

# Cryptanalysis against Symmetric-Key Schemes with Online Classical Queries and Offline Quantum Computations

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**Abstract.** In this paper, quantum attacks against symmetric-key schemes are presented in which adversaries only make classical queries but use quantum computers for offline computations. Our attacks are not as efficient as polynomial-time attacks making quantum superposition queries, while our attacks use the realistic model and overwhelmingly improve the classical attacks. Our attacks convert a type of classical meet-in-the-middle attacks into quantum ones. The attack cost depends on the number of available qubits and the way to realize the quantum hardware. The tradeoffs between data complexity  $D$  and time complexity  $T$  against the problem of cardinality  $N$  are  $D^2 \cdot T^2 = N$  and  $D \cdot T^6 = N^3$  in the best and worst case scenarios to the adversary respectively, while the classic attack requires  $D \cdot T = N$ . This improvement is meaningful from an engineering aspect because several existing schemes claim beyond-birthday-bound security for  $T$  by limiting the maximum  $D$  to be below  $2^{n/2}$  according to the classical tradeoff  $D \cdot T = N$ . Those schemes are broken when quantum computations are available to the adversaries. The attack can be applied to many schemes such as a tweakable block-cipher construction *TDR*, a dedicated MAC scheme *Chaskey*, an on-line authenticated encryption scheme *McOE-X*, a hash function based MAC *H<sup>2</sup>-MAC* and a permutation based MAC *keyed-sponge*. The idea is then applied to the FX-construction to discover new tradeoffs in the classical query model.

**keywords:** post-quantum cryptography, classical query model, meet-in-the-middle, tradeoff, Chaskey, TDR, keyed sponge, KMAC, FX

## 1 Introduction

Recent advancement of the development of quantum computers arises a lot of security concerns in cryptography. It is well-known that factoring can be solved with quantum computers much faster than classical computers, thus security of RSA cryptosystems significantly drops against quantum computers. The similar issue occurs in many other cryptosystems and post-quantum security is of great interest in the current cryptographic community.

Algorithmic speed-up using quantum computers can be applied to symmetric-key schemes as well. For example, Grover’s seminal result [Gro96] recovers the  $k$ -bit key  $K$  only with  $O(2^{k/2})$  quantum computations and finds preimages of an  $n$ -bit output of cryptographic hash function  $H$  only with  $O(2^{n/2})$  quantum computations. Moreover, Brassard *et al.* [BHT97] showed the algorithm to generate collision of  $H$  only with  $O(2^{n/3})$  quantum computations<sup>1</sup>.

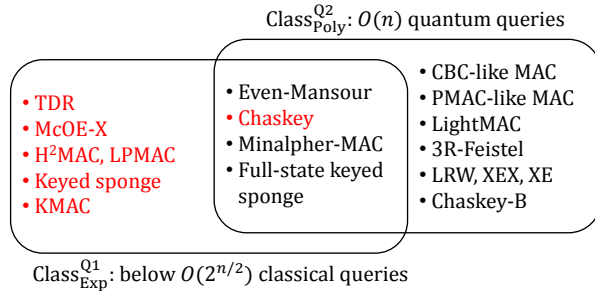
Besides the above improvements on generic attacks, quantum attacks against particular modes, constructions and primitives have been studied. Kuwakado and Morii proposed a distinguishing attack against 3-round Feistel cipher [KM10] and a key recovery attack against Even-Mansour construction [KM12]. Kaplan *et al.* proposed forgery attacks on various CBC-like MACs [KLLN16a] and proposed differential cryptanalysis in the quantum setting [KLLN16b]. Liu and Liu pointed out that existential forgery attacks in [KLLN16a] can be universal forgery attacks [LL17b] and proposed key recovery attacks against full keyed-sponge construction [LL17a]. Most of the attacks assume that all communications are done in superposition, and the attacker is allowed to make superposition queries. Although the assumption of quantum queries is strong, the attacks work only with  $O(n)$  queries and computational complexities where  $n$  is the size of the function output, say the size of the ciphertext block or the tag length.

As those attacks showed, security against quantum computations in symmetric-key schemes heavily depends on the construction. For example, the Even-Mansour construction can be attacked in polynomial-time in the quantum query model whereas block-ciphers resist attacks up to  $O(2^{k/2})$  quantum computations even with quantum queries. Similarly, CBC-like MACs can be attacked in polynomial-time in the quantum query model where HMAC resist attacks up to  $O(2^{k/2})$  quantum computations even with quantum queries. Those motivate researchers to classify various constructions depending on their post-quantum security. Indeed, the recent standardization activity for lightweight cryptosystems by NIST [MBTM17] explicitly mentions that the post-quantum security is taken into account during the selection process.

While the polynomial-time attacks in quantum query model are efficient, the model that requires all the users to implement quantum computers and data in the network is communicated in the form of superposition is strong. Of course, such environment may be feasible in future, and thus researchers should not stop researches in the quantum query model. However its strong assumption motivates us to investigate the security of symmetric-key schemes against attackers who make queries only in the classical manner and performs offline computations by using quantum computers. Many generic attacks e.g. key recovery attack with Grover’s algorithm, work in this model, while only a limited number of results are known for dedicated schemes e.g. the key recovery attack against Even-Mansour construction [KM12], which recovers the key only with  $O(2^{n/3})$  classical queries and  $O(2^{n/3})$  quantum computations.

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<sup>1</sup> While several concerns have been pointed out recently [Ber09,BB17], those works surely took important roles to the progress of this research topic in an early stage.



**Fig. 1.** Classification of Problems Attacked in Quantum Adversaries. Primitives colored in red are attacked in this paper.

**Our Contributions.** We present quantum attacks against symmetric-key schemes in which adversaries make queries only in the classical manner but use quantum computers for offline computations.

We first observe that many of previous quantum attacks can be classified into two classes; polynomial-time complexity in the quantum query model and exponential-time complexity (but significantly improves classical attacks) in the classical query model. We call the former class  $\text{Class}_{\text{Poly}}^{\text{Q2}}$  and the latter class  $\text{Class}_{\text{Exp}}^{\text{Q1}}$ . Most of the previous work focused on  $\text{Class}_{\text{Poly}}^{\text{Q2}}$ , yet [KM12] showed that attacks in  $\text{Class}_{\text{Poly}}^{\text{Q2}}$  may also belong to  $\text{Class}_{\text{Exp}}^{\text{Q1}}$ . The current community pays much attention to  $\text{Class}_{\text{Poly}}^{\text{Q2}}$ , while  $\text{Class}_{\text{Exp}}^{\text{Q1}}$  receives less attention. This motivates us to search for attacks in  $\text{Class}_{\text{Exp}}^{\text{Q1}}$  where the query model is more realistic. We will show many problems that belong to  $\text{Class}_{\text{Exp}}^{\text{Q1}}$  but not to  $\text{Class}_{\text{Poly}}^{\text{Q2}}$ . If researchers only focus on  $\text{Class}_{\text{Poly}}^{\text{Q2}}$ , those problems will be overlooked. The two classes and problems in each class are shown in Fig. 1.

Our attack converts a type of the classical meet-in-the-middle (MitM) attacks into quantum ones. In details, if the classical MitM attacks make  $D$  online queries and  $T$  offline computations such that  $D \cdot T = N$ , we replace the classical offline computations with quantum ones, while the classical online queries stay unchanged. Hence, we call the attack *online-offline MitM attack*.<sup>2</sup>

There are two issues about the evaluation of the cost of quantum computations. 1) Grover and Rudolph [GR04] pointed out that the equivalence between having  $Q$  quantum memory and  $Q$  quantum processors, which may affect the best choice of the quantum computations for offline computations. 2) Bernstein [Ber09] argued that quantum hardware architecture significantly impacts to the cost of the quantum computation. In this paper, the attacks are evaluated by taking into account those observations. As a result, the classical tradeoff of

<sup>2</sup> Kaplan [Kap14] proposed another type of quantum MitM attack for multiple encryptions. It computes two independent parts offline, thus is different from ours.

$D \cdot T = N$  can be improved to  $D^2 \cdot T^2 = N$ ,  $D^{3/2} \cdot T^2 = N$ ,  $D^4 \cdot T^6 = N^3$ , or  $D \cdot T^6 = N^3$ , depending on the assumption of the models.

This improvement is meaningful because several existing schemes claim beyond-birthday-bound (BBB) security for  $T$  by limiting the maximum  $D$  to be below  $2^{n/2}$  by following the classical tradeoff of  $D \cdot T = N$ . Those schemes are broken by our attacks. For example, a tweakable block-cipher (TBC) construction *tweak dependent rekey (TDR)* proposed by Minematsu [Min09] and a dedicated MAC scheme *Chaskey* [Mou15,MMH<sup>+</sup>14] are AES-based 128-bit output schemes. TDR and Chaskey claim 86-bit security and 80-bit security for  $T$  by limiting the maximum  $D$  to be  $2^{42}$  and  $2^{48}$ , respectively. Our attacks can break those schemes with  $T = D = 2^{32}$  using  $2^{32}$  qubits or with  $D = 2^{57}, T = 2^{42}$  using only  $128 \cdot c$  qubits where  $c$  is a small constant. Our attacks have more applications such as an on-line authenticated encryption scheme *McOE-X* [FFL12], a hash based MAC *H<sup>2</sup>-MAC* [Yas09], a permutation based MAC *keyed-sponge* [BDPA08] thus *KMAC* [NIS16] standardized by NIST.

We also discuss a tradeoff of the quantum attacks against the FX-construction proposed by Leander and May [LM17] in the classical query model, in which only the quantum query model is discussed in [LM17]. The attack is further extended to three constructions: 2-key variants of LRW, XE, and XEX constructions.

**Paper Outline.** The remaining part of this paper is organized as follows. Section 2 introduces quantum attack models and previous work. Section 3 gives general description of the quantum online-offline MitM attacks. Section 4 applies our attack to various schemes. Section 5 discusses the attack against the FX construction. Section 6 finally concludes the paper.

## 2 Preliminaries

We explain the models to evaluate cost of quantum computations in Sect. 2.1. We then summarize the cost of quantum multi-target preimage search in Sect. 2.2. Previous quantum attacks are reviewed in Sect. 2.3. As for attack model, we received several comments from other researchers, which can be found in the appendix.

### 2.1 Attack Models for Quantum Computations

**Cost of Quantum Computation.** Two important quantities to evaluate the cost of quantum computations are *time* complexity and number of *qubits*.

The complexity of qubits is measured by the quantum register size of a quantum computer. Although memory is cheaper than processor in the classical setting, they are physically equivalent in the quantum setting. As pointed out by Grover and Rudolph [GR04], executing an algorithm using  $Q$  quantum memory and parallelly processing  $Q$  threads of 1-qubit processor are equally difficult.

As for time complexity, we regard that the time required to operate encryption once as unit time, and also regard that time required for elementary

operations (memory look-up, XOR, and so on) is negligibly small compared to the time required for encryption once. If an encryption algorithm is implemented on both of classical and quantum circuits, we assume that running time of these circuits differ by a constant factor. Bernstein [Ber09] pointed out that the way of realizing quantum hardware significantly impacts to running time of algorithms. We consider the following two models by following the terminology in [Ber09].

**Free communication model.** A quantum hardware can operate elementary quantum gates, e.g. Toffoli gates, on an arbitrary tuple of small (constant) number of qubits.

**Realistic communication model.** Qubits in a quantum hardware are arranged in a square, and elementary operations can only be applied to the pair of qubits within a constant distance.

When the size of the qubits is only polynomial to the size of the problem to solve, restrictions from the hardware architecture has negligible impact in the evaluation of asymptotic time complexity. For example, suppose that a quantum hardware in realistic communication model with  $O(n)$ -qubits is available to solve the problem of size  $O(2^n)$ . Then, it can emulate a quantum hardware in free communication model with  $O(n)$ -qubits, only with time overhead of  $O(n)$  (see [BBG<sup>+</sup>13] for details). Similarly, even if the size of qubits is exponential, the evaluation of asymptotic time complexity is not significantly affected by the communication model if the hardware is composed of small (i.e. only polynomially many qubits) independent quantum processors which do not communicate with each other.

**Query Model.** In the classical setting, an adversary is given an oracle that is usually a black box to her and the oracle runs a keyed operation such as encryption, decryption, or MAC. There are two quantum attack models that naturally extend the classical attack models, which are called *Q1 model* and *Q2 model* in [KLLN16b].

**Q1 model:** The adversary is allowed to make *classical* online queries, similarly as in the classical settings.

**Q2 model:** The adversary is allowed to make *quantum superposition* online queries. That is, oracles allow queries in quantum superposition states and return the results as quantum superposition states.

Q2 model implicitly requires that all the data on the network must be communicated as quantum superposition states. Q1 model is relatively more realistic.

## 2.2 Quantum Multi-target Preimage Search

**Basics.** Grover's algorithm [Gro96] is a quantum algorithm for unstructured database search problem, which is mathematically modeled as follows:

*Problem 2.1.* Let  $f : \{0, 1\}^n \rightarrow \{0, 1\}$  be a binary function on the set of  $n$ -bit strings. The problem is to find an element  $x \in \{0, 1\}^n$  such that  $f(x) = 1$ .

Given  $f$  as a quantum circuit or a quantum oracle, and with the promise  $|f^{-1}(1)| = 1$ , the original algorithm [Gro96] solves this problem with  $O(2^{n/2})$  evaluations of  $f$ . The algorithm was later generalized by Boyer *et al.* [BBHT98] to solve the problem without promise, and it can solve the problem with  $O\left(\sqrt{2^n/\ell}\right)$  evaluations of  $f$ , here  $\ell = |f^{-1}(1)|$ . Hereafter, we also call this generalized version Grover's algorithm.

**Proposition 2.1** ([BBHT98] **Theorem 3**). *Let  $\ell = |f^{-1}(1)|$ . There is a quantum algorithm that can solve Problem 2.1 with an expected number of  $O(\sqrt{2^n/\ell})$  evaluations of  $f$ . If  $\ell = 0$ , then this algorithm will never abort.*

**Quantum Multi-target Preimage Search.** Let us consider to solve the following problem using quantum algorithms.

*Problem 2.2.* Fix a parameter  $t < n/2$ . Let  $H : \{0, 1\}^n \rightarrow \{0, 1\}^n$  be a random function, and  $L \subseteq \{0, 1\}^n$  be a subset of size  $2^t$  that is chosen uniformly at random. Given the list  $L$  and access to quantum oracle  $H$ , find  $x \in \{0, 1\}^n$  such that  $H(x) \in L$ .

**Naive Algorithm.** Naive way to solve the above problem is to apply Grover's algorithm as follows. Let us consider free communication model. First, we sort the list  $L$ . This requires  $O(t2^t)$  classical computations. Let  $f : \{0, 1\}^n \rightarrow \{0, 1\}$  be a function such that  $f(x) = 1$  if and only if  $H(x) \in L$ . Since  $H$  is a random function and  $L$  is chosen randomly,  $|f^{-1}(1)| \approx |L| = 2^t$ . Thus, using Grover's algorithm, we can find  $x \in \{0, 1\}^n$  such that  $f(x) = 1$ , which is equivalent to  $H(x) \in L$ , with  $O(2^{(n-t)/2})$  evaluation of  $f$ . One evaluation of  $f$  requires  $O(1)$  evaluations of  $H$ , and a search in the list  $L$ , which can be done in time  $O(t)$ . Therefore the total computational time is  $O(t2^{(n-t)/2})$ . We need  $O(2^t)$  qubits because  $L$  should be embedded to the quantum circuit of  $f$ . Eventually we obtain the following proposition.

**Proposition 2.2.** *In the free communication model, there is a quantum algorithm that can solve Problem 2.2 in time  $\tilde{O}(2^{(n-t)/2})$ , using  $O(2^t)$  qubits.*

**Combination of Grover's Algorithm with Parallel Rho Method.** Baneagas and Bernstein [BB17] presented a parallelized quantum multi-target preimage search that combines Grover's algorithm with a parallel rho method [VOW94]. The paper has two results, which takes into account the ways of realizing quantum hardware.

One result is that, in the free communication model, there exists a quantum algorithm that solves Problem 2.1 in time  $\tilde{O}(\sqrt{2^n/p2^t})$  using  $\tilde{O}(p)$  qubits, where  $p \geq 2^t$ . Another result is that, in the realistic communication model, there exists a quantum algorithm that solves Problem 2.1 in time  $\tilde{O}(\sqrt{2^n/p2^{t/2}})$  using  $\tilde{O}(p)$  qubits, where  $p \geq 2^t$ .

This paper assumes that the number of qubits available is at most the size of  $L$ , which is  $2^t$ . By setting  $p = 2^t$ , their results are summarized as follows.

**Proposition 2.3** ([BB17]). *In the free communication model, there exists a quantum algorithm that solves Problem 2.1 in time  $\tilde{O}(\sqrt{2^n/2^{2t}})$ , using  $\tilde{O}(2^t)$  qubits. In the realistic communication model, there exists a quantum algorithm that solves Problem 2.1 in time  $\tilde{O}(\sqrt{2^n/2^{3t/2}})$ , using  $\tilde{O}(2^t)$  qubits.*

**Algorithm with Small Number of Qubits.** Even if the number of available qubits is limited to polynomial in  $n$ , we can use the algorithm by Chailoux *et al.* [CNPS17]. Note that as discussed in Section 2.1, quantum hardware architecture does not impact to its complexity.

**Proposition 2.4** ([CNPS17], **Theorem 3**). *Assume that  $t < \frac{3n}{7}$  holds. Then, there exists a quantum algorithm that can solve Problem 2.2 in time  $\tilde{O}(2^{n/2-t/6})$ , using  $O(n)$  qubits and  $\tilde{O}(2^{t/3})$  classical memory.*

**Parallelized Algorithm with Small Independent Processors.** The above algorithm which uses only polynomially many qubits can be parallelized [CNPS17] with small independent quantum processors without communication. As described before, even if the size of qubits is exponential, the evaluation of asymptotic time complexity is not significantly affected by the communication model if the hardware is composed of small (i.e. only polynomially many qubits) independent quantum processors which do not communicate with each other.

**Proposition 2.5** ([CNPS17], **Theorem 5**). *Assume that  $2^s$  small quantum processors are available and  $t < \frac{3n+3t}{7}$  holds. Then, there exists a quantum algorithm that can solve Problem 2.2 in time  $\tilde{O}(2^{n/2-t/6-s/2})$ , using  $O(2^s)$  qubits and  $\tilde{O}(2^{t/3})$  classical memory.*

### 2.3 Previous Quantum Attacks

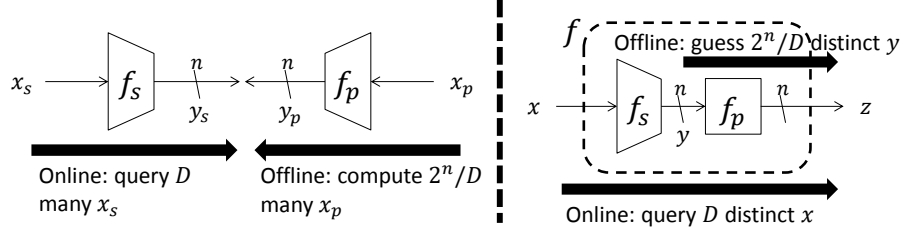
**Q2 Model.** There are many works on polynomial-time quantum attacks against symmetric-key schemes [Bon17,HA17,KM10,KM12,KLLN16a,KLLN16b,LL17b].

Those obtain *exponential speed-up* but requires Q2 model to adopt Simon’s algorithm [Sim97]. In short, Simon’s algorithm can find the secret period of a periodic function  $f : \{0, 1\}^n \rightarrow \{0, 1\}^n$  with time complexity of polynomial in  $n$ .

**Q1 Model.** To avoid relying on strong Q2 model, several previous researches discussed quantum attacks in Q1 model, i.e. adversaries only can make classical queries [KM12,KLLN16b,Kap14,MS17]. This kind of attacks has been less focused compared to the attacks in Q2 model.

## 3 General Framework

In this section, we present a general framework of the quantum online-offline MitM attack in Q1 model. We review the classical online-offline MitM attack in Sect 3.1. We then introduce quantum online-offline MitM attack in Q1 model in Sect 3.2. The impact of new tradeoffs is discussed in Sect. 3.3.



**Fig. 2.** General Settings for Online-Offline MitM

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**Algorithm 1** Classical Online-Offline MitM Attack

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**Classical Online Queries**

- 1: **for**  $i \leftarrow 1, 2, \dots, D$  **do**
- 2:   Choose distinct input  $x_i$ .
- 3:   Query  $x_i$  to  $f$ , and store the corresponding  $z_i$  in the classical memory  $L$ .
- 4: **end for**

**Classical Offline Computations**

- 5: **for**  $j \leftarrow 1, 2, \dots, \frac{2^n}{D}$  **do**
  - 6:   Guess internal state value  $y^j$
  - 7:   Compute  $z_j \leftarrow f_p(y_j)$  offline and check a match between  $z_j$  and  $L$ .
  - 8: **end for**
- 

### 3.1 Classical Online-Offline MitM Attack

Let  $f_s : \{0, 1\}^* \mapsto \{0, 1\}^n$  and  $f_p : \{0, 1\}^n \mapsto \{0, 1\}^n$  be a secret and public functions in which the attacker wants to find a collision between  $f_s$  and  $f_p$  (Fig. 2, left). This often occurs when the attack target  $f : \{0, 1\}^* \mapsto \{0, 1\}^n$  is a composition  $f_s$  followed by  $f_p$ , namely  $f = f_p \circ f_s$  (Fig. 2, right). Here, the input, the internal state and the output are denoted by  $x$ ,  $y$  and  $z$ , respectively.

The online-offline MitM attack is a type of the MitM attack, in which the adversary first makes  $D$  online queries to collect  $D$  output values with randomized  $n$ -bit internal state, and then makes  $2^n/D$  random guesses of the internal state and computes  $f_p$  offline. The match of the  $n$ -bit output suggests the correct value of the  $n$ -bit internal state. The attack is described in Algorithm 1.

The number of possible pairs from online and offline phases is  $2^n$ , thus a match of the  $n$ -bit value is expected with a reasonably high probability. The classical online-offline MitM attack provides the tradeoff of

$$D \cdot T = N, \tag{1}$$

where  $D$  and  $T$  are balanced when  $D = T = N^{1/2}$ .



### 3.2 Quantum Online-Offline MitM Attack

We now introduce the quantum online-offline MitM attack in Q1 model. Queries can only be made in the classical manner. Hence, the online phase in Algorithm 1 stays unchanged, and we replace the offline phase with quantum computations.

**Insufficiency of Multi-target Preimage Search.** The simplest way is applying the naive multi-target preimage search in section 2.2 instead of the random guess in Algorithm 1. When  $D$  targets are available in the quantum list, as in Proposition 2.1, the multi-target preimage search runs with  $T = O(\sqrt{N/D})$  quantum computations. Hence, the tradeoff becomes  $D \cdot T^2 = N$ , in which  $T$  and  $D$  are balanced when  $T = D = N^{1/3}$ . This achieves a good improvement over the classical setting. However, this method has the crucial drawback;  $D = N^{1/3}$  qubits are exploited only for storing the data. If we apply Grover's algorithm (for key search) in parallel with  $N^{1/3}$  qubits, the offline phase for  $D = 1$  can finish in  $O(N^{1/3})$ , which is better than applying the multi-target preimage search in terms of the data complexity.

**Case Analysis Depending on Quantum Hardware.** Let  $Q$  be the number of qubits available to the attacker. We use those  $Q$  qubits to process quantum operations rather than to store the data. Here, the time complexity of quantum algorithms relies on  $Q$ . Hence we do the case analysis; the first case assumes that  $Q$  is an exponential size, while the second case assumes that  $Q$  is a limited size.

Bernstein [Ber09], and also Banegas and Bernstein [BB17], pointed out that the hardware architecture, i.e. how to positioning qubits in quantum hardware, significantly impacts to the computational cost of quantum algorithms. As discussed in Sect 2.1, we consider the free and realistic communication models. The former allows any qubit to interact with any other qubit. The latter assumes that each qubits is arranged in a square and the range to interact is limited. The gap between two models is big when  $Q$  is an exponential size. While for a sufficiently small  $Q$ , say polynomial in  $\log N$ , the way of realizing hardware does not significantly effect on the time complexity. Similarly, even if  $Q$  is an exponential size, the evaluation of asymptotic time complexity is not significantly affected by communication model if the hardware is composed of small (i.e. only polynomially many qubits) independent quantum processors which do not communicate with each other. In summary, we analyze the following four cases.

1.  $Q$  is exponential (more advantageous to the attacker).
  - (a) free communication model
  - (b) realistic communication model
  - (c) independent small processors without communication
2.  $Q$  is not exponential (more challenging to the attacker).

In the following case analysis, we assume that the classical online queries collect  $D$  targets and those are stored in the classical memory  $M$ .

**Tradeoff for Case 1a.** It assumes that  $Q$  qubits are available in the free-communication model, where  $O(Q) \geq D$ . Banegas and Bernstein [BB17] showed that the computational cost  $T$  of the multi-target preimage search in the free communication model is  $T = \tilde{O}\left(\sqrt{\frac{N}{Q \cdot D}}\right)$ . By setting  $Q = D$ , the tradeoff for Case 1a becomes

$$D^2 \cdot T^2 = N, \quad (2)$$

where  $D$  and  $T$  are balanced when  $D = T = N^{1/4}$ .  $Q$  and  $M$  are also  $N^{1/4}$ .

**Tradeoff for Case 1b.** It assumes that  $Q$  qubits are available in the realistic-communication model, where  $O(Q) \geq D$ . Banegas and Bernstein [BB17] showed that the computational cost  $T$  of the multi-target preimage search in the realistic communication model is  $T = \tilde{O}\left(\sqrt{\frac{N}{Q \cdot D^{1/2}}}\right)$ . By setting  $Q = D$ , the tradeoff for Case 1b becomes

$$D^{3/2} \cdot T^2 = N, \quad (3)$$

where  $D$  and  $T$  are balanced when  $D = T = Q = M = N^{2/7}$ .

**Tradeoff for Case 1c.** It assumes that  $Q$  qubits are divided to  $Q$  independent small quantum processors. Chailoux et al. [CNPS17] showed that the computational cost  $T$  of the multi-target preimage search with  $Q$  qubits is  $T = \tilde{O}\left(\sqrt{\frac{N}{Q \cdot D^{1/3}}}\right)$ . By setting  $Q = D$ , the tradeoff for Case 1b becomes

$$D^4 \cdot T^6 = N^3, \quad (4)$$

where  $D$  and  $T$  are balanced when  $D = T = Q = M = N^{3/10}$ .

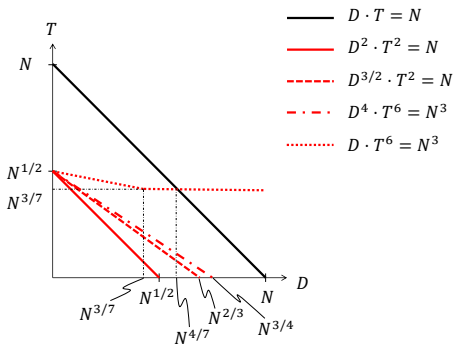
**Tradeoff for Case 2.** It assumes that  $Q = O(\log N)$  qubits are available. Chailoux *et al.* [CNPS17] showed that  $T$  of the multi-target preimage search with  $O(\log N)$  qubits is  $T = \tilde{O}\left(\frac{N^{1/2}}{D^{1/6}}\right)$  for  $D < N^{3/7}$ , using  $D^{1/3}$  classical memory. The tradeoff for  $D < N^{3/7}$  in Case 2 becomes

$$D \cdot T^6 = N^3, \quad (5)$$

where  $D$  and  $T$  are balanced when  $D = T = N^{3/7}$ . Note that  $T = N^{3/7}$  even with  $D > N^{3/7}$ . The number of qubits  $Q = O(\log N)$  is sufficiently small when  $N$  in practical functions are considered. For example,  $N = 2^{128}$ ,  $D = 2^{42}$ , and  $Q = 128 \cdot c$  for a small constant  $c$  in an example discussed in Sect. 4.

**Table 1.** Tradeoff of Online-Offline MitM Attack in Various Models

reference	Sect. 3.1	Case 1a	Case 1b	Case 1c	Case 2
query model	classic	classic	classic	classic	classic
num of qubits	0	$O(D)$	$O(D)$	$O(D)$	$O(\log N)$
comm model	-	free	realistic	any	any
algorithm	Algorithm 1	[BB17]	[BB17]	[CNPS17]	[CNPS17]
tradeoff	$D \cdot T = N$	$D^2 \cdot T^2 = N$	$D^{3/2} \cdot T^2 = N$	$D^4 \cdot T^6 = N^3$	$D \cdot T^6 = N^3$
$\min\{D, T\}$	$N^{1/2}$	$N^{1/4}$	$N^{2/7}$	$N^{3/10}$	$N^{3/7}$



**Fig. 3.** Illustration of Tradeoff Curves (plotted in logarithmic scale)

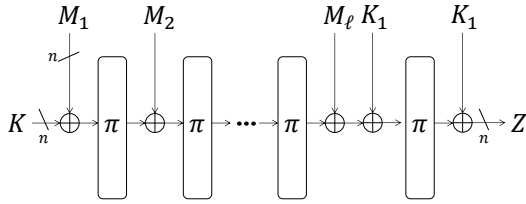
### 3.3 Impact

The tradeoffs of the online-offline MitM attacks are compared in Table 1. The tradeoff curves are plotted in Fig. 3. As long as  $Q$  is an exponential size, the complexities of the quantum attacks are exponentially smaller than ones in the classical online-offline MitM. When  $Q$  is  $O(\log N)$ , the quantum attack improves  $T$  as long as  $D \leq N^{4/7}$ .

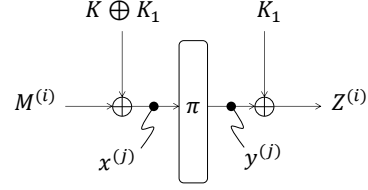
As we later discuss in Sect. 4, several existing schemes claim BBB security by setting the number of maximum queries to be less than  $N^{1/2}$  to ensure the minimum number of computational cost is more than  $N^{1/2}$  according to the classic tradeoff in Eq (1). Such security claims collapse against attackers with quantum computers even in Q1 model.

## 4 Applications of Online-Offline MitM Attacks

In this section, we discuss that the online-offline MitM attack can be applied to a lot of existing symmetric-key schemes. Section 4.1 focuses on the two schemes that claim BBB security by limiting the maximum number of queries per key. Section 4.2 shows a few more applications.



**Fig. 4.** Computation Structure of Chaskey



**Fig. 5.** Online-Offline MitM Attack against Chaskey

#### 4.1 Applications to Schemes with Beyond-Birthday-Bound Security

**Chaskey.** Chaskey [Mou15,MMH<sup>+</sup>14] is a light-weight MAC scheme. The construction follows CBC-MAC but the  $n$ -bit block cipher in CBC-MAC is replaced with Even-Mansour construction with a public  $n$ -bit permutation.

It uses an  $n$ -bit key  $K$ , and generates the second key  $K_1$  by  $K_1 \leftarrow 2 \cdot K$ , where ‘ $\cdot$ ’ is a multiplication over a finite field. Suppose that the size of the input message  $M$  is a multiple of  $n$ .  $M$  is then divided into  $n$ -bit blocks such that  $M_1 \| M_2 \| \dots \| M_\ell \leftarrow M$ . Let  $\pi$  be an  $n$ -bit public permutation. Then, a tag  $Z$  for  $M$  is computed as follows, which is illustrated in Fig. 4.

1.  $\text{State} \leftarrow K$
2.  $\text{State} \leftarrow \pi(\text{State} \oplus M_i)$  for  $i = 1, 2, \dots, \ell - 1$ .
3.  $\text{State} \leftarrow \pi(\text{State} \oplus M_\ell \oplus K_1)$
4.  $Z \leftarrow \text{State} \oplus K_1$ .

Security of Chaskey is the same level as the Even-Mansour construction. Indeed, when the input message length is 1-block, the construction becomes Even-Mansour construction with the first key  $K \oplus K_1$  and the second key  $K_1$ . It is known that, even by the classical adversaries, Even-Mansour construction can be attacked with  $D$  queries and  $T$  offline computations satisfying  $D \cdot T = 2^n$ .

The size of  $\pi$  is 128 bits. Hence it can be attacked with  $D = T = 2^{64}$  by the classical adversaries, while 64-bit security is sometimes too small. To avoid this problem, the number of MACs generated under a single key is limited to  $2^{48}$ . Then, it offers 80-bit security against offline computations.

*Attack Procedure.* The online-offline MitM attack can be directly applied to Chaskey. The attack in [KM12] targets the two-key Even-Mansour construction, hence the attack uses two pairs of ciphertexts and takes their difference to eliminate the impact of the second key  $K_2$ . In our 1-block attack in Chaskey illustrated in Fig. 5,  $K_1$  is linearly derived from  $K$ . Hence, we make a small optimization for Chaskey to improve the constant factor of 2.

We first revisit the attack in the classical model. The adversary chooses  $D$  distinct messages  $M^{(i)}$  and obtains the corresponding tag  $Z^{(i)}$  via encryption

queries. In the offline phase, the adversary makes  $T$  guesses  $x^{(j)}$  of the input value to  $\pi$  and calculates its output  $y^{(j)}$  offline. Here, we have

$$M^{(i)} \oplus x^{(j)} \oplus y^{(j)} \oplus Z^{(i)} = K, \quad K_1 = y^{(j)} \oplus Z^{(i)} = 2 \cdot K.$$

Hence,  $2 \cdot (M^{(i)} \oplus x^{(j)} \oplus y^{(j)} \oplus Z^{(i)}) = y^{(j)} \oplus Z^{(i)}$ , which is converted to the match between values computed online and offline:  $2 \cdot M^{(i)} \oplus 3 \cdot Z^{(i)} = 2 \cdot x^{(j)} \oplus 3 \cdot y^{(j)}$ . The match suggests the key  $K$ . Hence, with  $DT = 2^n$ , the key is recovered. In other words, we simply run Algorithm 1 by defining  $f$  and  $f_p$  as

$$\begin{aligned} f(m) &: \{0, 1\}^n \mapsto \{0, 1\}^n \triangleq 2 \cdot m \oplus 3 \cdot \text{Chaskey}(m), \\ f_p(x) &: \{0, 1\}^n \mapsto \{0, 1\}^n \triangleq 2 \cdot x \oplus 3 \cdot \pi(x). \end{aligned}$$

As discussed in Sect 3.2, the complexity of the quantum algorithm depends on the assumptions of the quantum hardware architecture.

**Case 1a (exponential qubits, free communication).** The internal state (and then both keys) are recovered at the balanced point of the tradeoff curve, in which  $D = T = Q = M = 2^{128/4} = 2^{32}$ .

**Case 1b (exponential qubits, realistic communication).** The attack is performed at the balanced point;  $D = T = Q = M = 2^{2 \cdot 128/7} \approx 2^{36.6}$ .

**Case 1c (exponential qubits, any communication).** The attack is performed at the balanced point;  $D = T = Q = M = 2^{3 \cdot 128/10} \approx 2^{38.4}$ .

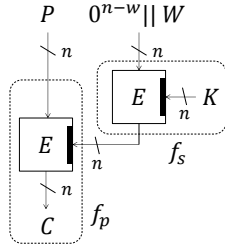
**Case 2 (non-exponential qubits).** The balanced point  $2^{3 \cdot 128/7} \approx 2^{54.9}$  cannot be reached due to the limitation of the number of queries. When  $D = 2^{48}$ ,  $Q$  is  $O(\log N) = 128 \cdot c$  for a small constant  $c$  and  $M = D^{1/3} = 2^{16}$ . The tradeoff curve becomes  $2^{48} \cdot T^6 = 2^{3 \cdot 128}$ , which leads to  $T = 2^{56}$ .

In any case,  $T$  is overwhelmingly smaller than  $2^{80}$  of the classical attack.

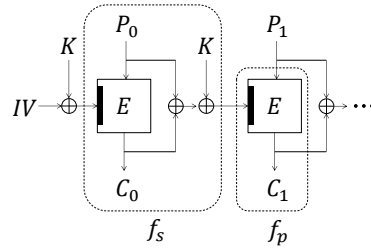
**Remarks on Chaskey-B.** The original paper of Chaskey [MMH<sup>+</sup>14] proposes a block-cipher variant of Chaskey, called Chaskey-B. Roughly speaking, it replaces a public permutation  $\pi$  of Chaskey with block-cipher  $E_k$ , which makes the construction identical with a standard CBC-MAC.

As shown by Kaplan *et al.* [KLLN16a] and Liu and Liu [LL17b], (universal) forgery can be applied in Q2 model, while no method is known to break birthday bound in Q1 model. This indicates that Chaskey and Chaskey-B have very different security level against quantum adversaries in Q1 model.

**Tweak-Dependent Rekeying (TDR).** Minematsu proposed a block cipher mode called *tweak-dependent rekeying (TDR)*, which constructs a TBC with BBB security [Min09]. Let  $E_K$  be a block cipher of which both the block size and key size are  $n$  bits. Let  $E_K^w$  be a construction in which the first  $n - w$  bits of the plaintext for  $E_K$  are fixed to 0, which reduces the plaintext space from  $n$  bits to  $w$  bits. TDR builds a TBC (using  $w$ -bit tweak) with two  $E_K$  calls;  $K' \leftarrow E_K^w(W)$  then  $C \leftarrow E_{K'}(P)$ . The construction is illustrated in Fig. 6.



**Fig. 6.** Tweak Dependent Rekeying (TDR)



**Fig. 7.** McOE-X

Minematsu proved that TDR achieves the security curve  $D \cdot T = 2^n$  against classical adversaries. This bound is tight. The online-offline MitM attack in Algorithm 1 can be applied by fixing  $P$  to an arbitrary value, defining  $f$  as a oracle query to TDR and defining  $f_p$  as the offline computation of  $E_{K'}$  with guessing  $K'$ . The attack reveals  $K'$ . Although  $K$  is not recovered, knowledge of  $K'$  allows the adversary to convert any  $P$  to  $C$  or  $C$  to  $P$ , thus confidentiality is broken.

AES is considered as an underlying cipher, thus  $n = 128$ . When  $w < n/2$ , BBB security is proved against the offline computational cost. Minematsu recommended  $w = n/3$  to ensure  $2n/3$ -bit security. For the AES instantiation,  $w$  is set to 42 bits, thus security for the offline computation is up to 86 bits.

Similarly to Chaskey, the quantum online-offline MitM can directly be applied with about  $2^{32}$ ,  $2^{36.6}$  and  $2^{38.4}$  complexities for Case 1a, Case 1b, and Case 1c, respectively. For Case 2,  $D = 2^{42}$ ,  $Q = 128 \cdot c$  qubits for a small constant  $c$ ,  $M = 2^{14}$  classical memory, and  $T = 2^{57}$ .

Comparison with other TBC constructions is of interest. On one hand, some TBC constructions such as LRW and XEX can be broken with  $O(n)$  complexity in Q2 model [KLLN16a], while no attack is known in Q1 model (though we will propose another type of tradeoff for 2-key variants in Sect. 5). On the other hand TDR resists  $O(n)$  attack in Q2 model, while security in Q1 model is worse than LRW and XEX. As shown in Fig. 1, those TBC constructions essentially belong to different classes. We again believe that such knowledge will help cryptographers to design new schemes with post-quantum security.

## 4.2 Application to Other Schemes

We show more applications that online-offline MitM attack in Q1 model can be applied while the attack with  $O(n)$  complexity in Q2 model cannot be applied.

**McOE-X.** Fleischmann *et al.* proposed the McOE family of online authenticated encryption schemes [FFL12]. Their idea is to use a TBC to process each message block, where the tweak is an XOR of plaintext and ciphertext in the previous block. Let  $E_{K,W}$  be a TBC under a key  $K$  and a tweak  $W$ . Then, the

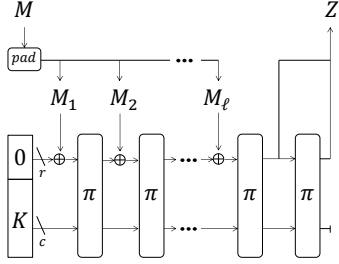


Fig. 8. Keyed Sponge

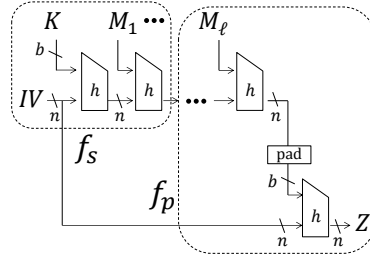


Fig. 9. H<sup>2</sup>MAC

ciphertext  $C_i$  of the  $i$ -th message block  $P_i$  is computed by

$$W_i \leftarrow P_{i-1} \oplus C_{i-1}, \quad C_i \leftarrow E_{K,W_i}(P_i).$$

Among several instances to compute  $E_{K,W}$ , McOE-X defines that  $E_{K,W} = E_{K \oplus W}$ . The construction is illustrated in Fig. 7.

Mendel *et al.* [MMRT12] showed that the key of McOE-X can be recovered with  $D \cdot T = N$ , by applying the meet-in-the-middle attack. According to the framework in Algorithm 1, we fix  $P_1$  to arbitrary chosen one, define  $f$  as the query of  $P_0 \| P_1$  and define  $f_p$  as the second block with guessing the key input.

By replacing the classical offline computation in Algorithm 1 with quantum one, the attack complexity is significantly improved as discussed in Sect 3.

**Keyed Sponge.** The sponge construction and its application to keyed usages were introduced by Bertoni *et al.* [BDPA08]. It is based on a permutation  $\pi : \{0, 1\}^b \mapsto \{0, 1\}^b$  and has two parameters; rate  $r$  and  $c$ , where  $r + c = b$ . The keyed sponge construction takes as input a key  $K \in \{0, 1\}^k$  where  $k < c$  and an arbitrary length message  $M$  to produce an  $n$ -bit tag  $Z$ . The  $b$ -bit state  $S$  is initialized to  $0^{b-k} \| K$ . The message  $M$  is separated into  $r$ -bit blocks as  $M_1 \| M_2 \| \dots \| M_\ell$  and is absorbed to the state block-by-block by  $S \leftarrow \pi(S \oplus M_i \| 0^c)$  for  $i = 1, 2, \dots, \ell$ . After all  $M_\ell$  is absorbed, it starts to squeeze the output by  $r$  bits from each state. Let  $\text{trunc}_r$  denote a truncation of  $r$  bits. When  $n$  is a multiple of  $r$ ,  $Z$  is generated by  $Z_i \leftarrow \text{trunc}_r(S), S \leftarrow \pi(S)$  for  $i = 1, 2, 3, \dots$ , until the size of  $Z = Z_1 \| Z_2 \| Z_3 \| \dots$  reaches  $n$  bits. See Fig. 8.

Liu and Liu [LL17a] found that the full-state keyed sponge ( $c = 0$  during the absorption) can be attacked with  $O(c)$  in Q2 model by applying Simon's algorithm. This paper analyzes more popular case; attacks in Q1 model on ordinary keyed sponge in Fig. 8. For example, KMAC standardized by NIST [NIS16] adopts the keyed sponge in a slightly different way; first initializes the state to a constant and processes  $K \| M$ . This difference does not impact to our attack.

With the classical environment, key recovery attack with a complexity  $2^{c/2}$  is known that works as follows. Here, we assume that the tag size  $n$  is 1-block.

1. Iterate the following two steps  $D$  times.
  - (a) Choose a random 1-block message  $M$  and query it to obtain  $Z_1$ .
  - (b) Query a 2-block message  $M' = M||Z_1$  to obtain  $Z'$ , and store it in  $L$ .
2. Make  $2^c/D$  guesses of  $c$ -bit capacity and compute  $S \leftarrow \pi(0^r||c)$  offline. Check whether  $\text{trunc}_r(S)$  matches one of the values in  $L$ .

Step 1b ensures that the rate of the state after  $Z_0$  is 0. Hence,  $L$  collects tag values for  $D$  randomly generated capacity values while the rate is 0. Step 2 corresponds to  $f_p$  in Algorithm 1. The match recovers the entire state value, thus key  $K$  can be recovered by backtracking the computation with  $\pi^{-1}$ .

As the procedure clearly suggests, this is an offline-online MitM and thus by replacing Step 2 with quantum algorithm, the keyed sponge construction can be attacked in Q1 model with complexity discussed in Sect 3.

**H<sup>2</sup>-MAC.** H<sup>2</sup>-MAC, a variant of HMAC without second key, was proposed by Yasuda [Yas09] with birth-bound security proved. It takes a key  $K$  and a message  $M = M_1||M_2||\dots||M_\ell$  as input and computes an  $n$ -bit MAC tag. Let  $h : \{0, 1\}^{b+n} \mapsto \{0, 1\}^n$  be a compression function. Let also  $IV$  and  $H_i$  be an  $n$ -bit constant and  $n$ -bit variable, respectively. The scheme first computes  $H_1 \leftarrow h(IV, K)$ , then iteratively process message blocks by  $H_{i+1} \leftarrow h(H_i, M_i)$  for  $i = 1, 2, \dots, \ell$ . Finally, the tag  $Z$  is computed by  $Z \leftarrow h(IV, \text{pad}(H_{\ell+1}))$  with a proper padding scheme “pad.” See Fig. 9 for its illustration.

The forgery attack in the classical setting was proposed by Liu *et al.* [LXS11] by online-offline MitM, which runs Algorithm 1 by defining  $f$  as the entire query and  $f_p$  as the offline computation from the second block with guessing  $H_2$ . As discussed in Sect 3, the quantum offline computation can be applied in Q1 model.

We stress that the same attack can be applied to other secret-prefix MACs [Tsu92], for example, LPMAC attacked by Sasaki [Sas12].

## 5 Attacks on the FX Construction in Q1 Model

This section, inspired by the Q2-model attack by Leander and May [LM17], gives a Q1-model attack on the FX construction by applying our general framework. The FX construction proposed by Killian and Rogaway [KR96, KR01] is a block cipher adopting a similar structure as the Even-Mansour construction, where its public random permutation is replaced with a block cipher. Let  $E$  be an  $n$ -bit block cipher with  $m$ -bit key. Then the FX construction using  $E$  is an  $n$ -bit block cipher with  $m + 2n$ -bit key, of which encryption of  $M$  is defined as

$$FX_{k_0, k_1, k_2}^E(M) = E_{k_0}(M \oplus k_1) \oplus k_2.$$

Since  $k_0$  is secret, the quantum key recovery attack against the Even-Mansour construction in [KM12] can no longer be used.

Leander and May cleverly combined Grover’s algorithm and Simon’s algorithm to make a quantum key recovery attack on the FX construction [LM17].



Their attack requires Q2 model. In short, it runs Simon's algorithm in parallel to recover  $k_1$  and runs Grover's algorithm to guess  $k_0$ . The time complexity is  $\tilde{O}(2^{m/2})$  by using  $O(m + n^2)$  qubits. Although the attack requires strong Q2 model, it costs exponential time owing to Grover's algorithm.

Here, we describe a *classical* key recovery attack against the FX construction with a cost of  $D$  queries and  $T$  computations satisfying  $D \cdot T = 2^{m+n}$ . Set  $\alpha := \lceil \frac{m}{n} \rceil$ . Let  $H : \{0, 1\}^m \times \{0, 1\}^n \rightarrow \{0, 1\}^{(\alpha+1)n}$  be a function defined by

$$H(k, x) := E_k(x) \oplus E_k(x \oplus 1) \parallel \cdots \parallel E_k(x) \oplus E_k(x \oplus (\alpha + 1)).$$

1. Choose  $D$  distinct values of message  $M^{(i)}$ , query  $M^{(i)}, M^{(i)} \oplus 1, \dots, M^{(i)} \oplus (\alpha + 1)$  to the encryption oracle, and obtain the corresponding ciphertexts  $C_0^{(i)}, C_1^{(i)}, \dots, C_{\alpha+1}^{(i)}$ . Store  $M^{(i)}$  in a table  $L$  along with  $C_0^{(i)} \oplus C_1^{(i)} \parallel \cdots \parallel C_{\alpha+1}^{(i)}$ . (Note that  $C_0^{(i)} \oplus C_1^{(i)} \parallel \cdots \parallel C_{\alpha+1}^{(i)} = H(k_0, M^{(i)} \oplus k_1)$  holds.)
2. Make exhaustive  $2^m$  guesses of  $k_0$ , denoted by  $k'$ ,  $T$  guesses of  $M \oplus k_1$ , and compute  $H(k', M \oplus k_1)$ . Check for a match of the value  $H(k', M \oplus k_1) = C_0 \oplus C_1 \parallel \cdots \parallel C_{\alpha+1}$  with  $L$ .

The above attack succeeds with high probability, since  $H$  is an almost random function, and  $H(k, x) = H(k', y) \Leftrightarrow (k, x) = (k', y)$  with high probability.

From a different point of view, the above attack procedure is essentially equal to running Algorithm 1 for  $N = 2^{m+n}$  by defining  $f$  and  $f_p$  as

$$\begin{aligned} f(M) &: \{0, 1\}^n \mapsto \{0, 1\}^n \triangleq H(k_0, M \oplus k_1), \\ f_p(k, x) &: \{0, 1\}^m \times \{0, 1\}^n \mapsto \{0, 1\}^n \triangleq H(k, x). \end{aligned}$$

While the strategy of attacks in Sect. 4 is simply to find a collision of two functions  $f$  and  $f_p$ , here we additionally need to guess  $m$ -bit key  $k_0$ . Moreover, there is a limitation that  $D \leq N/2^m$  since  $D$  cannot exceed  $2^n$ .

Next, we convert the above classical attack to a quantum attack only with classical online queries. We again consider three cases. Due to the condition  $D \leq N/2^m$ , we set upper limit of  $m$  for each case.

**Case 1a (exponential qubits, free-communication).** Assume  $m \leq 3n$ . The attack is performed at the balanced point;  $D = T = Q = M = 2^{\frac{(m+n)}{4}}$ .

**Case 1b (exponential qubits, realistic-communication).** Assume  $m \leq 5n/2$ . The attack is performed at the balanced point;  $D = T = Q = M = 2^{\frac{2(m+n)}{7}}$ .

**Case 1c (exponential qubits, any communication).** Assume  $m \leq 7n/3$ . The attack is performed at the balanced point;  $D = T = Q = M = 2^{\frac{3(m+n)}{10}}$ .

**Case 2 (non-exponential qubits).** Assume  $m \leq 4n/3$ . The attack is performed at the balanced point;  $D = T = 2^{\frac{3(m+n)}{7}}$ , using  $O(n)$  qubits and  $M = \tilde{O}(2^{\frac{m+n}{7}})$  classical memory.

**Applications to Two-Key Variants of LRW, XEX and XE.** The LRW construction [LRW11] is a TBC construction based on a block cipher proposed

by Liskov *et al.* It replaces whitening keys  $k_1, k_2$  of the FX construction with a single value  $h(w)$ , where  $w$  is a tweak and  $h$  is a secret function:  $LRW_{k_0, w}^E(M) = E_{k_0}(M \oplus h(w)) \oplus h(w)$ . Kaplan *et al.* [KLLN16a] proposed polynomial-time attacks in Q2 model against LRW, XEX and XE constructions.

Typically,  $h$  is dependent on the secret key  $k_0$ , though it may be of interest to consider a two-key variant of these constructions, i.e.  $h$  is independent from  $k_0$ . For the two-key variant, the structure becomes essentially the same as the FX construction, and thus we can apply the above attack in Q1 model with the same complexities.

## 6 Concluding Remarks

We presented quantum attacks against symmetric-key schemes in Q1 model, that has not received much attention. We converted the classical online-offline MitM attacks into quantum ones in Q1 model. The complexity depends on the number of qubits available and communication models. We derived the new tradeoff in four models. Some existing schemes claim BBB security on  $T$  by limiting the maximum number of  $D$  by following the classical tradeoff  $D \cdot T = N$ . Such claims are broken if adversary can access to quantum computers.

Efficiency of the quantum attacks depend on the constructions. Possible future directions are looking for more instances of  $\text{Class}_{\text{Exp}}^{\text{Q1}}$  and  $\text{Class}_{\text{Poly}}^{\text{Q2}}$ , or searching for a class of schemes with different cryptanalysis approaches.

## A Further Discussion on Quantum Computation Models

Regarding attack models for quantum computations, we received several comments from other researchers. Below we introduce two issues which are pointed out by them.

### A.1 Flying Qubits

As discussed in [BGG+13], if each qubit (or each small quantum processor) in a quantum hardware of size  $O(2^n)$  can communicate with  $O(n)$  qubits (or small quantum processors), then the hardware can simulate a hardware in free communicational model with the time overhead  $O(n^2)$ . Thus, if we can modify a quantum hardware in realistic communication model so that each qubit in the hardware can communicate with a little more qubits (which is called “flying qubits” in [BBG<sup>+</sup>13]), then the hardware can simulate free communication model with a small overhead. However, realization of “flying qubits” fully depends on future development of quantum hardware, and here we give no argument about realizability of it.

## A.2 Feasibility of Q2 Model

Q1 model is more realistic than Q2 model, though Q2 model should not be regarded as “non-realistic model.” In the main body of this paper, we described that Q2 model assumes that all the users implement algorithms on quantum computers and the network is communicated in the form of superposition. However, if an adversary attacks some kind of cryptosystems like “disk encryption” which is implemented on a quantum computer, then the notion of network becomes abstract. In addition, if white-box encryption algorithm is implemented on a quantum computer, then network becomes irrelevant.

Q2 model is simple and non-trivial. It ensures security in any intermediate scenario including hybrid ones like classical machines with quantum modules, where Q1 model could not really apply. We do not know how fast technologies on quantum computation and communication will develop, and using primitives not known to be secure in Q2 model would be challenging in the future.

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