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Scaling Up Crowd-Sourcing to Very Large Datasets: A Case for Active Learning

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ABSTRACT

Crowd-sourcing has become a popular means of acquiring labeled data for many tasks where humans are more accurate than computers, such as image tagging, entity resolution, and sentiment analysis. However, due to the time and cost of human labor, solutions that rely solely on crowd-sourcing are often limited to small datasets (i.e., a few thousand items). This paper proposes algorithms for integrating machine learning into crowd-sourced databases in order to combine the accuracy of human labeling with the speed and cost-effectiveness of machine learning classifiers. By using *active learning* as our optimization strategy for labeling tasks in crowd-sourced databases, we can minimize the number of questions asked to the crowd, allowing crowd-sourced applications to *scale* (i.e., label much larger datasets at lower costs).

Designing active learning algorithms for a crowd-sourced database poses many practical challenges: such algorithms need to be generic, scalable, and easy to use, even for practitioners who are not machine learning experts. We draw on the theory of nonparametric bootstrap to design, to the best of our knowledge, the first active learning algorithms that meet all these requirements.

Our results, on 3 real-world datasets collected with Amazon's Mechanical Turk, and on 15 UCI datasets, show that our methods on average ask 1–2 orders of magnitude fewer questions than the baseline, and 4.5–44× fewer than existing active learning algorithms.

1. INTRODUCTION

Crowd-sourcing marketplaces, such as Amazon's Mechanical Turk, have made it easy to recruit a crowd of people to perform tasks that are difficult for computers, such as entity resolution [3, 4, 25, 30, 37], image annotation [35], and sentiment analysis [27]. Many of these tasks can be modeled as database problems, where each item is represented as a row with some missing attributes (labels) that the crowd workers supply. This has given rise to a new generation of database systems, called *crowd-sourced databases* [17, 19, 25], that enable users to issue more powerful queries by combining human-intensive tasks with traditional query processing techniques. Figure 1 provides a few examples of such queries, where part of the query is processed by the machine (e.g., whether the word “iPad” appears in a tweet) while the *human-intensive* part is sent to the

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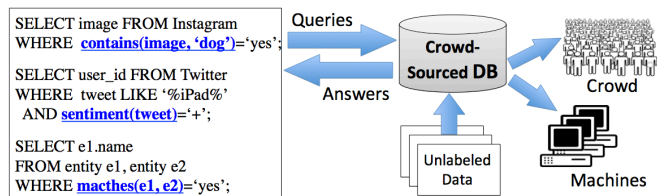


Figure 1: Examples of Labeling Queries in a Crowd-sourced DB.

crowd for labeling (e.g., to decide if the tweet has a positive sentiment).

While query optimization techniques [17, 24, 28] can reduce the number of items that need labeling, any crowd-sourced database that relies solely on human-provided labels will eventually suffer from scalability issues when faced with web-scale datasets and problems (e.g., daily tweets or images). This is because labeling each item by humans can cost several cents and take several minutes. For instance, given the example of Figure 1, even if we filter out tweets that do not contain “iPad”, there could still be millions of tweets with “iPad” that require sentiment labels (‘positive’ or ‘non-positive’).

To enable crowd-sourced databases to scale up to large datasets, we advocate combining humans and machine learning algorithms (e.g., classifiers), where (i) the crowd labels items that are either inherently *difficult* for the algorithm, or if labeled, will form the *best training data* for the algorithm, and (ii) the (trained) algorithm is used to label the remaining items much more quickly and cheaply. In this paper, we focus on *labeling* algorithms (classifiers) that assign one of several discrete values to each item, as opposed to predicting numeric values (i.e., regression), or finding missing items [33], leaving these other settings for future work.

Specifically, given a large, unlabeled dataset (say, millions of images) and a classifier that can attach a label to each unlabeled item (after sufficient training), our goal is to determine *which* questions to ask the crowd in order to (1) achieve the best training data and overall accuracy, given time or budget constraints, or (2) minimize the number of questions, given a desired level of accuracy.

Our problem is closely related to the classical problem of *active learning (AL)*, where the objective is to select statistically optimal training data [11]. However, in order for an AL algorithm to be a practical optimization strategy for labeling tasks in a crowd-sourced database, it must satisfy a number of *systems* challenges and criteria that have not been a focal concern in traditional AL literature, as described next.

1.1 Design Criteria

An AL algorithm must meet the following criteria to be used as the default optimization strategy in a crowd-sourced database:

1. **Generality.** Our system must come with a few built-in AL

algorithms that are applicable to arbitrary classification and labeling tasks, as crowd-sourced systems are used in a wide range of different domains. In Figure 1, for example, one query involves sentiment analysis while another seeks images containing a dog. Clearly, these tasks require drastically different classifiers. Although our system will allow expert users to provide their own custom-designed AL for their classification algorithm, most users may only have a classifier. Thus, to support general queries, our AL algorithm should make minimal or no assumptions about the classifiers that users provide for their labeling tasks.

2. Black-box treatment of the classifier. Many AL algorithms that provide theoretical guarantees need to *access* and *modify* the internal logic of the given classifier (e.g., adding constraints to the classifier’s internal loss-minimization step [5]). Such modifications are acceptable in theoretical settings but not in real-world applications, as state-of-the-art classifiers used in science and industry are rarely a straightforward implementation of textbook algorithms. Rather, these finely-tuned classifiers typically use thousands of lines of code to implement many intertwined steps (e.g., data cleaning, feature selection, parameter tuning, heuristics, etc.). In some cases, moreover, these codebases use proprietary libraries that cannot be modified. Thus, to make our crowd-sourcing system useful to a wide range of practitioners and scientists (who are not necessarily machine learning experts), we need an AL algorithm that treats the classifier as a black-box, i.e., does not modify the internals of the classifier.

3. Batching. Many (theoretical) AL algorithms are designed for online (a.k.a. streaming) scenarios in which items are revealed *one at a time*. This means that the AL algorithm decides whether to request a label for the current item, and if so, awaits the label before proceeding to the next item. While these settings are theoretically attractive, they are unrealistic in practice. First, we often have access to a large pool of unlabeled data to choose from (not just the next item), which should allow us to make better choices. Second, online AL settings typically perform an (expensive) analysis for each item [5, 6, 12], rendering them computationally prohibitive. Thus, for efficiency and practicality, the AL algorithm must support *batching*,¹ so that (i) the analysis is done only once for each *batch of multiple items*, and (ii) items are sent to the crowd in batches (to be labeled in parallel).

4. Parallelism. We aim to achieve *human-scalability* (i.e., asking the crowd fewer questions) through AL. However, we are also concerned with *machine-scalability*, because AL often involves repeatedly training a classifier and can thus be computationally expensive. While AL has been historically applied to small datasets, increasingly massive datasets (such as those motivating this paper) pose new computational challenges. Thus, a design criterion in our system is that our AL algorithm must be amenable to parallel execution on modern many-core processors and distributed clusters.

5. Noise management. AL has traditionally dealt with expert-provided labels that are often taken as ground truth (notable exceptions are agnostic AL approaches [12]). In contrast, crowd-provided labels are subject to a greater degree of noise, e.g., innocent errors, typos, lack of domain knowledge, and even deliberate spamming.

AL has a rich literature in machine learning [31]. However, the focus has been largely theoretical, with concepts from learning theory used to establish bounds on sample complexity but leaving a significant gap between theory and practice. Rather than aiming at such theoretical bounds, this paper focuses on a set of practical design

criteria and provides sound heuristics for the AL problem; the origin of these criteria is in real-world and systems considerations (in particular, issues of scale and ease-of-use).

In this paper, we design the the first AL algorithms that meet *all* the aforementioned requirements, which to the best of our knowledge no existing AL algorithm has completely satisfied. For example, existing AL algorithms that are general [5, 6, 12] do not support batching or parallelism and often require modifications to the classifier. (See Section 7 for a detailed literature review.) Thus, our proposal paves the way towards a scalable and generic crowd-sourcing system that can be used by a wide range of practitioners.

1.2 Our Contributions

Our main contributions are two AL algorithms, called *MinExpError* and *Uncertainty*, along with a noise-management technique, called *partitioning-based allocation (PBA)*. The *Uncertainty* algorithm requests labels for the items that the classifier is *most uncertain* about. We also design a more sophisticated algorithm, called *MinExpError*, that combines the current quality (say, accuracy) of the classifier with its uncertainty in a mathematically sound way, in order to choose the best questions to ask. *Uncertainty* is faster than *MinExpError*, but it also has lower overall accuracy, especially in the *upfront* scenario, i.e., where we request all labels in a single batch. We also study the *iterative* scenario, i.e., where we request labels in multiple batches and refine our decisions after receiving each batch.

A major novelty of our AL algorithms is in their use of bootstrap theory,² which yields several key advantages. First, bootstrap can deliver consistent estimates for a large class of estimators,³ making our AL algorithms general and applicable to nearly any classification task. Second, bootstrap-based estimates can be obtained while treating the classifier as a complete black-box. Finally, the required bootstrap computations can be performed independently from each other, hence allowing for an embarrassingly parallel execution.

Once *MinExpError* or *Uncertainty* decides which items to send to the crowd, dealing with the inherent noise in crowd-provided labels is the next challenge. A common practice is to use *redundancy*, i.e., to ask each question to multiple workers. However, instead of applying the same degree of redundancy to all items, we have developed a novel technique based on integer linear programming, called *PBA*, which dynamically partitions the unlabeled items based on their degree of difficulty for the crowd and determines the required degree of redundancy for each partition.

Thus, with a careful use of bootstrap theory as well as our batching and noise-management techniques, our AL algorithms meet *all* our requirements for building a practical crowd-sourced system, namely generality, scalability (batching and parallelism), and ease-of-use (black-box view and automatic noise-management).

We have evaluated the effectiveness of our algorithms on 18 real-world datasets (3 crowd-sourced using Amazon Mechanical Turk, and 15 well-known datasets from the UCI KDD repository). Experiments show that our AL algorithms achieve the same quality as several existing approaches while significantly reducing the number of questions asked of the crowd. Specifically, on average, we reduce the number of questions asked by:

- $100 \times (7 \times)$ in the upfront (iterative) scenario, compared to passive learning, and

¹Batch support is challenging because AL usually involves case-analysis for different combinations of labels and items. These combinations grow exponentially with the number of items in the batch, *unless* most of the analysis can be shared among different cases.

²A short background on bootstrap theory is provided in Section 3.1.

³Formally, this class includes any function that is Hadamard differentiable, including *M*-estimators (which include most machine learning algorithms and maximum likelihood estimators [22]).

- 44× (4.5×) in the upfront (iterative) scenario, compared to IWAL [2, 5, 6] which is the state-of-the-art general-purpose AL algorithm.

Interestingly, we also find that our algorithms (which are general-purpose) are still competitive with, and sometimes even superior to, some of the state-of-the-art domain-specific (i.e., less general) AL techniques. For example, our algorithms ask:

- 7× fewer questions than CrowdER [37] and an order of magnitude fewer than CVHull [4], which are among the most recent AL algorithms for entity resolution in database literature,
- 2–8× fewer questions than Bootstrap-LV [29] and MarginDistance [34], and
- 5–7× fewer questions for SVM classifiers than AL techniques that are specifically designed for SVM [34].

2. OVERVIEW OF APPROACH

Our approach in this paper is as follows. The user provides (i) a pool of unlabeled items (possibly with some labeled items as initial training data), (ii) a classifier (or “learner”) that improves with more and better training data, and a specification as to whether learning should be *upfront* or *iterative*, and (iii) a budget or goal, in terms of time, quality, or cost (and a pricing scheme for paying the crowd).

Our system can operate in two different scenarios, *upfront* or *iterative*, to train the classifier and label the items (Section 2.2). Our proposed AL algorithms, Uncertainty and MinExpError, can be used in either scenario (Section 3). Acquiring labels from a crowd raises interesting issues, such as how best to employ redundancy to minimize error, and how many questions to ask the crowd (Sections 4 and 5). Finally, we empirically evaluate our algorithms (Section 6).

2.1 Active Learning Notation

An active learning algorithm is typically composed of (i) a *ranker* \mathcal{R} , (ii) a *selection strategy* \mathcal{S} , and (iii) a budget allocation strategy Γ . The ranker \mathcal{R} takes as input a classification algorithm⁴ θ , a set of labeled items L , and a set of unlabeled items U , and returns as output an “effectiveness” score w_i for each unlabeled item $u_i \in U$. Our proposed algorithms in Section 3 are essentially ranking algorithms that produce these scores. A selection strategy then uses the scores returned from the ranker to choose a subset $U' \subseteq U$ which will be sent for human labeling. For instance, one selection strategy is picking the top k items with the largest (or smallest) scores, where k is determined by the budget or quality requirements. In this paper, we use weighted sampling to choose k unlabeled items, where the probability of choosing each item is proportional to its score. Finally, once U' is chosen, a budget allocation strategy Γ decides how to best acquire labels for all the items in U' : $\Gamma(U', B)$ for finding the most accurate labels given a fixed budget B , or $\Gamma(U', Q)$ for the cheapest labels given a minimum quality requirement Q . For instance, to reduce the crowd noise, a common strategy is to ask each question to multiple labelers and take the majority vote. In Section 4, we introduce our *Partitioning Based Allocation (PBA)* algorithm, which will be our choice of Γ in this paper.

2.2 Active Learning Scenarios

This section describes how learning works in the *upfront* and the *iterative* scenarios. Suppose we are given a budget B for asking questions (e.g., in terms of money or time) or a quality requirement Q

⁴For ease of presentation, in this paper we assume binary classification (i.e., $\{0, 1\}$), but our work applies to arbitrary classifiers.

Upfront Active Learning (B or Q , L_o , U , θ , \mathcal{R} , \mathcal{S} , Γ)

Input: B is the total budget (money, time, or number of questions),
 Q is the quality requirement (e.g., minimum accuracy, F1-measure),
 L_o is the initial labeled data,
 U is the unlabeled data,
 θ is a classification algorithm to (imperfectly) label the data,
 \mathcal{R} is a ranker that gives “effectiveness” scores to unlabeled items,
 \mathcal{S} is a selection strategy (specifying which items should be labeled by the crowd, given their effectiveness scores).
 Γ is a budget allocation strategy for acquiring labels from the crowd
Output: L is the labeled version of U
1: $W \leftarrow \mathcal{R}(\theta, L_o, U)$ // $w_i \in W$ is the effectiveness score for $u_i \in U$
2: Choose $U' \subseteq U$ based on $\mathcal{S}(U, W)$ such that U' can be labeled with
3: budget B or $\theta^{L_o}(U - U')$ satisfies Q
4: $ML \leftarrow \theta^{L_o}(U - U')$ // **train θ on L_o to automatically label $U - U'$**
5: Immediately display ML to the user // **Early query results**
6: $CL \leftarrow \Gamma(U', B$ or $Q)$ // **Ask (and wait for) the crowd to label U'**
7: $L \leftarrow CL \cup ML$ // **combine crowd and machine provided labels**
Return L

Figure 2: The upfront scenario in active learning.

Iterative Active Learning (B or Q , L_o , U , θ , \mathcal{R} , \mathcal{S} , Γ)

Input: Same as those in Figure 2
Output: L is the labeled version of U
1: $CL \leftarrow \emptyset$ // **labeled data acquired from the crowd**
2: $L \leftarrow \theta^{L_o}(U)$ // **train θ on L_o & invoke it to label U**
3: While our budget B is not exhausted or L 's quality does not meet Q :
4: $W \leftarrow \mathcal{R}(\theta, L_o \cup CL, U)$ // $w_i \in W$ is the effective score for $u_i \in U$
5: Choose $U' \subseteq U$ based on $\mathcal{S}(U, W)$ (subject to B or Q)
6: $L' \leftarrow \Gamma(U', B$ or $Q)$ // **Ask (and wait for) the crowd to label U'**
7: $CL \leftarrow CL \cup L'$, $U \leftarrow U - U'$ // **remove crowd labels from U**
8: $L \leftarrow CL \cup \theta^{L_o \cup CL}(U)$ // **train θ on $L_o \cup CL$ to label remaining U**
Return L

Figure 3: The iterative scenario in active learning.

(e.g., accuracy or F1-measure⁵) that our classifier must achieve. The choice of the scenario depends on user’s preference and needs.

Figure 2 is the pseudocode of the *upfront* scenario. In this scenario, the ranker computes effectiveness scores solely based on the initial labeled data⁶, L_o . Then, a subset $U' \subseteq U$ is chosen and sent to the crowd (based on \mathcal{S} and B or Q). While waiting for the crowd to label U' (based on Γ and B or Q), we train our classifier θ on L_o to label the remaining items, namely $U - U'$, and immediately send their labels back to the user. Once crowd-provided labels arrive, they are also sent to the user. Thus, the final result consists of the union of these two labeled sets.

Figure 3 shows the pseudocode of the *iterative* scenario. In this scenario, we ask for labels in several iterations. We ask the crowd to label a few items, adding those labels to the existing training set, and retrain. Then, we choose a new set of unlabeled items and iterate until we have exhausted our budget B or met our quality goal Q . At each iteration, our allocation strategy (Γ) seeks the cheapest or most accurate labels for the chosen items (U'), then our ranker uses the original training data L_o as well as the crowd labels CL collected thus far to decide how to score the remaining unlabeled items.

Note that the upfront scenario is *not* an iterative scenario with a single iteration, because the former does not use the crowd-sourced labels in training the classifier. This difference is important as different applications may call for different scenarios. When early answers are strictly preferred, say in an interactive search interface, the upfront scenario can immediately feed users with model-provided labels until the crowd’s answers arrive for the remaining items. The

⁵F1-measure is the harmonic mean of precision and recall and is frequently used to assess the quality of a classifier.

⁶In the absence of initially labeled data, we can first spend part of the budget to label a small, random sample of the data.

upfront scenario is also preferred when the project has a stringent accuracy requirement that only *gold* data (here, L_o) be used for training the classifier, say to avoid the potential noise of crowd labels in the training phase. In contrast, the iterative scenario is computationally slower, as it has to repeatedly retrain a classifier and wait for crowd-sourced labels. However, it can adaptively adjust its scores in each iteration, thus achieving a smaller error for the same budget than the upfront one. This is because the upfront scenario must choose all the items it wants labeled at once, based only on a limited set of initial labels.

3. RANKING ALGORITHMS

This paper proposes two novel AL algorithms, Uncertainty and MinExpError. AL algorithms consist of (i) a ranker \mathcal{R} that assigns scores to unlabeled items, (ii) a selection strategy \mathcal{S} that uses these scores to choose which items to label, and (iii) a budget allocation strategy Γ to decide *how* to acquire crowd labels for those chosen items. As explained in Section 2.1, our AL algorithms use weighted sampling and PBA (introduced in Section 4) as their selection and budget allocation strategies, respectively. Thus, for simplicity, we use Uncertainty and MinExpError to refer to our AL algorithms as well as their corresponding rankers. Both rankers can be used in either *upfront* or *iterative* scenarios. Section 3.1 provides brief background on nonparametric bootstrap theory, which we use in our rankers. Our rankers are introduced in Sections 3.2 and 3.3.

3.1 Background: Nonparametric Bootstrap

Our ranking algorithms rely on *nonparametric bootstrap* (or simply the *bootstrap*) to assess the benefit of acquiring labels for different unlabeled items. Bootstrap [15] is a powerful statistical technique traditionally designed for estimating the uncertainty of estimators. Consider an estimator θ (say, a classifier) that can be learned from data L (say, some training data) to estimate some value of interest for a data point u (say, the class label of u). This estimated value, denoted as $\theta^L(u)$, is a point-estimate (i.e., a single value), and hence, reveals little information about how this value would change if we used a different data L' . This information is critical as most real-world datasets are noisy, subject-to-change, or incomplete. For example, in our active learning context, missing data means that we can only access part of our training data. Thus, L should be treated a random variable drawn from some (unknown) underlying distribution \mathbb{D} . Consequently, statisticians are often interested in measuring *distributional information* about $\theta^L(u)$, such as variance, bias, etc. Ideally, one could measure such statistics by (i) drawing many new datasets, say L_1, \dots, L_k for some large k , from the same distribution \mathbb{D} that generated the original L ,⁷ (ii) computing $\theta^{L_1}(u), \dots, \theta^{L_k}(u)$, and finally (iii) inducing a distribution for $\theta(u)$ based on the observed values of $\theta^{L_i}(u)$. We call this true distribution $D^\theta(u)$ or simply $D(u)$ when θ is understood. Figure 4(a) illustrates this computation. For example, when θ is a binary classifier, $D(u)$ is simply a histogram with two bins (0 and 1), where the value of the i 'th bin for $i \in \{0, 1\}$ is $Pr[\theta^L(u) = i]$ when L is drawn from \mathbb{D} . Given $D(u)$, any distributional information (e.g., variance) can be obtained.

Unfortunately, in practice the underlying distribution \mathbb{D} is often unknown, and hence, direct computation of $D(u)$ using the procedure of Figure 4(a) is impossible. This is where bootstrap [15] becomes useful. The main idea of bootstrap is simple: treat L as a proxy for its underlying distribution \mathbb{D} . In other words, instead of drawing L_i 's directly from \mathbb{D} , generate new datasets S_1, \dots, S_k by resampling from L itself. Each S_i is called a (bootstrap) replicate or

simply a bootstrap. Each S_i is generated by drawing $n = |L|$ I.I.D. samples *with replacement* from L , and hence, some elements of L might be repeated or missing in S_i . Note that all bootstraps have the same cardinality as L , i.e. $|S_i| = |L|$ for all i . By computing θ on these bootstraps, namely $\theta^{S_1}(u), \dots, \theta^{S_k}(u)$, we can create an empirical distribution $\hat{D}(u)$. This is the bootstrap computation, which is visualized in Figure 4(b).

The theory of bootstrap guarantees that for a large class of estimators θ and sufficiently large k , we can use $\hat{D}(u)$ as a consistent approximation of $D(u)$. The intuition is that, by resampling from L , we emulate the original distribution \mathbb{D} that generated L . Here, it is sufficient (but not necessary) that θ be relatively smooth (i.e., Hadamard differentiable [15]) which holds for a large class of machine learning algorithms [22] such as M -estimators, themselves including maximum likelihood estimators and most classification techniques. In our experiments (Section 6), $k=100$ or even 10 have yielded reasonable accuracy (k can also be tuned automatically; see [15]).

Both of our AL algorithms use bootstrap to estimate the classifier's uncertainty in its predictions (say, to stop asking the crowd once we are confident enough). Employing bootstrap has several key advantages. First, as noted, bootstrap delivers consistent estimates for a large class of estimators, making our AL algorithms general and applicable to nearly any classification algorithm.⁸ Second, the bootstrap computation uses a "plug-in" principle; that is, we simply need to invoke our estimator θ with S_i instead of L . Thus, we can treat θ (here, our classifier) as a complete black-box since its internal implementation does not need to be modified. Finally, individual bootstrap computations $\theta^{S_1}(u), \dots, \theta^{S_k}(u)$ are independent from each other, and hence can be executed in parallel. This embarrassingly parallel execution model enables scalability by taking full advantage of modern many-core and distributed systems.

Thus, by exploiting powerful theoretical results from classical nonparametric statistics, we can estimate the uncertainty of complex estimators and also scale up the computation to large volumes of data. Aside from Provost et al. [29] (which is limited to probabilistic classifiers and is less effective than our algorithms; see Sections 6 and 7), no one has exploited the power of bootstrap in AL, perhaps due to bootstrap's computational overhead. However, with recent advances in speeding up bootstrap computation [20, 38, 39] and increases in RAM sizes and the number of CPU cores, bootstrap is now a computationally viable approach, motivating our use of it in this paper.

3.2 Uncertainty Algorithm

Our Uncertainty algorithm aims to ask the crowd the questions that are hardest for the classifier. Specifically, we (i) find out how uncertain (or certain) our given classifier θ is in its label predictions for different unlabeled items, and (ii) ask the crowd to label items for which the classifier is least certain. The intuition is that, the more uncertain the classifier, the more likely it will mislabel the item.

While focusing on uncertain items is one of the oldest ideas in AL literature [11], previous proposals either (i) require a *probabilistic* classifier that produces highly accurate class probabilities along with its label predictions [29, 35], or (ii) are limited to a particular classifier. For example, for probabilistic classifiers, the class distribution's entropy is a common measure of uncertainty [40]. While entropy-based AL can be effective in many situations [36, 40], in many other situations, when the classifiers do not produce accurate probabilities, entropy does not guarantee an unbiased estimate of the uncertainty (see Section 6.3). Other heuristics, such as the distance from the separator, are also used as a measure of uncertainty

⁷A common assumption in nonparametric bootstrap is that L and L_i datasets are independently and identically drawn (I.I.D.) from \mathbb{D} .

⁸The only known exceptions are lasso classifiers (i.e., L_1 regularization). Interestingly, even for lasso classifiers, there is a modified version of bootstrap that can produce consistent estimates [10].

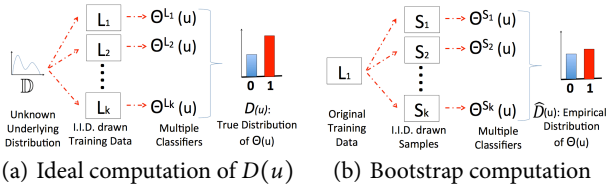


Figure 4: Bootstrap approximation $\hat{D}(u)$ of true distribution $D(u)$.

(e.g., SVM classifiers [34, 36]). However, these heuristics cannot be applied to arbitrary classifiers. In contrast, our Uncertainty algorithm applies to both probabilistic and non-probabilistic classifiers, and is also guaranteed by bootstrap theory to produce unbiased estimates. (In Section 6, we also empirically show that our algorithm is more effective.) Next, we describe how Uncertainty uses bootstrap to estimate the classifier’s uncertainty.

Let l be the predicted label for item u when we train θ on our labeled data L , i.e., $\theta^L(u) = l$. As explained in Section 3.1, L is often a random variable, and hence, $\theta^L(u)$ has a distribution (and variance). We use the variance of θ in its prediction, namely $\text{Var}[\theta^L(u)]$, as our formal notion of *uncertainty*. Our intuition behind this choice is as follows. A well-established result from Kohavi and Wolpert [21] has shown that the classification error for item u , say e_u , can be decomposed into a sum of three terms:

$$e_u = \text{Var}[\theta^L(u)] + \text{bias}^2[\theta^L(u)] + \sigma^2(u)$$

where $\text{bias}[\cdot]$ is the bias of the classifier and $\sigma^2(u)$ is a noise term.⁹ Our ultimate goal in AL is to reduce the sum of e_u for all u . The $\sigma^2(u)$ is an error inherent to the data collection process, and thus cannot be eliminated through AL. Thus, requesting labels for u ’s that have a large variance, indirectly reduces e_u for u ’s that have a large classification error. Hence, our Uncertainty algorithm assigns $\text{Var}[\theta^L(u)]$ as the score for each unlabeled item u , to ensure that items with larger variance are sent to the crowd for labels. Thus, in Uncertainty algorithm, our goal is to measure $\text{Var}[\theta^L(u)]$.

Since the underlying distribution of the training data (\mathbb{D} in Figure 4) is unknown to us, we use bootstrap. In other words, we bootstrap our current set of labeled data L , say k times, to obtain k different classifiers that are then invoked to generate labels for each item u , as shown in Figure 4(b). The output of these classifiers form an *empirical distribution* $\hat{D}(u)$ that approximates the true distribution of $\theta^L(u)$. We can then estimate $\text{Var}[\theta^L(u)]$ using $\hat{D}(u)$ which is guaranteed, by bootstrap theory [15], to quickly converge to the true value of $\text{Var}[\theta^L(u)]$ as we increase k .

Let S_i denote the i ’th bootstrap, and $\theta^{S_i}(u) = l_u^i$ be the prediction of our classifier for u when trained on this bootstrap. Define $X(u) := \sum_{i=1}^k l_u^i / k$, i.e., the fraction of classifiers in Figure 4(b) that predict a label of 1 for u . Since $l_u^i \in \{0, 1\}$, the uncertainty score for instance u is given by its variance, which can be computed as:

$$\text{Uncertainty}(u) = \text{Var}[\theta^L(u)] = X(u)(1 - X(u)) \quad (1)$$

We evaluate our Uncertainty algorithm in Section 6.

3.3 MinExpError Algorithm

Consider the toy dataset of Figure 5(a). Initially, only a few labels (+ or -) are revealed to us, as shown in Figure 5(b). With these initial labels we train a classifier, say a linear separator (shown as a solid line).

The intuition behind Uncertainty is that, by requesting labels for the most *uncertain* items (here being those closer to the separator),

⁹Squared bias $\text{bias}^2[\theta^L(u)]$ is defined as $[f(u) - E[\theta^L(u)]]^2$, where $f(u) = E[l_u|u]$, i.e., expected value of true label given u [21].

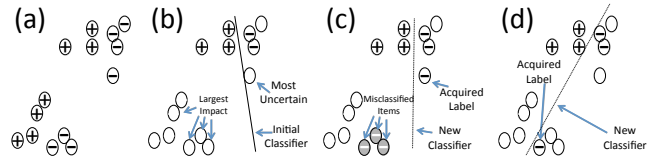


Figure 5: (a) Fully labeled dataset, (b) initial labels, (c) asking hardest questions, and (d) asking questions with high impact.

the crowd essentially handles items that are *hardest* (or ambiguous) for the classifier. However, this might not always be the best strategy. By acquiring a label for the item closest to the separator and training a new classifier, as shown in Figure 5(c), our overall accuracy does not change much: despite acquiring a new label, the new classifier still misclassifies three of the items at the lower-left corner of Figure 5(c). This observation shows that labeling items with most uncertainty (i.e., asking the hardest questions) may not have the *largest impact* on the classifier’s prediction power for other data points. In other words, an alternative strategy would be to acquire human labels for items that, if their labels differ from what the current classifier thinks, would have a huge impact on the classifier’s future decisions. In Figure 5(d), the lower-left points exemplify such items, which we refer to as having the *potential for largest impact* on the classifier’s accuracy.

Note that we cannot completely ignore uncertainty and choose items based only on their *potential* impact on the classifier. When the classifier is highly confident of its predicted label, no matter how much impact an opposite label could have on the classifier, acquiring a label for that item wastes resources because the crowd label will most likely agree with that of the classifier anyway. Thus, our MinExpError algorithm combines these two strategies in a mathematically sound way, described next.

Let $l = \theta^L(u)$ be the current classifier’s predicted label for u . If we magically knew that l was the correct label, we could simply add $\langle u, l \rangle$ to L and retrain the classifier. Let e_{right} be this new classifier’s error. On the other hand, if we somehow knew that l was the incorrect label, we would instead add $\langle u, 1 - l \rangle$ to L and retrain the classifier accordingly. Let e_{wrong} denote the error of this new classifier. The problem is that (i) we do not know what the true label is, and (ii) we do not know the classifier’s error in either case.

Solving (ii) is relatively easy: in each case we assume those labels and use cross validation on L to estimate both errors, say \hat{e}_{right} and \hat{e}_{wrong} . To solve problem (i), we can again use bootstrap to estimate the probability of our prediction l being correct (or incorrect), say $p(u) := \text{Pr}[l = l_u|u]$, where l_u is u ’s true label. Since we do not know l_u (or its distribution), we bootstrap L to train k different classifiers (following Figure 4’s notation). Let l_1, \dots, l_k be the labels predicted by these classifiers for u . Then, $p(u)$ can be approximated as

$$\hat{p}(u) = \frac{\sum_{i=1}^k \mathbf{1}(l_i = l)}{k} \quad (2)$$

Here, $\mathbf{1}(c)$ is the decision function which is 1 when condition c holds and is zero otherwise. Intuitively, equation (2) says that the probability of the classifier’s prediction being correct can be estimated by the fraction of classifiers that agree on that prediction, *if* those classifiers are each trained on a bootstrap of the training set L .

Our MinExpError algorithm aims to compute the classifier’s expected error if we use the classifier itself to label each item, and ask the crowd to label those items for which this expected error is larger. This way, the overall expected error will be minimized. To compute the classifier’s expected error, we can average over different la-

bel choices:

$$\text{MinExpError}(u) = \hat{p}(u)\hat{e}_{\text{right}} + (1 - \hat{p}(u))\hat{e}_{\text{wrong}} \quad (3)$$

We can break down equation (3) as:

$$\text{MinExpError}(u) = \hat{e}_{\text{wrong}} - \hat{p}(u)(\hat{e}_{\text{wrong}} - \hat{e}_{\text{right}}) \quad (4)$$

Assume that $\hat{e}_{\text{wrong}} - \hat{e}_{\text{right}} \geq 0$ (an analogous decomposition is possible when it is negative). Eq. (4) tells us that if the question is too hard (small $\hat{p}(u)$), we may still ask for a crowd label to avoid a high risk of misclassification on u . On the other hand, we may want to ask a question for which our model is fairly confident (large $\hat{p}(u)$), but having its true label can still make a big difference in classifying other items (\hat{e}_{wrong} is too large). This means that, however unlikely, if our classifier happens to be wrong, we will have a higher overall error if we do *not* ask for the true label of u . Thus, our MinExpError scores naturally combine both the difficulty of the question and how much knowing its answer can improve our classifier.

3.4 Complexity and Scalability

Besides its generality, a major benefit of bootstrap is that each replicate can be shipped to a different node, enabling parallel training. The time complexity of each iteration of Uncertainty is $O(k \cdot T(|U|))$, where $|U|$ is the number of unlabeled items in that iteration, $T(\cdot)$ is the classifier’s training time (e.g., this is $O(|U|^3)$ for SVM), and k is the number of bootstraps. Thus, we only need k nodes to achieve the same run-time as training a single classifier.

MinExpError is more expensive than Uncertainty as it requires a case analysis for each unlabeled item. For each iteration, its time complexity is $O((k+|U|) \cdot T(|U|))$. Since unlabeled items can be independently analyzed, the algorithm is still parallelizable. However, MinExpError requires more nodes (i.e., $O(|U|)$ nodes) to achieve the same performance as Uncertainty. This additional overhead is justified in the upfront scenario, given MinExpError’s superior performance on a limited set of initially labeled data (see Section 6).

4. HANDLING CROWD UNCERTAINTY

Crowd-provided labels are subject to a great degree of uncertainty: humans may make innocent (or deliberate errors), or give incorrect answers to ambiguous questions. This section proposes an algorithm called *Partitioning Based Allocation (PBA)* that reduces this uncertainty by strategically allocating different degrees of redundancy to different subgroups of the unlabeled items. PBA is our proposed instantiation of Γ in the upfront and iterative scenarios (Figures 2 and 3) which given a fixed budget B maximizes the labels’ accuracy, or given a required accuracy, minimizes cost.

Optimizing Redundancy for Subgroups. Most previous AL approaches assume that labels are provided by domain experts and thus perfectly correct (see Section 7). In contrast, incorrect labels are common in a crowd database — an issue conventionally handled by using *redundancy*, e.g., asking each question to multiple workers and combining their answers for the best overall result. Standard techniques, such as asking for multiple answers and using majority vote or the techniques of Dawid and Skene (DS) [13] can improve answer quality when the crowd is mostly correct, but will not help much if users do not converge to the right answer or converge too slowly. In our experience, crowd workers can be quite imprecise for certain classification tasks. For example, we removed the labels from 1000 tweets with hand-labeled (“gold data”) sentiment (dataset details in Section 6.1.3), and asked Amazon Mechanical Turk workers to label

them again, then measured the workers’ agreement. We used different redundancy ratios (1, 3, 5) and different voting schemes (majority and DS), then computed the crowd’s ability to agree with the hand-produced labels. The results are shown in Table 1.

Voting Scheme	Majority Vote	Dawid & Skene
1 worker/label	67%	51%
3 workers/label	70%	69%
5 workers/label	70%	70%

Table 1: The effect of redundancy (using both majority voting and Dawid and Skene voting) on the accuracy of crowd labels.

In this case, increasing redundancy from 3 to 5 labels does not significantly increase the crowd’s accuracy. Secondly, we have noticed that crowd accuracy varies for different subgroups of the unlabeled data. For example, in a different experiment, we asked Mechanical Turk workers to label facial expressions in the CMU Facial Expression dataset,¹⁰ and measured agreement with hand-supplied labels. This dataset consists of 585 head-shots of 20 users, each in 32 different combinations of head positions (straight, left, right, and up), sunglasses (with and without), and facial expressions (neutral, happy, sad, and angry). The crowd’s accuracy was significantly worse when the faces were looking up versus other positions:

Facial orientation	Avg. accuracy
straight	0.6335%
left	0.6216%
right	0.6049%
up	0.4805%

Similar patterns appear in several other datasets, where crowd accuracy is considerably lower for certain subgroups. To exploit these two observations, we developed our PBA algorithm, which computes the optimal number of questions to ask about each subgroup by estimating the probability p_g with which the crowd correctly classifies items of a given subgroup g , and then solves an integer linear program (ILP) to choose the optimal number of questions (i.e., degree of redundancy) for labeling each item from that subgroup, given these probabilities.

Before introducing our algorithm, we make the following observation. Using majority voting to combine an odd number of answers, say $2v + 1$, for an unlabeled item u with a true label l , the probability of the crowd’s combined answer l^* being correct is the probability that at most v or fewer workers get the answer wrong. Denoting this probability with $P_{g,(2v+1)}$, we have:

$$P_{g,(2v+1)} = Pr(l = l^* | 2v + 1 \text{ votes}) = \sum_{i=0}^v \binom{2v+1}{i} \cdot p_g^{2v+1-i} \cdot (1-p_g)^i \quad (5)$$

where p_g is the probability that a crowd worker will correctly label an item in group g .

Next, we describe our PBA algorithm, which partitions the items into subgroups and optimally allocates the budget to different subgroups by computing the optimal number of votes per item, V_g , for each subgroup g . PBA consists of three steps:

Step 1. Partition the dataset into G subgroups. This can be done either by partitioning on some low-cardinality field that is already present in the dataset to be labeled (for example, in an image recognition dataset, we might partition by photographer ID or the time of day when the picture was shot), or by using an unsupervised clustering algorithm such as k -means. For instance, in the CMU facial

¹⁰<http://kdd.ics.uci.edu/databases/faces/faces.data.html>

expression dataset, we partitioned the images based on user IDs, leading to $G = 20$ subgroups, each with roughly 32 images.

Step 2. Randomly pick $n_o > 1$ different data items from each subgroup, and obtain v_o labels for each one of them. Estimate p_g for each subgroup g , either by choosing data items for which the label is known and computing the fraction of labels that are correct, or by taking the majority vote for each of the n_o items, assuming it is correct, and then computing the fraction of labels that agree with the majority vote. For example, for the CMU dataset, we asked for $v_o = 9$ labels for $n_o = 2$ random images¹¹ from each subgroup, and hand-labeled those $n_o * G = 40$ images to estimate p_g for $g = 1, \dots, 20$.

Step 3. Solve an ILP to find the optimal V_g for every group g . We use b_g to denote the budget allocated to subgroup g , and create a binary indicator variable x_{gb} whose value is 1 iff subgroup g is allocated a budget of b . Also, let f_g be the number of items that our learner has chosen to label from subgroup g . Our ILP formulation depends on the user’s goal:

Goal 1. Suppose we are given a budget B (in terms of the number of questions) and our goal is to acquire the most accurate labels for the items requested by the learner. We can then formulate an ILP to minimize the following objective function:

$$\sum_{g=1}^G \sum_{b=1}^{b^{max}} x_{gb} \cdot (1 - P_{g,b}) \cdot f_g \quad (6)$$

where b^{max} is the maximum number of votes that we are willing to ask per item. This goal function captures the expected weighted error of the crowd, i.e., it has a lower value when we allocate a larger budget ($x_{gb}=1$ for a large b when $P_g > 0.5$) to subgroups whose questions are harder for the crowd ($P_{g,b}$ is small) or the learner has chosen more items from that group (f_g is large). This optimization is subject to the following constraints:

$$\forall 1 \leq g \leq G, \quad \sum_{b=1}^{b^{max}} x_{gb} = 1 \quad (7)$$

$$\sum_{g=1}^G \sum_{b=1}^{b^{max}} x_{gb} \cdot b \cdot f_g \leq B - v_o \cdot n_o \cdot G \quad (8)$$

Here, constraint (7) ensures that we pick exactly one b value for each subgroup and (8) ensures that we stay within our labeling budget (we subtract the initial cost of estimating p_g ’s from B).

Goal 2. If we are given a minimum required accuracy Q , and our goal is to minimize the total number of questions asked, we turn (8) into a goal and (6) into a constraint, i.e., minimizing $\sum_{g=1}^G \sum_{b=1}^{b^{max}} x_{gb} \cdot b \cdot f_g$ while ensuring that $\sum_{g=1}^G \sum_{b=1}^{b^{max}} x_{gb} \cdot (1 - P_{g,b}) \cdot f_g \leq 1 - Q$.

Note that one could further improve the p_g estimates and the expected error estimate in (6) by incorporating the past accuracy of individual workers. We leave such extensions to future work. We evaluate PBA in Section 6.2.

5. OPTIMIZING FOR THE CROWD

The previous section described our algorithm for handling noisy labels. In practice, other optimization questions arise. In Section 5.1, we address how to decide when our accuracy is “good enough”. Given that a crowd can label multiple items in parallel, in Section 5.2 we discuss the effect of batch size (number of simultaneous questions) on our learning performance.

¹¹Larger v_o and n_o (and accounting for each worker’s accuracy) yield more reliable p_g estimates. Here, we use a small budget to show that even with a rough estimate we can improve on uniform allocations. We study the effect of n_o on PBA performance in our report [26].

5.1 When To Stop Asking

As mentioned in Section 2.2, users may either provide a fixed budget B or a minimum quality requirement Q (e.g., F1-measure). Given a fixed budget, we can ask questions until the budget is exhausted. However, to achieve a quality level Q , we must estimate the current error of the trained classifier. The easiest way to do this is to measure the trained classifier’s ability to classify the gold data accurately, according to the desired quality metric. We can then continue to ask questions until a specific accuracy on the gold data is achieved (or until the rate of improvement of accuracy levels off).

In the absence of (sufficient) gold data, we adopt the standard *k-fold cross validation* technique, randomly partitioning the crowd-labeled data into test and training sets, and measuring the ability of a model learned on training data to predict test values. We repeat this procedure k times and take the average as an overall assessment of the model’s quality. Section 6.2 shows that this method provides more reliable estimates of the model’s current quality than relying on a small amount of gold data.

5.2 Effect of Batch Sizes

At each iteration of the iterative scenario, we must choose a subset of the unlabeled items according to their effectiveness scores, and send it to the crowd for labeling. We call this subset a “batch” (denoted as U' in Line 5 of Figure 3). An interesting question is how to set this *batch size*, say β .

Intuitively, a smaller β increases opportunities to improve the AL algorithm’s effectiveness by incorporating previously requested labels before deciding which labels to request next. For instance, best results are achieved when $\beta=1$. However, larger batch sizes reduce the overall run-time substantially by (i) allowing several workers to label items in parallel, and (ii) reducing the number of iterations. This is confirmed by our experiments in Section 6.2, which show that the impact of increasing β on the effectiveness of our algorithms is less dramatic than its impact on the overall run-time. Thus, to find the optimal β , a reasonable choice is to start from a smaller batch size and continuously increase it (say, double it) until the run-time becomes reasonable, or the quality metric falls below the minimum requirement. For a detailed discussion, refer to [26].

6. EXPERIMENTAL RESULTS

This section evaluates the effectiveness of our AL algorithms compared to the state-of-the-art AL strategies, in terms of speed, cost, and accuracy with which they can label a dataset.

Overview of the Results. Overall, our experiments show the following: (i) our AL algorithms require several orders of magnitude fewer questions to achieve the same quality than the random baseline, and substantially fewer questions ($4.5 \times - 44 \times$) than the best general-purpose AL algorithm (IWAL [2, 5, 6]), (ii) our MinExpError algorithm works better than Uncertainty in the upfront setting, but the two are comparable in the iterative setting, (iii) Uncertainty has a much lower computational overhead than MinExpError, and (iv) surprisingly, even though our AL algorithms are generic and widely applicable, they still perform comparably to and sometimes much better than AL algorithms designed for specific tasks, e.g., $7 \times$ fewer questions than CrowdER [37] and an order of magnitude fewer than CVHull [4] (two of the most recent AL algorithms for entity resolution), competitive results to Brew et al [9], and also $2-8 \times$ fewer questions than less general AL algorithms (Bootstrap-LV [29] and MarginDistance [34]).

Experimental Setup. All algorithms were tested on a Linux server with dual-quad core Intel Xeon 2.4 GHz processors and 24GB of RAM. Throughout this section, unless stated otherwise, we repeated

each experiment 20 times and reported the average result, every task cost 1€, and the size of the initial training and the batch size were 0.03% and 10% of the unlabeled set, respectively.

Methods Compared. We ran experiments on the following learning algorithms in both the upfront and iterative scenarios:

1. *Uncertainty*: Our method from Section 3.2.
2. *MinExpError*: Our method from Section 3.3.
3. *IWAL*: A popular AL algorithm [6] that follows Importance Weighted Active Learning [5], recently extended with batching [2].
4. *Bootstrap-LV*: Another bootstrap-based AL that uses the model’s class probability estimates to measure uncertainty [29]. This method only works for probabilistic classifiers (e.g., we exclude this in our experiments with SVMs).
5. *CrowdER*: One of the most recent AL techniques specifically designed for entity resolution tasks [37].
6. *CVHull*: Another state-of-the-art AL specifically designed for entity resolution [4]. We found several other entity resolution algorithms [3, 30] to be less effective than [4] and thus omit those from our comparisons here. (For details see [26].)
7. *MarginDistance*: An AL algorithm specifically designed for SVM classifiers, which picks items that are closer to the margin [34].
8. *Entropy*: A common AL strategy [31] that picks items for which the entropy of different class probabilities is higher, i.e., the more uncertain the classifier, the more similar the probabilities of different classes, and the higher the entropy.
9. *Brew et al. [9]*: a domain-specific AL designed for sentiment analysis, which uses clustering to select an appropriate subset of articles (or tweets) to be tagged by users.
10. *Baseline*: A passive learner that randomly selects unlabeled items to send to the crowd.

In the plots, we prepend the scenario name to the algorithm names, e.g., UpfrontMinExpError or IterativeBaseline. We have repeated our experiments with different classifiers as the underlying learner, including SVM, Naïve-Bayes classifier, neural networks, and decision trees. For lack of space, we only report each experiment for one type of classifier. When unspecified, we used linear SVM.

Evaluation Metrics. AL algorithms are usually evaluated based on their *learning curve*, which plots the quality measure of interest (e.g., accuracy or F1-measure) as a function of the number of data items that are labeled [31]. To compare different learning curves quantitatively, the following metrics are typically used:

1. Area under curve (AUC) of the learning curve.
2. AUCLOG, which is the AUC of the learning curve when the X-axis is in log-scale.
3. Questions saved, which is the ratio of number of questions asked by an active learner to those asked by the baseline to achieve the same quality.

Higher AUCs indicate that the learner achieves a higher quality for the same cost/number of questions. Due to the diminishing-return of learning curves, the average quality improvement is usually in the 0–16% range.

AUCLOG favors algorithms that improve the metric of interest early on (e.g., with few examples). Due to the logarithm, the improvement of this measure is typically in the 0–6% range.

To compute question savings, we average over all the quality levels that are achievable by both AL and baseline curves. For competent active learners, this measure should (greatly) exceed 1, as a ratio < 1 indicates a performance worse than that of the random baseline.

6.1 Crowd-sourced Datasets

We experiment with several datasets labeled using Amazon Mechanical Turk. In this section, we report the performance of our

algorithms on each of them.

6.1.1 Entity Resolution

Entity resolution (ER) involves finding different records that refer to the same entity, and is an essential step in data integration/cleaning [3, 4, 25, 30, 37]. Humans are typically more accurate at ER than classifiers, but also slower and more expensive [37].

We used the Product (<http://dbs.uni-leipzig.de/file/Abt-Buy.zip>) dataset, which contains product attributes (name, description, and price) of items listed on the `abt.com` and `buy.com` websites. The task is to detect pairs of items that are identical but listed under different descriptions on the two websites (e.g., “iPhone White 16 GB” vs “Apple 16GB White iPhone 4”). We used the same dataset as [37], where the crowd was asked to label 8315 pairs of items as either identical or non-identical. This dataset consists of 12% identical pairs and 88% non-identical pairs. In this dataset, each pair has been labeled by 3 different workers, with an average accuracy of 89% and an F1-measure of 56%. We also used the same classifier used in [37], namely a linear SVM where each pair of items is represented by their Levenshtein and Cosine similarities. When trained on 3% of the data, this classifier has an average accuracy of 80% and an F1-measure of 40%. Figure 8 shows the results of using different AL algorithms. As expected, while all methods eventually improve with more questions, their overall F1-measures improve at different rates. MarginDistance, MinExpError, and CrowdER are all comparable, while Uncertainty improves much more quickly than the others. Here, Uncertainty can identify the items about which the model has the most uncertainty and get the crowd to label those earlier on. Interestingly, IWAL, which is a generic state-of-the-art AL, performs extremely poorly in practice. CVHull performs equally poorly, as it internally relies on IWAL as its AL subroutine. This suggests opportunities for extending CVHull to rely on other AL algorithms in future work.

This result is highly encouraging: even though CrowdER and CVHull are recent AL algorithms highly *specialized* for improving the recall (and indirectly, F1-measure) of ER, our *general-purpose* AL algorithms are still quite competitive. In fact, Uncertainty uses $6.6\times$ fewer questions than CrowdER and an order of magnitude fewer questions than CVHull to achieve the same F1-measure.

6.1.2 Image Search

Vision-related problems also utilize crowd-sourcing heavily, e.g., in tagging pictures, finding objects, and identifying bounding boxes [35]. All of our vision experiments employed a relatively simple classifier where the PHOW features (a variant of dense SIFT descriptors commonly used in vision tasks [8]) of a set of images are first extracted as a bag of words, and then a linear SVM is used for their classification. Even though this is not the state-of-the-art image detection algorithm, we show that our AL algorithms still greatly reduce the cost of many challenging vision tasks.

Gender Detection. We used the faces from Caltech101 dataset [16] and manually labeled each image with its gender (266 males, 169 females) as our ground truth. We also gathered crowd labels by asking the gender of each image from 5 different workers. We started by training the model on a random set of 11% of the data. In Figure 6, we show the accuracy of the crowd, the accuracy of our machine learning model, and the overall accuracy of the model plus crowd data. For instance, when a fraction x of the labels were obtained from the crowd, the other $1 - x$ labels were determined from the model, and thus, the overall accuracy was $x * a_c + (1 - x) * a_m$, where a_c and a_m are the crowd and model’s accuracy, respectively. As in our entity resolution experiments, our algorithms improve the quality of the labels provided by the crowd, i.e., by asking questions for which

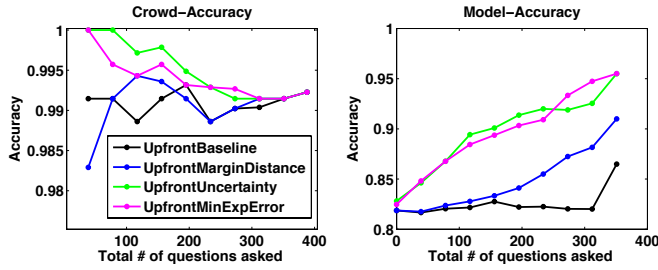


Figure 6: The object detection task (detecting the gender of the person in an image): accuracy of the (a) crowd, (b) model, (c) overall.

the crowd tends to be more reliable. Here, though, the crowd produces higher overall quality than in the entity resolution case and therefore its accuracy is improved only from 98.5% to 100%. Figure 6 shows that both MinExpError and Uncertainty perform well in the upfront scenario, respectively improving the baseline accuracy by 4% and 2% on average, and improving its AUCLOG by 2-3%. Here, due to the upfront scenario, MinExpError saves the most number of questions. The baseline has to ask 4.7 \times (3.7 \times) more questions than MinExpError (Uncertainty) to achieve the same accuracy. Again, although specifically designed for SVM, MarginDistance achieves little improvement over the baseline.

Image Search. We again mixed 50 human faces and 50 background images from Caltech101 [16]. Because differentiating human faces from background clutter is easy for humans, we used the crowd labels as ground truth in this experiment. Figure 7 shows the upfront scenario with an initial set of 10 labeled images, where both Uncertainty and MinExpError lift the baseline’s F1-measure by 16%, while MarginDistance provides a lift of 13%. All three algorithms increase the baseline’s AUCLOG by 5-6%. Note that the baseline’s F1-measure degrades slightly as it reaches higher budgets, since the baseline is forced to give answers to hard-to-classify questions, while the AL algorithms avoid such questions, leaving them to the last batch (which is answered by the crowd).

6.1.3 Sentiment Analysis

Microblogging sites such as Twitter provide rich datasets for sentiment analysis [27], where analysts can ask questions such as “how many of the tweets containing ‘iPhone’ have a positive or negative sentiment?” Training accurate classifiers requires sufficient accurately labeled data, and with millions of daily tweets, it is too costly to ask the crowd to label all of them. In this experiment, we show that our AL algorithms, with as few as 1K-3K crowd-labeled tweets, can achieve very high accuracy and F1-measure on a corpus of 10K-100K unlabeled tweets. We randomly chose these tweets from an online corpus¹² that provides ground truth labels for the tweets, with equal numbers of positive- and negative-sentiment tweets. We obtained crowd labels (*positive, negative, neutral, or vague/unknown*) for each tweet from 3 different workers. Figure 9 shows the results for using 1K initially labeled tweets with the 10K dataset in the iterative setting. The results confirm that the iterative scenario is best handled by our Uncertainty algorithm, which even improves on the Brew et al. algorithm [9], which is a domain-specific AL designed for sentiment analysis. Here, the Uncertainty, MinExpError, and Brew et al. algorithms improve the average F1-measure of the baseline model by 9%, 4% and 4%, respectively. Also, Uncertainty increases baseline’s AUCLOG by 4%. In comparison to the baseline, Uncertainty, MinExpError, and Brew et al. reduce the number of questions by factors of 5.38 \times , 1.76 \times , and 4.87 \times , respectively. Again, the savings are expectedly modest compared to the upfront scenario, where savings are between 27–46 \times (see [26] for details of the upfront experiments with the 100K corpus).

¹²<http://twittersentiment.appspot.com>

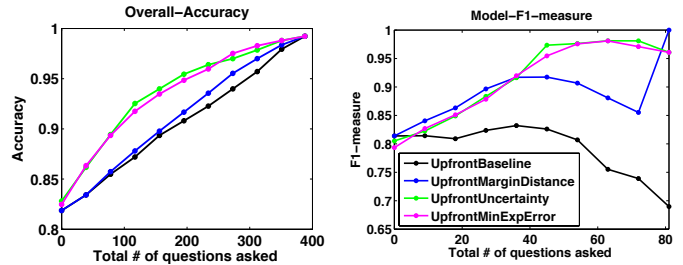


Figure 7: Image search task: whether a scene contains a human.

6.2 PBA and other Crowd Optimizations

In this section, we present results for our crowd-specific optimizations described in Section 5.

PBA Algorithm: We first report experiments on the *PBA* algorithm. Recall that this algorithm partitions the items into subgroups and optimally allocates the budget amongst them. In the CMU facial expressions dataset, the crowd had a particularly hard time telling the facial expression of certain individuals, so we created subgroups based on the *user* column of the dataset, and asked the crowd to label the expression on each face. Choosing $v_o = 9$, $b^{max} = 9$, and $n_o = 2$, we compared *PBA* against a uniform budget allocation scheme, where the same number of questions are asked about all items uniformly, as done in previous research (see Section 7). The results are shown in Figure 10. Here, the X axis shows the normalized budget, e.g., a value of 2 means the budget was twice the total number of unlabeled items. The Y axis shows the overall (classification) error of the crowd using majority voting under different allocations. Here, the solid lines show the actual error achieved under both strategies, while the blue and green dotted lines show our estimates of their performance before running the algorithms. Figure 10 shows that although our estimates of the actual error are not highly accurate, since we only use them to solve an ILP that would favor harder subgroups, our *PBA* algorithm (solid green) still reduces the overall crowd error by about 10% (from 45% to 35%). We also show how *PBA* would perform if it had an oracle that provided access to exact values of $P_{g,b}$ (red line).

k-Fold Cross Validation for Estimating Accuracy: We use k-fold cross validation to estimate the current quality of our model (details in [26]). Figure 11 shows our estimated F1-measure for an SVM classifier on UCI’s cancer dataset. Our estimates are reasonably close to the true F1 values, especially as more labels are obtained from the crowd. This suggests that k-fold cross validation allows us to effectively estimate current model accuracy, and to stop acquiring more data once model accuracy has reached a reasonable level.

The Effect of Batch Size: We now study the effect of batch size on result quality, based on the observations in Section 5.2. The effect is typically moderate (and often linear), as shown in Figure 12. Here we show that the F1-measure gains can be in the 8–10% range (see Section 5.2). However, larger batch sizes reduce runtime substantially, as Figure 13 shows. Here, going from batch size 1 to 200 significantly reduces the time to train a model, by about two orders of magnitude (from 1000’s of seconds to 10’s of seconds).

6.3 UCI Classification Datasets

In Section 6.1, we validated our algorithms on crowd-sourced datasets. This section also compares our algorithms on datasets from the UCI KDD [1], where labels are provided by experts; that is, ground truth and crowd labels are the same. Thus, by excluding the effect of noisy labels, we can compare different AL strategies in isolation. We have chosen 15 well-known datasets, as shown in Figures 14 and 15. To avoid bias, we have avoided any dataset-specific tuning or prepro-

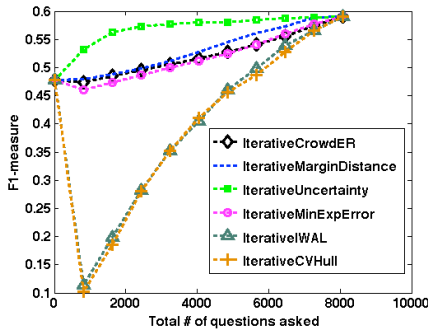


Figure 8: Comparison of different AL algorithms for entity resolution: the overall F1-measure in the iterative scenario.

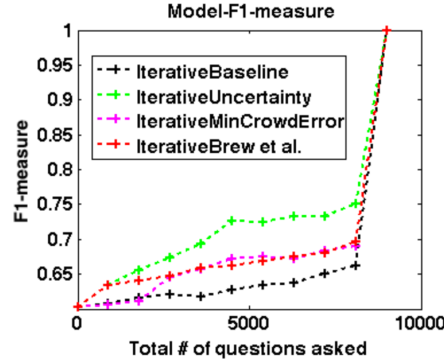


Figure 9: Sentiment analysis task: F1-measure of the model for 10K tweets.

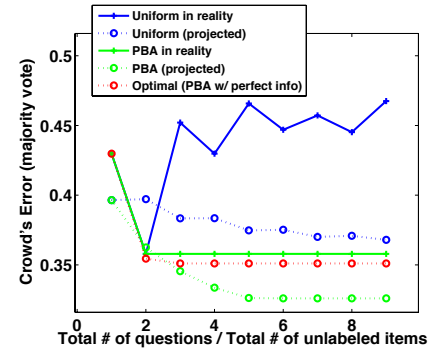


Figure 10: The crowd's noise under different budget allocation schemes.

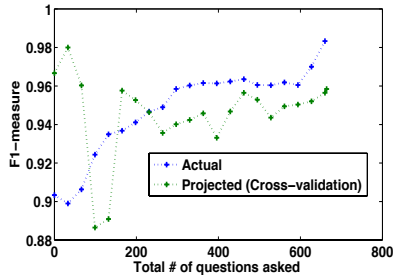


Figure 11: Estimating quality with k -fold cross validation.

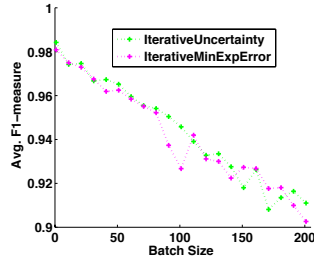


Figure 12: Effect of batch size on our algorithms' F1-measure (vehicle dataset, w/ a budget of 400 questions).

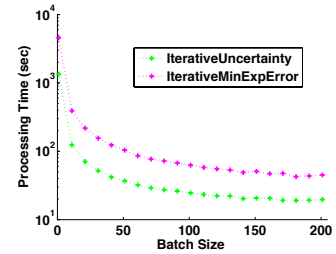


Figure 13: Effect of batch size on our algorithms' processing times (vehicle dataset, w/ a budget of 400 questions).

cessing steps, and applied the same classifier with the same settings to all datasets. In each case, we experimented with 10 different budgets of 10%, 20%, ..., 100% (of total number of labels), each repeated 10 times, and reported the average. Also, to compute the F1-measure for datasets with more than 2 classes, we have either grouped all non-majority classes into a single class, or arbitrarily partitioned all the classes into two new ones (details in [26]).

Here, besides the random baseline, we compare Uncertainty and MinExpError against four other AL techniques, namely IWAL, MarginDistance, Bootstrap-LV, and Entropy. IWAL is as general as our algorithms, MarginDistance only applies to SVM classification, and Bootstrap-LV and Entropy are only applicable to probabilistic classifiers. For all methods (except for MarginDistance) we used MATLAB's decision trees as the classifier, with its default parameters except for the following: no pruning, no leaf merging, and a 'minparent' of 1 (impure nodes with 1 or more items can be split).

Figures 14 and 15 show the reduction in the number of questions under both upfront and iterative settings for Entropy, Bootstrap-LV, Uncertainty, and MinExpError, while Table 2 shows the average AUCLOG, F1, and reduction in the number of questions asked across all 15 datasets for all AL methods. The two figures omit detailed results for MarginDistance and IWAL, as they performed poorly (as indicated in Table 2). We report all the measures of different AL algorithms in terms of their performance improvement relative to the baseline (so higher numbers are better). For instance, consider Figure 14. On the *yeast* dataset, MinExpError reduces the number of questions asked by 84 \times , while Uncertainty and Bootstrap-LV reduce it by about 38 \times and Entropy does not improve the baseline.

In summary, these results are consistent with those observed with crowd-sourced datasets. In the upfront setting, MinExpError significantly outperforms other AL techniques, with over 104 \times savings in the total number of questions on average. MinExpError also improves the AUCLOG and average F1-measure of the baseline on

average by 5% and 15%, respectively. After MinExpError, the Uncertainty and Bootstrap-LV are most effective, producing 55-69 \times savings, improving the AUCLOG by 3%, and lifting the average F1-measure by 11-12%. Bootstrap-LV performs well here, which we expect is due to its use of bootstrap (similar to our algorithms). However, recall that Bootstrap-LV only works for probabilistic classifiers (e.g., decision trees). Here, MarginDistance is only moderately effective, providing around 13 \times savings. Finally, the least effective algorithms are IWAL and Entropy, which perform quite poorly across almost all datasets. IWAL uses learning theory to establish worst-case bounds on sample complexity (based on VC-dimensions), but these bounds are known to leave a significant gap between theory and practice, as seen here. Entropy relies on the classifier's own class probability estimates [31], and thus can be quite ineffective when these estimates are highly inaccurate. To confirm this, we used bootstrap to estimate the class probabilities more accurately (similarly to our Uncertainty algorithm), and then computed the entropy of these estimates. The modified version, denoted as Uncertainty (Entropy), is significantly more effective than the baseline (73 \times), which shows that our idea of using bootstrap in AL not only achieves generality (beyond probabilistic classifiers) but can also improve traditional AL strategies by providing more accurate probability estimates.

For the iterative scenario, Uncertainty actually works better than MinExpError, with an average saving of 7 \times over the baseline in questions asked and an increase in AUCLOG and average F1-measure by 1% and 3%, respectively. Note that savings are generally more modest than in the upfront case because the baseline receives much more labeled data in the iterative setting and therefore, its average performance is much higher, leaving less room for improvement. However, given the comparable (and even slightly better) performance of Uncertainty compared to MinExpError in the iterative scenario, Uncertainty becomes a preferable choice here due to its considerably smaller processing overhead (see Section 6.4).

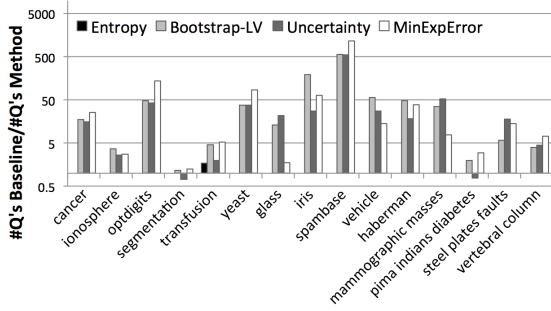


Figure 14: The ratio of the num. of questions asked by the random baseline to those asked by different AL algorithms in the Upfront scenario.

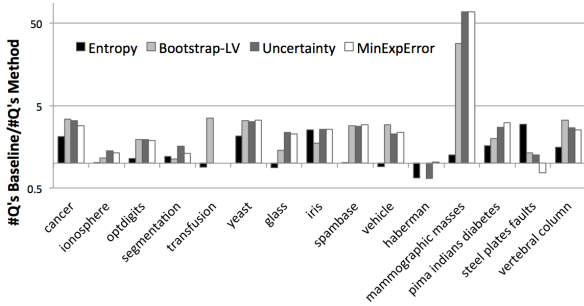


Figure 15: The ratio of the num. of questions asked by the random baseline to those asked by different AL algorithms in the Iterative scenario.

6.4 Run-time, Scalability, and Monetary Cost

To measure algorithm runtime, we experimented with multiple datasets. Here, we only report the results for the vehicle dataset. Figure 13 shows that training runtimes range from a few seconds to about 5,000 seconds and depend heavily on batch size, which determines how many times the model is re-trained.

We also studied the effect of parallelism on our algorithms’ runtimes. Here, we compared different AL algorithms in the upfront scenario on Twitter dataset (10K tweets) as we enabled cores on a multicore machine. The results are shown in Figure 16. For Uncertainty, the run-time only improves until we have as many cores as we have bootstrap replicas (here, 10). After that, improvement is marginal. In contrast, MinExpError scales extremely well, achieving nearly linear speedup because it re-trains the classifier once for every training point.

Finally, we perform a monetary comparison between our AL algorithms and two different baselines. Figure 17 shows the combined

Upfront			
Method	AUCLOG(F1)	Avg(Q's Saved)	Avg(F1)
Uncertainty	1.03x	55.14x	1.11x
MinExpError	1.05x	104.52x	1.15x
IWAL	1.05x	2.34x	1.07x
MarginDistance	1.00x	12.97x	1.05x
Bootstrap-LV	1.03x	69.31x	1.12x
Entropy	1.00x	1.05x	1.00x
Uncertainty (Entropy)	1.03x	72.92x	1.13x
Iterative			
Method	AUCLOG(F1)	Avg(Q's Saved)	Avg(F1)
Uncertainty	1.01x	6.99x	1.03x
MinExpError	1.01x	6.95x	1.03x
IWAL	1.01x	1.53x	1.01x
MarginDistance	1.01x	1.47x	1.00x
Bootstrap-LV	1.01x	4.19x	1.03x
Entropy	1.01x	1.46x	1.01x
Uncertainty (Entropy)	1.01x	1.48x	1.00x

Table 2: Average improvement in AUCLOG, Questions Saved, and Average F1, across all 15 UCI datasets, by different AL algorithms.

monetary cost (crowd+machines) of achieving different levels of quality (i.e., the Model’s F1-measure for Twitter dataset from Section 6.1.3). The crowd cost is $(0.01+0.005)*3$ per labeled item, which includes 3x redundancy and Amazon’s commission. The machine cost for the baseline (passive learner) only consists of training a classifier while for our algorithms we have also included the computation of the AL scores. To compute the machine cost, we measured the running time in core-hours using c3.8xlarge instances of Amazon EC2 cloud, which is currently \$1.68/hour. The overall cost is clearly dominated by crowd cost, which is why our AL learners can achieve the same quality with a much lower cost (since they ask much fewer questions to the crowd). We also compared against a second baseline where all the items are labeled by the crowd (i.e., no classifiers). As expected, this ‘Crowd Only’ approach is significantly more expensive than our AL algorithms. Figure 18 shows that the crowd can label all the items for \$363 with an accuracy of 88.1–89.8%, while we can easily achieve a close accuracy of 87.9% with only \$36 (for labeling 807 items and spending less than \$0.00014 on machine computation). This order of magnitude in saved dollars will only become more dramatic over time, as we expect machine costs to continue dropping according to Moore’s law, while human worker costs will presumably remain the same or even increase.

7. RELATED WORK

Crowd-sourced Databases. These systems [17, 19, 24, 25, 33] use optimization techniques to reduce the number of *unnecessary* questions asked to humans (e.g., number of pair-wise comparisons in a join or sort query). However, the crowd must still provide at least as many labels as there are unlabeled items directly requested by the user. It is simply unfeasible to label millions of items in this fashion. To scale up to large datasets, we use machine learning to avoid obtaining crowd labels for a significant portion of the data.

Active Learning. AL has a rich literature in machine learning (see [31]). However, to the best of our knowledge, no existing AL algorithm satisfies *all* of the desiderata required for a practical crowd-sourced system, namely generality, black-box approach, batching, parallelism, and label-noise management. For example, many AL algorithms are designed for a specific classifier (e.g., neural networks [11], SVM [34], or probabilistic classifiers [23]) or a specific domain (e.g., entity resolution [3, 4, 30, 37], vision [35], or medical imaging [18]). However, our algorithms work for arbitrary classifiers and do not require any domain knowledge. Surprisingly, we are also competitive with (and sometimes even superior to) these domain-specific algorithms. E.g., we compared against MarginDistance [34], CrowdER [37], CVHull [4], and Brew et al. [9].

The popular IWAL algorithm [5] is generic (except for hinge-loss classifiers such as SVM), but does not support batching or parallelism, and requires adding new constraints to the classifier’s internal loss-minimization step. In fact, most AL proposals that provide theoretical guarantees (i) are not black-box, as they need to *know and shrink* the classifier’s hypothesis space at each step, and (ii) do not support batching, as they rely on IID-based analysis. Notable exceptions are [12] and its IWAL variant [6]; they are black-box but do not support batching or parallelism. Bootstrap-LV [29] and ParaActive [2] support parallelism and batching, but both [2, 6] are based on VC-dimension bounds [7], which are known to be too loose in practice. They cause the model to request many more labels than needed, leading to negligible savings over passive learning (as shown in Section 6). Bootstrap-LV also uses bootstrap, but unlike our algorithms, it is not general. Noisy labelers are handled in [14, 24, 28, 32], but [24, 28, 32] assume the same quality for all labelers and [14] assumes that each labeler’s quality is the same across all items. Moreover, in Section 6, we empirically showed that our

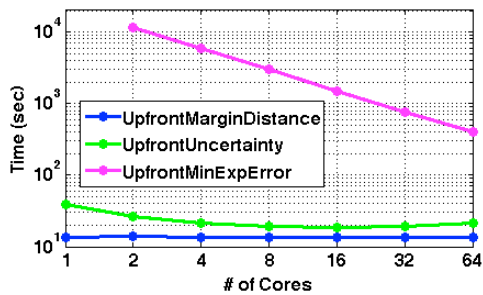


Figure 16: Effect of parallelism on processing time: 100K tweets.

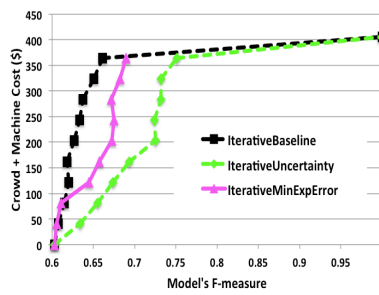


Figure 17: The combined monetary cost of the crowd and machines for Twitter dataset.

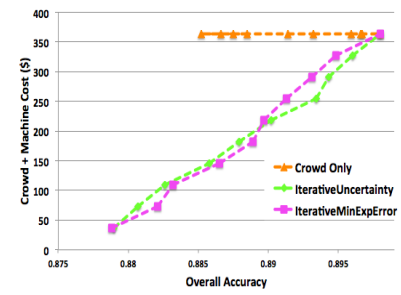


Figure 18: The monetary cost of AL vs. using the crowd for all labels.

algorithms are superior to generic AL algorithms (IWAL +ParaActive [2, 5, 6] and Bootstrap-LV [29]).

8. CONCLUSIONS

In this paper, we proposed two AL algorithms, Uncertainty and MinExpError, to enable crowd-sourced databases to scale up to large datasets. To broaden their applicability to different classification tasks, we designed these algorithms based on the theory of non-parametric bootstrap and evaluated them in two different settings. In the *upfront* setting, we ask all questions to the crowd in one go. In the *iterative* setting, the questions are adaptively picked and added to the labeled pool. Then, we retrain the model and repeat this process. While iterative retraining is more expensive, it also has a higher chance of learning a better model. Additionally, we proposed algorithms for choosing the number of questions to ask different crowd-workers, based on the characteristics of the data being labeled. We also studied the effect of batching on the overall runtime and quality of our AL algorithms. Our results, on three datasets collected with Amazon’s Mechanical Turk, and on 15 datasets from the UCI KDD archive, show that our algorithms make substantially fewer label requests than state-of-the-art AL techniques. We believe that these algorithms will prove to be immensely useful in crowd-sourced database systems.

9. ACKNOWLEDGEMENT

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