On the Complexity of Scheduling in Wireless Networks

ABSTRACT

We consider the problem of throughput-optimal scheduling in wireless networks subject to interference constraints. We model the interference using a family of K-hop interference models. We define a K-hop interference model as one for which no two links within K hops can successfully transmit at the same time (Note that IEEE 802.11 DCF corresponds to a 2-hop interference model). For a given K, a throughput-optimal scheduler needs to solve a maximum weighted matching problem subject to the K-hop interference constraints. For $K = 1$, the resulting problem is the classical Maximum Weighted Matching problem (MWMP), which can be solved in polynomial time. However, we show that for $K > 1$, the resulting problems are NP-Hard and cannot be approximated within a factor that grows polynomially with the number of nodes in the network. Interestingly, we show that for specific kinds of graphs, which can be used to model the underlying connectivity graph of a wide range of wireless networks, the resulting problems admit polynomial time approximation schemes. We also show that a simple greedy matching algorithm provides a constant factor approximation to the scheduling problem for all K in this case. We then show that under a setting with single-hop traffic and no rate control, the maximal scheduling policy considered in recent related works can achieve a constant fraction of the capacity region for networks whose connectivity graph can be represented using one of the above classes of graphs. These results are encouraging as they suggest that one can develop distributed algorithms to achieve near optimal throughput in case of a wide range of wireless networks.

Categories and Subject Descriptors

C.2.1 [**Computer-Communication Networks**]: Network Architecture and Design—*Wireless communication*; F.2.2 [**Analysis of Algorithms and Problem Complexity**]: Nonnumerical Algorithms and Problems—*Sequencing and scheduling*; G.2.2 [**Discrete Mathematics**]: Graph Theory—*Graph algorithms*

General Terms

Theory, Algorithms

Keywords

Scheduling, computational complexity, capacity, wireless networks, throughput, maximum weighted matching, interference

1. INTRODUCTION

Scheduling link transmissions in a wireless network so as to optimize one or more of the performance objectives (e.g. throughput, delay, or energy) has been the topic of much interest over the past several decades. In their seminal work, Tassiulas and Ephremides [32] characterized the *capacity region* of constrained queuing systems, such as a wireless network. They developed a queue length based scheduling scheme that is *throughput-optimal*, i.e. it stabilizes the network if the user rates fall within the capacity region of the network. Unlike wireline networks, where all links have fixed capacities, the capacity of a wireless link can be influenced by channel variation due to fading, changes in power allocation or routing, changes in network topology etc. Thus, the capacity region of a wireless network can vary due to changes in power allocation or routing. To efficiently utilize the wireless resources, one must therefore develop algorithms that can perform jointly optimal routing, link scheduling, and power control under possibly varying channel conditions and network topology. This has spurred recent interest in developing cross-layer optimization algorithms (see, for example, [36, 26, 25, 31, 7]).

Motivated by the works on fair resource allocation in wireline networks [17, 28, 22, 3, 37], researchers have also incorporated congestion control into the cross-layer optimization framework [4, 21, 20, 24, 35, 30, 38, 27]. The congestion control component controls the rate at which users inject data into the network so as to ensure that the user rates fall within the capacity region of the network.

Most of the above cross-layer optimization problems have been shown to exhibit a nice decoupling property (see, for example, [36, 21]). More precisely, a cross-layer optimization problem can often be decomposed into multiple subproblems, where each subproblem corresponds to optimization across a single layer. The subproblems are coupled through parameters that correspond to congestion prices or queue lengths at the individual links.

The main component of all these cross-layer optimization schemes is the optimal scheduler that solves a very difficult global optimization problem of the form:

$$\text{maximize } \sum_{l \in \mathcal{L}} p_l r_l \qquad (1)$$

$$\text{subject to } \mathbf{r} \in \Delta$$

where $\mathcal{L}$ denotes the set of wireless links; $\mathbf{r}$ is the vector of link rates r_l, $l \in \mathcal{L}$; p_l, $l \in \mathcal{L}$, is the congestion price or possibly some function of queue length at link l; and Δ is the capacity region of the network.

The main difficulty in solving the above optimization problem is that the capacity region Δ depends on the complete network topology and, in general, has no easy representation in terms of the power constraints at the individual links or nodes. The above optimization problem is, in general, NP-Complete and Non-Approximable.

In this paper, we consider a wide class of scheduling problems that we term Maximum Weighted K-Valid Matching Problems (MWKVMPs). These problems arise as simplifications to the scheduling problem specified by (1). The basic idea is to limit the interference to only K hops, where K is a positive integer. By varying K, one can capture the interference characteristics of a broad range of wireless networks.

The rest of the paper is organized as follows. The model, problem formulation, related works, and main contributions of this work are presented in the next section. Some hardness and approximability results for the class of scheduling problems we consider are presented in Section 3. We then restrict our attention to specific graphs that naturally model the connectivity graph of wireless networks in Section 4. We develop some approximation algorithms and schemes for our scheduling problems restricted to these specific graphs. We complement our analytical results with some numerical results in Section 5. Finally, we provide some concluding remarks in Section 6.

2. SYSTEM MODEL AND PROBLEM FORMULATION

We consider a set V of wireless nodes, each communicating over a single wireless interface. We assume that all transmissions are carried out over the same wireless channel, and therefore interfere with each other. We assume that all transmissions from a node are carried out at the same power level (which can be different for different nodes). We connect two nodes with an (undirected) edge if each of them can successfully receive from the other, provided no other node in the network transmits at the same time. The set of (undirected) edges so formed is denoted by E. Note that the existence of an edge between two nodes depends on the power allocated to the nodes, noise variances at the nodes, as well as coding and modulation schemes used at the nodes. Our emphasis on bidirectional edges stems from the fact that most network and transport layer protocols assume bidirectional edges between the nodes. Our main results as well as algorithms we develop can easily be extended to settings where directed edges are allowed between the nodes.

We next introduce the class of scheduling problems we consider in this paper. We first need to introduce some notation. Let $G = (V, E)$ be an undirected graph (connectivity graph of a wireless network, in our case) having V as the set of nodes and E as the set of edges. A *matching* is a set of edges no two of which share a common vertex. We now generalize this concept of matching to K-Valid matchings for $K = 1, 2, ...$

Let $d_S(x, y)$ denote the shortest distance (in terms of the number of edges) between nodes $x, y \in V$. Define a function $d : (E, E) \to \mathbb{N}^1$ as follows: For $e_u = u_1 u_2$, $e_v = v_1 v_2 \in E$, let

$$d(e_u, e_v) = \min_{i,j \in \{1,2\}} d_S(u_i, v_j).$$

We call a set of edges M a "K-valid matching" if for all $e_1, e_2 \in M$ with $e_1 \neq e_2$, we have $d(e_1, e_2) \geq K$. Observe that the concept of matching discussed before is equivalent to the concept of 1-Valid matching in this new terminology. Let S_K denote the set of K-Valid matchings of the graph G. We consider the following scheduling problems:

$$\text{maximize } \sum_{l \in M} w_l \qquad (2)$$

$$\text{subject to } M \in S_K$$

where w_l, $l \in \mathcal{L}$, denotes the weight of edge l. Note that the weight of each edge l is a positive, but otherwise arbitrary, number that can possibly depend on many factors (e.g., congestion price, supported rate, queue length). The above class of problems will henceforth be referred to as Maximum Weighted K-Valid Matching Problems (MWKVMPs). When all edge weights are set to unity, we obtain the following class of problems:

$$\text{maximize } |M| \qquad (3)$$

$$\text{subject to } M \in S_K$$

where $|M|$ denotes the cardinality of the set M. In the sequel, we refer to these problems as Maximum K-Valid Matching Problems (MKVMPs).

We note that the scheduling problems specified by (2) are natural simplifications of the complex scheduling problem specified by (1). This is because for a given K, by satisfying the K-hop interference constraint one can guarantee a certain fixed data rate at a given edge. The weight of each edge can then be determined as some function of the rate it supports and the congestion price at the edge. The scheduling problem specified by (1) then corresponds to MWKVMP for that particular value of K. For simplicity of notation, we did not explicitly show the dependence of edge weights on K in (2).

From the above discussion, it is not surprising to see that MWKVMPs can represent the scheduling problem specified by (1) under a wide variety of interference models. Below we discuss two widely used interference models that can be obtained as special cases of the interference constraints in (2).

Node Exclusive Interference Model: This is a commonly used model for Bluetooth and FH-CDMA networks [23, 2, 12]. The only constraint on the set of edges scheduled to transmit in this case is that it must constitute a matching. The scheduling problems specified by (2) and (3) correspond to the classical Maximum Weighted Matching and Maximum Matching, respectively, in this case. Both these problems can be solved in polynomial time [8].

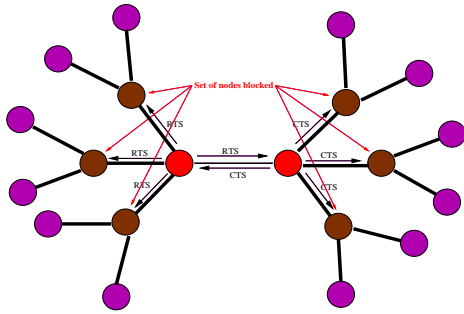[1]$\mathbb{N}$ denotes the set of non-negative integers.

Figure 1: The 2-hop interference set of a given edge for RTS/CTS based communication model of IEEE 802.11 DCF - $K = 2$ case.

IEEE 802.11 Based Interference Model: This is a commonly used model for IEEE 802.11 based wireless networks [11, 35]. Under this model, the chosen set of edges must constitute a 2-Valid matching. This models the communication under the RTS/CTS based scheme of IEEE 802.11 DCF. Note that the exchange of RTS and CTS messages between the sender and the receiver ensures that nodes within one hop of the sender or the receiver cannot participate in a communication, which is equivalent to saying that the chosen set of node pairs must constitute a 2-Valid matching (see Figure 1).

2.1 Related Work

The node exclusive interference model has been studied in many different contexts due to its simplicity [12, 2, 5, 21, 4, 32, 31, 27]). In [12], the authors developed a polynomial time link scheduling algorithm under the node exclusive interference model. The works in [21, 4, 5] have developed distributed schemes that guarantee a throughput within a constant factor of the optimal.

In [35], the performance of a greedy scheduling scheme (referred to as "maximal scheduling scheme" in [5]) is studied under the IEEE 802.11 based interference model ($K = 2$ case). It is shown that the greedy scheduling scheme achieves a throughput within a factor of N_ϵ of the optimal, where

$$N_\epsilon = \max_{(i,j) \in E} deg(i) + deg(j) - 1.$$

In [5], the maximal scheduling scheme is shown to achieve a throughput within a factor of $K(N)$ of the optimal, where $K(N)$ is the interference degree of the connectivity graph. We refer the reader to [5] for a definition of the interference degree of a graph.

The MKVMP for $K = 2$ is more commonly known as the induced matching problem. In [29], it is shown to be NP-Hard. The work in [11] is closest in spirit to our work. The authors consider the induced matching problem (they refer to it as distance-2 matching problem) from the perspective of carrying out maximum number of simultaneous transmissions in an IEEE 802.11 based wireless network. They study the approximability of the induced matching problem for general as well as specific kinds of graphs. They also develop PTAS and distributed constant factor polynomial time approximation algorithm for the induced matching problem restricted to unit disk graphs.

To the best of our knowledge, MWKVMP for $K \geq 2$ has

not been considered in the literature. We next highlight the main contributions of this work.

2.2 Main Contributions

A main contribution of our work is the formulation of the cross-layer scheduling problem as weighted matching problem under a wide class of K-hop interference models. Our formulation generalizes and significantly extends the formulations considered in recent related works [5, 21, 32, 4, 35].

From a theoretical perspective, we provide several results on hardness and approximability of MWKVMP and MKVMP for $K > 1$. Although, some of these results have previously been obtained for $K = 2$, to the best of our knowledge no prior work has studied MWKVMP or MKVMP for $K > 2$. Since weighted matching problems arise in a variety of contexts, these results might find applications in other fields (e.g., VLSI) as well.

From a wireless networking perspective, we develop a PTAS for MWKVMP restricted to specific kind of graphs that can be used to represent the connectivity graph of a wide range of wireless networks. We also show that the "natural" greedy scheme yields a constant factor approximation to MWKVMP in this case. Note that a γ-approximation algorithm, when run over each slot separately, results in a S_γ-scheduling policy in the terminology of [21]. According to the results in [21], such a policy is guaranteed to achieve at least $1/\gamma$ of the capacity region under a K-hop interference model. Thus, both greedy algorithm and PTAS for MWKVMP can be used to construct scheduling policies that achieve a constant fraction of the capacity region under K-hop interference models. The schemes mentioned thus far require centralized control, and can therefore be implemented in a limited class of wireless networks (e.g., wireless mesh networks). We complement these results by showing that the maximal scheduling policy considered in [5, 35] achieves a constant fraction of the capacity region. Note that the maximal scheduling policy is amenable to distributed implementation. These results are encouraging as they indicate that one can develop distributed algorithms to achieve near optimal throughput in case of a wide range of wireless networks.

Determining the optimal value of K for specific networks is a challenging issue. We study this issue numerically in case of IEEE 802.11 DSSS and EDGE networks. Our results clearly show that the physical layer has a strong impact on the optimal value of K (the optimal value was found to vary between $1 - 3$ for the physical layers considered in our experiments), and that the optimal value of K can in fact be larger than 2. All earlier works have considered only $K = 1$ or 2.

3. HARDNESS AND APPROXIMABILITY RESULTS

We now formulate the decision problems KVMP and WKVMP corresponding to MKVMP and MWKVMP, respectively, and prove that they are NP-Complete. We have the following definitions:

DEFINITION 1. $KVMP = \{< G, m >: G$ is a graph with a K-valid matching of size $m\}$.

DEFINITION 2. $WKVMP = \{< G, m >: G$ is a graph with a K-valid matching of size m and total weight $W_M\}$.

We start by showing that WKVMP ∈ NP; which implies that KVMP ∈ NP.

THEOREM 1. *WKVMP ∈ NP for all K.*

PROOF. Given a certificate in the form of a list of edges, it can easily be verified in polynomial time whether that list corresponds to a set of m edges that are at a distance of K or more from each other and have a total weight of W_M or not. Thus, whether the set of edges constitute a K-valid matching of size m with a total weight of W_M can be verified in polynomial time. Hence, WKVMP ∈ NP. □

We next show that KVMP is NP-Hard; which implies that the decision problem WKVMP is NP-Hard as well.

THEOREM 2. *KVMP is NP-Hard for $K \geq 2$.*

PROOF. The proof uses a novel reduction from 3-CNF-SAT problem to KVMP, and is omitted for brevity. In Theorem 3, we provide a stronger result which shows that MKVMP, and therefore MWKVMP, is Non-Approximable for $K \geq 2$. □

We now analyze the approximability of MKVMP for $K \geq 2$. We have the following result:

THEOREM 3. *Let η be such that $(|V| + K|E|)^\eta = \Theta(|V|)$. Then, MKVMP (and therefore, MWKVMP) for $K \geq 2$ is not approximable within $|V|^{\eta/2-\epsilon}$ for any $\epsilon > 0$, unless NP = P. Further, it is not approximable within $|V|^{\eta-\epsilon}$ for any $\epsilon > 0$, unless NP = ZPP.*

Note that the complexity class ZPP denotes the class of Zero-error Probabilistic Polynomial time problems. We refer the reader to the original work of Gill [9] for a rigorous definition of the complexity class ZPP.

Now, since $K = O(V)$ and $E = O(V^2)$, the following result follows from Theorem 3:

COROLLARY 1. *MKVMP (and therefore, MWKVMP) for $K \geq 2$ is not approximable within $|V|^{1/6-\epsilon}$ for any $\epsilon > 0$, unless NP = P. Further, it is not approximable within $|V|^{1/3-\epsilon}$ for any $\epsilon > 0$, unless NP = ZPP.*

In order to prove Theorem 3, we need to introduce some terminology. Consider a graph $G = (V, E)$. A set of vertices $S \subseteq V$ is termed "independent" provided no two vertices in S have an edge between them. The classical maximum independent set problem (MISP) is to find an independent set of vertices of maximum possible cardinality. Hastad [14] has shown that MISP is not approximable within $|V|^{1/2-\epsilon}$ for any $\epsilon > 0$, unless NP = P; and it is not approximable within $|V|^{1-\epsilon}$ for any $\epsilon > 0$, unless NP = ZPP. We are now ready to prove Theorem 3.

PROOF. We show that given an instance of MISP, i.e., a graph $G = (V, E)$, we can construct a graph $G' = (V', E')$ in polynomial time such that the graph G' has a K-valid matching of cardinality no smaller than the cardinality of a maximum independent set of G. Both $|V'|$ and $|E'|$ will be shown to be $\Theta(|V| + K|E|) = O(|V||E|)$. Further, we will show that given a K-valid matching in G', one can obtain an independent set of vertices in G with the same cardinality in polynomial time.

Suppose MKVMP admits a polynomial time ρ-approximation scheme (PTAS). Given an instance G of the MISP, one can
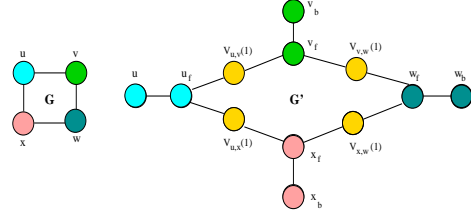

Figure 2: A graph G along with the graph G' constructed as specified in the proof of Theorem 3 for $K = 4$.

construct the corresponding graph G' in polynomial time; use the PTAS for MKVMP to obtain a K-valid matching of size at least $1/\rho$ times the cardinality of any maximum independent set of G in polynomial time; an then map it back to an independent set of vertices in G with the same cardinality, in polynomial time. This would then result in a ρ-approximation scheme for MISP, which, in view of the results in [14], would imply Theorem 3.

We next discuss how to construct the graph G' from G in polynomial time. We first consider even K. For each vertex v in V, we add a pair of vertices v_f, v_b in G', and connect them with an edge. For each edge uv in E, we connect the vertices u_f, v_f through a sequence of $K/2$ edges and $(K-2)/2$ vertices. Let the vertices be numbered

$$V_{u,v}(1), ..., V_{u,v}((k-2)/2),$$

with $V_{u,v}(1)$ being the vertex adjacent to vertex u. Figure 2 shows a graph G along with the graph G' constructed using the above procedure for $K = 4$. It is straightforward to see that the graph G' can be constructed in polynomial (in $|V|$ and $|E|$) time. Also, note that

$$|V'| = 2|V| + \left(\frac{K-2}{2}\right)|E| = O(|E||V|),$$

$$|E'| = |V| + \left(\frac{K}{2}\right)|E| = O(|E||V|).$$

Now, suppose $\{v^1, v^2, ..., v^m\}$ constitutes an independent set of vertices in G. It is then clear that $\{v_b^i v_f^i\}_{i=1,2,...,m}$ constitutes a K-valid matching in G'. To see this, observe that since $\{v^1, v^2, ..., v^m\}$ constitutes an independent set of vertices in G, we have $d_S(v_i, v_j) \geq 2$ for all $i, j \in \{1, 2, ..., m\}$ with $i \neq j$. Hence, for $i \neq j$, we have $d(v_f^i v_f^i, v_b^j v_f^j) \geq 2\left(\frac{K}{2}\right) = K$. Therefore, it follows that the graph G' has a K-valid matching of cardinality no smaller than the cardinality of the maximum independent set of G.

It remains to show that given a K-valid matching in G', one can, in polynomial time, obtain an independent set of vertices in G with the same cardinality. Consider the algorithm given in Table 1. It is easy to see that the running time of the above algorithm is bounded above by a polynomial in $|V|$ and $|E|$. Now, suppose v, u are two arbitrary vertices in L. Then, we claim that $d_S(u, v) \geq 2$; which implies that L is an independent set in G. For if not, then $d_S(u, v) = 1$. It then follows that there must exist edges $e_1, e_2 \in M$ such that $d(e_1, e_2) \leq \frac{K}{2} + 2\left(\frac{K-2}{4}\right) < K$; contradicting our initial hypothesis, namely, M is a K-valid matching.

Next, we discuss how to construct the graph G' for $K \geq 5$ and odd. There are only minor differences in the construction of the graph in this case: (i) instead of adding a pair of

Table 1: Algorithm for constructing independent set for even K.

Step	Construct Independent Set K-Even $(G' = (V', E'), M, L)$
1	$L := \phi$
2	**while** $M \neq \phi$ **do**
3	Pick and edge $e \in M$
4	**if** e is of the form $v_b v_f$
	then $L := L \cup v$
5	**else if** e is of the form $u V_{u,v}(1)$
	then $L := L \cup u$
6	**else if** e is of the form $V_{u,v}((K-2)/2)v$
	then $L := L \cup v$
7	**else if** e is of the form $V_{u,v}(i)V_{u,v}(i+1)$
	then
8	**if** $i \leq \frac{K-2}{4}$ **then** $L := L \cup u$
9	**else** $L := L \cup v$
10	**end**
11	$M := M - e$
12	**end**

Table 2: Algorithm for constructing independent set for odd K.

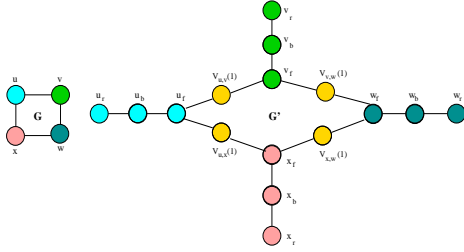| Step	Construct Independent Set K-Odd $(G' = (V', E'), M, L)$
1	$L := \phi$
2	**while** $M \neq \phi$ **do**
3	Pick and edge $e \in M$
4	**if** e is of the form $v_b v_f, v_r v_b$
	then $L := L \cup v$
5	**else if** e is of the form $u V_{u,v}(1)$
	then $L := L \cup u$
6	**else if** e is of the form $V_{u,v}((K-3)/2)v$
	then $L := L \cup v$
7	**else if** e is of the form $V_{u,v}(i)V_{u,v}(i+1)$
	then
8	**if** $i \leq \frac{K-3}{4}$ **then** $L := L \cup u$
9	**else** $L := L \cup v$
10	**end**
11	$M := M - e$
12	**end**

Figure 3: A graph G along with the graph G' constructed as specified in the proof of Theorem 3 for $K = 5$.

vertices to G' for each vertex $v \in V$, we now add a triplet of vertices v_f, v_b, v_r to G' (see Figure 3); (ii) for each edge $uv \in E$, we now connect the vertices u_f, v_f through a sequence of $(K-1)/2$ edges and $(K-3)/2$ vertices. For each $v \in V$, we connect the pairs of vertices v_f, v_b and v_b, v_r with an edge. We now have

$$|V'| = 3|V| + \left(\frac{K-3}{2}\right)|E| = O(|E||V|),$$

$$|E'| = 2|V| + \left(\frac{K-1}{2}\right)|E| = O(|E||V|).$$

Suppose $\{v^1, v^2, ..., v^m\}$ constitutes an independent set of vertices in G. It is then clear that $\{v_b^i v_r^i\}_{i=1,2,...,m}$ constitutes a K-valid matching in G'. To see this, observe that since $\{v^1, v^2, ..., v^m\}$ constitutes an independent set of vertices in G, we have $d_S(v_i, v_j) \geq 2$ for all $i, j \in \{1, 2, ..., m\}$ with $i \neq j$. Hence, for $i \neq j$, we have $d(v_b^i v_r^i, v_b^j v_r^j) \geq 2 + 2\left(\frac{K-1}{2}\right) = K + 1 > K$. Therefore, it follows that the graph G' has a K-valid matching of cardinality no smaller than the cardinality of the maximum independent set of G.

It remains to show that given a K-valid matching in G', one can, in polynomial time, obtain an independent set of vertices in G with the same cardinality. Consider the algorithm given in Table 2, which is a simple modification of the algorithm given in Table 1.

It is easy to see that the running time of the above algorithm is bounded above by a polynomial in $|V|$ and $|E|$. Now, suppose v, u be any two arbitrary vertices in L. Then, we claim that $d_S(u,v) \geq 2$, which implies that L is an independent set in G. For if not, then $d_S(u,v) = 1$. It then follows that there must exist edges $e_1, e_2 \in M$ such that

$$d(e_1, e_2) \leq \max\left(\frac{K-1}{2} + 2\left(\frac{K-3}{4}\right), \frac{K-1}{2} + 2\right)$$

$$= \max\left(K - 2, \frac{K+3}{2}\right) \leq K - 1 < K$$

for $K \geq 5$; contradicting our initial hypothesis, namely, M is a K-valid matching.

The construction of the graph G' and proof of the related results for $K = 3$ is similar to the $K = 4$ case, and therefore omitted. $\square$

Theorem 3 gives a lower bound on the approximation ratio of any polynomial time approximation algorithm for MWKVMP or MWKVMP. The next result we have is opposite in flavor: it shows that there exists a polynomial time algorithm which for MWKVMP which has an approximation ratio no worse than $\Theta\left(\frac{|E|}{(\log|E|)^2}\right)$.

THEOREM 4. *MWKVMP can be approximated within a factor of* $\Theta\left(\frac{|E|}{(\log|E|)^2}\right)$.

The following Corollary is an immediate consequence of Theorem 4:

COROLLARY 2. *MKVMP can be approximated within a factor of* $\Theta\left(\frac{|E|}{(\log|E|)^2}\right)$.

The following terminology will be useful in the proof of Theorem 4. Consider a graph $G = (V, E)$. The vertex weighted maximum independent set problem (VWMISP) is a variation of the maximum independent set problem (MISP), in which the vertices are weighted. Let $w(v)$ denote the weight of vertex v. The goal of VWMISP is to find

an independent set of vertices S that maximizes $\sum_{v \in S} w(v)$. In [13], it is shown that VWMISP is approximable within $\Theta\left(\frac{|V|}{(\log |V|)^2}\right)$. We are now ready to prove Theorem 4.

PROOF. Given an instance of MWKVMP, i.e., a graph $G = (V, E)$, construct the graph $G' = (V', E')$ as follows: For each edge $e \in E$, add a vertex v_e to V' with weight $w(v_e) = w(e)$. For two edges $e_1, e_2 \in E$ with $d(e_1, e_2) \leq K - 1$, connect the corresponding vertices v_{e_1}, v_{e_2} with an edge. Clearly, the graph G' can be constructed in polynomial time. Observe that $|V'| = |E|$.

The way graph G' is constructed it follows that for a K-valid matching in G there exists an independent set of vertices in G', having the same weight, and vice versa. To see this, suppose $\{e_1, ..., e_m\}$ be a K-valid matching in G. Then, for all $i, j \in \{1, 2, ..., m\}$ with $i \neq j$, we have $d(e_i, e_j) \geq K > K - 1$. And therefore, v_{e_i} and v_{e_j} do not have an edge between them in G'. Thus, $\{v_{e_1}, ..., v_{e_m}\}$ constitutes an independent set of vertices in G'. Further, since $w(v_{e_i}) = w(e_i)$, we have $\sum_{i=1}^{m} w(e_i) = \sum_{i=1}^{m} w(v_{e_i})$. Similarly, it can be shown that for an independent set of vertices in G' there exists a K-valid matching in G, having the same weight. Observe that the weight of an optimal K-valid matching in G is the same as the weight of an optimal independent set in G'.

Now, given an instance of MWKVMP, we can construct an instance of $VWMISP$ in polynomial time. From the results in [13], an independent set in G' with weight at least $\Theta\left(\frac{(\log |V'|)^2}{|V'|}\right) = \Theta\left(\frac{(\log |E|)^2}{|E|}\right)$ times the weight of an optimal independent set can then be found in polynomial time; from which a K-valid matching in G with weight at least $\Theta\left(\frac{(\log |E|)^2}{|E|}\right)$ times the weight of an optimal K-valid matching can be found in polynomial time. $\square$

4. MWKVMP FOR SPECIFIC GRAPHS

In this section, we consider MWKVMP restricted to certain specific graphs. In particular, we consider the following graphs:

Geometric graphs: The vertices are placed on a plane and two vertices are connected if and only if the distance between them is $\leq r$, for some $r > 0$.

Disk graphs: The vertices are placed on a plane and a disk of radius $D(v)$ is placed around each vertex v. The vertices u and v are connected if and only if the distance between them is $\leq \min\{D(u), D(v)\}$.

(r,s)-civilized graphs: Graphs whose vertices can be mapped to points on the plane such that the length of each edge is $\leq r$ and distance between any two points is $\geq s$.

The above graphs have been used quite extensively in the literature for modeling the connectivity graph of wireless networks [10, 18, 33]. In the next subsection, we show that MWKVMP can be approximated within a constant factor in case of geometric graphs. Due to space limitations, similar results for other graphs will be presented in a companion paper. We note that the results we derive in the sequel can also be extended to quasi unit disk graphs [19].

4.1 Greedy Approach for MWKVMP

We first study the performance of the following greedy approach:

Greedy Weighted K-Valid Matching Algorithm$(G = (V, E), w : E \to \mathbb{R}, M)$

1. $M := \phi$ and $i := 1$.
2. Arrange edges of E in descending order of weight, starting with $e_1, e_2, ...$
3. If $M \cup e_i$ is a valid K-valid matching, then $M := M \cup e_i$. $i := i + 1$.
4. Repeat Step 3 for all edges in E.

It is well known that the above greedy approach yields a 2-approximation algorithm for MWMP ($K = 1$ case). Interestingly, for $K \geq 2$, the performance of the above greedy approach can be much worse. We now show that in this case the performance of the greedy approach depends on certain properties of the graph G; and can be arbitrarily bad for certain graphs. Finally, we show that the greedy approach performs quite well in case of geometric graphs. The following definitions are now in order:

DEFINITION 3. *The K-hop interference set of an edge $e \in E$, denoted by $I_K(e)$, is the set of edges $u \in E$ such that $d(e, u) \leq K$.*

We call a subset S of $I_K(e)$ "K-maximal" if no other edge $u \in I_K(e)$ can be added to S such that we have $d(u, v) > K$ for all $v \in I_K(e)$.

DEFINITION 4. *The K-hop interference degree of an edge $e \in E$, denoted by $d_K(e)$, is defined as*

$$d_K(e) = \max_{S \subseteq I_K(e): S \text{ is } K-maximal} |S|.$$

DEFINITION 5. *The K-hop interference degree of the graph $G = (V, E)$, denoted by $d_K(G)$, is defined as*

$$d_K(G) = \max_{e \in E} d_K(e).$$

We are now ready to show the main result of this subsection:

THEOREM 5. *The weight of the matching returned by the greedy algorithm is always within a factor $d_K(G)$ of the weight of an optimal matching. Further, there exists a graph G for which the above ratio is exactly $d_K(G)$.*

PROOF. Let e_1 be the edge added to the matching during the first step by the greedy algorithm. Then, we have $w(e_1) \geq w(e)$ for all $e \in E$. Now, the optimal matching can contain at most $d_K(G)$ edges belonging to $I_K(e_1)$, each with a weight no larger than $w(e_1)$. Let e_2 be the edge added to the matching during the second step by the greedy algorithm. Then, we have $w(e_2) \geq w(e)$ for all $e \in E \backslash I_K(e_1)$, where $A \backslash B$ denotes the set consisting of elements of A that are not in B. Moreover, the optimal matching can contain at most $d_K(G)$ edges belonging to $I_K(e_2) \backslash I_K(e_1)$, each with a weight no larger than $w(e_2)$.

For $i \geq 1$, let $L_K(e_i) = I_K(e_1) \cup \cdots \cup I_K(e_i)$. Arguing as above, it can be shown that during the ith step the greedy algorithm adds an edge e_i to the matching that satisfies:

$$w(e_i) = \max_{e \in E \backslash L_K(e_{i-1})} w(e),$$

and the optimal matching contains no more than $d_K(G)$ edges belonging to $I_K(e_i) \backslash L_K(e_{i-1})$. Let e_m be the last edge added to the matching by the greedy algorithm. Observe
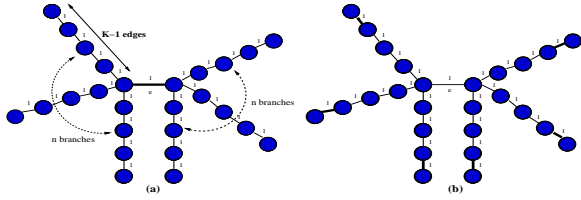
Figure 4: A graph for which the greedy approach does not perform well: (a) A possible matching returned by the greedy algorithm (shown in dark); (b) An optimal matching (shown in dark).
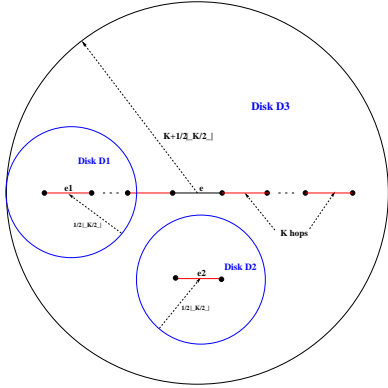

Figure 5: The disks $D1$, $D2$ and $D3$ used in the proof of Theorem 6.

that, we have $E = L_K(e_m)$. From the above discussion, it is clear that for $1 \leq i \leq m$, we have

$$\sum_{e \in O \cap I_K(e_i) \backslash L_K(e_{i-1})} w(e) \leq d_K(G)w(e_i),$$

where O is an optimal matching. Note that by convention $I_K(e_0) = \phi$. Summing over i, we obtain

$$\sum_{e \in O} w(e) \leq d_K(G) \sum_{i=1}^{m} w(e_i),$$

proving our first claim in the statement of Theorem 5.

To prove the second claim, consider the graph G shown in Figure 4. Observe that, we have $d_K(G) = d_K(e) = 2n$. One possible matching obtained using the greedy algorithm is shown in Figure 4(a). Note that the weight of this matching is 1; whereas, the weight of an optimal matching, as shown in Figure 4(b), is $2n$. Thus, we see that the greedy algorithm can at times return a matching whose weight is off by a factor of $d_K(G)$ in comparison to the optimal matching. $\square$

As the graph in Figure 4 clearly shows, $d_K(G)$ can be of the order of $|E|$; and correspondingly, the performance of the above greedy algorithm can be far from optimal. We next show that $d_K(G)$ is bounded by a constant in case of geometric graphs.

THEOREM 6. *The weight of the matching returned by the greedy algorithm is always within a factor of 49 of the weight of an optimal matching in case of geometric graphs.*

PROOF. In view of Theorem 5, we only need to show that $d_K(G) \leq 49$ for all geometric graphs G. Without loss of

generality, we may restrict our attention to geometric graphs with $r = 1$. Consider any arbitrary edge $e \in E$; in the sequel, we will show that $d_K(e) \leq 49$. Since the edge $e \in E$ is arbitrary, it will follow that $d_K(G) \leq 49$.

Let $S \subseteq I_K(e)$ be $K - maximal$ and consider any two arbitrary edges $e_1, e_2 \in S$ with $e_1 \neq e_2$. If there does not exist such a pair of edges, then $d_K(e) \leq 1$ and we are done. Otherwise, it is easy to see that disks D_1, D_2 of radius $\frac{1}{2}\lfloor K/2 \rfloor$, centered at the mid-points of e_1 and e_2, respectively, are disjoint (see Figure 5). To see this, first consider $K = 2$. In this case, any two edges that are at a distance of two or more hops from each other, must be at a Euclidean distance of more than 1 from each other. For if not, then there will be an edge connecting the two, contradicting our initial hypothesis that the edges are at a distance of two or more hops from each other. A repeated use of such an argument shows that any two edges that are at a distance of K or more hops from each other, must be at a Euclidean distance of at least $\lfloor K/2 \rfloor$ from each other. Thus, disks of radius $\frac{1}{2}\lfloor K/2 \rfloor$ centered at their mid-points must be disjoint.

Now, clearly the disks D_1, D_2 will both be contained inside disk D_3 of radius $K + \frac{1}{2}\lfloor K/2 \rfloor$, centered at the mid-point of edge e (see Figure 5). Thus, S contains no more than

$$\frac{\pi \left(K + \frac{1}{2}\lfloor K/2 \rfloor \right)^2}{\frac{\pi}{4} \left\lfloor \frac{K}{2} \right\rfloor^2} \leq 49$$

such edges, for all $K \geq 2$. Hence, $d_K(e) \leq 49$. $\square$

It is worth noting that the above proof is valid even for graphs that are disconnected. In which case, all edges in $I_K(e)$ are part of the connected component of G that contains the edge e. The above proof carries forward by considering only those edges that are contained in the same connected component as e.

4.1.1 PTAS for MWKVMP

Several NP-complete problems are known to admit PTAS when restricted to planar or geometric graphs. In [1], PTASs are developed for various NP-complete problems restricted to planar graphs. NC-approximation schemes for various NP-Hard and PSPACE-Hard problems restricted to geometric graphs are developed in [16]. Following the approach in [16], we now show that MWKVMP and, therefore, MKVMP admits a constant factor PTAS when restricted to geometric graphs.

Consider a geometric graph $G = (V, E)$ with $r = 1$; specified using the coordinates of its vertices in the plane. We now present an algorithm that yields a K-valid matching with weight at least $(1 + \epsilon)^{-1}$ times the weight of an optimal K-valid matching in polynomial time, where $\epsilon > 0$ is a constant, and can be chosen to be arbitrarily small.

The basic technique is the following: Given any $\epsilon > 0$, we calculate the smallest possible m that satisfies $\left(\frac{m+1}{m} \right)^2 \leq 1 + \epsilon$. We divide the plane into horizontal strips of width $K + 2$. For each $i \in \{0, 1, ..., m\}$, we partition the set of edges E into $s_i \geq 1$ disjoint sets $E_{i,1}, ..., E_{i,s_i}$ by removing each edge that connects a pair of vertices that lie within a strip congruent with $i \mod (m+1)$. Each strip is left (top) closed and right (bottom) open. For $1 \leq j \leq s_i$, let $V_{i,j}$ be the smallest subset of V such that all edges in $E_{i,j}$ are of the form uv for some $u, v \in V_{i,j}$. Also, let $G_{i,j} = (V_{i,j}, E_{i,j})$, $1 \leq j \leq s_i$. For each subgraph G_j, we find a K-valid matching of size at least $\frac{m}{m+1}$ times the size of the optimal K-valid

matching in $G_{i,j}$. Observe that the above choice of the width of the strips ensures that the union of K-valid matchings for subgraphs $G_{i,1}, ..., G_{i,s_i}$ is a K-valid matching for the graph G. Using arguments similar to [16, 15], we then show that the iteration in which the partition yields a K-valid matching of maximum possible weight returns a K-valid matching with weight at least $\left(\frac{m}{m+1}\right)^2$ times the weight of an optimal K-valid matching in G. Our algorithm is described in detail in Table 3.

We next show that our algorithm returns a K-valid matching with weight at least $\left(\frac{m}{m+1}\right)^2$ times the weight of an optimal K-valid matching. For each subgraph G' of G, let $O(G')$ be an optimal K-valid matching in G'. We start by showing that at least one out of the $m+1$ iterations for i has the property that the aggregate weight of the edges not considered in the K-valid matching computation during that iteration is a small fraction of $w(O(G))$.

LEMMA 1. We have
$$\max_{0 \le i \le m} w(O(G_i)) \ge \frac{m}{m+1} w(O(G)).$$

PROOF. For $i \in \{0, 1, ..., m\}$, let $S_i \triangleq E \backslash E_i$. Observe that S_i, $i \in \{0, 1, ..., m\}$, is the set of edges that are not considered in the computation of the K-valid matching during iteration i, and satisfy:

$$S_i \cap S_j = \phi \text{ for } 0 \le i, j \le m, \ i \ne j; \text{ and } \cup_{i=0}^m S_i \subseteq E.$$

For $i \in \{0, 1, ..., m\}$, let $M_i \triangleq S_i \cap O(G)$. From the above set of equations it is clear that

$$\sum_{i=0}^m w(M_i) \le w(O(G)),$$

and therefore,

$$\min_{0 \le i \le m} w(M_i) \le \frac{w(O(G))}{m+1}.$$

Thus, we have

$$\max_{0 \le i \le m} w(O(G_i)) \ge w(O(G)) - \min_{0 \le i \le m} w(M_i) \ge \frac{m}{m+1} w(O(G)).$$

□

We next show that the weight of a K-valid matching returned by the above algorithm (denoted by $KVM(G)$) is within a factor $\left(\frac{m}{m+1}\right)^2$ of an optimal K-valid matching.

THEOREM 7. We have
$$w(KVM(G)) \ge \left(\frac{m}{m+1}\right)^2 w(O(G)).$$

PROOF. We first claim that
$$w(KVM(G_{i,j})) \ge \frac{m}{m+1} w(O(G_{i,j})).$$

To see this, observe that by applying Lemma 1 to $G_{i,j}$ we have that there exists a $p \in \{0, 1, ..., m\}$ such that

$$w(O(G_{i,j}^p)) \ge \frac{m}{m+1} w(O(G_{i,j})).$$

Therefore, we have

$$
\begin{aligned}
w(KVM(G_{i,j})) &= \max_{0 \le p \le m} w(KVM(G_{i,j}^p)) \\
&= \max_{0 \le p \le m} \sum_{l=1}^{s_{i,j}} w(KVM(G_{i,j}^{p,l})) \\
&= \max_{0 \le p \le m} \sum_{l=1}^{s_{i,j}} w(O(G_{i,j}^{p,l})) \\
&= \max_{0 \le p \le m} w(O(G_{i,j}^p)) \\
&\ge \frac{m}{m+1} w(O(G_{i,j})).
\end{aligned}
$$

We now chose an $i \in \{0, 1, ..., m\}$ such that $w(O(G_i)) \ge \left(\frac{m}{m+1}\right) w(O(G))$. The existence of such an i follows from Lemma 1. Now, we have

$$
\begin{aligned}
w(KVM(G)) &= \max_{0 \le i \le m} w(KVM(G_i)) \\
&= \max_{0 \le i \le m} \sum_{j=1}^{s_i} w(KVM(G_{i,j})) \\
&\ge \left(\frac{m}{m+1}\right) \max_{0 \le i \le m} \sum_{j=1}^{s_i} w(O(G_{i,j})) \\
&= \left(\frac{m}{m+1}\right) \max_{0 \le i \le m} w(O(G_i)) \\
&\ge \left(\frac{m}{m+1}\right)^2 w(O(G)),
\end{aligned}
$$

proving the claim. □

We now analyze the running time of the above algorithm. First, we claim that the cardinality of any K-valid matching of the graph $G_{i,j}^{p,l}$, $i, p \in \{0, 1, ..., m\}$, $j \in \{1, 2, ..., s_i\}$, $l \in \{1, 2, ..., s_{i,j}\}$, is $O(m^2)$ for all $K \ge 2$. This follows easily by observing that (i) the vertices inside $G_{i,j}^{p,l}$ are contained inside a square of size $(K + 2)m + 2$; and (ii) for any two edges $e_1 = u_1 u_2$ and $e_2 = v_1 v_2$ that are part of a K-valid matching, we must have

$$\min_{i,j=1,2} \eta(u_i, v_j) \ge \lfloor K/2 \rfloor,$$

where $\eta(u, v)$ denotes the Euclidean distance between u and v. The time required to obtain the K-valid matching of $G_{i,j}$ is therefore $n^{O(m^2)}$. And since the outer loop is executed $m + 1$ times, the overall running time is $n^{O(m^2)}$.

REMARK 1. Using a dynamic programming approach, it is possible to improve both the running time as well the performance guarantee of the above algorithm. In particular, by solving the $MWKVMP$ for each graph $G_{i,j}$ optimally using the dynamic programming approach one can reduce the running time from $n^{O(m^2)}$ to $n^{O(m)}$, and at the same time improve the performance guarantee to $\frac{m}{m+1}$. For more discussion on such techniques and their analysis, we refer the reader to [16].

4.2 Throughput Guarantees using Maximal Scheduling Policy

As discussed in Section 2.2, the greedy algorithm and PTAS developed in earlier subsections can both be used to

Table 3: A (1+ε)-Approximation Scheme for MWKVMP

Step	$(1 + \epsilon)$-**Approximation Scheme for MWKVMP**$(G' = (V, E), M)$
1	Find the smallest m such that $\left(\frac{m+1}{m}\right)^2 \leq 1 + \epsilon$.
2	Divide the plane into horizontal strips of width $K + 2$.
3	Divide each horizontal strip into vertical strips of width $K + 2$.
4	**for** each $i \in \{0, 1, ..., m\}$ **do**
5	Partition the set of edges into $s_i \geq 1$ disjoint sets $E_{i,1}, ..., E_{i,s_i}$ by removing each edge that connects a pair of vertices within a horizontal strip congruent with $i \mod (m + 1)$. Let $G_{i,j} = (V_{i,j}, E_{i,j})$, $j \in \{1, 2, ..., s_i\}$, be the subgraph induced by the set of edges $E_{i,j}$ (see the above discussion).
6	$E_i := \cup_{1 \leq j \leq s_i} E_{i,j}$. Let $G_i = (V_i, E_i)$ be the subgraph induced by the set of edges E_i.
7	**for** each $j \in \{1, ..., s_i\}$ **do**
8	**for** each $p \in \{0, 1, ..., m\}$ **do**
9	Partition the set of edges $E_{i,j}$ into $s_{i,j}$ disjoint sets $E_{i,j}^{p,1}, ..., E_{i,j}^{p,s_{i,j}}$ by removing each edge that connects a pair of vertices within a vertical strip congruent with $p \mod (m + 1)$. Let $G_{i,j}^{p,l} = (V_{i,j}^{p,l}, E_{i,j}^{p,l})$, $l \in \{1, 2, ..., s_{i,j}\}$, be the subgraph induced by the set of edges $E_{i,j}^{p,l}$.
10	$E_{i,j}^p := \cup_{1 \leq l \leq s_{i,j}} E_{i,j}^{p,l}$. Let $G_{i,j}^p = (V_{i,j}^p, E_{i,j}^p)$ be the subgraph induced by the set of edges $E_{i,j}^p$.
11	For each $G_{i,j}^{p,l}$, $l \in \{1, 2, ..., s_{i,j}\}$, obtain an optimal K-valid matching $KVM(G_{i,j}^{p,l})$.
12	$KVM(G_{i,j}^p) := \cup_{1 \leq l \leq s_{i,j}} KVM(G_{i,j}^{p,l})$.
13	**end**
14	$KVM(G_{i,j}) := G_{i,j}^p$, where $p = \arg\max_{0 \leq p \leq m} w(KVM(G_{i,j}^p))$.
15	**end**
16	$KVM(G_i) := \cup_{1 \leq j \leq l_i} KVM(G_{i,j})$.
17	$KVM(G) := G_i$, where $i = \arg\max_{0 \leq i \leq m} w(KVM(G_i))$. $M := KVM(G)$.
18	**end**

construct scheduling policies that achieve a constant fraction of the capacity region under K-hop interference models. However, they both require centralized control, and can therefore be implemented in a limited class of wireless networks (e.g., wireless mesh networks).

In this section, we focus on wireless networks in which all transmissions are carried out at certain fixed rate (i.e., rate control is not exercised), and show that the maximal scheduling policy considered in [5, 28, 21][2] achieves a constant fraction of the capacity region for such networks. The main motivation for looking at the maximal scheduling policy is that it is a simple scheduling scheme and is amenable to distributed implementation. We start with the definition of the maximal scheduling policy.

DEFINITION 6. *A scheduling policy is said to be a maximal scheduling policy if it chooses a subset M of edges for transmission (during each packet transmission slot) in such a way that for each edge $e = uv \in E$, one or more of the following conditions are satisfied:*

- *$I_K(e) \cap M \neq \phi$.*

- *$q_{uv} + q_{vu} = 0$, where q_{uv} denotes the number of packets waiting to be transmitted from node u to node v.*

In words, the maximal scheduling policy ensures that if there are any packets waiting to be transmitted over an edge, then either that edge or one of the edges which interfere with that edge must be scheduled to transmit. Now, consider a network with one or more single-hop (MAC layer) sessions.

[2]In [35], authors refer to it as greedy scheduling and analyze its performance for $K = 2$ case. In [21], the authors refer to it as maximal matching and analyze its performance for $K = 1$ case.

Let Δ denote the capacity region of the network, i.e. the set of session arrival rates for which the network can be stabilized under some scheduling policy. It was shown in [5, Theorem 1] that the maximal scheduling policy achieves at least $1/K(N)$ fraction of the capacity region, i.e. it stabilizes the network for any set of arrival rates that are within $\Delta/K(N)$, where $K(N)$ is the interference degree of the graph (see [5, Definition 13]). Under the K-hop interference model, the interference degree of a graph as defined in [5], is the same as the K-hop interference degree of the graph. We therefore obtain the following result as a Corollary to Theorem 1 in [5] and Theorem 6 in this paper.

THEOREM 8. *Consider a wireless network whose connectivity graph can be modeled as a geometric graph and interference constraints can be modeled using a K-hop interference model, for some value of K. If all transmissions are carried out at some fixed rate then the maximal scheduling policy stabilizes the network for any set of session arrival rates within $\Delta/49$.*

The above result can be generalized to accommodate rate control as well as multi-hop nature of the traffic in wireless networks using techniques similar to the ones used in [21, 34]. The details will be presented in a companion paper.

5. EXPERIMENTAL RESULTS

We now present some experimental results to complement our analytical results. The experiments had the following goals:

- to study the aggregate capacity of wireless networks for different values of K and demonstrate the effect of physical layer on the optimal value of K;

- to study the effect of transmit power diversity on the aggregate capacity of a wireless network under different values of K;

- and to compare the performance of weighted and unweighted greedy matching.

All our experiments were performed with nodes placed randomly and uniformly within a square of size 1km. Two different sets of physical layer parameters corresponding to IEEE 802.11 DSSS and EDGE networks were considered (see Table 5). These parameter values were taken from [6]. The path loss model used in our experiments was also taken from [6]. Unless otherwise specified, the transmit power level of each node was set to -10dB (100mW). The communication range of each node was set equal to the distance at which the SNR=SINR Threshold. A pair of nodes was connected with an edge if and only if each of them was within the communication range of the other. We refer the reader to [6] for details of these calculations. We next describe our results in detail. A transmission between a pair of nodes was considered to be successful, if the SINR at both the nodes was above the SINR Threshold.

Table 4: Physical Layer Parameters

	DSSS	EDGE
Noise Spectral Density (dB/Hz)	-204	-204
Channel BW (dB)	73	53
Noise Factor (dB)	5	5
Transmit Power (dB)	-10, -20	-10
Shadowing Margin (dB)	8	8
Building Penetration (dB)	15	15
Path gain @ 100m (dB)	-73	-73
Propagation exponent	3.5	3.5
SINR Threshold (dB)	0	10

The first set of experiments were performed on 802.11 DSSS networks. The goal was to study the aggregate capacity of these networks for different values of K and node densities. All edge weights were set equal to 1 and the greedy matching algorithm was used to obtain a K-valid matching. The values reported in Figures 6(a) and 6(b) were averaged across 100 random network realizations for each value of node density. Further, for each network realization, the greedy algorithm was run ten times and the matching of maximum cardinality across those ten runs was taken as an approximation to an optimal K-valid matching. The results suggest that $K = 2$ is optimal for moderate and large node densities, whereas $K = 1$ is optimal for small node densities. These results are consistent with the fact that at small node densities the number of potential interferers is small and therefore a small value of K suffices to limit the interference.

To study the impact of the physical layer on the optimal value of K, our next set of experiments were performed on EDGE networks. The most important distinguishing feature of these networks is that they have significantly higher SINR Threshold than the 802.11 DSSS networks. One would therefore expect that the optimal value of K for such networks must be higher than 802.11 DSSS networks. This is indeed the case; results indicate that (see Figure 6(c)) for a

wide range of node densities $K = 3$ outperforms $K = 2$ in case of EDGE networks.

To further study the impact of physical layer on the optimal value of K, we modified our earlier set up for 802.11 DSSS networks by setting the rate of transmission at each edge to be

$$B \log (1 + SINR_{min}),$$

(the Shannon rate, assuming Gaussian noise and interference) where the $SINR_{min}$ is the minimum of the SINR at the receiver and the sender; and B is the channel bandwidth. The results are shown in Figures 6(d) and 6(e). The optimal value of K now becomes 2 and 1 for small and large node densities, respectively. Intuitively, this happens because at small node densities the cost of decreased spatial reuse caused by an increase in K is more than compensated by the enhanced rate at each of the scheduled edges. However, as the node density increases the cost of decrease in spatial reuse increases and therefore it becomes more beneficial to simultaneously schedule a large number of edges, each supporting a small rate.

From the above results, we conclude that physical layer has a significant impact on the optimal value of K.

For the rest of the experiments, we only show the results for 802.11 DSSS networks; the results for EDGE networks are similar in flavor and therefore omitted.

The next set of experiments were performed to determine whether transmit power diversity has an effect on the optimal value of K. The transmit power of each node was set to -10 dB (100mW) or -20 dB (10mW), each value being equally likely. As shown in Figures 6(f) and 6(g), our results suggest that transmit power diversity does not have any impact on the optimal value of K.

The next set of results compare the performance of weighted and unweighted greedy matching. In the weighted case, the weight of an edge was set in proportion to the SNR for that edge. The idea was to give preference to node pairs that have a higher chance of communicating successfully (in a SINR threshold based setting) or transmitting at a higher rate (in a setting with rate control). The results clearly indicate (see Figures 6(h) and 6(i)) that the performance of weighted greedy matching is much better than the unweighted one.

We conclude from the above results that the weighted greedy matching performs much better in practice than the unweighted greedy matching. Moreover, the edge weights are quite simple to calculate as they can be locally determined at each edge (note that both SNR and congestion price can be locally determined).

6. CONCLUDING REMARKS

We considered the problem of throughput-optimal scheduling in wireless networks subject to interference constraints. The interference constraints were modeled using a family of K-hop interference models. Under the assumption that each node transmits at a fixed power level (which can be different for different nodes), the optimal scheduling problems were shown to be weighted matching problems with constraints determined by the K-hop interference model. These weighted matching problems were termed Maximum Weighted K-Valid Matching problems (MWKVMPs).

For $K = 1$, MWKVMP corresponds to the well studied Maximum Weighted Matching problem (MWMP) in the literature, which can be solved in polynomial time. Interest-
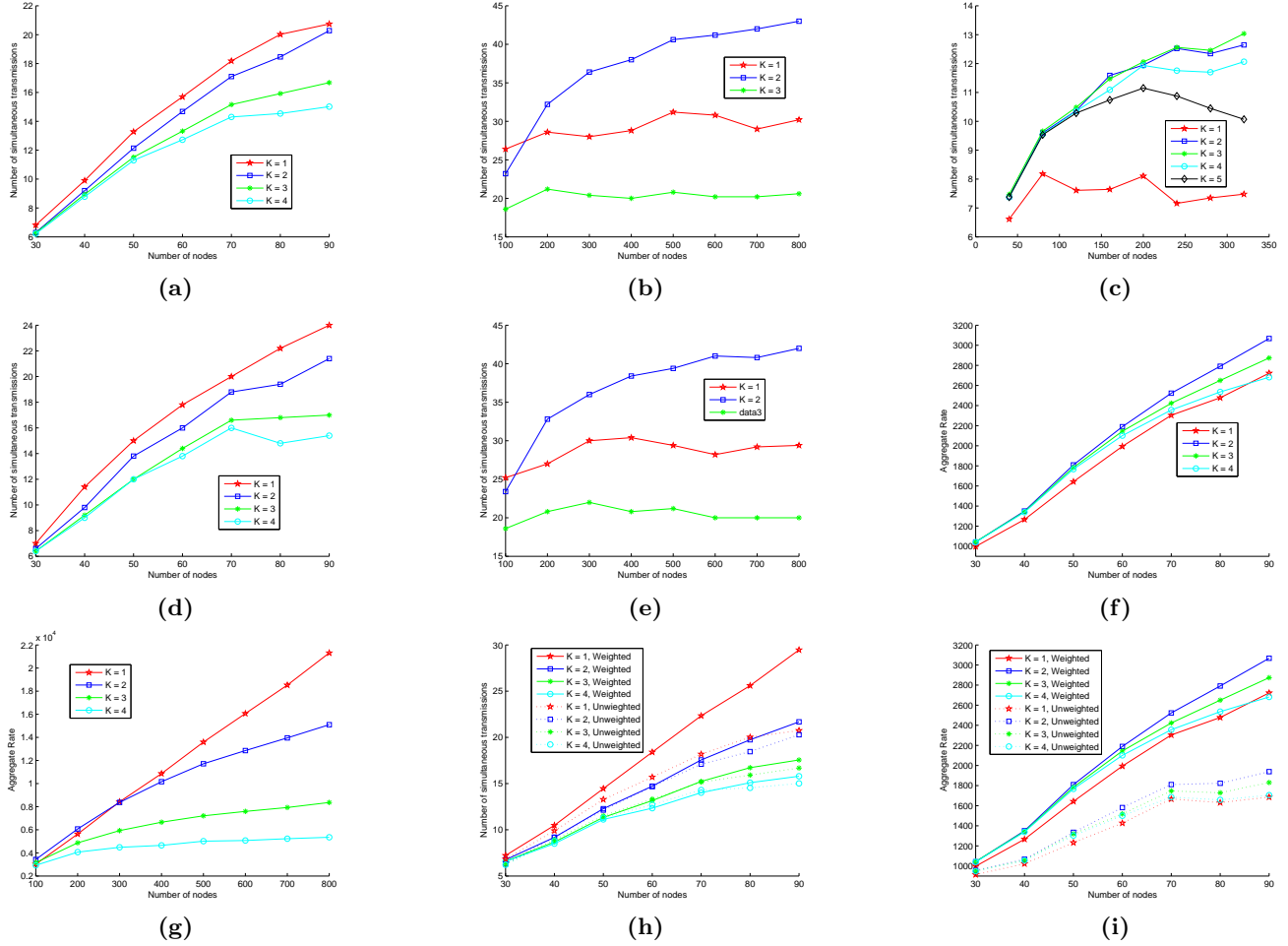
Figure 6: The variation of aggregate capacity with the number of nodes is shown under different settings: (a) and (b) are for 802.11 DSSS networks ; (c) is for EDGE networks; (d) and (e) are for 802.11 DSSS networks with variable transmit power; (f) and (g) are for 802.11 DSSS type of networks with additional rate control mechanism; and (h) and (i) compare the performance of weighted and unweighted greedy matching algorithms for 802.11 DSSS networks.

ingly, we showed that KWKVMP is NP-Hard for $K \geq 2$ and provided upper and lower bounds on its approximability.

We then considered a restriction of MWKVMP to geometric graphs and showed that it admits a PTAS. We also showed that the "natural" greedy matching algorithm yields a 49-approximation to MWKVMP restricted to geometric graphs for all K . Note that a γ-approximation algorithm, when run over each slot separately, results in a S_γ-scheduling policy in the terminology of [21]. Using our results in conjunction with the results in [21], it follows that both greedy algorithm and PTAS can be used to construct scheduling policies that achieve a constant fraction of the capacity region under K-hop interference models.

The schemes mentioned thus far required centralized control; we complemented these results by showing that for networks whose connectivity graph is a geometric graph, the maximal scheduling policy achieves 1/49th fraction of the capacity region under a setting with single-hop traffic and fixed rate transmissions. The maximal scheduling policy is amenable to distributed implementation; we plan to address this issue in our future work. These results are encouraging

as they indicate that one can develop distributed polynomial time algorithms to achieve near optimal throughput in case of a wide range of wireless networks.

The problem of determining the optimal value of K for specific networks is a challenging problem. We numerically studied this problem in case of IEEE 802.11 DSSS and EDGE networks. Our results indicate that the optimal value of K is strongly dependent on the physical layer and may not necessarily be 1 or 2, which have been the only cases studied in the literature. In our future work, we plan to study this issue in more detail using both experimental and analytical tools.

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