Securing Lattice-Based KEMs with Code-Based Masking: A Theoretical Approach*

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Abstract. The recent technological advances in Post-Quantum Cryptography (PQC) raise the questions of robust implementations of new asymmetric cryptographic primitives in today's technology. This is the case for the lattice-based Module Lattice-Key Encapsulation Mechanism (ML-KEM) algorithm which is proposed by the NIST as the first standard for Key Encapsulation Mechanism (KEM), taking inspiration from CRYSTALS-Kyber. We have notably to make sure the ML-KEM implementation is resilient against physical attacks like Side-Channel Analysis (SCA) and Fault Injection Attacks (FIA). To reach this goal, we propose to adapt a masking countermeasure, more precisely the generic Direct Sum Masking method (DSM). By taking inspiration of a previous paper on AES, we extend the method to finite fields of characteristic prime other than 2 and even-length codes. We also investigate its application to Keccak, which is the hash-based function used in ML-KEM. We propose masked conversions and use cost-amortization to perform this hash. We provide the first masked implementation of ML-KEM with both SCA and FIA resilience able of correcting errors. Our FIA resilience allows for fault correction even within the multiplicative gadget. Finally, we adapt a polynomial evaluation method to compute masked polynomials with public coefficients over finite fields of characteristic different from 2.

Keywords: Post-Quantum Cryptography · ML-KEM · Side-Channel Analysis · Fault Injection Attack · Code-Based Masking · Conversion

1 Introduction

Since the dawn of cryptology, cryptanalysis has focused on the theoretical background used to perform cryptography. However, since the late 1990s and the publication of Kocher on Side-Channel Analysis [22], physical attacks try to take advantage of leakages or faults within the implementation rather than breaking the algorithm in itself. For this reason, the software and hardware designers of cryptographic primitives have to take into account this threat. The recent Post-Quantum Cryptographic algorithms are particularly targeted as their implementation still requires secure architectures and analysis to make them robust against physical attacks.
Quantum computing is an active research field which progresses monthly and the likelihood of

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an efficient quantum computer in the coming 30 years is almost certain [24]. Such a computer would be able to break current asymmetric cryptography primitives by taking advantage of the Shor quantum algorithm [34]. In order to assure a continuity in asymmetric cryptography, the NIST has launched a standardization process of PQC in 2016 [12] resulting in an international competition to create the future digital signature and KEM protocols which must be secure against quantum and classical computer. The end of the third and final round was announced the 5th of July 2022 [1] and 3 signatures and one KEM were selected while 4 other KEMs are heading for a final round to serve as alternatives in case of a cryptanalysis breakthrough [3].
The selection process focused first on quantum resilience, cost and performance, and then on the algorithm and its implementation. Most of the candidates claimed to be secure against time-based SCA as they provide constant time implementation and no conditional branching depending on sensitive data. But they do not make them secure against power-based SCA, like like Correlation Power Analysis (CPA), and Fault Injection Attacks (FIA). Even more, some of the candidates contain functions that can not be easily secured using generic defenses and require specific mechanisms to ensure their Side-Channel resilience.

1.1 Background on Masking

One of the most efficient and proven countermeasure against power-based SCA is masking [11]. The core idea is to avoid manipulating the sensitive data but instead *shares* of it that are reassembled once the computations are done. The shares being a combination of the sensitive data and a number of random variables called *masks*. Thus, an attacker can only observe leakages from the shares and might not be able to recover the secret data. The order of masking is determined by the number of independent shares used. A high-order masking means a better security against differential attacks but it generally comes at the cost of performances and space. Classical masking involves either arithmetical masking, where the random shares are subtracted or added to the secret, and boolean masking where the random shares are XORed with the secret. Conversions from one type of masking to the other do exist but have to be performed carefully. Here we use a variant of the Direct Sum Masking (DSM) introduced by Bringer et al. [8], namely a Code-Based Masking.
In this paper we focus on ML-KEM [27], inspired by CRYSTALS-Kyber [6], a post-quantum PKE/KEM. There have been already several publications on how to mask it on several platforms. Most noticeably, the work from Heinz et al. [19] proposed the first open-source implementation of a masked Kyber on microprocessor while relying on the work of Oder et al. [28] on previous lattice-based primitives. Bos et al. [7] proposed a masked software implementation of Kyber while Bronchain and Cassiers [9] proposed new gadgets for Arithmetic to Boolean (A2B) and B2A conversions and tested them in an open-source masked implementation of CRYSTALS-Kyber for microprocessors. When it comes to other platforms, Fritzmann et al. [15] worked on masking HW/SW codesign. Beckwith et al. [4] worked on a shared FPGA implementation of CRYSTALS-Kyber and CRYSTALS-Dilithium while masking the CRYSTALS-Kyber.

Remark 1. It is important to note that masking at the first order alone is not a sufficient defense. The PhD thesis work of Kalle Ngo [25] and master's thesis of Linus Backlund [3] proved that novel methods relying on deep-learning were able to thwart attempts of protecting Kyber with

[3] One of the KEMs of the 4th round fell victim to such a breakthrough in August 2022, stressing the need for alternative standards and hybridization

first order masking and/or shuffling. Hence, it is important to either mix defense mechanism (shuffling, blinding, hiding...) or use higher order masking. Also note that these attack methods have not been tested yet against Code-Based Masking.

When it comes to SCA and FIA resilient implementations, Pöppelmann and Heinz [20] proposed a combined fault and DPA protection for lattice-based cryptography. However, they only secured the arithmetic parts of the algorithm and were not able to correct faults, only detecting a few. Fault attacks against masked implementations of ML-KEM are a real concern, with work from Delvaux [13] and Kundu et al. [23] successfully using FIA to break masked implementations of ML-KEM on microcontroller. Thus, there is a clear need for solutions with better resilience to combined attacks using SCA and FIA. Our work aims not only to provide such a solution with better error detection, but also to add an error correcting capability that does not yet exist to this day in the state-of-the-art literature.

1.2 Our Contributions

The first contribution of this paper is related to the use of Code-Based Masking in a finite field of characteristic other than 2, which is more common in asymmetric cryptography compared to symmetric cryptography. In this work, we extend the masking method from [10] and prove it can also be applied to finite fields of characteristic other than 2. We also prove that we are able to correct several faults within the multiplicative gadget.

A second contribution of this work is the application of Code-Based Masking on a post-quantum cryptography primitive, namely ML-KEM. Not only do we propose a new any-order masking method for this algorithm, but we also propose one that has a built-in solution against FIA. There is currently no other solution in the State-of-the-Art capable of providing resilience against both SCA and FIA for the entirety of the algorithm. The only existing solution from Pöppelmann and Heinz [20] only covers the Number Theoretic Transform (NTT) and cannot correct errors.

A third contribution of this paper is the study of conversions between different Code-Based Maskings. While conversions between arithmetic and boolean maskings has been widely studied, there is currently no literature on conversions between Code-Based Maskings. We use this conversion alongside cost-amortization methods first introduced in [35] to improve the performances of our design.

The final contribution is the adaptation of the Paterson and Stockmeyer method of evaluating polynomials [29] to the evaluation of masked polynomials in a finite field of characterisitc different from 2. This method aims at reducing the amount of multiplication between two sensitive data when evaluating a polynomial, as such multiplications are noticeably more costly to perform than additions and scalar multiplications. We provide details on the exact complexity of performing such an evaluation in a secure manner.

The paper is structured as follows: in Section 2, we introduce notations and ML-KEM. In Section 3, we present our Code-Based Masking and our adaptation of the Horner method in Section 3.9. In Section 4, we explain how to adapt our masking method to ML-KEM. Finally, in Section 5, we discuss performances. Section 6 concludes our paper.

2 Preliminaries

2.1 Notations

Let $n \in \mathbb{N}^*$ the length of a code and d its dimension. We denote *odm* the masking order. We consider the finite field $\mathbb{F}_q$ with q a prime integer. Let ν a primitive element of $\mathbb{F}_q$. We assume that $n \neq 0 \mod q$ divides $q-1$, then we have

$$\omega = \nu^{\frac{q-1}{n}} \Rightarrow \omega^n = 1.$$

We must distinguish the case n odd and n even, then we set $d = \lfloor n/2 \rfloor$. For any vector $(u_0, ..., u_{n-1}) \in \mathbb{F}_q^n$, we can associate the polynomial $U(X) = u_0 + u_1 X + \ldots + u_{n-1}X^{n-1}$ and the discrete Fourier transform is defined by

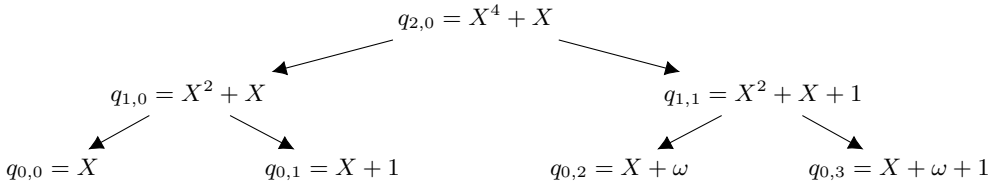
$$\texttt{DFT}_\omega(u_0, ..., u_{n-1}) = \left(\sum_{i=0}^{n-1} u_i \omega^{ij}\right)_{j\in[0\ldots n-1]} = \left(U(\omega^j)\right)_{j\in[0\cdots n-1]}.$$

Then the $\texttt{DFT}_\omega$ inverse is defined by:

$$\texttt{IDFT}_\omega(U(1), \ldots, U(\omega^{n-1})) = n^{-1}\left(\sum_{i=0}^{n-1} U(\omega^i)\omega^{-ij}\right)_{j\in[0\cdots n-1]} = (u_0, ..., u_{n-1}).$$

Remark 2. We have clearly made the hypothesis "n divides $q-1$" to find the condition of application of the Fast Fourier Transform but the procedure that we are going to develop obviously works by considering respectively $\texttt{DFT}_\omega$ and $\texttt{IDFT}_\omega$ as a Vandermonde multiplication and its inverse. The impact is just in term of complexity which cost n^2 multiplications over $\mathbb{F}_q$ against $\mathcal{O}(n \log n)$ for a $\texttt{DFT}_\omega$ in the most favourable cases.

For the ML-KEM algorithm, the considered $\texttt{DFT}_\omega$ is coming from methods described in [10,36]. It consists in building a tree (see section 4) of polynomials and to compute input vector interpreted as a polynomial modulo these polynomials. In particular cases, for example over finite fields of even characteristic and $n+1$ a power of two, the tree is composed of linearized polynomials (up to constant) which are sparse by nature [36]. For example in the figure below, the tree is defined over the finite field $\mathbb{F}_{2^4}$ with $n = 3$ and ω satisfying $\omega^3 = 1$:



Then, to calculate $\texttt{DFT}_\omega(C)$ with $C = (c_0, c_1, c_2, c_3)$ and $C(X) = c_0 + c_1X + c_2X^2 + c_3X^3$, we first compute $C_{1,0} = C(X) \mod q_{1,0}$ and $C_{1,1} = C(X) \mod q_{1,1}$, finally we get the result by performing $C_{1,0} \mod q_{0,0}$, $C_{1,0} \mod q_{0,1}$, $C_{1,1} \mod q_{0,2}$ and $C_{1,1} \mod q_{0,3}$. We show in the section 4 that we have the same principles with the ML-KEM parameters.

We have seen that the $\texttt{DFT}_\omega$ operation is equivalent to a Vandermonde matrix multiplication $V(\omega)$ with $V(\omega) = (\omega^{ij})_{i,j\in[\![0,2d-1]\!]}$ and

$$\texttt{DFT}_\omega(u_0, ..., u_{n-1}) = (u_0, ..., u_{n-1}) \times V(\omega).$$

For length n vectors of the form $(u_0, ..., u_{d-1}, 0, \ldots, 0)$, the $\texttt{DFT}_\omega$ operation corresponds to an encoding procedure by the Reed-Solomon code denoted: $\mathrm{RS}[n, d, n-d+1]$. A generator matrix of this code is given by the shortened matrix $(\omega^{ij})_{i\in[\![0,d-1]\!],j\in[\![0,2d-1]\!]}$. We recall some results that can be found in [30]: This error correcting code is classic, it is a MDS (maximum distance separable) code, which means that its minimal distance is optimal and equals $n-d+1$ where n is code length and d is its dimension. Among the good properties of these codes, we have, if R is a generator matrix of MDS code $\mathcal{R}S$ of length n and dimension d that:

- If $\mathcal{R}S$ is MDS, then $\mathcal{R}S^\perp$ is MDS where $\mathcal{R}S^\perp$ is the code defined by kernel(R);
- If $\mathcal{R}S$ is MDS, then all set of d columns are free.

We recall that any $[n, d, n-d+1]$-linear code can detect until $n-d$ errors.

2.2 ML-KEM

ML-KEM or FIPS 203 [27] is the first post-quantum KEM standard by the NIST. It is a slight modification of CRYSTALS-Kyber [2,6], a Module-Lattice-Based KEM which has been selected at the end of the 3^{rd} round of the NIST Standards Post-Quantum Competition in July 2022 [1]. It relies on several instances of the Module-LWE/LWR problems for its key generation, encapsulation and decapsulation procedures.
At its core, ML-KEM is a CPA-Secure PKE. To ensure CCA-level of security and a KEM status, a modified version of the Fujisaki-Okamoto Transform [16] is used.
ML-KEM has three levels of security, with different parameter sets (see [27] Table 2 page 33 or Table 5 in Appendix B). All sets use the same modulo, namely $q = 3329$. We also denote $\mathbb{Z}_q[X]/(X^{256}+1)$ by R_q and $S_\eta := \{P \in R_q, \|P\|_\infty \leq \eta\}$ a subset of R_q.
Amongst other notations defined by ML-KEM, we have $\lceil \rfloor$, the nearest integer with ties rounded up used in the compression functions, defined as follow:

$$Compress_q(\alpha, d_i) = \left\lceil \frac{2^{d_i}}{q} \cdot \alpha \right\rfloor \ mod\ 2^{d_i}, \alpha \in \mathbb{Z}_q \tag{1}$$

$$Decompress_q(\beta, d_i) = \left\lceil \frac{q}{2^{d_i}} \cdot \beta \right\rfloor, \beta \in \mathbb{F}_{2^{d_i}} \tag{2}$$

When applied to a vector of polynomials, those two functions are applied to each coefficient of each polynomial separately.

Remark 3. It is interesting to note that, for $d_i = 1$, the $Decompress_q$ function can be seen as a simple multiplication by a scalar, as the value β in the equation 2 can be extracted from the rounding as it can only be 0 or 1. Thus, we have $\lceil \frac{q}{2} \cdot \beta \rfloor = 1665 \cdot \beta$. This does not apply to $Compress_q$ (Equation 1) however.

Remark 4. It is also important to note that the compression functions are lossy:

$$\text{If } m' = Decompress_q(Compress_q(m, d_i), d_i), \text{ then } |m - m'| \leq \lceil q/2^{d_i+1} \rfloor \tag{3}$$

In ML-KEM, the distribution used for random sampling of sensitive values is the Center Binomial Distribution:

$$CBD_\eta(\beta) = \sum_{i=0}^{255}(\sum_{j=0}^{\eta}\beta_{2i\eta+j} - \sum_{j=0}^{\eta}\beta_{2i\eta+\eta+j})X^i \text{ with } \beta \in \{0,1\}^{512\eta} \tag{4}$$

This function is fed with a pseudo-random input β, generated by

$$PRF(seed, N) = SHAKE256(seed\|N). \tag{5}$$

$SHAKE256$ is a hash function described in the FIPS 202 standard [14]. The counter here allows seed reuse for the multiple values sampled during the PKE algorithms of ML-KEM. We use the $\hookleftarrow$ notation for sensitive value sampling. Keep in mind this is a call to Equation 4 where the input is $PRF(seed, N)$. The N counter is incremented after each call to CBD.
Non sensitive values are sampled a bit differently but this is out of the scope of this paper and we simply denote this sampling by $\leftarrow$.

We only present the KEM Decapsulation of ML-KEM here. For more details, we invite you to consult Appendix B where are described the PKE and KEM algorithms as well as figures showing the sensitiveness of the different operations within ML-KEM. You can also consult the reference paper of ML-KEM [27] for the algorithms and [31] (slide 76), [33] (slide 32-35) for the sensitiveness.

Algorithm 1 KEM Decapsulation

Input: Ciphertext $c = (c_u, c_v)$
Input: Secret Key $sk = (\vec{s}, pk, h = H(pk), z)$
Output: Shared key K
$\vec{u}, v = Decompress_q(c_u, d_u), Decompress_q(c_v, d_v)$
$m' := PKE.Decrypt(\vec{s}, \vec{u}, v)$ ▷ (see Appendix B, Figure 4 or [27])
$(K', seed') := G(m'\|h)$
$\tilde{K} = J(z\|c, 32)$
$\vec{u'}, v' := PKE.Encrypt(pk, m', seed')$ ▷ (see Appendix B, Figure 3 or [27])
$c' = (Compress_q(\vec{u'}, d_u), Compress_q(v', d_v))$
if $c \neq c'$ **then**
    $K' = \tilde{K}$
end if
return K'

Remark 5. H, G and J are all different Keccak [14] instances.

Remark 6. Keep in mind that PKE.Encrypt always results in the same outputs for a given set of inputs, as the seed for the sampling is one of the inputs. Thus, tampering with the ciphertexts results in tampering with the seed and leads to a completely different result out of the re-encapsulation.

If you are interested in knowing more about ML-KEM, we invite you to read the FIPS 203 (draft) standard from the NIST [27] and the CRYSTALS-Kyber specification papers [2,6].

3 Code-Based Masking, a DSM Example

The DSM encoding [8] consists in mapping the information x in a masked information (x, r) where r is a random mask such that:

$$x \mapsto (x, r) \mapsto x\mathbf{G} + r\mathbf{H}, \tag{6}$$

where $\mathbf{G}$ and $\mathbf{H}$ are two generator matrices of the two complementary codes $\mathcal{C}$ and $\mathcal{D}$ with $\mathcal{C} \cap \mathcal{D} = \{0\}$.
We propose to describe a masking method based on Reed Solomon encoding. This method is described in [10] for the characteristic 2 and odd length. We show in this section that it works for the characteristic prime q. We want to mask an information of size t and we assume that $\omega \in \mathbb{F}_q$ is a n-square root of unity and we consider a free family $u_0, u_1, u_2, ..., u_{d-1}$ of $\mathbb{F}_q^{d-1}$ with $u_i \neq \omega^j$ for any $0 < i \leq t-1$ and $0 < j \leq n-1$. We want now to mask the vector $\vec{x} = (x_0, \ldots, x_{t-1}) \in \mathbb{F}_q^t$ with $t < d$ and $d = \lfloor n/2 \rfloor$.

3.1 Encoding Procedure

First we pick randomly $\vec{r} = (r_t, r_{t+1}, \ldots, r_{d-1})$ in $\mathbb{F}_q^{d-t}$. It is well known that there exist a vector $\vec{a} = (a_0, a_1, \ldots, a_{d-1})$ and the associate polynomial $P_{\vec{x}}(X) = a_0 + a_1 X + \cdots + a_{d-1}X^{d-1}$ of degree at most $d-1$ that satisfies $P_{\vec{x}}(u_i) = x_i$ for $i \in \{0, \ldots, t-1\}$ and $P_{\vec{r}}(u_i) = r_i$ for $i \in \{t, \ldots, d-1\}$.
Let us denote the matrix $A \in \mathbb{F}_q^{(d)\times(d)}$, where $A_{i,j} = u_j^i$ for any i, j in $\{0, \ldots, d-1\}$. We have:

$$\vec{a} = (\vec{x} \mid \vec{r}) \times A^{-1}$$

The second step of our masking procedure consists in evaluating the $P_{\vec{r}}$ over the set $1, \omega, \omega^2, \ldots, \omega^{n-1}$. By construction, the second step of encoding consists in computing $\texttt{DFT}_\omega(a_0, \ldots, a_{d-1}, 0, \ldots, 0)$. Thus finally:

$$\texttt{Mask}(\vec{x}) = \texttt{DFT}_\omega(a_0, \ldots, a_{d-1}, 0, \ldots, 0).$$

Algorithm 2 `SeveralByteMasking` — Complexity : d^2

1: **Input:** a sensitive vector $\vec{x} \in \mathbb{F}_q^t$
2: **Output:** $\texttt{Mask}(\vec{x}) \in \mathbb{F}_q^n$
3: $\vec{r} \xleftarrow{\$} \mathbb{F}_q^{d-t}$
4: $\vec{a} \leftarrow (\vec{x} \mid \vec{r}) \times A^{-1}$ ▷ A^{-1} is a precomputed value
5: **return** $\texttt{DFT}_\omega(\vec{a} \mid \vec{0})$

We have presented a $\mathcal{O}(d^2)$ complexity encoding procedure, but we can do better with the following method: We can construct $P(X) = T_t(X) + R_t(X)$ by first picking randomly the polynomial $T_t(X) = a_t X^t + \cdots + a_{d-1}X^{d-1}$. Then we evaluate T_t over $1, u, \ldots, u_{t-1}$ which cost $t(d-1-t)$ multiplications over $\mathbb{F}_q^{d-1}$. We want now constructing $R_t(X) = a_0 + a_1 X + \ldots + a_{t-1}X^{t-1}$ which leads to solve the linear system

$$\underbrace{\begin{bmatrix} 1 & u_0 & \dots & u_0^{t-1} \\ & & \vdots & \\ 1 & u_i & \dots & u_i^{t-1} \\ & & \vdots & \\ 1 & u_{t-1} & \dots & u_{(t-1)}^{t-1} \end{bmatrix}}_{A^\top} \times \underbrace{\begin{bmatrix} a_0 \\ \vdots \\ a_i \\ \vdots \\ a_{t-1} \end{bmatrix}}_{\vec{a}\,'} = \underbrace{\begin{bmatrix} x_0 + T_t(u_0) \\ \vdots \\ x_i + T_t(u_i) \\ \vdots \\ x_{t-1} + T_t(u_{t-1}) \end{bmatrix}}_{\vec{y}\,'}.$$

The matrix inversion of A is a precomputation, thus, the calculation of:

$$\vec{a}\,' = (A^{-1})^\top \vec{y}\,'$$

costs $(t+1)^2$ multiplications over $\mathbb{F}_q$. Hence, the total cost of this encoding (including the $T_t(u_i)$ calculation) does not exceed $t(d-1-t)+t^2 = t(d-1)$ multiplications over $\mathbb{F}_q$. Again, the second step of encoding consists in computing $\texttt{DFT}_\omega(a_0, \dots, a_{d-1}, 0, \dots, 0)$ which can be achieved with not more than $(2d-1)\log(2d-1)$ multiplications over $\mathbb{F}_q$.

Remark 7. All the aforementioned operations are obviously reversible and we denote by `Unmask` the reverse operation. A tedious calculation gives a complexity in $t(d-1) + (2d-1)\log(2d-1)$ multiplications over $\mathbb{F}_q$.

3.2 Error Correcting Code Interpretation

We note that by construction, there exists an invertible matrix R that satisfies:

$$\begin{pmatrix} a_0 \\ \vdots \\ a_{t-1} \\ a_t \\ \vdots \\ a_{d-1} \end{pmatrix} = R \times \begin{pmatrix} x_0 \\ \vdots \\ x_{t-1} \\ P(u_t) \\ \vdots \\ P(u_{d-1}) \end{pmatrix}$$

We note that this `DFT` computation corresponds to the encoding in the Reed-Solomon code defined by the evaluation of $1, X, \dots, X^{d-1}$ over $1, \omega, \omega^2, ..., \omega^{n-1}$, and represented by a Vandermonde matrix $V(\omega)$. Hence, we get that

$$\texttt{Mask}(\vec{x}) = (\vec{x}, \vec{r})R^\top V(\omega) \quad (= \vec{x}G + \vec{r}H \text{ in the DSM model}).$$

We deduce that our masking algorithm corresponds to encoding procedure with a generalized Reed-Solomon code of minimal distance $n-d+1$, dimension d and length n.

3.3 Masking Addition, Subtraction and Scaling

Let us denote: $\vec{z} = \texttt{Mask}(x)$ and $\vec{z}\,' = \texttt{Mask}(x')$. The following properties are obviously satisfied:

- $\texttt{Mask}(x + x') = \vec{z} + \vec{z}\,'$,
- $\texttt{Mask}(x - x') = \vec{z} - \vec{z}\,'$,
- $\texttt{Mask}(\lambda x) = \lambda \cdot \vec{z}$ for any $\lambda \in \mathbb{F}_q$.

3.4 Masking the Multiplication

Let's denote: $\vec{z} = \texttt{Mask}(\vec{x})$ and $\vec{z}\,' = \texttt{Mask}(\vec{x}')$. Obviously,

$$\vec{z} \odot \vec{z}\,' = \texttt{DFT}_\omega(a_0, \ldots, a_{d-1}, 0, \ldots, 0) \odot \texttt{DFT}_\omega(a'_0, \ldots, a'_{d-1}, 0, \ldots, 0).$$

The polynomial obtained by performing $\texttt{DFT}_\omega^{-1}(\texttt{DFT}_\omega(P_{\vec{x}}) \times \texttt{DFT}_\omega(P_{\vec{x}'})) = P_{\vec{x}}(X) \times P_{\vec{x}'}(X) = C(X)$ is a $2d-2$ degree polynomial, which satisfies $C(u_i) = P_{\vec{x}}(u_i) \times P_{\vec{x}'}(u_i) = x_i x'_i$ for i in $\{0, \ldots, t-1\}$.
Now we have to propose a method that associates a degree $d-1$ polynomial $D(X)$ to $C(X)$. This polynomial must satisfies the same properties: $D(u_i) = C(u_i)$ for all $0 \leq i \leq t-1$.
The authors of [17] proposed the following construction for $t = 1$:

$$\begin{aligned} D(X) &= c_0 + c_1 X + \ldots + c_{d-1} X^{d-1} + u_0^{d-1}(c_d X + \ldots + c_{2d-2} X^{d-1}) \\ &= c_0 + (c_1 + u_0^{d-1} c_d) X + \cdots + (c_{d-1} + u_0^{d-1} c_{2d-2}) X^{d-1} \ . \end{aligned}$$

Obviously, in this case $D(u_0) = C(u_0) = x_0 x'_0$. This construction can be generalized and let:

$$U_j(X) = u_j^{d-1} \frac{(X-u_0)\cdots(X-u_{j-1})(X-u_{j+1})\cdots(X-u_{t-1})}{(u_j-u_0)\cdots(u_j-u_{j-1})(u_j-u_{j+1})\cdots(u_j-u_{t-1})}.$$

Hence, by construction, $U_j(u_j) = u_j^{d-1}$ and $U_j(u_i) = 0 \; \forall \, i \in \{0, \ldots, t-1\} \backslash \{j\}$ and $\deg(U_j(X)) = t-1$.
Then we set:

$$\begin{aligned} D(X) = c_0 + c_1 X + \cdots + c_{d-1} X^{d-1} &+ \textstyle\sum_{j=1}^{t} U_j(X)(c_d X + \cdots + c_{2d-t-1} X^{d-t}) \\ &+ \textstyle\sum_{j=1}^{t} U_j(X) \sum_{i=1}^{t-1} c_{2d-t-1+i} u_j^{d-t+i}. \end{aligned}$$

The degree $d-1$ polynomial $D(X)$ satisfies $D(u_i) = C(u_i) = x_i x'_i$ of $i \in \{0, \ldots, t-1\}$.
In order to build efficiently $\texttt{DFT}_\omega(D(X))$, let's write:

$$\begin{aligned} D(X) = c_0 + c_1 X + \cdots + c_{d-1} X^{d-1} &+ (c_d X + \cdots + c_{2d-t-1} X^{d-t}) \textstyle\sum_{j=1}^{t} U_j(X) \\ &+ \textstyle\sum_{i=1}^{t-1} c_{2d-t-1+i} \sum_{j=1}^{t} U_j(X) u_j^{d-t+i} \end{aligned}$$

Thus:

$$\begin{aligned} \texttt{DFT}_\omega(D(X)) = \quad & \texttt{DFT}_\omega(C(X)) \\ & - \texttt{DFT}_\omega(c_d X^d + \cdots + c_{2d-2} X^{2d-2}) \\ & + \texttt{DFT}_\omega(c_d X + \cdots + c_{2d-t-1} X^{d-t}) \odot \vec{u} \\ & + \textstyle\sum_{i=1}^{t-1} c_{2d-t-1+i} \cdot G_i. \\ = \quad & \texttt{Mask}(\vec{x} \odot \vec{x}') \ , \end{aligned}$$

where: $G_i = \texttt{DFT}_\omega(\sum_{j=1}^{t} U_j(X) u_j^{d-t+i})$ for $i \in \{1, \ldots, t-1\}$ and $\vec{u} = \texttt{DFT}_\omega(\sum_{j=1}^{t} U_j(X))$ are precomputed values, and $c_d, \ldots, c_{2d-2} = \texttt{extractLastCoefficients}(\vec{z} \odot \vec{z}\,')$. We remind that *extractLastCoefficients* has been defined in [10]:

We have seen that $\texttt{IDFT}_\omega(\vec{z} \odot \vec{z}\,') = (c_i)_{i\in\{0,\dots,n-1\}} = C(X)$, then if we denote $\vec{y} = \vec{z} \odot \vec{z}\,'$, by definition $c_{j+d} = \sum_{i=0}^{n-1} y_i \omega^{-i(j+d)} = \sum_{i=0}^{n-1} (y_i \omega^{-id}) \omega^{-ij}\ \forall\ 0 \leq j \leq d-1$ and $(c_{j+d})_{j\in\{0,\dots,d-1\}}$ is obtained from $\texttt{IDFT}\left((y_i\omega^{-id})_{0\leq i\leq n-1}\right)$.

If we denote $\phi(C,\omega) = -\texttt{DFT}_\omega(c_{d+1}X^d + \cdots + c_{2d-2}X^{2d-2}) + \texttt{DFT}_\omega(c_dX + \cdots + c_{2d-t-1}X^{d-t}) \odot \vec{u} + \sum_{i=1}^{t-1} c_{2d-t-1+i} \cdot G_i$ where C represents the $d-1$ last coefficients of $\texttt{IDFT}(\texttt{Mask}(\vec{x}) \odot \texttt{Mask}(\vec{x}'))$, then we get that

$$\texttt{Mask}(\vec{x} \odot \vec{x}') = \texttt{Mask}(\vec{x}) \odot \texttt{Mask}(\vec{x}') + \phi(C,\omega)$$

3.5 Security Proof

One way of proving the security of an implementation is through a theoretical approach using formal security models. Such a model for SCA resilience is the d-probing security model, first introduced by Ishai et al. [21]. To prove the security of our design in this model, we first introduce some definitions.

Definition 1. *[35] (Private circuit compiler [21]). A private circuit compiler for a circuit C with input in $\mathbb{F}_q^\phi$ and output in $\mathbb{F}_q^{\phi'}$ is defined by a triple (I,T,O) where*

- *$I : \mathbb{F}_q^\phi \to \mathbb{F}_q^\varphi$ is an input encoder that randomly maps the input in $\mathbb{F}_q^\phi$ to the input sharing in $\mathbb{F}_q^\varphi$.*
- *T is a circuit transformation whose input is circuit C, and output is a randomized circuit C', whose input is the input sharing $\mathbb{F}_q^\varphi$, and the output in $\mathbb{F}_q^{\varphi'}$ is called outputsharing.*
- *$O : \mathbb{F}_q^{\varphi'} \to \mathbb{F}_q^{\phi'}$ is a decoder that maps the output sharing in $\mathbb{F}_q^{\varphi'}$ to the output of C in $\mathbb{F}_q^{\phi'}$.*

We say that (I,T,O) is a private circuit compiler and C' is a $d-$private circuit (or $d-$probing secure, where d is called the security order) if the following requirements hold:

- *Correctness: for any input $a \in \mathbb{F}_q^\phi$, $Pr[O(C'(I(a)))] = C(a) = 1$.*
- *Privacy: for any input $a \in \mathbb{F}_q^\phi$ and any set of probes $\mathcal{P}$ such that $|\mathcal{P}| \leq d$, $C'_{\mathcal{P}}(I(a))$ are independent of the input a.*

Definition 2. *[35]* (Encoder, Codeword, Sharing, Valid Sharing and Share). *An encoder, Enc : $\mathbb{F}_{q^k} \to \mathbb{F}_{q^n}$ is a probabilistic algorithm that maps a vector in $\mathbb{F}_{q^k}$ to a vector in $\mathbb{F}_{q^n}$. The latter vector in $\mathbb{F}_{q^n}$ is called codeword or valid sharing. A sharing is a vector in $\mathbb{F}_{q^n}$, and the elements of a codeword or sharing are called shares. Moreover, an encoder is called $d-$private encoder if and only if the joint distribution of any d shares are independent of the input of the encoder, where the probability is over the random coins from the encoder.*

We propose to show in the following paragraphs that our method corresponds to $(d-t)$-probing order for the security with a discussion around more sophisticate security models. For fault injection resilience, we assume that we are in the random fault model with a reasonable number of injected faults.

SCA Resilience We showed that this construction is identical to the original construction of [10] up to the sign and up to the parity of n. The proof is coming from the property of this masking that can be written as a DSM encoding [8]:

$$x \mapsto (x, r) \mapsto xG + rH.$$

For this model, the masking order is provided by the minimal distance of the code $H^{\perp}$. It is proven in [10] that the probing order depends of code $H^{\perp}$ which can be MDS (i.e $d_{min} = d+2-t$) or AMDS (i.e $d_{min} = d+1-t$). We show in the subsection 4 (with $t = 1$) that we are in the MDS favourable case. It means that *odm* equals $d+1-t$, i.e $d_{min} = d+2-t$. The gadget multiplication is also in this case $(d+1-t)$-probing secure due to the MDS property of H. Let's remind the main steps of the proof:
We can rewrite our encoding procedure as follows:

$$\texttt{Mask}(\vec{x}) = \big((\vec{x}, \vec{0}) \times A^{-1} \times R\big) + \big((\vec{0}, \vec{r}) \times A^{-1} \times R\big) = \vec{x}G + \vec{r}H,$$

where $G = (Id_t, 0)A^{-1}R$ and $H = (0, Id_{d-t})A^{-1}R$.

Proposition 1. *The masking operation* $\texttt{Mask}(\vec{x})$ *is a generic encoder.*

Proof. We have seen that $\texttt{Mask}(\vec{x}) = \vec{x}G+\vec{r}H$. By construction, $\text{rank}(G) = t$ and $\text{rank}(H) = d-t$. If we denote $\mathcal{C}_G$, $\mathcal{C}_H$ and $\mathcal{C}_{H^{\perp}}$ the codes respectively generated by the generator matrix G, H and the kernel of H, then $\mathcal{C}_G \cap \mathcal{C}_H = \{0\}$. If we denote $B = \begin{pmatrix} G \\ H \end{pmatrix}$, then we have:

$$\texttt{Mask}(\vec{x}) = (\vec{x}, \vec{r}) \times B$$

and matrix B satisfies the definition of a generic encoder denoted enc_B.

If we denote by d' the minimal distance of $\mathcal{C}_{H^{\perp}}$: $d' = d_{min}(\mathcal{C}_{H^{\perp}})$, then, as explained in [35], a direct consequence is that the encoding procedure enc_B is d'-private. Our task consists now in evaluating d'.

Theorem 1. *Let an integer* t, $1 \leq t \leq d-1$, *a Vandermonde matrix* A *of the form* $(u_j^i)_{i,j\in[\![0,d-1]\!]}$ *with* $u_i \neq u_j$. *Let* R *the generator matrix of the Reed-Solomon code* $RS[n, d, n-d+1]$ *of the form* $(\omega^{ij})_{i\in[\![0,d-1]\!],j\in[\![0,n-1]\!]}$. *We denote*

$$H = (0_t, Id_{d-t}) \times A^{-1} \times R.$$

Let $\mathcal{C}_H$ *the code generated by* H, *then,* $d_{min}(\mathcal{C}_{H^{\perp}})$ *the minimal distance of* $\mathcal{C}_{H^{\perp}}$ *satisfies*

$$d - t \leq d_{min}(\mathcal{C}_{H^{\perp}}) \leq d + 1 - t.$$

Proof. We denote by K the matrix which corresponds to the last $d-t$ rows of A^{-1}, then

$$H = (0_t, Id_{d-t})A^{-1}R = K \times R$$

where R is a generator matrix of $RS[n, d, n-d+1]$. By construction, H is $(d-t) \times n$ matrix since $(0_t, Id_{d-t})A^{-1}$ .is a full rank matrix.

By construction, the parity check matrix of R that we can denote T is a generator matrix of the Reed-Solomon code $RS[n, d, n-d+1]$ and we have $H^tT = 0$. Hence, $H^tT = K \times .R \times {}^tT = 0$ and the subspace generated by the rows of T are included in the kernel of H.

Study of K: We remind that $K = (0_t, Id_{d-t})A^{-1}$. First of all, A^{-1} is a Reed-Solomon generator matrix as any invertible square matrix because it is equivalent (up to an invertible matrix) to a Reed-Solomon code. Hence K is a generator matrix of a sub code of a RS$[d, d]$ code. We would like to determine now the dual code of K and we observe the equation $A^{-1} \times A = Id_d$. By setting

$$A^{-1} = \begin{pmatrix} K'_{t\times(d)} \\ K_{(d-t)\times(d)} \end{pmatrix} \text{ and } A = \left(B_{d\times t}, B'_{(d)\times(d-t)}\right),$$

we get that

$$\begin{pmatrix} K'_{t\times(d)} \\ K_{(d-t)\times(d)} \end{pmatrix} \times \left(B_{(d)\times t}, B'_{(d)\times(d-t)}\right) = \begin{pmatrix} Id_t & 0_{t\times(d-t)} \\ 0_{(d-t)\times t} & Id_{d-t} \end{pmatrix}.$$

We deduce that $K_{(d-t)\times d} \times B_{d\times t} = 0_{(d-t)\times t}$ and we know that

$$K = K_{(d-t)\times d} \text{ and } B = Kernel(K) = B_{d\times t} = (u_i^j)_{i\in[\![0..d-1]\!],j\in[\![0..t-1]\!]}.$$

By construction ${}^t(B_{d\times t}) = {}^tB$ is a generator matrix of a code generated by the polynomials $1, X, X^2, \ldots, X^{t-1}$ defined over the set $u_0, \ldots, u_{d-1}$: this is a Reed-Solomon code $RS[d, t, d+1-t]$ of minimal distance $d+1-t$. We deduce that the encoder $(x, r) \mapsto (x, r)A^{-1}$ is a generic encoder of probing order $d-t$.

We want now to describe the kernel of $K \times R$. We can repeat the same construction for R. If we denote V_ω the Vandermonde matrix associated to DFT_ω:

$$V_\omega \times V_\omega^{-1} = \begin{pmatrix} R_{d\times n} \\ R'_{(n-d)\times n} \end{pmatrix} \times \left(Ri_{n\times d}, Ri'_{n\times(n-d)}\right), \text{ and}$$

$$V_\omega \times V_\omega^{-1} = \begin{pmatrix} Id_d & 0_{d\times(n-d)} \\ 0_{(n-d)\times d} & Id_{n-d} \end{pmatrix}.$$

We deduce that $R_{d\times n} \times Ri_{n\times d} = Id_d$ with $R = R_{d\times n}$. The matrix V_ω^{-1} is Vandermonde matrix associated to $IDFT_\omega$, then $R_i = Ri_{n\times d} = (\omega^{-ij})_{i\in[\![0..n-1]\!],j\in[\![0..d-1]\!]}$. We remark that $K \times R \times {}^tT = 0$ and $K \times R \times R_i \times B = K \times Id \times H = 0$. Hence we can build a vector space included in the kernel of $H = K \times R$ with T which is the generator matrix of a RS$[n, d]$ code and $D = {}^tB \times {}^tR_i$.

We note that ${}^tR_i = (\omega^{(n-i-1)j})_{i\in[\![0..d-1]\!],j\in[\![0..n-1]\!]}$ is a generator matrix of a code generated by d polynomials of degree more than $n-d$. Then ${}^tB = (u_i^j)_{i\in[\![0..t-1]\!],j\in[\![0..d-1]\!]}$. Hence the code generated by D is an evaluation code generated by t independent polynomials of degree more than d whereas T is a generator matrix of a code generated by $d-1$ polynomials of degree strictly less than d, then these two codes are linearly independent and we deduce that we have built the kernel of H. We have now to evaluate the minimal distance of this code $(T \cup D)$.

Hence, we have

$$D = {}^tB \times {}^tRi = \left(\sum_{k=0}^{d-1} u_i^k \omega^{(n-1-k)j} \right)_{i \in [\![0..t-1]\!], j \in [\![0..n-1]\!]}.$$

Let

$$D_{i,j} = \sum_{k=0}^{d-1} u_i^k \omega^{(n-1-k)j} = \omega^{(n-d)j} \sum_{k=0}^{d-1} u_i^k \omega^{(d-1-k)j}$$

and

$$D_{i,j} = \omega^{(n-d)j} \sum_{k=0}^{d-1} u_i^{(d-1-k)} \omega^{kj}.$$

Then

$$D_{i,j} = u_i^{d-1} \omega^{(n-d)j} \sum_{k=0}^{d-1} \left(\frac{\omega^j}{u_i} \right)^k = u_i^{d-1} \omega^{(n-d)j} \frac{1 - \left(\frac{\omega^j}{u_i} \right)^d}{1 - \frac{\omega^j}{u_i}}.$$

For $i = 0$ (i.e $t = 1$), it means that the vector D_0 corresponds to the evaluation of the fraction

$$\frac{u_0^d X^{n-d} - X^{n-2d}}{u_0 - X} \tag{7}$$

over $\{1, \omega, \ldots, \omega^{n-1}\}$ and we are looking for a degree $d-1$ polynomial $P(X)$ that cancels the maximum of positions of D_0, i.e. such that $Q(X) = (X - u_0)P(X) - X^{n-2d} + u_0^d X^{n-d}$ admits the maximum of zeros. We remark that degree$(Q) \leq d$, then the number of zero is less than $n - d$ which is equivalent to a minimal distance greater than d. In the same time, the Singleton bound states that $d_{\min}(T \cup D_0) \leq d + 1$. We deduce that for $D = D_0$,

$$d + 1 - t \leq d_{\min}(T \cup D) \leq d + 2 - t.$$

For $t = 2$, the Singleton bound states that $d_{\min}(T \cup D_0 \cup D_1) \leq d + 2 - t$. We want to evaluate now the minimal distance of a codeword built from a linear combination of $D_{0,j}$, $D_{1,j}$ and T. It means that for a fixed element $\theta \in \mathbf{F}_q$ we are looking for a degree d polynomial $P(X)$ such that for a maximum of input we have

$$P(X) = \frac{u_0^d X^{n-d} - X^{n-2d}}{u_0 - X} + \theta \frac{u_1^d X^{n-d} - X^{n-2d}}{u_1 - X}.$$

This is equivalent of studying the number of zero of the function $T(X) = (X - u_0)(X - u_1)P(X) + (X - u_1)(u_0^d X^{n-d} - X^{n-2d}) + \theta(X - u_0)(u_1^d X^{n-d} - X^{n-2d})$. The degree of $T(X)$ is less or equal to $n - d + 1$ then $T(X)$ has $n - d + 1$ roots maximum which is equivalent to a minimal distance greater than $n - (n - d + 1) = d - 1$ and we deduce:

$$d + 1 - t \leq d_{\min}(T \cup D) \leq d + 2 - t.$$

By induction we have that for any t, $d + 1 - t \leq d' \leq d + 2 - t$ and the probing security order is between $d - t$ and $d + 1 - t$, thus we have demonstrated the theorem 1.

Gadget multiplication resilience The steps of the proof can be summarized by the hypothesis that has been done in [18] and proven in [10]:

Theorem 2 (Hypothesis (FFT Probing Security)). *The circuits processing*

$$DFT_\omega(x\|0) \mapsto r \text{ and } DFT_\omega^{-1}$$

are t_n^{DFT}-probing secure with $t_n^{DFT} \geq d - t$.

Proof. In fact the application $DFT_\omega(\vec{x}\|0) \mapsto r$ corresponds exactly to our masking operation $\texttt{Mask}(\vec{x}) = (\vec{x}, \vec{r}) \times A^{-1} \times R$ except that A is more general than simply a Matrix of the form $(\alpha^{ij})_{i,j}$. We deduce that $t_n^{DFT} \geq d - t$ in this case since it corresponds to the theorem 1.
Regarding $\texttt{DFT}_\omega^{-1} : u' \mapsto tt$: in fact, $u' = refresh(\texttt{Mask}(\vec{x}) * \texttt{Mask}(\vec{y}))$ where $*$ represents here the multiplication term by term and not the mask multiplication. In our masking, by definition, we have $u' = \texttt{Mask}(\vec{0}) + \texttt{Mask}(\vec{x}) * \texttt{Mask}(\vec{y})$. $\texttt{Mask}(\vec{0}) = \vec{r}H$ where $\vec{r}$ is a $d + 1 - t$ dimension vector which is random, then building $\vec{r}$ requires at least $d + 1 - t$ positions from the vector $\vec{r}H$. By construction, $\texttt{DFT}_\omega^{-1}(\texttt{Mask}(\vec{0})) = (a_0(r), a_1(r), \ldots, a_d(r), 0, \ldots, 0) = (0, r)A^{-1}$. Then $\texttt{DFT}_\omega^{-1}(\texttt{Mask}(\vec{x}) * \texttt{Mask}(\vec{y})) = (c_0, \ldots, c_{2d})$. We deduce that:

$$tt = (c_0 + a_0(r), c_1 + a_1(r), \ldots, c_d + a_d(r), c_{d+1}, \ldots, c_{2d}).$$

We prove below this proof that we cannot construct a sensitive information from $(c_{d+1}, \ldots, c_{2d})$. The coefficients a_i of the vector $(c_0 + a_0(r), c_1 + a_1(r), \ldots, c_d + a_d(r))$ depends linearly of r. We have already proven that the encoder $(x, r)A^{-1}$ is $d + 1 - t$ probing secured, thus getting information from $(c_0 + a_0(r), c_1 + a_1(r), \ldots, c_d + a_d(r))$ requires to capture at least $d + 1 - t$ positions. We deduce the final result, the hypothesis is correct with $t_n^{DFT} = d - t$.

Then, due to the previous demonstrated hypothesis, the following lemma is deduced in [18]:

Lemma 1. *[18] The circuit processing $(\texttt{Mask}(x), \texttt{Mask}(y)) \mapsto u = \texttt{Mask}(x) * \texttt{Mask}(y)$ is at least $(d - t)$-probing secure.*

For simplicity, we denote $d_n =$.To conclude about the security of the gadget multiplication, we remind that the Mask multiplication (gadget) is obtained from the following computation:

$$\begin{aligned}
\texttt{DFT}_\omega(D(X)) = \quad & \texttt{DFT}_\omega(C(X)) \\
& + \texttt{DFT}_\omega(c_d X^d + \cdots + c_{2d-2} X^{2d-2}) \\
& + \texttt{DFT}_\omega\left((c_d X + \cdots + c_{2d-t-1} X^{d-t}) * \vec{U}\right) \\
& + \textstyle\sum_{i=1}^{t-1} c_{2-t+i} \cdot G_i \\
= \quad & \texttt{Mask}(\vec{x} * \vec{x}\,')
\end{aligned}$$

where $G_i = \texttt{DFT}_\omega(\sum_{j=0}^{t-1} U_j(X) u_j^{d-t+i})$ for $i \in \{1, \ldots, t-1\}$ and $\vec{U} = \sum_{j=1}^{t} U_j(X)$ are precomputed values. Then, it is clear that the computation of $\texttt{DFT}_\omega(c_d X^d + \cdots + c_{2d-2} X^{2d-2})$, $\texttt{DFT}_\omega\left((c_d X + \cdots + c_{2d-t-1} X^{d-t}) * \vec{U}\right)$ and $\sum_{i=1}^{t-1} c_{2d-t-1+i} \cdot G_i$ involves only the variables $c_d, \ldots, c_{2d-t-1}$ related to the sensitive information. Hence, the weakest side is obtained with the vector

$$(c_d, \ldots, c_{2d-2}) = \texttt{ExtractLastCoefficients}(\vec{z} * \vec{z}').$$

Then the question is: can we get information from $d-t$ position of the vector $(c_{d+1},\dots,c_{2d})$. Our claim is that our gadget is at least $d-t$ probing secure, then we must assume that in the model of attack, maximum $d-t$ values can be guessed from some measures. From $d-t$ pieces of knowledge from the vector $(c_d,\dots,c_{2d-2})$, $x=\texttt{unmask}(z)$ and $x'=\texttt{unmask}(z')$ cannot be reconstructed: if an attacker has access to the following system of equations

$$\begin{cases} c_{2d-2} &= a_{d-1}a'_{d-1} \\ c_{2d-3} &= a_{d-2}a'_{d-1}+a_{d-1}a'_{d-2} \\ c_{2d-4} &= a_{d-3}a'_{d-1}+a'_{d-3}a_{d-1}+a'_{d-2}a_{d-2} \\ &\vdots \\ c_{2d-2-k} &= \sum_{i=0}^{k} a_{d-i-1}a'_{d-(k-i)-1} \\ &\vdots \\ c_d &= \sum_{i=0}^{d-1} a_{d-i-1}a'_{i+1}. \end{cases}$$

We can evaluate the number of potential solutions for $(a_i)_{i\in[\![0..d-1]\!]}$: by assuming that $c_{2d}\neq 0$, then the equation $c_{2d-2}=a_{d-1}a'_{d-1}$ admits $q-1$ solutions. If $c_{2d}=0$, then $a_da'_d$ admits q solutions. By setting $a_d\neq 0$ and $a'_d\neq 0$ we get the equation $c_{2d-3}=a_{d-2}a'_{d-1}+a_{d-1}a'_{d-2}$ admits q solutions. By induction, we get the same property at any step $k\leq d-1$. Thus totally this system admits at least $q\times q^{1+2+\dots+d-2}>q^{1+\frac{(d-2)^2}{2}}$ solutions for the set of d variables a_i. Hence, this system of equation does not give information about the a_i with $i\in[\![0..d-1]\!]$. By symmetry, we get the same property for a'_i, but we know according to the proof of the theorem 1 that we must get at least $d-t$ values of a_i to expect reconstructing the encoded sensitive information, thus we conclude that the gadget multiplication is at least $d-t$ probing secured.

Remark 8. While d-probing security guarantees that a gadget is secure, it does not prove that the composition of d-probing gadgets are secure. Thus, for more sophisticate security models like IOS, SNI or t-region probing, we refer to [18,5] for a similar method involving the DFT or Vandermonde computation and [35] which gives proves for a Code-Based Masking. According to the proof of [10], the masking method of this paper satisfies the main properties of [18,35,5]. However proving rigorously that the presented masking method is $(d-t)$-SNI is out of the scope of this paper. Note however that we believe that adding a refresh after each gadget call should be sufficient. While not optimal, this strategy enables gadget composition security. A refresh is performed by adding a mask of 0.

Faults Injection By construction, everywhere a codeword C is present, the integrity of C can be checked by computing the syndrome of C, i.e by computing $\texttt{IDFT}(C)=c$ and checking that c corresponds to a degree $d-1$ polynomial. If not, it means that some errors have been introduced. According to our parameters, C belongs to the Reed-Solomon code RS$[2d,d,d+1]$ and can detect d errors.

The difficult question concerns the gadget multiplication between two vectors $\texttt{Mask}(x)$ and $\texttt{Mask}(y)$. For this computation we must perform $\texttt{Mask}(x)\odot\texttt{Mask}(y)$ where "$\odot$" corresponds to the multiplication term by term. We showed that $\texttt{Mask}(x)\odot\texttt{Mask}(y)=\texttt{DFT}_\omega(C(X))$ where $C(X)$ is a degree $2d-2$ polynomial. However, our codewords have length $n=2d$ and $\texttt{DFT}_\omega(C(X))\in$ RS$[2d,2d-1,2]$. Hence we can check with a syndrome calculation (i.e $\texttt{IDFT}(\texttt{Mask}(x)\odot\texttt{Mask}(y))$)

that $C(X)$ is degree $2d-2$ polynomial. If not, it means that at least one error has been injected. Then an attacker may inject faults on the vector $(c_d, \ldots, c_{2d-2})$, however in this case we remind that

$$\texttt{Mask}(\vec{x} \odot \vec{y}) = \texttt{Mask}(\vec{x}) \odot \texttt{Mask}(\vec{y}) + \phi((c_d, \ldots, c_{2d-2}), \omega),$$

with $\texttt{Mask}(\vec{x} \odot \vec{y})$ and $\texttt{Mask}(\vec{x}) \odot \texttt{Mask}(\vec{y})$ that can be verified, then the injected fault is detected.

We showed here that our gadget supports one fault injection. As shown in [10], to support more injections we could modify our encoder by reducing the dimension of $\vec{r}$. As a direct consequence, the degree of the resulting polynomial $C(X)$ from a multiplication has a degree strictly less than $2d-2$ and more errors can be detected. In the same time this modification decreases the security probing order, thus, it is a question of balance. However, a trick used in [5] to reduce the degree of the polynomials which is compliant with our scheme while the number of information symbols $t < d/2$: indeed, we can set $P_x(X) = \texttt{IDFT}_\omega(\texttt{Mask}(\vec{x})) = P_0(X) + X^{d/2}P_1(X)$ and $P_{x'}(X) = \texttt{IDFT}_\omega(\texttt{Mask}(\vec{x'})) = P'_0(X) + X^{d/2}P'_1(X)$. The P_i and P'_i can be computed because the encoder $x \mapsto (x, r)A^{-1}$ is $d+1-t$ probing secure. We have:

$$P_{x'}(X)P_x(X) = P_0(X)P'_0(X) + X^{d/2}\left(P'_0(X)P_1(X) + P_0(X)P'_1(X)\right) + X^d P_1(X)P'_1(X),$$

with $T(X) = P'_0(X)P_1(X) + P_0(X)P'_1(X) = T_0(X) + x^{d/2}T_1(X)$. Then we observe that d errors can be detected on the vectors $\vec{C_0} = \texttt{DFT}_\omega(P_0(X)P'_0(X))$, $\vec{C_1} = \texttt{DFT}_\omega(X^{d/2}T_0(X))$, $\vec{C_2} = \texttt{DFT}_\omega(X^d T_1(X))$ and $\vec{C_3} = \texttt{DFT}_\omega(X^d P_1(X)P'_1(X))$, just by remarking that at least d identified coefficients must be null for each corresponding polynomial. Finally our cost amortization trick can be applied for each vectors $\vec{C_i}$, $i \in \{0, 1, 2, 3\}$ in order to get 4 degree d polynomials D_0, D_1, D_2 and D_3 that satisfies $D = D_0 + D_1 + D_2 + D_3$. Hence we avoid the degree $2d$ polynomial in $C(X)$ and consequently, d errors can be detected by applying our detection method.

Remark 9. We assume that faults are random and do not directly affect the syndrome computations that detect them. We could also assume that a tamper-resistant component performs the fault detection task.

3.6 Complexity

It is shown in [10] that the complexity of the multiplication is quasi-linear as it requires $\mathcal{O}(4d\log(2d))$ multiplications in $\mathbb{F}_q$. This is a standard complexity, but regarding real performances and applicability a study must be performed over different platforms (hardware and software) with different strategies: parallel computation, pipeline, bitslicing... From now on, we set $t = 1$ as it seems us difficult to take benefit of several symbol encoding due to the design of CRYSTALS-Kyber. Taking $t > 1$ may be interesting if we manage to compute simultaneously several KEM computation but it affects the probing security order.

In terms of randomness, we require $d - t$ random symbols to mask t sensitive ones. As the multiplication includes a refresh done by adding the mask of $\vec{0}$, it requires another batch of $d-t$ random symbols.

3.7 Masking a Polynomial Function

By induction, we can compute $\texttt{Mask}(x^n)$ for an arbitrary n value in $\mathbb{Z}$. We can write $\texttt{Mask}(x^n) = \texttt{Mask}(x^{n-1} * x) = \texttt{Mask}(x) \odot \texttt{Mask}(x^{n-1}) + \phi(C, \omega)$ thus if we assume that $\texttt{Mask}(x^{n-1})$ has been computed, then the property is demonstrated. The same proof holds for Horner (polynomial evaluation) algorithm.

3.8 Masking a Formal Polynomial

Let $s(X), u(X) \in (\mathbb{F}_q^{(deg)}[X])^2$. We define

$$\texttt{Mask}(s(X)) = \sum_{i=0}^{deg} \texttt{Mask}(s_i) X^i \tag{8}$$

Also, we have $\texttt{Mask}(s(X)) \odot \texttt{Mask}(u(X)) = \texttt{Mask}(s(X) * u(X))$. We deduce that we can perform $\texttt{Mask}(s(X) * u(X))$ using the Fast Fourier Transform based on Cooley-Turkey algorithm with a n-root of unity since the scalar multiplication is well defined over the linear codes.

3.9 Adapting the Horner method to masked polynomials

The Horner method evaluates a polynomial of degree *deg* in *deg* multiplications and *deg* additions. However, masked multiplications between two sensitive data, that we denote as sensitive multiplications, are costly. On the other hand, masked multiplications between a sensible data and a public one are cheap as they can be considered as scaling (see Section 3.3). We take inspiration from the work of Paterson and Stockmeyer [29] where they perform a variation of the Horner method. We adapt it to the specific case of masked evaluation of public polynomials in characteristic other than 2.

Remark 10. Note that characteristic 2 has already been covered by Roy and Vivek in [32].

Theorem 3. *Let P be a public polynomial and deg its degree. Let's assume first that $s = \sqrt{deg}$ is an integer for simplicity. We notice that $P(X)$, where X is the sensitive data we mask, can be written as:*

$$\begin{gathered} P(X) = P_0(X) + X^s P_1(X) + \ldots + X^{deg-s} P_{s-1}(X), \\ \text{with } \forall i \in [\![0, s-2]\!], \deg(P_i(X)) \leq s-1 \text{ and } \deg(P_{s-1}(X)) = s \end{gathered} \tag{9}$$

The complexity of evaluating this polynomial in X is $2s-2$ sensitive multiplications, $(s-1)^2$ scalar multiplications and $deg - s$ additions.

Proof. 1. *Sensitive multiplications.* First we evaluate all the X^j up to $j = s$. They are re-used inside all the P_i. Thus, we have $s-1$ sensitive multiplications to perform. Once it is done for the P_i, we can consider the polynomial in Equation 9 as a polynomial of degree $s-1$ in $Y = X^s$. Thus, by applying the normal Horner method, it requires only $s-1$ sensitive multiplications. For a total of $(s-1) + (s-1) = 2s-2$.
2. *Scalar multiplications.* $\forall i \in [\![0, s-2]\!], \deg(P_i(X)) \leq s-1$. Thus, we have $(s-1) * (s-2)$ scalar multiplications to perform. For P_{s-1}, we have a polynomial of degree s and thus $s-1$ scalar multiplications. For a total of $(s-1)(s-2) + (s-1) = (s-1)^2$.
3. *Additions* Regarding the evaluation of the P_i, there is as much additions as scalar multiplications, so $(s-1)^2$. However, we must add to that count the additions performed during the final Horner on $Y = X^s$. For a total of $(s-1)^2 + (s-1) = deg - s$.

Remark 11. If *deg* is not a square, let e be the greatest square lower than *deg* and $r = \sqrt{e}$. The complexity of evaluating P becomes $2r-1$ sensitive multiplications, $r(r-2)+(deg-e) = deg-2r$ scalar multiplications and $(deg - 2r) + r = deg - r$ additions.

3.10 Conversion Between Code-Based Maskings

Current state of the art regarding masking ML-KEM [7,9] use conversions between different masking methods before and after the use of Keccak in order to benefit from a fast boolean masked implementation of Keccak. However, converting is costly and can create vulnerabilities. For instance, the work from Bronchain and Cassiers [9] was a patch for leakage in the conversion used in [7]. Kundu et al. [23] successfully targets the conversion with a fault attack to recover sensitive data. Thus, using conversion between masking methods without proper FIA countermeasures can be risky. We propose a conversion method for Code-Based Masking.

Converting from arithmetic to Boolean representation can be interpreted as a converting from a field of characteristic different from 2 to another field of characteristic 2. Both representations must be equipped of masking algorithm with a similar d probing order. We have previously seen that $\mathbb{F}_q$ can be equipped of Reed-Solomon code based masking. According the parameters, we can benefit of the Fast Fourier calculation or simply a Vandermonde matrix multiplication. The results of [10] states that we have a such encoder over characteristic 2 fields. We consider now the case $\mathbb{F}_{2^6}$ with a $RS[7,4,4]$ code which leads to a 3-probing order encoder.

There is no common subfield between $\mathbb{F}_{2^6}$ and $\mathbb{F}_q$, however it is possible to build $(\mathbb{F}_2, \otimes, \oplus)$ as a subset of $(\mathbb{F}_q, \times, +)$ by using the interpolation of the following operators

$$\forall (x,y) \in \{0,1\}^2 \in (\mathbb{F}_q)^2, \begin{cases} x \otimes y = x \times y; \\ x \oplus y = x + y - 2 \times x \times y. \end{cases}$$

We deduce that $\texttt{Mask}_q(a \oplus b) = \texttt{Mask}_q(a) + \texttt{Mask}_q(b) - 2 \cdot \texttt{Mask}_q(a) \star \texttt{Mask}_q(b)$ and $\texttt{Mask}_q(a \otimes b) = \texttt{Mask}_q(a) \star \texttt{Mask}_q(b)$.
In the ML-KEM context, the conversion from $\mathbb{Z}_q$ to $\mathbb{F}_{2^6}$ is favourable since the input of the Keccak function comes from a polynomial in $\mathbb{Z}_q[X]/ < X^{256} + 1 >$ which coefficients are either 0 or 1.

Remark 12. In case the value masked is not certain to be either 0 or 1, it is necessary to obtain its binary decomposition before performing the conversion. This can be performed in a secure manner using Lagrange's interpolations. This is however costly, with an estimated complexity of $\lceil log(q) \rceil \times (2\lfloor \sqrt{q} \rfloor - 1)$ masked multiplications in the worst case[4]. As a result we have $\lceil q \rceil$ conversions to perform.

From $\mathbb{F}_q$ To $\mathbb{F}_{2^6}$. Before the evaluation by Keccak, we have 256 masked bits $b_i \in \mathbb{F}_q$, $i \in [1..256]$: $\texttt{Mask}_q(b_i)$. Then we pick randomly $0 \leq r_j \leq 63 \in \mathbb{F}_q$, $j \in [1..3]$ and compute $\texttt{Mask}_q(r_1), \texttt{Mask}_q(r_2), \texttt{Mask}_q(r_3)$ in order to form the vector

$$\vec{v} = (\texttt{Mask}_q(b_i), \texttt{Mask}_q(r_1), \texttt{Mask}_q(r_2), \texttt{Mask}_q(r_3)) = \texttt{Mask}_q(b_i, r_1, r_2, r_3).$$

If we denote M_2 the Reed-Solomon encoder over $\mathbb{F}_{2^6}$, then we would like to evaluate

$$\texttt{Mask}_q\left((b_i, r_1, r_2, r_3) \otimes M_2\right) = \texttt{Mask}_q\left((0, r_1, r_2, r_3) \otimes M_2 \oplus (b_i, 0, 0, 0) \otimes M_2\right).$$

[4] Some interpolations might have a structure and thus the possibility of reducing the degree of the evaluated polynomial through variable changes.

$(0, r_1, r_2, r_3) \otimes M_2$ can be calculated and then $\texttt{Mask}_q\left((0, r_1, r_2, r_3) \otimes M_2\right) = \vec{F}$ is deduced. Then we suggest to considere $((0, r_1, r_2, r_3) \otimes M_2)$ in its binary form: $\vec{F_2} \in \mathbb{F}_2^{6\times 7}$. The vector $(b_i, 0, 0, 0) \otimes M_2 = b_i \otimes (M_2[0][0], \ldots, M_2[0][6])$ admits a binary representation $b_i \otimes B_2$ with $B_2 \in \mathbb{F}_2^{6\times 7}$. Finally, if we denote $BM = (b_i, r_1, r_2, r_3) \otimes M_2 = \texttt{Mask}_2(b_i)$, then

$$C = \texttt{Mask}_q\left(BM\right) = \texttt{Mask}_q\left(b_i \otimes B_2 \oplus \vec{F_2}\right)$$

where $C[s][j] = \texttt{Mask}_q(BM[s][j])$ and

$$C[s][j] = (B_2[s][j] \times \texttt{Mask}_q(b_i)) + \texttt{Mask}_q(F_2[s][j]) - 2 \times \texttt{Mask}_q(F_2[s][j]) \times (B_2[s][j] \times \texttt{Mask}_q(b_i))$$

for $s \in [0..5]$ and $j \in [0..6]$. We can now unmask C, and get $BM = \texttt{Mask}_2(b_i) = \texttt{Unmask}_q(C)$.

Algorithm 3 CBM Conversion from $\mathbb{F}_q$ to $\mathbb{F}_{2^6}$ in the case of ML-KEM

Input: $in = \texttt{Mask}_q(b_i)$ a $\mathbb{F}_q-$CBM of b_i
Output: $out = \texttt{Mask}_{2^6}(b_i) = 6 \times \texttt{Mask}_2(b_i)$ a $\mathbb{F}_{2^6}-$CBM of b_i
Internal vars: $F[7], Rout[6][7][8]$
Tabs used for optimization : Mul, Vec_{b2}
$r_1, r_2, r_3 \leftarrow$ random 8bits words
for i from 1 to 7 **do**
    $F[i] \leftarrow Mul[r_1][V^{-1}[1][i]] - Mul[r_2][V^{-1}[2][i]] - Mul[r_3][V^{-1}[3][i]]$
end for
for i from 1 to 7 **do**
    for j from 1 to 6 **do**
        $Rout[j][i] \leftarrow (F[i] >> j) \otimes 1$
    end for
end for
for i from 1 to 6 **do**
    for j from 1 to 7 **do**
        if $Vec_{b2}[i][j] == 1$ **then**
            $m \leftarrow 2 \cdot (\texttt{Mask}_q(b_i) \star Rout[i][j])$
            $a \leftarrow \texttt{Mask}_q(b_i) + Rout[i][j]$
            $Rout[i][j] \leftarrow a - m$
        end if
    end for
end for
$out = \{0\}^7$
for i from 1 to 6 **do**
    for j from 1 to 7 **do**
        $out[j] \leftarrow out[j] \oplus (\texttt{Unmask}_q(Rout[i][j]) << i)$
    end for
end for
return *out*

From $\mathbb{F}_{2^6}$ To $\mathbb{F}_q$. After the evaluation of Keccac, we have some masked bits $\texttt{Mask}_2(c_i)$ and we must convert it in $\texttt{Mask}_q(c_i)$ without demasking c_i. If we denote M_q the Reed-Solomon encoder over $\mathbb{F}_q$, We propose to compute $\texttt{Mask}_q(c_i) = (c_i, r_1, r_2, r_3) \times M_q$ from $\texttt{Mask}_2(c_i)$. $\texttt{Mask}_q(c_i) =$

$(c_i, 0, 0, 0) \times M_q + (0, r_1, r_2, r_3) \times M_q = (c_i, 0, 0, 0) \times M_q + \texttt{Mask}_q(0)$. $\texttt{Mask}_q(0)$ admits a binary representation $F \in \mathbb{F}_2^{12\times 8}$. $(c_i, 0, 0, 0) \times M_q = c_i \times (M_q[0][0], \ldots, M_q[0][7])$ admits a binary representation $c_i \times B_q$ with $B_q \in \mathbb{F}_2^{12\times 8}$. We must compute now $\texttt{Mask}_2(c_i \times B_q + F)$. We have seen above that $x + y = (x \oplus y, x \otimes y)$, then $\texttt{Mask}_2(c_i \otimes B_q[s][j] + F[s][j]) = (\texttt{Mask}_2(c_i) \otimes \texttt{Mask}_2(B_q[s][j]) \oplus \texttt{Mask}_2(F[s][j]), \texttt{Mask}_2(c_i) \otimes \texttt{Mask}_2(B_q[s][j]) \otimes \texttt{Mask}_2(F[s][j]))$. By propagating the carry, we get finally $\texttt{Mask}_q(c_i)$ written in a masked binary form (i.e. belonging to $\left(\mathbb{F}_{2^6}^7\right)^{13\times 8}$).

Algorithm 4 Carry(out,y,z)

1: **Input:** y, z two $\mathbb{F}_{2^6}-$CBM
2: **Output:** *out* a $\mathbb{F}_{2^6}-$CBM
3: $u \leftarrow y \oplus z$
4: $v \leftarrow y \otimes z$
5: $out \leftarrow out \otimes u \otimes v$

Algorithm 5 CBM Conversion from $\mathbb{F}_{2^6}$ to $\mathbb{F}_q$ in the case of ML-KEM

Input: *in* a $\mathbb{F}_{2^6}-$CBM of b_i
Output: *out* a $\mathbb{F}_q-$CBM of b_i
Internal vars: $F[8], Rout[13][8][7]$
Tabs used for optimization : Vec_{bq}
$F \leftarrow \texttt{Mask}_q(0)$
for i from 1 to 8 **do**
  for j from 1 to 12 **do**
    $Rout[i,j] \leftarrow \texttt{Mask}_2((F[i] >> j) \otimes 1)$
  end for
end for
for i from 1 to 8 **do**
  $ret \leftarrow \texttt{Mask}_2(0)$
  $vet \leftarrow \texttt{Mask}_2(0)$
  for j from 1 to 12 **do**
    if $Vec_{bq}[j][i] == 1$ **then**
      $Carry(vet, in, Rout[j][i])$
      $Rout[j][i] \leftarrow Rout[j][i] \oplus in \oplus ret$
      $ret \leftarrow vet$
    else
      $vet \leftarrow ret \otimes Rout[j][i]$
      $Rout[j][i] \leftarrow Rout[j][i] \oplus ret$
      $ret \leftarrow vet$
    end if
  end for
  $Rout[13][i] \leftarrow ret$
end for
$out \leftarrow \{0\}^8$
for i from 1 to 13 **do**
  for j from 1 to 8 **do**
    $out[j] \leftarrow out[j] \oplus (\texttt{Unmask}_2(Rout[i][j]) << i)$
  end for
end for
for j from 1 to 8 **do**
  $out[j] \leftarrow out[j] \% q$
end for

4 ML-KEM Example

4.1 Discussion on Parameters

We have several possibilities when it comes to which code we can use and for specific masking orders. In this section, we propose several examples of parameters for different masking orders.

First Order of Masking We propose to consider the parameters $n = 4 = 2d'$, $d = 2$, $\omega = \nu^{\frac{q-1}{4}} = 1729$ and $\alpha = \nu^{\frac{q-1}{13}} = 2970$. We chose to these parameters,

$$A = \begin{pmatrix} 1 & 1 \\ \alpha & \alpha^2 \end{pmatrix}, \; V(\omega) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & \omega & \omega^2 & \omega^3 \end{pmatrix}$$

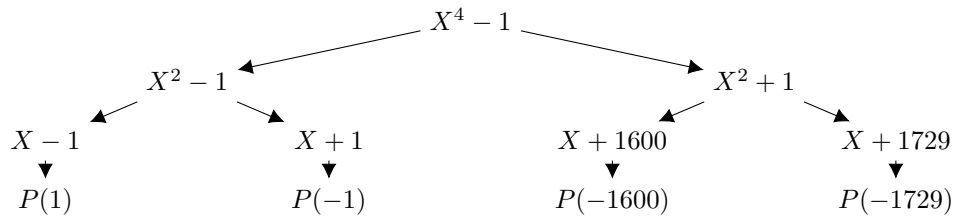
Then,

$$\texttt{Mask}(x) = (x, r) \times (A^{-1}) \times V(\omega),$$

with $r \in \mathbb{Z}_q$ picked randomly.

$$\texttt{Mask}(x) = (x, r) \begin{pmatrix} 103 & 2590 & 1545 & 2387 \\ 3227 & 740 & 1785 & 943 \end{pmatrix} = xG + rH,$$

It is possible to check (cf: MAGMA online) that the minimal distance of $H^{\perp}$ is 2 as predicted by the theory, then $odm = 1$. Furthermore $V(\omega)$ is a Reed Solomon code and 2 faults can be detected. The complexity of the detection corresponds to complexity of the syndrome computation that can be achieved with a DFT and for $n = 4$ we have the following tree decomposition:



It is shown in [36] that this representation is favourable to hardware implementation and complexity does not exceed $nlog(n)$ multiplications over $\mathbb{F}_q$.

Remark 13. The current state-of-the-art [20] on combined SCA and FIA resilient implementations is able to mask at the first order[5] and to detect at most 1 fault for the lowest parameter and 3 for a higher parameter at the cost of performance. Our method masks at the first order and detects 2 faults. We improve the state-of-the-art by being able to correct 1 fault. We also propose higher orders of masking.

Third Order of Masking For higher order, we make the following choice: $n = 8 = 2d'$, $d = 4$, $\omega = \nu^{\frac{q-1}{8}} = 749$ and $\alpha = \nu^{\frac{q-1}{13}} = 2970$. We chose to these parameters,

$$A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 2970 & 2379 & 1492 & 341 \\ 2379 & 341 & 2292 & 3095 \\ 1492 & 2292 & 781 & 1812 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ \alpha & \alpha^2 & \alpha^3 & \alpha^4 \\ \alpha^2 & \alpha^4 & \alpha^6 & \alpha^8 \\ \alpha^3 & \alpha^8 & \alpha^9 & \alpha^{12} \end{pmatrix}, \; V(\omega) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & \omega & \omega^2 & \omega^3 & \omega^4 & \omega^5 & \omega^6 & \omega^7 \\ 1 & \omega^2 & \omega^4 & \omega^6 & 1 & \omega^2 & \omega^4 & \omega^6 \\ 1 & \omega^3 & \omega^6 & \omega & \omega^4 & \omega^7 & \omega^2 & \omega^5 \end{pmatrix}$$

Then,

$$\texttt{Mask}(x) = (x, r_1, r_2, r_3) \times A^{-1} \times V(\omega),$$

and

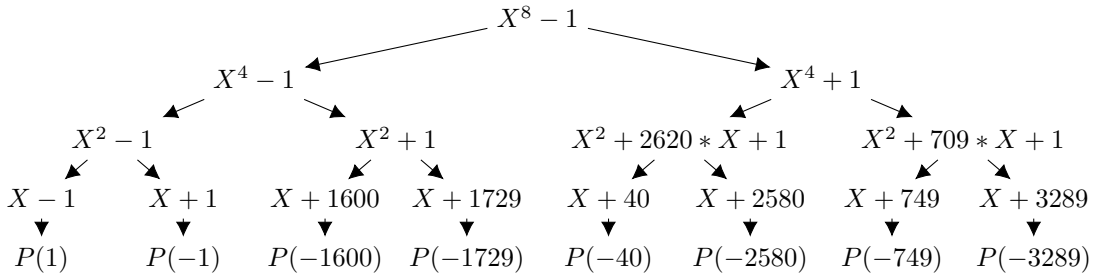
$$\texttt{Mask}(x) = (x, r_1, r_2, r_3) \times \begin{pmatrix} 3212 & 747 & 3112 & 897 & 1801 & 1931 & 428 & 1649 \\ 1893 & 2178 & 3029 & 3078 & 1546 & 491 & 3239 & 631 \\ 2127 & 2027 & 2130 & 1062 & 1354 & 3312 & 3206 & 2416 \\ 2756 & 1707 & 1717 & 1622 & 1958 & 925 & 3115 & 1963 \end{pmatrix},$$

[5] Their method is not masking per say but it offers the same level of protection than first order masking

with $\vec{r} = (r_1, r_2, r_3) \in \mathbb{Z}_q$ picked randomly.

$$\texttt{Mask}(x) = xG + \vec{r}H, \text{ with } H = \begin{pmatrix} 1893 & 2178 & 3029 & 3078 & 1546 & 491 & 3239 & 631 \\ 2127 & 2027 & 2130 & 1062 & 1354 & 3312 & 3206 & 2416 \\ 2756 & 1707 & 1717 & 1622 & 1958 & 925 & 3115 & 1963 \end{pmatrix}.$$

We have a minimal distance of $H^{\perp}$ equal to 4 and consequently, $odm = 3$. We can detect 4 faults. These parameters lead also to a very fast DFT with the following tree decomposition:



Remark 14. We present the first and third order of masking in this paper. While second order is possible, the code chosen to do so does not have the same complexity when it comes to detecting faults as we cannot use the DFT for the syndrome computation.

Higher Orders of Masking As stated in Section 2.2, ML-KEM operations are defined over $\mathbb{Z}_q$ with $q = 3329$ satisfying $q - 1 = 2^8 \times 13$. If $\nu = 3$ is a primitive element of $\mathbb{F}_q$, we could set $n = 13 = 2d' + 1$, $d = 6$, with $\omega = \nu^{\frac{q-1}{13}}$ and $\alpha = \nu^{\frac{q-1}{16}}$. The masking method is working for these parameters but we have not found a better way to compute the `DFT` than using a Vandermonde matrix multiplication which costs $\mathcal{O}(n^2)$. However, with these parameters, for $t = 1$, we get $odm = d + 1 - 1 = 6$ and 6 faults can be detected on codewords.

Code-Based Masking offers a high flexibility in terms of which code we can use. We can choose either to improve the FIA resilience or the SCA resilience.

4.2 Masking Strategy

First we focus on securing the PKE.Decrypt part of ML-KEM (see [27] Algorithm 14 page 28 or Figure 4 in Appendix B). Then we discuss how we can extend our masking method to the entirety of the KEM Decapsulation procedure (see Algorithm 1) and ML-KEM itself.

You can use the following map (Figure 1) of the KEM Decapsulation to navigate between the different parts we had to mask.

Graph legend:

- : Non-sensitive operation
- : Non-sensitive input/output of the algorithm
- : Non-sensitive intermediate data
- : Sensitive operation
- : Sensitive input/output of the algorithm

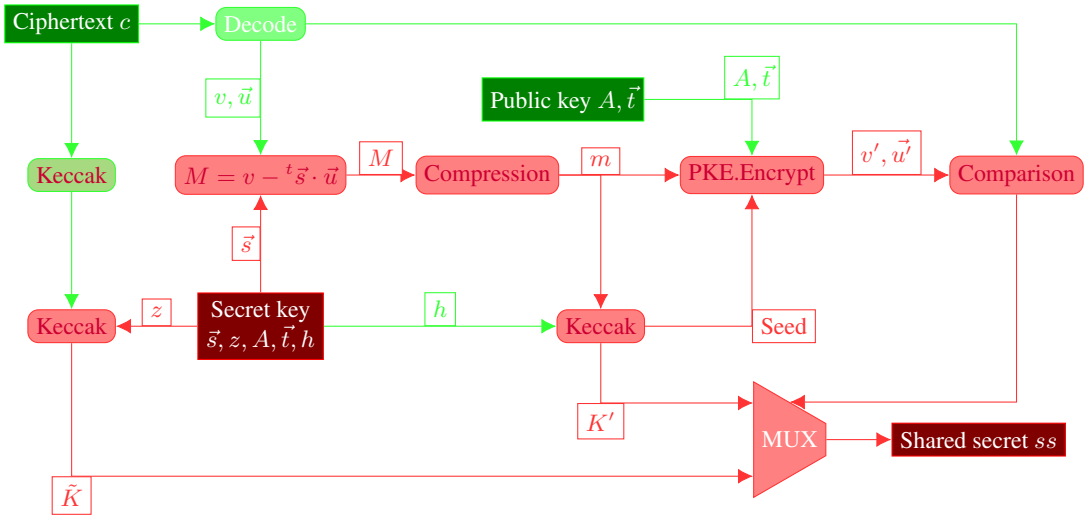


Fig. 1. Interactive map (links) of our masking strategy

– ⟶ : Sensitive intermediate data

Remark 15. We choose to add a hash function ● to hash the ciphertext before using it with z in $\tilde{K} = J(z\|c, 32)$. The reason is performances. By prehashing the ciphertext, we significantly reduce the size of the input of a masked Keccak while the prehashing is done on a public data and thus with a non-masked Keccak. This modification only affects the output of KEM Decapsulation in the event of a FO Transform failure. This does not affect the theoretical security of ML-KEM and we still are fully compatible with other implementations of ML-KEM.

We propose to mask the following operation: $v - {}^t\vec{s} \cdot \vec{u}$ with $v \in R_q$ public, $\vec{u} \in R_q^{sec}$ public, and $\vec{s} \in R_q^{sec}$ secret. First, we have to discuss how to multiply a sensitive data and a public one.

Point-Wise Multiplication between a Sensitive and a Public Polynomials To multiply two sensitive polynomials we choose to rely on the NTT algorithm as described in ML-KEM but in a masked manner. However, most of the multiplications between two polynomials in ML-KEM involve a sensitive and a public one. To avoid the cost of masking a public polynomial, we instead consider its coefficients as scalars.
The NTT is computed with regular calls to "butterflies units". Those units perform an addition a subtraction as well as a scalar multiplication. The inversed NTT uses the same operations but in reverse. We use our gadgets for each operation for the sensitive polynomial and the reference NTT for the public one.

Remark 16. While we proved our gadgets security, we recommend to use a refresh after each call to a gadget. This ensure the composition of those gadgets remains secure.

Thanks to the homomorphic properties of the `Mask` procedure we have:

$$\texttt{Mask}(v - {}^t\vec{s} \cdot \vec{u}) = \texttt{Mask}(v) - \texttt{Mask}NTT^{-1}\left(\sum_{i=0}^{sec} \texttt{Mask}NTT(\texttt{Mask}(s_i)) \odot_{\texttt{Mask}} NTT(u_i)\right),$$

where ${}^t\vec{s} \cdot \vec{u} = NTT^{-1}\left(\sum_{i=0}^{sec}(NTT(s_i) \odot NTT(u_i))\right)$ with $s_i \in R_q$ and $u_i \in R_q$.

Compression The next step is to apply the $Compress_q$ function while staying masked. We have the following theorem:

Theorem 4. *$Compress_q$ can be computed using a polynomial function.*

Proof. We can rewrite the $Compress_q$ function from Equation 1 as

$$\forall \alpha \in \mathbb{Z}_q,\ Compress_q(\alpha) = \begin{cases} 1 & \text{if } \lceil \frac{q}{4} \rfloor < \alpha < \lceil \frac{3q}{4} \rfloor \\ 0 & \text{otherwise} \end{cases} \tag{10}$$

As $\mathbb{Z}_q$ is a finite field, we can simply enumerate all the values of α resulting in 1 and those resulting in 0 and thus we can rewrite Equation 10 as

$$\forall \alpha \in \mathbb{Z}_q,\ Compress_q(\alpha) = \begin{cases} 1 & \text{if } \alpha \in \{\alpha_0, \alpha_1, \ldots, \alpha_h\} \\ 0 & \text{if } \alpha \in \{\beta_0, \beta_1, \ldots, \beta_l\} \end{cases} \text{ with } h + l = q \tag{11}$$

A simple Lagrange interpolation of Equation 11 thus give us the following:

$$\forall \alpha \in \mathbb{Z}_q,\ Compress_q(\alpha) \Leftrightarrow P(X = \alpha) = \prod_{i=0}^{l}(X - \beta_i) \sum_{j=0}^{h} \prod_{k=0, k\neq j}^{h} \frac{(X - \alpha_k)}{(\alpha_j - \alpha_k)} \tag{12}$$

We proved $Compress_q$ can be seen as a polynomial function and we can further extend the reach of our masking method within ML-KEM.

We deduce from this theorem that we can mask $Compress_q$ as a polynomial function. Evaluating this polynomial has a complexity of $2\lfloor\sqrt{3328}\rfloor - 1 = 113$ masked multiplications using the method from Section 3.9. However, this polynomial has a structure. All its exponents are odd. Thus, by setting $Y = X^2$ we can gain some complexity and evaluate this polynomial in $2\lfloor\sqrt{1665}\rfloor - 1 + 1 = 80$ masked multiplications.

Using the adaptation of the Paterson-Stockmeyer [29] method as in Section 3.9, we are able to perform the masked message compression in just 80 sensitive multiplications instead of the 3328 required by a straight application of the Horner method.

From PKE to KEM PKE Decrypt is used in KEM Decapsulation as shown in Algorithm 1. In order to mask the rest of the KEM Decapsulation procedure of ML-KEM and the other KEM procedures, we have to address a few points.

- **How do we hash the message output of PKE Decrypt?** The message is a polynomial where each term is either 0 or 1 masked. As stated in Subsection 3.10, we need the input to be masked bits, which is the case here. Thus, we can directly apply our DSM Keccak implementation on the output of PKE Decrypt.

– **How do we mask PKE Encrypt?** The homomorphic properties of `Mask` can also be applied to both the PKE encryption and key generation of ML-KEM. By taking into account Remark 3 regarding the $Decompress_q$ function, we can secure most of the computations using the sensitive data $\vec{s}, m, \vec{r}, \vec{e}, \vec{e_1}$ and e_2 in PKE Key Gen (see [27] page 26 Algorithm 12) and PKE Encrypt (see [27] page 27 Algorithm 13 or Figure 4 Appendix B). For instance, to compute v we do

$$\texttt{Mask}(v) = \left(\sum_{i=0}^{sec} t_i * \texttt{Mask}(r_i)\right) + \texttt{Mask}(e_2) + 1665 * \texttt{Mask}(m) \tag{13}$$

Note that the product of t_i and $\texttt{Mask}(r_i)$ uses the NTT as described in 4.2. Here we use "$*$" for readibility.
The next step to mask PKE Encrypt is to secure the sampling of these sensitive data.

– **How do we use our masking method to perform sensitive data sampling?** To sample sensitive values in the PKE Encrypt procedure, we use the CBD from Equation 4 fed by two chained Keccak instances. To chain those instances, we do not convert back from $\mathbb{F}_{2^6}$ to $\mathbb{Z}_q$ between each call to the hash function. Thus, we have as input for CBD a Code-Based Mask in $\mathbb{F}_{2^6}$. As conversions are costly, we perform the CBD before converting back to $\mathbb{Z}_q$. Adding the masked bits is performed with a η-bit boolean addition, using the following formula: $c = \sum_{i=0}^{\eta-1} G_i + \neg G_{i+\eta}$ where G is the output of the hash function. The result is a masked sample in $[\![0; 2\eta]\!]$ and will be re-centered on 0 after the conversion to $\mathbb{Z}_q$. We use boolean formulas to get each bits of c. For $\eta = 2$ we have:

$$\begin{aligned}
\alpha_0 &= a_0 \oplus \neg b_0 & \alpha_1 &= a_1 \oplus \neg b_1 \\
\beta_0 &= a_0 \otimes \neg b_0 & \beta_1 &= a_1 \otimes \neg b_1 \\
c_0 &= \alpha_0 \oplus \alpha_1 && (14)\\
c_1 &= (\alpha_0 \otimes \alpha_1) \oplus \beta_0 \oplus \beta_1 && (15)\\
c_2 &= \beta_0 \otimes \beta_1 && (16)
\end{aligned}$$

With similar notations, for $\eta = 3$ we have:

$$\begin{aligned}
\alpha_2 &= a_2 \oplus \neg b_2 \\
\beta_2 &= a_2 \otimes \neg b_2 \\
c_0 &= \alpha_0 \oplus \alpha_1 \oplus \alpha_2 && (17)\\
c_1 &= (\alpha_0 \otimes \alpha_1) \oplus (\alpha_0 \otimes \alpha_2) \oplus (\alpha_1 \otimes \alpha_2) \oplus \beta_0 \oplus \beta_1 \oplus \beta_2 && (18)\\
c_2 &= (\beta_0 \otimes \beta_1) \oplus (\beta_0 \otimes \beta_2) \oplus (\beta_1 \otimes \beta_2) \oplus (\beta_2 \otimes \alpha_0 \otimes \alpha_1) \oplus \\
&\quad (\beta_1 \otimes \alpha_0 \otimes \alpha_2) \oplus (\beta_0 \otimes \alpha_1 \otimes \alpha_2) && (19)
\end{aligned}$$

Instead of performing 4 conversions for $\eta = 2$ (resp. 6 for $\eta = 3$) per polynomial coefficient, we only perform 3 (resp. 4).
An interesting property of DSM and thus our Code-Based Masking is the possibility of masking several sensitive data within a single mask. This method was first introduced in [35] by Wang et al. under the name of Cost-Amortization and is used in [10]. Applied to ML-KEM, it helps to "parallelize" the different CBD instances and further reduce the amount

of conversions we are forced to perform.

– **How do we compare ciphertexts in the Fujisaki-Okamoto Transform without unmasking them?** One of the biggest issue with masking ML-KEM is the lossy nature of the $Compress_q$ function, as stated in Remark 4. As the ciphertext in the KEM Decapsulation (Algorithm 1) is given as input in a compressed state, the NIST draft paper [27] simply compresses the generated ciphertext into c' and compares it with the input ciphertext c. However, we have already seen that masking the $Compress_q$ function can be costly. Thus, papers masking CRYSTALS-Kyber [7,9] use a different approach: They compare the generated ciphertexts $\vec{u'}, v'$ with the decompression of c. This can also be applied to ML-KEM. We went a step further and relied on the property stated in Remark 4 Equation 3. A key point here is we want a function that returns 0 when the ciphertexts are good and not 0 when the comparison fails. Which means that, instead of performing a Lagrange interpolation[6] like for the message compression, here we can just list the values of $y = x - x'$ such as $|y| \leq \lceil q/2^{d_i+1} \rfloor \bmod q$ and consider them as the roots of the polynomial we are looking for. Thus, for $d_i = d_u = 10$, we have $\lceil q/2^{d_i+1} \rfloor = \lceil q/2^{10+1} \rfloor = 2$, thus $y \in [\![-2, 2]\!]$, 5 roots and a polynomial of degree 5:

$$P(X) = X^5 + 3324X^3 + 4X = X(Y-4)(Y-1) \text{ with } Y = X^2 \tag{20}$$

This Equation 20 only requires 3 masked multiplications. For $d_i = d_v = 4$, we have $\lceil q/2^{d_i+1} \rfloor = \lceil q/2^{4+1} \rfloor = 104$, thus $y \in [\![-104, 104]\!]$, 209 roots and a polynomial of degree 209. However, we know that X is a factor of this polynomial. We use the symmetric nature of the set of roots to have $(X-a)(X+a) = X^2 - a^2$, thus allowing us to have two polynomials, X and one of degree 104 in $Y = X^2$. By applying our adaption of the Paterson-Stockmeyer method, we can evaluate this polynomial in just 20 masked multiplications.

We demonstrated that our masking method can be applied completely to ML-KEM[7] to secure computations on sensitive data, without requiring any conversion to a different masking method and providing error detection and error correcting capabilities.

5 Implementation and Performances

5.1 *Proof of Concept* Implementation for ML-KEM

We made a *Proof of Concept* (*PoC*) implementation of our masking method in the C language on a desktop. The intent of this *PoC* is to validate the feasibility of our masking method at an algorithmical level. However, we think it is important to share some performances[8] and results to give an idea of the costs of our method and to highlight the effectiveness of some of our rationales. Figure 2 gives the performances of our masked implementations of each of the three key algorithms of ML-KEM, highlighting the efficiency of using conversions and cost-amortization compared to arithmetizing the Keccak calls.

[6] All the mathematical optimizations in this paper were computed using PARI GP.
[7] Note that we mask ML-KEM-512 but our method works for other security levels as well.
[8] Averages in milliseconds over 1000 iterations.

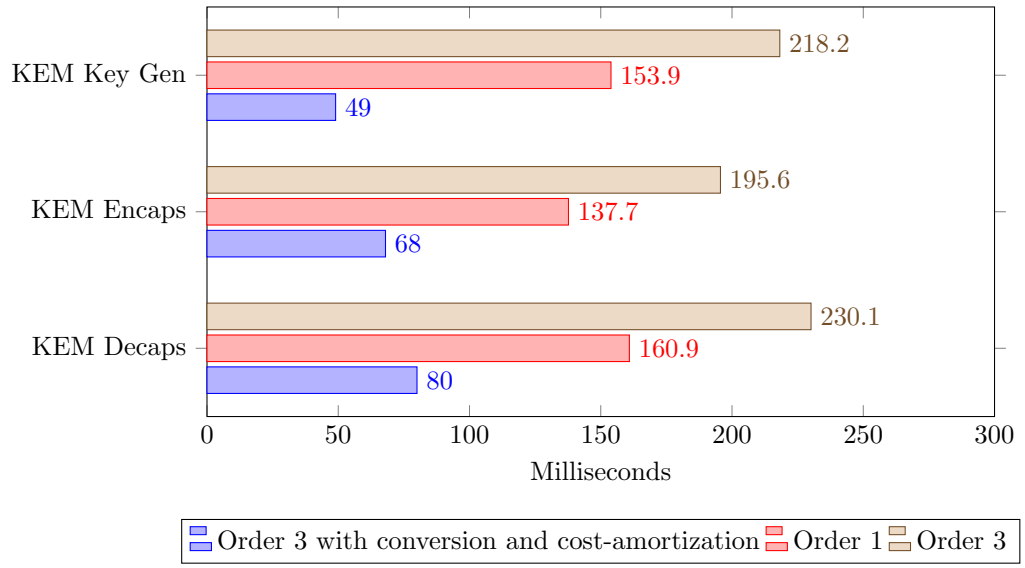


Fig. 2. Performances in milliseconds of our masked ML-KEM-512 at different masking orders

Table 1 shows the performances of our solutions to the several obstacles encountered while masking ML-KEM.

Table 1. Performances in milliseconds of several important functions

Masking order	1	3	3 Conv + CA
Ciphertexts comparison	0.713	1.01	1.01
Message compression	4.88	8.01	8.01
Hash function	17.1	24.3	11.71[9]

5.2 On the Impact of the Conversion and of the Use of Cost-Amortization on Keccak

Table 2 shows the performances of different masking strategies for Keccak. Note that performances involving Cost-Amortization implies two Keccak are performed into one. The results highlight the effectiveness of using the conversion and the cost-amortization in the specific case of CBM applied to ML-KEM.

Table 2. Performances of different masking strategies for Keccak in milliseconds at masking order 3

Keccak Strategy	Performances
Reference (using the CPU Keccak)	0.0007
Reference (without the CPU Keccak)	0.27
$\mathbb{F}_{2^6}$-CBM	3.16
$\mathbb{F}_{2^6}$-CBM + Cost-Amortization	3.24/2
$\mathbb{F}_q$-CBM	20.69
$\mathbb{F}_q$-CBM + Cost-Amortization	21.07/2
Conversion	7.39
Conversion + Cost-Amortization	11.71/2

5.3 Fast Evaluation of Masked Polynomials

The next Table 3 highlights the efficiency of our adaptation of the Paterson-Stockmeyer method [29] compared to a simple application of the Horner method or the use of the structure of the message compression polynomial. Indeed, it can be factorized in several smaller polynomials. As the biggest irreducible factor is of degree 599, this sets an evaluation complexity of slightly more than 600 sensitive multiplications. Please note that all those evaluations use the variable change $Y = X^2$ and thus are evaluated in Y.

Table 3. Impact of several evaluation methods on the message compression in milliseconds

Masking order	1	3
Message compression using the Horner method	37.25	56.65
Message compression using factorisation	12.77	17.68
Message compression using the Paterson-Stockmeyer method	4.88	8.01

Those experimental results clearly validate the theoretical results we highlighted in Section 3.9.

5.4 Comparison with Reference Implementation

We intend to port our method on a FPGA platform for experimental validation in a future work. This would allow us to compare our work with [20]. Keep in mind that the goal of this *PoC* is to prove the algorithmical feasibility of our masking method. This paper aims at laying the theoretical foundations on which future works and implementations on better targets (Micro-controller, FPGA, ASIC...) will be able to rely on. We compare our results with the reference C source code of CRYSTALS-Kyber in Table 4:

Table 4. Performances factors compared to CRYSTALS-Kyber reference implementation

Masking order	0 (reference time)	1	3	3 Conv + CA
KEM Key Gen	0.03 ms	×5130	×7273	×1634
KEM Encapsulation	0.03 ms	×4590	×6520	×2267
KEM Decapsulation	0.03 ms	×5363	×7670	×2667

The performances shown in this Section 5 were realized on a laptop computer equipped with a 11th Gen Intel(R) Core(TM) i7-11850H processor operating at 2.50 GHz with 16 GB of RAM. The source code was compiled and executed using gcc version 11.3.0. A particularity of our setup is the use of Ubuntu 22.04.1 through WSL2 (Windows Subsystems for Linux) on a computer operating Windows 11.

Remark 17. Due to the intellectual property regulation in place in our working environment, we are not able to share the source code of our *PoC* for the moment.

6 Conclusion and Future Work

In this paper we proved in Section 3 that Code-Based masking can be used with finite fields of prime characteristic other than 2 and with codes of even length. We adapted an evaluation method from Paterson and Stockmeyer [29] to evaluate a masked polynomial in characteristic different from 2 in Section 3.9. We proposed a design rationale to mask a post-quantum KEM using the Code-Based Masking method in Section 4. We also provided better security against Fault Injection Attacks (FIA) on ML-KEM compared to the current state-of-the-art [20] by being able to correct faults. The fault correction is enabled within the multiplicative gadget, which is an improvement on the DSM method proposed in [35].

The next step of our work will be to implement this solution on a FPGA/Microcontroler hardware platform and verify its robustness experimentally. This will also ease the comparison with state-of-the-art and future implementations. As for other asymmetric cryptography primitives, we expect our method to work on ML-DSA [26] with some minor tweaks, as ML-DSA and ML-KEM have a lot in common. Our work on enabling this masking method for finite fields of characteristic different from 2 should also allow us to further explore solutions with resilience to both SCA and FIA for current pre-quantum cryptography primitives.

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References

1. Alagic, G., Apon, D., Cooper, D., Dang, Q., Dang, T., Kelsey, J., Lichtinger, J., Miller, C., Moody, D., Peralta, R., et al.: Status report on the third round of the nist post-quantum cryptography standardization process. US Department of Commerce, NIST (2022). https://doi.org/10.6028/NIST.IR.8413-upd1
2. Avanzi, R., Bos, J., Ducas, L., Kiltz, E., Lepoint, T., Lyubashevsky, V., Schanck, J.M., Schwabe, P., Seiler, G., Stehlé, D.: Crystals-kyber algorithm specifications and supporting documentation. pq-crystals (2021)
3. Backlund, L.: A side-channel attack on masked and shuffled implementations of m-lwe and m-lwr cryptography: A case study of kyber and saber (2023)
4. Beckwith, L., Abdulgadir, A., Azarderakhsh, R.: A flexible shared hardware accelerator for nist-recommended algorithms crystals-kyber and crystals-dilithium with sca protection. In: Cryptographers' Track at the RSA Conference. pp. 469–490. Springer (2023). https://doi.org/10.1007/978-3-031-30872-7_18
5. Berndt, S., Eisenbarth, T., Faust, S., Gourjon, M., Orlt, M., Seker, O.: Combined fault and leakage resilience: Composability, constructions and compiler. Cryptology ePrint Archive, Paper 2023/1143 (2023), https://eprint.iacr.org/2023/1143
6. Bos, J., Ducas, L., Kiltz, E., Lepoint, T., Lyubashevsky, V., Schanck, J.M., Schwabe, P., Seiler, G., Stehlé, D.: Crystals-kyber: a cca-secure module-lattice-based kem. In: 2018 IEEE European Symposium on Security and Privacy (EuroS&P). pp. 353–367. IEEE (2018). https://doi.org/10.1109/EuroSP.2018.00032
7. Bos, J.W., Gourjon, M., Renes, J., Schneider, T., Van Vredendaal, C.: Masking kyber: First-and higher-order implementations. IACR Transactions on Cryptographic Hardware and Embedded Systems pp. 173–214 (2021). https://doi.org/10.46586/tches.v2021.i4.173-214
8. Bringer, J., Carlet, C., Chabanne, H., Guilley, S., Maghrebi, H.: Orthogonal direct sum masking: A smartcard friendly computation paradigm in a code, with builtin protection against side-channel and fault attacks. In: IFIP International Workshop on Information Security Theory and Practice. pp. 40–56. Springer (2014). https://doi.org/10.1007/978-3-662-43826-8_4
9. Bronchain, O., Cassiers, G.: Bitslicing arithmetic/boolean masking conversions for fun and profit: with application to lattice-based kems. IACR Transactions on Cryptographic Hardware and Embedded Systems pp. 553–588 (2022). https://doi.org/10.46586/tches.v2022.i4.553-588
10. Carlet, C., Daif, A., Guilley, S., Tavernier, C.: Quasi-linear masking against sca and fia, with cost amortization. IACR Transactions on Cryptographic Hardware and Embedded Systems **2024**(1), 398–432 (2024). https://doi.org/10.46586/tches.v2024.i1.398-432
11. Chari, S., Jutla, C.S., Rao, J.R., Rohatgi, P.: Towards sound approaches to counteract power-analysis attacks. In: Advances in Cryptology—CRYPTO'99: 19th Annual International Cryptology Conference Santa Barbara, California, USA, August 15–19, 1999 Proceedings 19. pp. 398–412. Springer (1999). https://doi.org/10.1007/3-540-48405-1_26
12. Chen, L., Chen, L., Jordan, S., Liu, Y.K., Moody, D., Peralta, R., Perlner, R.A., Smith-Tone, D.: Report on post-quantum cryptography, vol. 12. US Department of Commerce, National Institute of Standards and Technology ... (2016)
13. Delvaux, J.: Roulette: A diverse family of feasible fault attacks on masked kyber. Cryptology ePrint Archive, Paper 2021/1622 (2021), https://eprint.iacr.org/2021/1622
14. Dworkin, M.J.: Sha-3 standard: Permutation-based hash and extendable-output functions. NIST FIPS (2015). https://doi.org/10.6028/NIST.FIPS.202
15. Fritzmann, T., Van Beirendonck, M., Roy, D.B., Karl, P., Schamberger, T., Verbauwhede, I., Sigl, G.: Masked accelerators and instruction set extensions for post-quantum cryptography. Cryptology ePrint Archive (2021). https://doi.org/10.46586/tches.v2022.i1.414-460
16. Fujisaki, E., Okamoto, T.: Secure integration of asymmetric and symmetric encryption schemes. In: Annual international cryptology conference. pp. 537–554. Springer (1999). https://doi.org/10.1007/3-540-48405-1_34

17. Goudarzi, D., Joux, A., Rivain, M.: How to securely compute with noisy leakage in quasilinear complexity. In: International Conference on the Theory and Application of Cryptology and Information Security. pp. 547–574. Springer (2018). https://doi.org/10.1007/978-3-030-03329-3_19
18. Goudarzi, D., Prest, T., Rivain, M., Vergnaud, D.: Probing security through input-output separation and revisited quasilinear masking. IACR Transactions on Cryptographic Hardware and Embedded Systems pp. 599–640 (2021). https://doi.org/10.46586/tches.v2021.i3.599-640
19. Heinz, D., Kannwischer, M.J., Land, G., Pöppelmann, T., Schwabe, P., Sprenkels, D.: First-order masked kyber on arm cortex-m4. Cryptology ePrint Archive, Paper 2022/058 (2022), https://eprint.iacr.org/2022/058
20. Heinz, D., Pöppelmann, T.: Combined fault and dpa protection for lattice-based cryptography. IEEE Transactions on Computers **72**(4), 1055–1066 (2022). https://doi.org/10.1109/TC.2022.3197073
21. Ishai, Y., Sahai, A., Wagner, D.: Private circuits: Securing hardware against probing attacks. In: Advances in Cryptology-CRYPTO 2003: 23rd Annual International Cryptology Conference, Santa Barbara, California, USA, August 17-21, 2003. Proceedings 23. pp. 463–481. Springer (2003). https://doi.org/10.1007/978-3-540-45146-4_27
22. Kocher, P.C.: Timing attacks on implementations of diffie-hellman, rsa, dss, and other systems. In: Advances in Cryptology—CRYPTO'96: 16th Annual International Cryptology Conference Santa Barbara, California, USA August 18–22, 1996 Proceedings 16. pp. 104–113. Springer (1996). https://doi.org/10.1007/3-540-68697-5_9
23. Kundu, S., Chowdhury, S., Saha, S., Karmakar, A., Mukhopadhyay, D., Verbauwhede, I.: Carry your fault: A fault propagation attack on side-channel protected lwe-based kem. Cryptology ePrint Archive, Paper 2023/1674 (2023), https://eprint.iacr.org/2023/1674
24. Mosca, M., Piani, M.: 2021 quantum threat timeline report global risk institute. Global Risk Institute (2022)
25. Ngo, K.: Side-Channel Analysis of Post-Quantum Cryptographic Algorithms. Ph.D. thesis, KTH Royal Institute of Technology (2023)
26. NIST: Module-lattice-based digital signature standard. NIST FIPS (2023). https://doi.org/10.6028/NIST.FIPS.204.ipd
27. NIST: Module-lattice-based key-encapsulation mechanism standard. NIST FIPS (2023). https://doi.org/10.6028/NIST.FIPS.203.ipd
28. Oder, T., Schneider, T., Pöppelmann, T., Güneysu, T.: Practical cca2-secure and masked ring-lwe implementation. Cryptology ePrint Archive (2016). https://doi.org/10.13154/tches.v2018.i1.142-174
29. Paterson, M.S., Stockmeyer, L.J.: On the number of nonscalar multiplications necessary to evaluate polynomials. SIAM Journal on Computing **2**(1), 60–66 (1973). https://doi.org/10.1137/0202007
30. Rains, E.M., Sloane, N.J.: Self-dual codes. arXiv preprint math/0208001 (2002). https://doi.org/10.48550/arXiv.math/0208001
31. Ravi, P., Roy, S.S.: Side-channel analysis of lattice-based pqc candidates. In: Round 3 Seminars, NIST Post Quantum Cryptography (2021)
32. Roy, A., Vivek, S.: Analysis and improvement of the generic higher-order masking scheme of fse 2012. Cryptology ePrint Archive, Paper 2013/345 (2013), https://eprint.iacr.org/2013/345
33. Saarinen, M.J.: Intro to side-channel security of nist pqc standards. In: PQC Seminars, NIST (2023)
34. Shor, P.W.: Algorithms for quantum computation: discrete logarithms and factoring. In: Proceedings 35th annual symposium on foundations of computer science. pp. 124–134. Ieee (1994). https://doi.org/10.1109/SFCS.1994.365700
35. Wang, W., Méaux, P., Cassiers, G., Standaert, F.X.: Efficient and private computations with code-based masking. IACR Transactions on Cryptographic Hardware and Embedded Systems pp. 128–171 (2020). https://doi.org/10.13154/tches.v2020.i2.128-171
36. Wang, Y., Zhu, X.: A fast algorithm for the fourier transform over finite fields and its vlsi implementation. IEEE Journal on Selected Areas in Communications **6**(3), 572–577 (1988). https://doi.org/10.1109/49.1926

A Graph legend

- : Non-sensitive operation
- : Non-sensitive input/output of the algorithm
- : Non-sensitive intermediate data
- : Sensitive operation
- : Sensitive input/output of the algorithm
- : Sensitive intermediate data

B ML-KEM

B.1 PKE

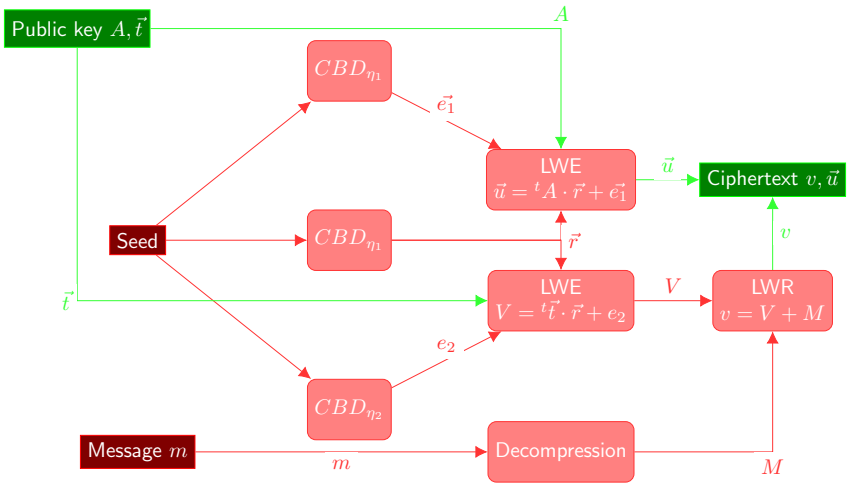

Fig. 3. Overview of the sensitive operations within PKE Encrypt

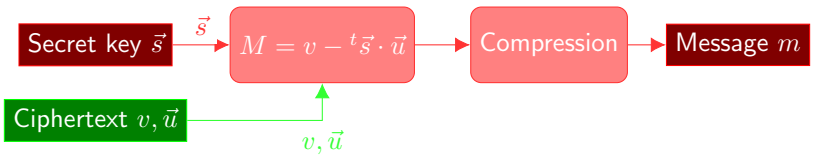

Fig. 4. Overview of the sensitive operations within PKE Decrypt

B.2 KEM

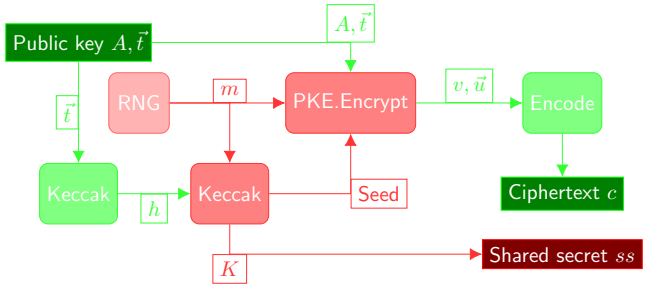

Fig. 5. Overview of the sensitive operations within the KEM Encapsulation

Algorithm 6 KEM Encapsulation

1: **Input:** Public key $pk = (A, \vec{t})$
2: **Output:** Ciphertext c
3: **Output:** Shared secret $K \in \{0,1\}^{256}$
4: m (256 random bits from system)
5: $m = H(m)$
6: $(K, seed) := G(m \| H(pk))$
7: $\vec{u}, v := PKE.Encrypt(pk, m, seed)$
8: $c = Encode(\vec{u}, v, d_u, d_v)$
9: **return** (c, K)

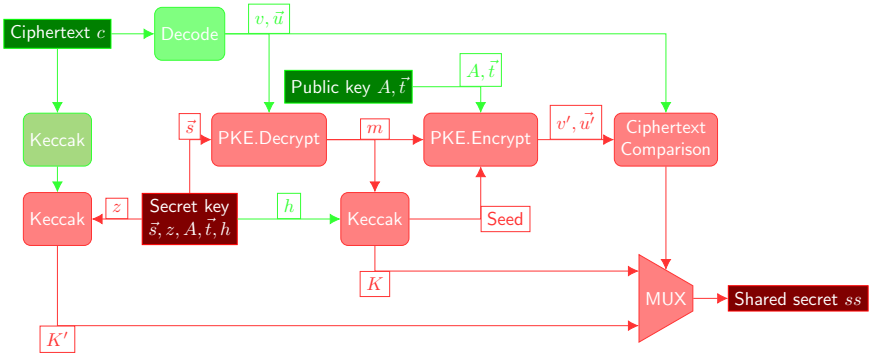

Fig. 6. Overview of the sensitive operations within the KEM Decapsulation

Algorithm 7 KEM Decapsulation

Input: Ciphertext c
Input: Secret Key $sk = (\vec{s}, pk, H(pk), z)$
Output: Shared key K
$\vec{u}, v = Decode(c, d_u, d_v)$
$m' := PKE.Decrypt(\vec{s}, \vec{u}, v)$
$(K', seed') := G(m' \| H(pk))$
$\tilde{K} = J(z \| c, 32)$ ▷ Our variation: $\tilde{K} = J(z \| H(c), 32)$
$\vec{u'}, v' := PKE.Encaps(pk, m', seed')$
$c' = (Compress_q(\vec{u'}, d_u), Compress_q(v', d_v))$
if $Compare(\vec{u'}, \vec{u}, v', v)$ **then** ▷ $Compare = 0$ if successful
    $K' = \tilde{K}$
end if
return K'

B.3 Parameters

Table 5. Parameter sets for ML-KEM

	NIST security level	n	q	sec	η_1	η_2	d_u	d_v
ML-KEM-512	I	256	3329	2	3	2	10	4
ML-KEM-768	III	256	3329	3	2	2	10	4
ML-KEM-1024	V	256	3329	4	2	2	11	5