difficult to attain with

202   slope-area analysis.

203

204   *Comparisons among profiles*

205

206   Another common application of river profile analysis is to identify topographic

207   differences among rivers that are thought to experience different uplift or precipitation

208   rates or that have eroded through different rock types (e.g., Kirby and Whipple, 2001;

209   Kirby et al., 2003). These effects are modelled by the uplift rate U and the erodibility

210   coefficient K. The coefficient of the power law in equation (2), which includes the ratio

211   of these two parameters, is often referred to as a steepness index, because, all else being

212   equal, the steady-state relief of the river profile is higher when U/K is larger. The

213   steepness index is usually determined from the intercept of a linear fit to log-transformed

214   slope and area data. The uncertainty in this intercept can be substantial due to scatter in

215   the slope-area data (Harkins et al., 2007). In our coordinate transformation, the steepness

216   index is simply the slope of the transformed profile, $dz/d\chi$, which provides a means of

217   estimating U/K that is less subject to uncertainty (Royden et al., 2000; Sorby and

218   England, 2004; Harkins et al., 2007; Whipple et al., 2007) as well as an intuitive visual

219   assessment of differences among profiles.

220

221   To illustrate this point, we analysed 18 of the profiles from the MTJ region studied by

222   Snyder et al. (2000) (Fig. 3a). The profiles span an inferred increase in uplift rate

223   northward along the coast from roughly 0.5 mm/yr to roughly 4 mm/yr associated with

224   the passage of the Mendocino Triple Junction (Fig. 3c; Merritts and Bull, 1989; Merritts

225   and Vincent, 1989; Merritts, 1996). The topographic data and procedures were the same

226   as in the Cooskie Creek example. We determined the best-fitting value of m/n for each

227   profile with the approach in Fig. 1b, and found a mean m/n of 0.46 ± 0.11 (s.d.).

228

229

230   When comparing the steepness of transformed profiles, it is important to use the same

231   values of A_0 and m/n to calculate χ. We therefore transformed all the profiles using $A_0 = 1$

232   km^2 and $m/n = 0.46$ (Fig. 1b). The goodness of linear fits to the profiles using this mean

233   value of m/n (average R^2 of 0.992) is nearly as good as when using the best-fitting m/n

234   for each profile (average R^2 of 0.995). (Note that these measures of R^2 are inflated by

235   serially correlated residuals – see Discussion section – but the comparison of their

236   relative values is valid.) With the profiles' concavity largely removed by the

237   transformation, the effect of uplift rate on profile steepness (the slopes of the profiles in

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238   Fig. 3b) is very apparent, whereas a very careful slope-area analysis is required to resolve

239   the steepness difference due to the noise in the elevation data (compare to Fig. 4 of

240   Wobus et al. (2006)).

241

242   The analysis in Fig. 3 also supports the conclusion of Snyder et al. (2000) that the

243   difference in steepness between the profiles in the zones of fast uplift (red profiles in Fig.

244   3b) and slower uplift (blue profiles in Fig. 3b) is less than expected if only uplift rate

245   differs between these two zones. The dimensionless slope of the transformed profiles is

246   0.21 ± 0.06 (mean ± s.d.) for those inferred to be experiencing uplift rates of 3 to 4 mm/yr

247   and 0.13 ± 0.01 for those inferred to be experiencing uplift rates of 0.5 mm/yr, a slope

248   ratio of only 1.62 ± 0.48 for a six- to eight-fold difference in uplift rate. If these uplift

249   rates are correct, and if n is less than ~2, as is typically inferred (Howard and Kerby,

250   1983; Seidl and Dietrich, 1992; Seidl et al., 1994; Rosenbloom and Anderson, 1994;

251   Stock and Montgomery, 1999; Whipple et al., 2000; van der Beek and Bishop, 2003),

252   there must be other differences among the profiles that affect the steepness. Given the

253   inferred uniformity of the lithology in the MTJ region (Snyder et al., 2003a, and

254   references therein), one possible explanation is that increased rainfall and associated

255   changes in weathering and erosion mechanisms have elevated the erodibility, K, in the

256   zone of faster uplift and higher relief (Snyder et al., 2000, 2003a,b).

257

258   The importance of variables other than uplift rate is most apparent in the chi plots of

259   Fourmile and Cooskie Creeks (orange profiles in Fig. 3b). These rivers have only slightly

260   slower inferred uplift rates than the red profiles in Fig. 3b, but they have much gentler

261 slopes. In fact, their slopes are comparable to those of the blue profiles in the slower

262 uplift zone. A possible explanation for this discrepancy is that local structural

263 deformation has rendered the bedrock more easily erodible in the Cooskie Shear Zone.

264 Whatever the reason for the reduced effect of uplift rate on profile steepness, this

265 example from the MTJ region demonstrates the ease of comparisons between

266 transformed river profiles believed to be in steady state with respect to different erosion

267 parameters or rates of tectonic forcing.

268

269

270 *Transient signals*

271

272 Even if a river is not in a topographic steady state, a chi plot of its longitudinal profile can

273 be useful. Just as transformed tributaries plot co-linearly with a transformed main stem,

274 transient signals with a common origin, propagating upstream through different channels,

275 plot in the same location in transformed coordinates (χ and z). Whipple and Tucker

276 (1999) noted that transient signals in river profiles governed by the stream power

277 equation propagate vertically at a constant rate, and exploited this property to calculate

278 timescales for transient adjustment of profiles in response to a step change in K or U. The

279 transformation presented in this paper removes the effect of drainage area, and therefore

280 shifts the transient signals in the profiles to the same horizontal position (χ), provided

281 that K and U are uniform.

282

283   We illustrate this property with an example from the Big Tujunga drainage basin in the

284   San Gabriel Mountains of southern California (Fig. 4a), where Wobus et al. (2006)

285   observed an apparent transient signal in multiple tributaries within the basin.

286   Longitudinal profiles (Fig. 4b) reveal steep reaches in the main stem and some tributaries

287   at elevations of roughly 900-1000 m but different streamwise positions. Differences in

288   drainage basin size and shape make it difficult to compare the profiles and determine if

289   and how these features are related. Transforming the profiles using $m/n = 0.4$, the value

290   that best collapses the tributaries to the main stem and linearizes the profiles, clarifies the

291   situation (Fig. 4c). The steep sections of the transformed profiles plot in nearly the same

292   location. The transformed profiles also have a systematically steeper slope downstream of

293   this knick point than upstream, suggesting an increase in uplift rate, the preferred

294   interpretation of Wobus et al. (2006), or a reduction in erodibility. It is difficult to tell

295   whether the knick point is stationary or migrating (Royden and Perron, 2012), but the

296   lack of an obvious fault or lithologic contact suggests that it may be a transient signal that

297   originated downstream of the confluence of the analysed profiles and has propagated

298   upstream to varying extents.

299

300   The horizontal overlap of the steep sections in the chi plot in Fig. 3c is compelling, but it

301   is not perfect. The residual offsets may have arisen from spatial variability in channel

302   incision processes, precipitation, or bedrock erodibility. This difference, which is not

303   obvious in the original longitudinal profiles (Fig. 3b) and would probably not be apparent

304   in a slope-area plot, highlights the sensitivity of the coordinate transformation technique.

305

306

Discussion

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Advantages of the integral approach to river profile analysis

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The approach described in this paper has several advantages over slope-area analysis.

The most significant advantage is that it obviates the need to calculate slope from noisy

topographic data. This makes it possible to perform useful analyses with elevation data

that would ordinarily be avoided. The landscape in Fig. 2, for example, has sufficiently

low relief that even elevation data derived from laser altimetry contains enough noise to

frustrate a slope-area analysis, but the transformed profiles are relatively easy to interpret.

317

The reduced scatter relative to slope-area plots provides better constraints on stream

power parameters estimated from topographic data. In addition, a chi plot can potentially

provide an independent constraint on both m/n and U/K, because the profile fits are

constrained two ways: by the requirement to linearize individual profiles (Fig. 1), and by

the requirement to align tributaries with the main stem (Fig. 2). Although steady state

tributary and main stem channels should also be co-linear on a logarithmic slope-area

plot, they typically have different drainage areas, and therefore do not usually overlap. In

contrast, the integral method produces transformed longitudinal profiles with overlapping

chi coordinates, making it easier to visually assess the match between tributaries and

main stem.

328

329   Removing the effect of drainage area through this coordinate transformation makes it

330   possible to compare river profiles independent of their spatial scale. This is useful both

331   for comparing different drainage basins (Fig. 3) and for comparing channels within a

332   drainage basin (Fig. 4). Transient erosional features, such as knick points, that originated

333   from a common source should plot at the same value of χ in all affected channels (Fig.

334   4). Transient features are also easier to identify in a chi plot because it is easy to see

335   departures from a linear trend with relatively little noise. Similarly, transformed profiles

336   should accentuate transitions from bedrock channels to other process zones within the

337   fluvial network, such as channels in which elevation changes are dominated by alluvial

338   sediment transport or colluvial processes and debris flows (Whipple and Tucker, 1999;

339   Tucker and Whipple, 2002; Stock et al., 2005).

340

341   Finally, the coordinate transformation presented here is compatible with the analytical

342   solutions of Royden and Perron (2012), which aid in understanding the transient

343   evolution of river profiles governed by the stream power equation. As noted above, the

344   integral quantity χ in equation (6) is similar to the dimensionless horizontal coordinate χ

345   used by Royden and Perron (2012) to derive analytical solutions for profiles adjusting to

346   spatial and temporal changes in uplift rate, erodibility, or precipitation. (For uniform K,

347   as is assumed in this paper, it is linearly proportional to their χ.) River profiles

348   transformed according to equation (6) can easily be compared with these solutions to

349   investigate possible scenarios of transient profile evolution.

350

351   *Disadvantages of the integral approach*

352

The main disadvantage of the integral approach is that the coordinate transformation

requires knowledge of m/n, which is usually not known *a priori*. However, we have

demonstrated a simple iterative approach for finding the best-fitting value of m/n that is

easy to implement (Fig. 1b). Moreover, the dependence of the transformation on m/n

provides an additional constraint on m/n, the co-linearity of main stem and tributaries,

which is not available in slope-area analysis.

359

Another drawback of the integral method is that chi plots, like slope-area plots, do not

account for variations on, or inadequacy of, the stream power/shear stress model.

Multiple studies have found that effects not included in equation (1), including erosion

thresholds (e.g., Snyder et al., 2003b; DiBiase and Whipple, 2011), discharge variability

(e.g., Snyder et al., 2003b; Lague et al., 2005; DiBiase and Whipple, 2011) and abrasion

and cover by sediment (e.g., Whipple and Tucker, 2002; Turowski et al. 2007), can

influence the longitudinal profiles of bedrock rivers. It may be possible to use an integral

approach to derive definitions of χ for channel incision models that include these effects,

but such an analysis is beyond the scope of this paper.

369

The form of the integral method presented here can, however, help to identify profiles

that are not adequately described by equation (1), because their chi plots should be non-

linear. Given the larger uncertainties in slope-area analyses, it is possible that some

profiles have incorrectly been identified as steady state, or otherwise consistent with the

stream power equation, with deviations from the model prediction concealed by the

375   scatter in the slope-area data. The integral method, which is less susceptible to noise in

376   elevation data, is a more sensitive tool for identifying such deviations.

377

378   *Evaluating uncertainty*

379

380   The transformation of river longitudinal profiles into linear profiles with little scatter

381   raises the question of how to estimate the uncertainty in stream power parameters

382   determined from topographic data. The most obvious approach is to use the uncertainties

383   obtained by fitting a model to an individual profile. Slope-area analysis of steady-state

384   profiles is appealing from this standpoint, because a least-squares linear regression of

385   log-transformed slope-area data provides an easy way of estimating the uncertainty in

386   m/n (the slope of the regression line) and $(U/K)^{1/n}$ (the intercept). However, the resulting

387   uncertainties mostly describe how precisely one can measure slope, not how precisely the

388   parameters are known for a given landscape.

389

390   The integral method presented in this paper makes this distinction more apparent. For

391   example, when the profile of Cooskie Creek in Fig. 1a is transformed with the best-fitting

392   value of m/n, there is a very small uncertainty (0.2% standard error) in the slope of the

393   best linear fit (Fig. 1c), but this small uncertainty surely overestimates the precision with

394   which $(U/K)^{1/n}$ can be measured for the bedrock rivers of the King Range. Statistically,

395   this uncertainty in the slope of the regression line is also an underestimate because the

396   transformed profile is a continuous curve, and therefore the residuals of the linear fit are

397   serially correlated. This property of the data does not bias the regression coefficients,

398 $z(x_b)$ and $(U/K)^{1/n}/A_0^{m/n}$, but it does lead to underestimates of their uncertainties. Thus, if a

399 chi plot is used to estimate the uncertainty of stream power parameters by fitting a line to

400 a single river profile, a procedure for regression with autocorrelated residuals must be

401 used (e.g., Kirchner, 2001).

402

403 There are better ways to estimate uncertainty in stream power parameters. One, which

404 can be applied to either slope-area analysis or the integral method, is to make multiple

405 independent measurements of different river profiles. This was the approach used to

406 estimate the uncertainty in steepness within each uplift zone in the MTJ region example.

407 The standard errors of the mean steepness among profiles within the fast uplift zone

408 (8.6%) and the slower uplift zone (3.4%) are considerably larger than the standard errors

409 of steepness for individual profiles. If it is possible to measure multiple profiles that are

410 believed to be geologically similar, this approach provides estimates of uncertainty that

411 are more meaningful than the uncertainty in the fit to any one profile. Alternatively, if

412 only one drainage basin is analysed, the integral method provides a new means of

413 estimating the uncertainty in m/n: comparing the value that best linearizes the main stem

414 profile (Fig. 1c) with the value that maximizes the co-linearity of the main stem with its

415 tributaries (Fig. 2c-e).

416

417

418 **Conclusions**

419

19

420 We have described a simple procedure that makes bedrock river profiles easier to

421 interpret than in slope-area analysis. The procedure eliminates the need to measure

422 channel slope from noisy topographic data, linearizes steady-state profiles, makes steady-

423 state tributaries co-linear with their main stem, and collapses transient erosional signals

424 with a common origin. The procedure is well suited to analysing both steady state and

425 transient profiles, and is useful for interpreting the lithology, tectonic histories, and

426 climate histories of river profiles, even from coarse or imprecise topographic data.

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436

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564 **Figure Captions**

565

566 **Figure 1.** Profile analysis of Cooskie Creek in the northern California King Range, USA. (a) Longitudinal

567 profile of the bedrock section of the Creek, as determined by Snyder et al. (2000), extracted from the 1/3

568 arcsecond (approximately 10 m) U. S. National Elevation Dataset using a steepest descent algorithm. (b) R^2

569 statistic as a function of m/n for least-squares regression based on equation (6). The maximum value of R^2,

570 which corresponds to the best linear fit, occurs at $m/n = 0.36$. (c) Chi plot of the longitudinal profile (black

571 line), transformed according to equation (6) with $A_0 = 1$ km^2, compared with the regression line for $m/n =$

572 0.36 (gray line). If the stream power equation is valid and the uplift rate U and erodibility coefficient K are

573 spatially uniform, the slope of the regression line is $(U/K)^{1/n}/A_0^{m/n}$.

574

575 **Figure 2.** Rush Run drainage basin in the Allegheny Plateau of northern West Virginia, USA. (a) Shaded

576 relief map with black line tracing the main stem and gray lines tracing nine tributaries. Digital elevation

577 data are from the 1/9 arcsecond (approximately 3 m) U.S. National Elevation Dataset. UTM zone 17 N. (b)

578 Longitudinal profiles of the main stem (black line) and tributaries (gray lines). (c-e) Chi plots of

579 longitudinal profiles, transformed according to equation (6), using $A_0 = 10$ km^2 and (c) $m/n = 0.55$, (d) m/n

580 $= 0.65$, (e) $m/n = 0.75$.

581

582 **Figure 3.** River profiles in the Mendocino Triple Junction region of northern California, USA. (a) Shaded

583 relief map showing locations of bedrock sections of the channels, as determined by Snyder et al. (2000),

584 extracted from the 1/3 arcsecond (approximately 10 m) National Elevation Dataset. Blue profiles have

585 slower uplift rates, red profiles have faster uplift rates, and orange profiles have faster uplift rates but are

586 located in the Cooskie Shear Zone. (b) Chi plot of longitudinal profiles transformed according to equation

587 (6), using $A_0 = 1$ km^2 and $m/n = 0.46$, the mean of the best-fitting values for all the profiles. Profiles have

588 been shifted so that their downstream ends are evenly spaced along the horizontal axis. Elevation is

589 measured relative to the downstream end of the bedrock section of each profile. (c) Uplift rate at the

590   location of each drainage basin, inferred from dating of marine terraces (Merritts and Bull, 1989; Merritts

591   and Vincent, 1989; Snyder et al., 2000).

592

593   **Figure 4.** Big Tujunga drainage basin in the San Gabriel Mountains of California, USA. (a) Shaded relief

594   map with black line tracing the main stem and gray lines tracing seven tributaries. Digital elevation data are

595   from the 1/3 arcsecond (approximately 10 m) U.S. National Elevation Dataset. UTM zone 11 N. (b)

596   Longitudinal profiles of the main stem (black line) and tributaries (gray lines). The gap in the main stem is

597   the location of Big Tujunga Dam and Reservoir. (c) Chi plot of longitudinal profiles, transformed according

598   to equation (6), using $A_0 = 10$ km^2 and $m/n = 0.4$, illustrating the approximate co-linearity of the tributaries

599   and main stem despite the fact that the profiles do not appear to be in steady state with respect to uniform

600   erodibility and uplift. Two straight dashed segments with different slopes are shown for comparison.

a

Northing (× 10⁶ m)

Easting (× 10⁵ m)

b

Elevation (m)

Upstream distance (m)

c

$m/n = 0.55$

d

Elevation (m)

$m/n = 0.65$

e

$m/n = 0.75$

χ (m)