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NEW p-TH MOMENT EXPONENTIAL STABILITY CRITERIA FOR FCNNS WITH TIME-VARYING DELAYS AND STOCHASTIC EFFECTS

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ABSTRACT. This paper is concerned with p-th moment exponential stability of fuzzy cellular neural networks(FCNNs) with time-varying delays and stochastic effects. Constructing some suitable Lyapunov functional and using stochastic analysis technique, some sufficient conditions are presented to guarantee p-th moment exponential stability of stochastic FCNNs with time-varying delays. The condition contains and improves some of the previous results in the earlier references. Moreover an example is provided to illustrate results obtained.

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1. INTRODUCTION

The dynamical behaviors of neural networks have appeared as a novel subject of research in theoretic and applications, including optimization, control, and image processing(see [1],[2],[3],[4],[5],[6],[7],[8],[9],[10],[11]). Obviously, finding stability criteria for these neural networks becomes an attractive research problem of importance. Some well results have appeared, for example, in [4],[5],[6],[7], for stochastic delayed Hopfield neural networks, stochastic cellular neural networks and stochastic Cohen-Grossberg neural networks, the linear matrix inequality approach is utilized to obtain the sufficient conditions on mean square exponential stability for the neural networks. In particular, in [4-5], by using linear matrix inequality and stochastic analysis, the sufficient conditions are given to guarantee the exponential stability of an equilibrium solution.

Fuzzy cellular neural networks (FCNNs) introduced by Yang and Yang [12],[13] is another type neural networks model. These models combined fuzzy operation (fuzzy AND and fuzzy OR) with cellular neural networks. Recently scholars have found that FCNNs are useful in image processing, optimization and control [14],[15],[16],[17],[18], [19],[20],[21],[22],[23],[24],[25]. The sufficient stability criteria have become one of research topic in these models. In particularly, some sufficient conditions to ensure

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global exponential stability for FCNNs with delays (including constant delay, time-varying delay, distributed delay and proportional delay) have been reported in recent years [14],[15],[16],[17],[18],[19],[20],[21],[22],[23],[24]. However, to the best of my knowledge, there are few results about stochastic effects to the stability property of fuzzy cellular neural networks with delays in the literature today [22],[24]. It is worth continuing to find new criteria on p-th moment exponential stability of fuzzy cellular neural networks with time-varying delays.

In this paper, we are concerned with the exponential stability of equilibrium point for FCNNs with time-varying delays and stochastic effects. Following [2], that activation functions require Lipschitz conditions and boundedness, by utilizing stochastic analysis, general Lyapunov function, Young inequality method and Poincare contraction theory are utilized to derive the conditions guaranteeing the existence of periodic solutions of stochastic delay cellular neural networks and the stability of periodic solutions. Different from the linear matrix inequality approach [2], [5] and variation parameter method, the Young inequality method is developed to investigate the stability of stochastic FCNNs with delays. These sufficient conditions extend the early works in Refs. [8], and they include those governing parameters of stochastic FCNNs, so they can be easily checked by simple algebraic methods, comparing with the results of [2], [3], [22],[24]. Furthermore, an example is given to demonstrate the usefulness of the results in this paper.

The structure of this paper is organized as follows. In Section 2, model formulation and preliminaries are given. In Section 3, some new results are given to ascertain p-th moment exponential stability of stochastic FCNNs with time-varying delays based on Lyapunov method and stochastic analysis. In Section 4, an example is given to illustrate the effectiveness of our results.

2. MODEL FORMULATION AND PRELIMINARIES

In this paper, we are concerned with the model of continuous-time neural networks

$$\begin{aligned} dx_i(t) &= \Bigg[-a_i x_i(t) + \sum_{j=1}^{n} b_{ij} f_j(x_j(t)) + \bigwedge_{j=1}^{n} \alpha_{ij} f_j(x_j(t-\tau_j(t))) \\ &\quad + \bigvee_{j=1}^{n} \beta_{ij} f_j(x_j(t-\tau_j(t))) + +E_i\Bigg] dt \\ &\quad + \sum_{j=1}^{n} \sigma_{ij}(x_j(t)) dw_j(t), i = 1, 2, \cdots, n. \end{aligned} \tag{2.1}$$

where n corresponds to the number of units in a neural network. $x_i(t)$ is the activations of the i-th neuron at the time t. $a_i > 0$ represents the amplification function. b_{ij} denotes the synaptic connection weight of the unit j on the unit i. $\bigwedge$ and $\bigvee$ denote the fuzzy AND and fuzzy OR operation, respectively. α_{ij} and β_{ij} are elements of fuzzy feedback MIN template and fuzzy feedback MAX template, respectively. the kernels $k_j : [0, +\infty) \to [0, +\infty)$. The time delay $\tau_j(t)$ is any nonnegative continuous function with $0 \le \tau_j(t) \le \tau$, where τ is a constant. E_i denotes the i-th component of an external input source introduced from outside the network to the ith cell. $f_j(\cdot)$ are the activation functions. $w(t) = (w_1(t), w_2(t), \cdots, w_n(t))^T$ is an n-dimensional Brownian motion defined on a complete probability space $(\Omega, F, \{F_t\}_t \ge 0, P)$. $\sigma_{ij}(t, x_j, y_j) : R^+ \times R \times R \to R$ is locally Lipschitz continuous and satisfies the linear growth condition.

Let $x(t) = (x_1(t), x_2(t), \cdots, x_n(t))^T \in C((-\tau, 0], R^n)$, $|x|, \|x\|$ denote the norms of the vector $x(t) = (x_1(t), x_2(t), \cdots, x_n(t))^T$, which are defined as

$$|x(t)| = \left[\sum_{i=1}^{n} |x_i(t)|^p\right]^{1/p}, \quad \|x(t)\| = \sup_{-\tau \le s \le 0} \left[\sum_{i=1}^{n} |x_i(t+s)|^p\right]^{1/p}.$$

The initial conditions of system (2.1) are given by

$$x_i(t) = \varphi_i(t),\ t \in (-\tau, 0], \quad i = 1, 2, \cdots, n.$$

Throughout this paper, we will employ that each $f_j, j = 1, 2, \cdots, n$ is bounded and satisfying the following condition.

(**A1**) There exists constant $L_j > 0, j = 1, 2, \cdots, n$, such that

$$0 \le \frac{f_j(u) - f_j(v)}{u - v} \le L_j, \quad \forall u, v \in R, u \ne v.$$

(**A2**) $\sigma(x(t)) = (\sigma_{ij}(x_j(t)))_{n\times n}, i, j = 1, 2, \cdots, n$, there exist nonnegative numbers $s_i, i = 1, 2, \cdots, n$ such that

$$\mathrm{trace}[\sigma^T(x)\sigma(x)] \le \sum_{i=1}^{n} s_i x_i^2.$$

Remark 2.1. In assumption (A1), the activation functions $f_j, j = 1, 2, \cdots, n$, are typically assumed to be bounded and Lipchtiz continuous and need not to be differential. This class of functions is clearly more general than both the usual sigmoid activation functions and the piecewise linear function.

Definition 2.2. Let $x^*(t) = (x_1^*(t), \cdots, x_n^*(t))^T$ be an equilibrium of system (2.1) with initial value $\varphi^*(t) = (\varphi_1^*(t), \cdots, \varphi_n^*(t))^T$ and $x(t) = (x_1(t), \cdots, x_n(t))^T$ be any solution of system (2.1) with any initial value $\varphi(t) = (\varphi_1(t), \cdots, \varphi_n(t))^T \in$

$C((-\infty,0],R^n)$. $x^*(t)$ is said to be pth moment exponentially stable, if there exist constants $M\geq 1$ and $\lambda>0$ such that

$$\mathbb{E}\|x_i(t)-x_i^*(t)\|^p\leq M\mathbf{E}\|\varphi-\varphi^*\|^pe^{-\lambda t},\quad t\geq 0.$$

where $\mathbf{E}$ stands for the mathematical expectation operator. In this case

$$\limsup_{t\to\infty}\frac{1}{t}\log(\mathbb{E}\|x_i(t)-x_i^*(t)\|^p)\leq -\lambda. \tag{2.2}$$

The right-hand side of (2.2) is called the pth moment Lyapunov exponent of the solution. It is usually called the mean square exponential stability as $p=2$.

In order to obtain our result, we need the following lemma.

Lemma 2.3. [12] *Let u and v be two states of system (2.1), then*

$$\left|\bigwedge_{j=1}^{n}\alpha_{ij}f_j(u)-\bigwedge_{j=1}^{n}\alpha_{ij}f_j(v)\right|\leq\sum_{j=1}^{n}|\alpha_{ij}||f_j(u)-f_j(v)|,$$

and

$$\left|\bigvee_{j=1}^{n}\beta_{ij}f_j(u)-\bigvee_{j=1}^{n}\beta_{ij}f_j(v)\right|\leq\sum_{j=1}^{n}|\beta_{ij}||f_j(u)-f_j(v)|.$$

Lemma 2.4. *Let $\gamma_i>0(i=1,2,\cdots,m)$, then*

$$\gamma_1\gamma_2\cdots\gamma_m\leq\frac{\gamma_1^p+\gamma_2^p+\cdots+\gamma_m^p}{p}, \tag{2.3}$$

where $p\geq 2$ is a positive integer. In particular, we have

$$\gamma_1^{p-1}\gamma_2\leq\frac{(p-1)\gamma_1^p}{p}+\frac{\gamma_2^p}{p}.$$

Lemma 2.5. [26] *If there exists a constant $0<u<1$ such that $0<H_2<uH_1$. Assume $Z(t)$ is nonnegative continuous function on $[t_0-\tau,t_0]$ and satisfies the following inequality*

$$D^+Z(t)\leq -H_1Z(t)+H_2\|Z_t\|,t\geq 0.$$

Then $Z(t)\leq\|Z_{t_0}\|e^{-\lambda(t-t_0)}$, where λ is the root of the equation $\lambda=H_1-H_2e^{\lambda\tau}$ and the upper right Dini derivative of $Z(t)$ is defined as

$$D^+Z(t)=\limsup_{\delta\to 0^+}\frac{Z(t+\delta)-Z(t)}{\delta}.$$

Let $C^{2,1}(R^n\times R^+;R^+)$ denote the family of all non-negative functions $V(x,t)$ on $R^n\times R^+$ which are continuously twice differentiable in x and once differentiable in t.

For each $V \in C^{2,1}(R^n \times R^+; R^+)$, define an operator $\mathbf{L}V$ associated with stochastic delayed neural networks (2.1) from $R^n \times R^+$ by

$$\begin{aligned} \mathbf{L}V(x,t) &= V_t(x,t) + \sum_{i=1}^{n} V_{x_i}(x,t) \Bigg\{ \Bigg[-a_i x_i(t) + \sum_{j=1}^{n} b_{ij} f_j(x_j(t)) \\ &\quad + \bigwedge_{j=1}^{n} \alpha_{ij} f_j(x_j(t-\tau_j(t))) + \bigvee_{j=1}^{n} \beta_{ij} f_j(x_j(t-\tau_j(t))) + E_i \Bigg] dt \Bigg\} \\ &\quad + \frac{1}{2} \text{trace}[\sigma^T V_{xx}(x,t)\sigma]. \end{aligned} \tag{2.4}$$

where

$$V_t(x,t) = \frac{\partial V(x,t)}{\partial t}, \quad V_{x_i}(x,t) = \frac{\partial V(x,t)}{\partial x_i}, \quad V_{xx}(x,t) = \left(\frac{\partial^2 V(x,t)}{\partial x_i \partial x_j} \right)_{n \times n}.$$

3. STABILITY OF EQUILIBRIUM POINT

In this section, using Lyapunov functional, stochastic analysis and differential inequality [27], we study pth moment exponential stability of system (2.1).

Theorem 3.1. *Suppose that assumptions (A1)-(A2) hold true. If there exist positive diagonal matrices $D = diag\{d_1, d_2, \cdots, d_n\}$ and a positive constant $0 < u < 1$ such that*

$$0 < H_2 \le u H_1.$$

where

$$\begin{aligned} H_1 &= \min_{1 \le i \le n} \Bigg\{ p a_i - \sum_{j=1}^{n} (p-1)|b_{ij}| L_j - \sum_{j=1}^{n} |b_{ji}| L_j - \sum_{j=1}^{n} (p-1)(|\alpha_{ij}| + |\beta_{ij}|) L_j \\ &\quad - \sum_{j=1}^{n} \frac{(p-1)(p-2)}{2} s_j - \sum_{j=1}^{n} \frac{d_j}{d_i} (p-1) s_i \Bigg\} \end{aligned}$$

$$H_2 = \max_{1 \le i \le n} \sum_{j=1}^{n} \frac{d_j}{d_i} (|\alpha_{ij}| + |\beta_{ij}|) L_j.$$

Then there exists unique equilibrium point of system (2.1) which is pth moment exponentially stable.

Proof. Assume that $x^*(t) = (x_1^*(t), x_2^*(t), \cdots, x_n^*(t))^T$ is an equilibrium of Eq. (2.1). Let $y_i(t) = x_i(t) - x_i^*(t)$ and $\widetilde{\sigma}_{ij}(y_j(t)) = \sigma_{ij}(y_j(t) + x_j^*) - \sigma_{ij}(x_j^*)$, then system (2.1) can become the following system

$$
\begin{aligned}
dy_i(t) &= \Bigg[-a_i y_i(t) + \sum_{j=1}^{n} b_{ij}[f_j(y_j(t) + x_j^*(t)) - f_j(x_j^*)] \\
&\quad + \bigwedge_{j=1}^{n} \alpha_{ij} f_j(y_j(t - \tau_j(t)) + x_j^*) - \bigwedge_{j=1}^{n} \alpha_{ij} f_j(x_j^*(t)) \\
&\quad + \bigvee_{j=1}^{n} \beta_{ij} f_j(y_j(t - \tau_j(t)) + x_j^*) - \bigvee_{j=1}^{n} \beta_{ij} f_j(x_j^*)\Bigg] dt \\
&\quad + \sum_{j=1}^{n} \sigma_{ij}(y_j(t)) dw_j(t)
\end{aligned}
\tag{3.1}
$$

To prove the stability of x^* of Eq. (2.1), it is sufficient to prove the stability of the trivial solution of Eq. (3.1). Consider the following Lyapunov function defined by

$$
V(y,t) = \sum_{i=1}^{n} d_i |y_i(t)|^p = \sum_{i=1}^{n} d_i |x_i(t) - x_i^*|^p, \quad p \geq 2. \tag{3.2}
$$

Applying Lemma 2.3, we calculate and estimate $\mathbb{L}V(y,t)$ along the trajectories of system (3.1) as follows

$$
\begin{aligned}
&\mathbf{L}V(y,t) = \\
&p\sum_{i=1}^{n} d_i|y_i(t)|^{p-1}\mathrm{sgn}\{y_i(t)\}\Bigg[-a_iy_i(t)+\sum_{j=1}^{n} b_{ij}[f_j(y_j(t)+x_j^*(t))-f_j(x_j^*(t))] \\
&+\bigwedge_{j=1}^{n}\alpha_{ij}f_j(y_j(t-\tau_j(t))+x_j^*(t))-\bigwedge_{j=1}^{n}\alpha_{ij}f_j(x_j^*(t)) \\
&+\bigvee_{j=1}^{n}\beta_{ij}f_j(y_j(t-\tau_j(t))+x_j^*(t))-\bigvee_{j=1}^{n}\beta_{ij}f_j(x_j^*(s))\Bigg] \\
&+\frac{p(p-1)}{2}\sum_{i=1}^{n}|y_i(t)|^{p-2}\sum_{j=1}^{n}\widetilde{\sigma}_{ij}(y_j(t)) \\
&\le -p\sum_{i=1}^{n} d_i|y_i(t)|^{p-1}a_iy_i(t)\mathrm{sgn}\{y_i(t)\} \\
&+p\sum_{i=1}^{n} d_i|y_i(t)|^{p-1}\sum_{j=1}^{n}|b_{ij}|L_j|y_j(t)|\mathrm{sgn}\{y_i(t)\} \qquad (3.3)\\
&+p\sum_{i=1}^{n} d_i|y_i(t)|^{p-1}\sum_{j=1}^{n}(|\alpha_{ij}|+|\beta_{ij}|)L_j|y_j(t-\tau_j(t))|\mathrm{sgn}\{y_i(t)\} \\
&+\frac{p(p-1)}{2}\sum_{i=1}^{n}|y_i(t)|^{p-2}\sum_{j=1}^{n}\sigma_{ij}^2\mathrm{sgn}\{y_i(t)\} \\
&\le -p\sum_{i=1}^{n} d_i|y_i(t)|^{p-1}a_i|y_i(t)|+p\sum_{i=1}^{n} d_i|y_i(t)|^{p-1}\sum_{j=1}^{n}|b_{ij}|L_j|y_j(t)| \\
&+p\sum_{i=1}^{n} d_i|y_i(t)|^{p-1}\sum_{j=1}^{n}(|\alpha_{ij}|+|\beta_{ij}|)L_j|y_j(t-\tau_j(t))| \\
&+\frac{p(p-1)}{2}\sum_{i=1}^{n}|y_i(t)|^{p-2}\sum_{j=1}^{n}\sigma_{ij}^2
\end{aligned}
$$

Using Lemma 2.4, we obtain that

$$
\begin{aligned}
&\mathbf{L}V(y,t) \le \\
&-\sum_{i=1}^{n} d_i\Bigg\{pa_i-\sum_{j=1}^{n}(p-1)|b_{ij}|L_j-\sum_{j=1}^{n}|b_{ji}|L_j-\sum_{j=1}^{n}(p-1)(|\alpha_{ij}|+|\beta_{ij}|)L_j \\
&-\sum_{j=1}^{n}\frac{(p-1)(p-2)}{2}s_j-\sum_{j=1}^{n}\frac{d_j}{d_i}(p-1)s_i\Bigg\}|y_i(t)|^p \qquad (3.4)
\end{aligned}
$$

$$
\begin{aligned}
&+\sum_{i=1}^{n} d_i \sum_{j=1}^{n} \frac{d_j}{d_i}(|\alpha_{ij}|+|\beta_{ij}|)L_j|y_j(t-\tau_j(t))|^p \\
&\leq -H_1 V(y(t),t) + H_2 \sup_{t-\tau\leq s\leq t} V(y(s),s)
\end{aligned}
$$

By Itö's formula, for $t \geq t_0$, we have

$$
\begin{aligned}
&V(y(t+\delta),t+\delta) - V(y(t),t) \\
&= \int_t^{t+\delta} \mathbb{L}V(y(s),s)ds + \int_t^{t+\delta} V_y(y(s),s)\sigma(y(s),s)d\omega(s)
\end{aligned}
\tag{3.5}
$$

Calculating the expectation of both sides of (3.5), since $\mathbb{E}[V_y(y(s),s)\sigma(y(s),s)d\omega(s)] = 0$, and noting (3.4), we have

$$
\begin{aligned}
&\mathbb{E}[V(y(t+\delta),t+\delta)] - \mathbb{E}[V(y(t),t)] \\
&\leq \int_t^{t+\delta} \left[-H_1\mathbb{E}[V(y(s),s)] + H_2\mathbb{E}\left(\sup_{s-\tau\leq\theta\leq s} V(y(\theta),\theta) \right) \right] ds
\end{aligned}
\tag{3.6}
$$

The Dini derivative D^+ of $\mathbb{E}V(y(t),t)$ is

$$
D^+\mathbb{E}V(y(t),t) = \lim_{\delta\to 0^+} \sup \frac{1}{\delta}\mathbb{E}[V(y(t+\delta),t+\delta)] - \mathbb{E}[V(y(t),t)]. \tag{3.7}
$$

Let $Z(t) = \mathbb{E}V(y(t),t)$, from (3.6), it can implies that

$$
D^+Z(t) \leq -H_1Z(t) + H_2\|Z_t\|^p \tag{3.8}
$$

By virtue of Lemma 2.5, we have

$$
Z(t) \leq \|Z(t_0)\|^p e^{-\lambda(t-t_0)}.
$$

Namely,

$$
\mathbb{E}[\|x(t)-x^*\|^p] \leq M\mathbb{E}[\|\varphi - x^*\|^p]e^{-\lambda(t-t_0)}, t \geq t_0.
$$

where $M = \frac{\max_{1\leq i\leq n}\{d_i\}}{\min_{1\leq i\leq n}\{d_i\}} > 1$. λ is the root of the equation $\lambda = H_1 - H_2e^{\lambda\tau}$. Therefore the equilibrium point x^* of system (2.1) is p-th moment exponentially stable. The proof is completed. □

Remark 3.2. The criteria of stochastic stability of system (1.1) can generalize stochastic fuzzy Cohen-Grossberg neural networks with time-varying delays.

Consider the following fuzzy Cohen-Grossberg neural networks with time-varying delays.

$$(3.9)\quad \begin{cases} dx_i(t) &= -a_i(x_i(t))\Big[c_i(x_i(t)) - \sum_{j=1}^n b_{ij} f_j(x_j(t)) + E_i \\ & \quad - \bigwedge_{j=1}^n \alpha_{ij} f_j(x_j(t-\tau_j(t))) - \bigvee_{j=1}^n \beta_{ij} f_j(x_j(t-\tau_j(t)))\Big]\, dt, \\ & \quad + \sum_{j=1}^n \sigma_{ij}(x_j(t)) dw_j(t) \\ x_i(t) &= \varphi_i(t),\ t \in (-\tau, t], \quad i = 1, 2, \cdots, n. \end{cases}$$

Similarly, we can obtain easily the following results on stochastic fuzzy Cohen-Grossberg neural networks with time-varying delays.

Theorem 3.3. *Suppose that assumptions (A1)-(A2) hold true, furthermore the following conditions satisfy*
(A3) there exist positive constants $\underline{a}_i, \overline{a}_i$ *such that*

$$0 < \underline{a}_i \le a_i(x) \le \overline{a}_i, \quad x \in R, i = 1, 2, \cdots, n.$$

(A4) For $c_i(x) \in C(R, R)$*, there exists* $k_i > 0$ *such that*

$$\frac{b_i(u) - b_i(v)}{u - v} \ge k_i, \quad u, v \in R, u \ne v,\ i = 1, 2, \cdots, n.$$

If there exist positive diagonal matrices $D = diag\{d_1, d_2, \cdots, d_n\}$ *and a positive constant* $0 < u < 1$ *such that*

$$0 < H_2 \le u H_1.$$

where

$$\begin{aligned} H_1 &= \min_{1 \le i \le n} \Big\{ p\underline{a}_i k_i - \sum_{j=1}^n \overline{a}_i (p-1)|b_{ij}| L_j - \sum_{j=1}^n \overline{a}_j |b_{ji}| L_j - \sum_{j=1}^n \overline{a}_i (p-1)(|\alpha_{ij}| + |\beta_{ij}|) L_j \\ & \quad - \sum_{j=1}^n \frac{(p-1)(p-2)}{2} s_j - \sum_{j=1}^n \frac{d_j}{d_i}(p-1) s_i \Big\} \end{aligned}$$

$$H_2 = \max_{1 \le i \le n} \sum_{j=1}^n \overline{a}_i \frac{d_j}{d_i} (|\alpha_{ij}| + |\beta_{ij}|) L_j.$$

Then there exists unique equilibrium point of system (3.9) which is pth moment exponentially stable.

4. AN ILLUSTRATIVE EXAMPLE

In this section, an example is used to demonstrate that the method presented in this paper is effective.

Example 4.1 Consider the following fuzzy cellular neural networks with time-varying delay and stochastic noise.

$$(4.1)\quad \begin{cases} dx_1(t) = \Big[-a_1x_1(t) + \sum_{j=1}^{2} b_{1j} f_j(x_j(t)) + \bigwedge_{j=1}^{2} \alpha_{1j} f_j(x_j(t-\tau_j(t))) \\ \qquad + \bigvee_{j=1}^{2} \beta_{1j} f_j(x_j(t-\tau_j(t))) + E_1\Big] dt + \sum_{j=1}^{2} \sigma_{ij}(x_j(t)) d\omega_j \\ dx_2(t) = \Big[-a_2x_2(t) + \sum_{j=1}^{2} b_{2j} f_j(x_j(t)) + \bigwedge_{j=1}^{2} \alpha_{2j} f_j(x_j(t-\tau_j(t))) \\ \qquad + \bigvee_{j=1}^{2} \beta_{2j} f_j(x_j(t-\tau_j(t))) + E_2\Big] dt + \sum_{j=1}^{2} \sigma_{2j}(x_j(t)) d\omega_j \end{cases}$$

where $a_1 = 3, a_2 = 4, f_j(x) = \frac{1}{2}(|x+1| - |x-1|)(j = 1, 2), \sigma_{11}(x) = 0.2x, \sigma_{12}(x) = 0.1x, \sigma_{21}(x) = 0.3x, \sigma_{22}(x) = 0.2x, \tau_1(t) = \tau_2(t) = \frac{1}{4}\sin t, E_1 = 0.05, E_2 = 0.06.$

$$(b_{ij})_{2\times 2} = \begin{pmatrix} 0.1 & -0.7 \\ -0.6 & 0.9 \end{pmatrix}, \quad (\alpha_{ij})_{2\times 2} = \begin{pmatrix} 0.4 & -0.5 \\ -0.8 & 0.2 \end{pmatrix}, \quad (\beta_{ij})_{2\times 2} = \begin{pmatrix} 0.6 & -0.7 \\ -0.8 & 0.9 \end{pmatrix}$$

There exist positive diagonal matrices $D = \mathrm{diag}(d_1, d_2) = (3, 2)$. and $L_j = 1(j = 1, 2), \tau = 0.25, s_1 = 0.05, s_2 = 0.13$. So assumptions $(A1) - (A2)$ are satisfied. Let $p = 2$, it is easy to get $H_1 = 1.47, H_2 = 0.83$, there exists $u = 0.8$. namely, we have $0 < H_2 \le uH_1$, Therefore, by virtue of Theorem 3.1, the equilibrium point (x_1^*, x_2^*) of system (4.1) is mean square exponentially stable.(see Fig. 1.)

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Fig.1. Numerical solution $x(t) = (x_1(t), x_2(t))^T$ of systems (4.1)
for initial value $\varphi(s) = (10, -8)^T, s \in (-2, 0)$.

5. Conclusion

In this paper, p-th moment exponential stability of fuzzy cellular neural networks with time-varying delays and stochastic noise is considered. Some sufficient conditions set up here are easily verified and these conditions are correlated with stochastic noise and parameters of the system (2.1), The criteria obtained can be applied to design exponential stability of fuzzy cellular neural networks.

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