A Stochastic Comparison-Grouping Model of Multialternative Choice: Explaining Decoy Effects

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Abstract

Based on Guo and Holyoak's (2002a, 2002b) work, we propose a stochastic comparison-grouping theory of multialternative decision making to explain three context-induced violations of rational choice. The attraction effect and the similarity effect are explained by stochastic comparison grouping, according to which similar alternatives are compared more frequently than dissimilar alternatives are. The compromise effect is explained by the assumption that attribute values are perceived according to a basic psychophysical function, in addition to the comparison grouping mechanism. Furthermore, this model explains individual differences in choice by assuming interpersonal differences in pre-existing attitude toward products.

Introduction

Rational theories of decision making suggest that choice is intrinsically determined by the utilities of individual alternatives, thereby unaffected by relationship among alternatives, which is a part of choice context. However, violations of this tenet have been found in many studies (e. g., Huber, Payne, & Puto, 1982; Simonson, 1989). Three much-studied findings of the so-called context-dependent choice warrants specific attention, as they constitute violations of axioms that are believed to be fundamental to rational choice. They are addressed together in this paper as they share important commonalities and can be explained by a unified framework. These findings include the attraction effect, the similarity effect, and the compromise effect (Huber, Payne, & Puto, 1982; Tversky, 1972; Simonson, 1989; Simonson & Tversky, 1992).

These effects all occur with the addition of a third alternative, called the decoy, to a two-alternative choice set and are all called *decoy effects*. Like in most research of the same line (e. g., Guo & Holyoak, 2002b; Roe, Busemeyer, & Townsend, 2001), they are examined in the present paper in a two-attribute form, which is schematized in Figure 1. The alternatives that constitute the core set are commonly referred to as the target and the competitor (also called the core alternatives in this paper), and the addition the decoy. The target and the competitor form a trade-off, that is, one is better than the other on one attribute but worse than the other on the other attribute. Depending on the position of the decoy relative to that of the target, three phenomena could occur. Two of them happen when the decoy is more similar to the target than to the competitor. If it is inferior to the target on all attributes, the choice probability of the target would increase relative to that of the competitor. This is called the attraction effect (Huber, Payne, & Puto, 1982). On the other hand, if a trade-off exists between the decoy and the target, the choice probability of the target would decrease relative to that of the competitor. This is called the similarity effect (Tversky, 1972). The third phenomenon occurs when the decoy sits between the target and the competitor, in which case the decoy, now constituting a compromise of the core alternatives, would be chosen most often. This is called the compromise effect. All three phenomena would potentially lead to violations of axioms of rational choice (will explain in detail later).

A number of explanations have been advanced for each of the three findings (e. g., Simonson & Tversky, 1992; Tversky, 1972; Tversky & Simonson, 1993), however, Roe et al. (2001) were the first to explain all three (in addition to other findings) with a single framework, implemented in a connectionist model derived from a previous stochastic mathematical theory (Busemeyer & Townsend, 1993). Their model accounts for these findings by variable lateral inhibition determined by similarity relations among alternatives and momentary shifting of attention from attribute to attribute. Subsequently, Guo and Holyoak (2002b) proposed a connectionist model accounting for the attraction effect and the similarity effect that is also based on inter-alternative similarity. They conceived the decision process as divided into two stages: the two more similar alternatives (i. e., the target and the decoy) are compared first, and joined by the competitor later. The first stage has an impact on the second stage and finally leads to these phenomena (will explain in detail later). The two-stage model derives its idea from perceptual grouping, according to which similar shapes are visually perceived as forming a unit. Analogously, similar alternatives are processed together at the early stage of decision process. In analogy to perceptual grouping, this mechanism is called *comparison grouping* in the present paper. Compared to Roe et al.'s model, the two-stage assumption is more consistent with some empirical studies that investigate decision processes of multialternative choice (Russo & Rosen, 1975; Satomura, Nakamura, & Sato, 1997). To explain the compromise effect, Guo and Holyoak (2002a) used another feature of the same model in addition to the two-stage assumption.

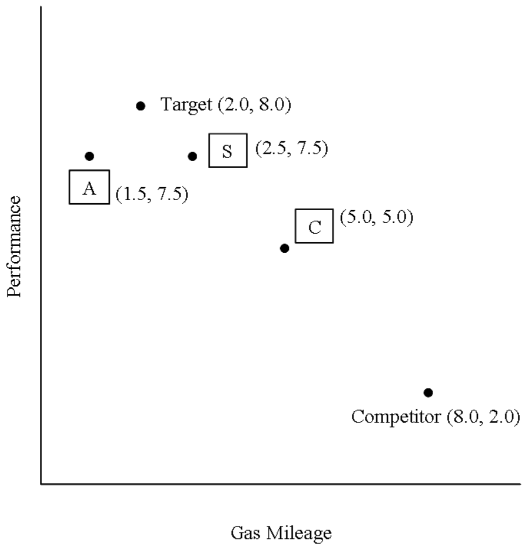

Figure 1: A summary of the phenomena simulated. The letters S, A, and C stand for the decoys for the similarity effect, the attraction effect, and the compromise effect respectively. The numbers in parentheses are attribute ratings.

Despite its explanatory simplicity and consistency with certain experimental data, the two-stage model seems oversimplified for describing human behavior – it is unlikely that people completely limit evaluation to just one pair of alternatives for a long period of time. Studies have shown that in multialternative choice tasks that resemble those giving rise to the three effects, people 1) momentarily shift attention across pairwise comparisons, and 2) similar pairs were compared *more frequently* than dissimilar pairs were (Russo & Rosen, 1975; Satomura et al., 1997). In addition, the second stage in that model was proposed to consist of triple-wise comparisons, whereas these studies suggested that choice predominantly consists of pairwise comparisons. Based on data from these studies, we propose a stochastic comparison-grouping model, in which all possible types of comparisons are performed momentarily with differential frequencies (Russo & Rosen, 1975; Satomura, Nakamura, & Sato, 1997). In addition, whereas Guo and Holyoak's model estimates choice probabilities from results of just one simulation by a mathematical conversion (Luce, 1959), the present model runs a large number of simulations to reflect decisions across individuals, thereby directly estimating choice probabilities. The psychophysical assumption (Guo & Holyoak, 2002a), proposed in conjunction with comparison grouping to explain the compromise effect, remains unchanged in the current model.

The Model

Decision Scenario and Model Architecture

The decision scenario used for simulation is adapted from that used by Roe et al. (2001). The decision maker has to choose one car from a set of two or three alternatives by evaluating two attributes; gas mileage and performance, which are measured on a 1 – 10 scale (see Figure 1). Accordingly, a connectionist model is constructed (see Figure 2). Each attribute or alternative is represented by one node (circle) in the network, with their relations represented by connections (lines with arrowheads). Each node has a certain degree of activation. For an alternative node, the activation stands for the valuation of the corresponding alternative; for an attribute node, it stands for the evaluative importance of that attribute. Node activations are within the range of 0.0 - 1.0.

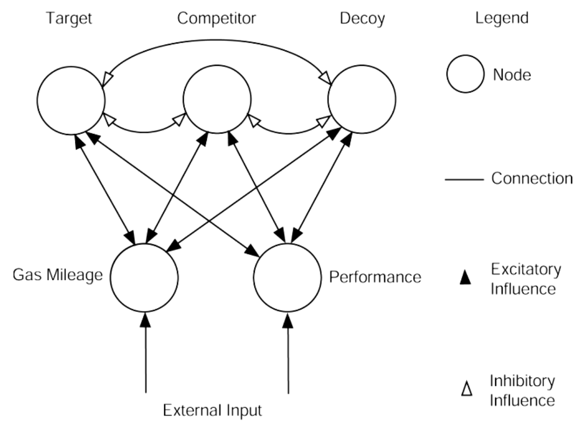

Figure 2: The architecture of the model. External Input represents the motivational and attentional sources that drive the decision process.

The connection between an attribute node and an alternative node, called the *attribute-alternative connection*, has an excitatory weight (i. e., when one node is more active, the other would be more active as well). The connection between each pair of alternative nodes has an inhibitory weight (i. e., when one node is more active, the other would be less active), also known as *lateral inhibition* in the literature. The lateral inhibition reflects the competitive relationship among alternatives. Via this mechanism, which would commonly result in one node achieving higher activation than the rest, the model "chooses" the winning alternative. All connections are bi-directional, reflecting the idea that influences can go either way between factors involved in decisions. The external inputs to the attribute nodes represent the motivational and attentional sources that drive the decision making.

This network representation was similar to Guo & Holyoak's (2002a, 2002b) model, and is consistent with common connectionist architecture used in decision modeling (e. g., Holyoak & Simon, 1999; Tsuzuki, Kawahara, & Kusumi, 2002).

Connection Weights and Initial Activations

The attribute-alternative weights reflect the perceived goodness of alternatives according to their attribute ratings, and were initially set to the corresponding attribute-alternative ratings. For example, the performance-target and gas mileage-target weights were first set to 8.0 and 2.0 respectively.

Recall that in Guo and Holyoak's model (2002a, 2002b) the perception of goodness follows a basic psychophysical function. In particular, this function reflects the idea that perceived goodness increases with negative acceleration with actual attribute value. Consistent with this idea, each attribute-alternative weight was further transformed by a logarithmic function:

$$w_{ij} = (\log_e (w_{ij} + \alpha) + \beta) / \gamma . \qquad (1)$$

Here, w_{ij} is the weight of the connection from node j to node i. α, β, and γ are set to 31.00, –3.35, and 0.905 respectively. Equation 1 reflects a weakly convex function. In addition, it serves as a normalization function that compresses these weights to values within a small range, which is comparable in magnitude to node activations (whereas the attribute values range from 0.0 to 10.0, the w_{ij}s range from 0.090 to 0.400.)

The lateral inhibitions are all set to -0.60. The initial activations of all nodes are conveniently set to 0.5, the middle point of the activation range, with the following qualification.

In reality, people usually have different pre-existing preferences regarding products. Accordingly, randomness was introduced to the initial activations of the alternative nodes, which were in the range of 0.5±0.25. The values follow a uniform distribution. As will be seen later, this randomness provides an explanation for individual differences in choice.

Running the Model

Connectionist models commonly run in an iterative fashion. In each iteration the activation of each node is updated according to the total influence it receives from the rest of the model – the activation increases if the influence is positive and decreases if otherwise. This influence can be understood as the overall reason for liking or disliking an alternative or attribute. A common activation function is used to specify this process (c. f., McClelland & Rumelhart, 1988).

$$a_i(t+1) = a_i(t) + \Delta a_i(t), \text{ where} \qquad (2)$$

$$\text{if } netinput_i > 0,$$
$$\Delta a_i = netinput_i\,(MAX - a_i) - decay \, . \, a_i$$
$$\text{otherwise}$$
$$\Delta a_i = netinput_i\,(a_i - MIN) - decay \, . \, a_i$$

$a_i(t+1)$ is the activation of node i at iteration (or time) $t + 1$; it is a function of $a_i(t)$, the activation of the same node at the previous iteration (or moment). $\Delta a_i(t)$ is the amount of activation change. The decay parameter reflects how much a neural signal decays over time (connectionist models draw analogies to neural processing), and is set to 0.04. The decay, however, does not play an important role in explaining the effects. *MAX* and *MIN* are the upper (1.0) and lower (0.0) limits of node activations. This equation specifies that node activation asymptotically approaches the upper or lower activation limit as a consequence of the total influence it receives from other components of the network. The total influence, $netinput_i$, is determined by the following equation.

$$netinput_i = istr \, . \, intinput_i + estr \, . \, extinput_i, \text{ where} \qquad (3)$$
$$intinput_i = \Sigma_j w_{ij}\, a_j(t)$$
$$extinput_i = 1$$

Intinput is the internal input that comes from all the attribute and alternative nodes, and depends on both the activation of the node feeding input to i and the connection strength that links the two nodes. *Extinput*, standing for the external input, should be a stable source of attention and motivation and is set to a constant. *Istr* and *estr*, set to 0.12 and 0.05 respectively, are constants that scale down activation changes so that the changes are not abrupt. Since the internal input is the source of these effects, *istr* is set to be larger than *estr*.

The model runs iteratively – in each iteration, the activation of each node in the model is updated according to Equation 2 and 3. The process reflects the evolution of valuation over time. This iterative process continues until an externally determined period of deliberation time, arbitrarily set to 100 iterations, is met.[1] The final winning choice is the alternative with the highest activation.

Stochastic Comparison Grouping

In a series of eye fixation studies, Russo & Rosen (1975) found that pairwise comparisons between similar options happen earlier and more frequently than other types of comparisons in multialternative choice. Consistent with that, Satomura et al. (1997) found that in decision tasks leading to the attraction effect, for the participants who chose the target (i. e., exhibited the attraction effect), 74% retrospectively reported that they compared the target to the decoy, while only 19% of them reported that they compared the target to the competitor. These studies give rise to the following modeling assumptions – All four kinds of possible comparisons (target-decoy, competitor-decoy, target-competitor, and target-decoy-competitor) are performed momentarily with different frequencies. To instantiate this in the model, for each iteration, a specific type of comparison is randomly chosen according to specified probabilities, and only the involved alternative nodes are updated.

[1] Other stopping criteria might be used. For example, the model can stop when the amount of activation difference across nodes is very large, or the amount of activation change becomes very small. However, according to our analysis, the type of criterion does not affect the qualitative pattern of the simulation results.

In simulating the attraction effect, the frequencies of these types of comparisons, in the order mentioned above, were set to the percentages of 74.0%, 10.4%, 10.4%, and 5.2% respectively. For example, for a random 74.0% of the iterations, only the activations of the target and the decoy were updated. These percentages were arbitrarily determined to roughly reflect previous experimental data (Russo & Rosen, 1975; Satomura et al, 1997), which suggested pairwise comparisons constitute the majority of the deliberation process, and the two similar alternatives were compared most often. To be consistent, the same decision process was employed for the similarity effect.

Simulations and Results

A total of 10,000 independent simulations, each standing for the deliberation of one individual, have been performed. For each simulation, the alternative with the highest final activation is the one chosen. Choice probability was obtained across all the simulations. The simulation results are presented both as choice probability (Table 1) and average node activation (Table 2). Note that choice probability is the criterion by which the modeling is judged.

Modeling Individual Differences

Note that node activation evolves as a continuous function of time. This means a node with high initial value tends to stay strong. For instance, if the initial value of the target is higher than that of the competitor, the target would tend to maintain relative advantage over the competitor in deliberation. Recall that initial activation values randomly vary across simulations. This randomness therefore leads to choice differences across simulations, and explains why sometimes the unlikely alternative was chosen. For example, the decoy was chosen with a slim chance in the attraction effect scenario – when the initial value of the decoy was very high relative to the target and the competitor, such initial advantage was too strong to be offset by the comparison-grouping mechanism. The modeling is consistent with the intuition that people have different pre-existing beliefs that randomly favor one alternative over another and tend to bias later decisions.

Binary Choice

The target and the competitor are set to equal additive attribute ratings: the target is rated 2.0 on gas mileage and 8.0 on performance, whereas the competitor is rated 8.0 on gas mileage and 2.0 on performance. The two attributes are assumed to be equally important. Consistent with the trivial prediction that their choice probabilities should be the same, the simulations yielded probabilities of 0.504 and 0.496 for them respectively. The slight inequality was due to the randomness in initial activations of the alternative nodes.

Attraction Effect

When the attraction effect occurs, the target benefits from the addition of the decoy more than does the competitor. Under some circumstances, this tendency leads to a higher choice probability for the target in the trinary set than in the core set. This constitutes a violation of the regularity principle of rational choice, according to which adding alternatives to a given choice set should not increase the probability of any alternative (Huber et al., 1982). In the simulation, the decoy was chosen to have attribute values of 1.5 and 7.5 for gas mileage and performance, respectively.

Comparison grouping, in conjunction with the competitive relationship among the alternatives, is able to explain this effect. Any time when the target is compared with the obviously inferior decoy (in modeling terms, this means the target node receives more input via the attribute-alternative connections than does the competitor), the activation of the target node increases whereas the activation the decoy node decreases. This differentiation is an intrinsic property of this type of connectionist model (c. f., McClelland & Rumelhart, 1988). Given that the deliberation process primarily consists of target-decoy comparisons, the target node would finally acquire higher activation than does the competitor.

The above analysis suggests that if the initial node activations were identical across the alternatives, the target would have been chosen for all simulations. However, with some randomness, it is possible that the competitor has a higher initial activation than does the target. If this initial difference, which has an impact on later comparisons, is large enough, the competitor would be chosen. This also suggests that with extreme initial values even the rather inferior decoy might be chosen. This seems consistent with the intuition that pre-existing beliefs regarding products carry a weight on later decisions.

The simulated choice probabilities of the target, the competitor, and the decoy were 58.7%, 36.6%, 4.8%. The probability of the target exceeds that in the binary choice scenario, thereby leading to a violation of the regularity principle.

Similarity Effect

In the decision situation that leads to the similarity effect, the target looks less attractive relative to the competitor once the decoy is introduced. Under certain situations, this would lead to a change of rank order of the target and the competitor. For example, in the simulated scenario, the core alternatives rank the same in the binary set, but the competitor would rank higher than the target if the similarity effect occurs. This constitutes a violation of the independence of irrelevant alternatives principle of rational choice, which states that adding a decoy to an original choice set should not alter the rank order of the alternatives (c. f., Tversky, 1972). Decoy was set to have attribute ratings of 2.5 and 7.5 for gas mileage and performance respectively. Note that its additive attribute rating is identical to that of the target.

Like in the case of the attraction effect, comparison grouping and inter-alternative competition are able to explain the similarity effect. Any time when the target is compared with the similarly attractive decoy, the activation of both the target and the decoy nodes decrease due to their mutual inhibition of equal strength. This again is an intrinsic property of this type of connectionist model (c. f., McClelland & Rumelhart, 1988). Because the target-decoy comparison is the predominant type of comparison, compared to the competitor node, the target node hurts more from the comparison with the decoy, and would finally acquire lower activation than does the competitor.

The simulated choice probabilities of the target, the competitor, and the decoy were 27.8%, 39.7%, and 32.6%. Note that the tie between the target and the competitor was broken, indicating a violation of the independence of irrelevant alternatives principle.

In summary, the similarity effect and the attraction effect can be explained by frequency difference between the target-decoy and competitor-decoy comparisons. This also suggests that simulations of these effects should not depend on a particular specification of frequency ratio for the four types of comparisons – so long as this frequency difference is substantial, the two effects should be observed. In fact, other frequency ratios were used in our simulations and the same pattern was obtained (not reported here due to space limit).

Table 1: Simulation results as choice probability (estimated from 10,000 simulations).

Choice scenarios	Choice probability		
	Target	Competitor	Decoy
Binary choice	0.504	0.496	-----
Attraction effect	0.587	0.366	0.048
Similarity effect	0.278	0.397	0.326
Compromise effect	0.213	0.219	0.568

Table 2: Simulation results as average node activation and *SD*.

Choice scenarios	Average Node Activation (*SD*)		
	Target	Competitor	Decoy
Binary choice	0.293 (0.060)	0.294 (0.060)	-----
Attraction effect	0.320 (0.034)	0.305 (0.046)	0.233 (0.038)
Similarity effect	0.286 (0.033)	0.300 (0.046)	0.291 (0.033)
Compromise effect	0.275 (0.021)	0.276 (0.021)	0.291 (0.022)

Note. The results are computed from the simulations summarized in Table 1.

Compromise Effect

When the decoy for the similarity effect moves toward the competitor and finally reaches the middle point between the target and the competitor, the similarity effect turns into the compromise effect – The decoy changes from the least popular to the most popular alternative. In a decision scenario slightly different from the present one, this effect can also lead to a violation of the regularity principle (see Simonson, 1989 for more detail).

The comparison grouping assumption suggests that frequency of pairwise comparison increases with inter-alternative similarity. Accordingly, the percentages of the target-decoy, competitor-decoy, and target-competitor comparisons have the ratio of 2 : 2 : 1, inversely proportional to psychological distance[1]. The triple-wise comparison, being the least frequent, was arbitrarily set to one half as frequent as the least frequent pairwise comparison. Hence, the percentages of the four types of comparisons were set to 36.36%, 36.36%, 18.18%, and 9.10%.

The psychophysical assumption implemented in Equation 1 gives rise to this phenomenon. (This mechanism was still at work in the simulations of other two phenomena, but it did not play a causal role in producing them.) Take the target-decoy comparison for an example. The advantage of the decoy over the target (ratings of 5 versus 2 on gas mileage) looms larger than the advantage of the target over the decoy (ratings 8 versus 5 on performance) after the attribute ratings have been transformed into connection weights. (Calculated by Equation 1, the sum of the two attribute-alternative weights is 0.512 for the decoy, higher than the 0.505 for the core alternatives.) Hence the total input the decoy node receives via the attribute-alternative connections is the largest among the alternative nodes, making the decoy the winner.

The simulated choice probabilities of the target, the competitor, and the decoy were 21.3%, 21.9%, and 56.8%. Note that the specification of comparison percentages is not unique – so long as there is no frequency difference between the target-decoy and the competitor-decoy comparisons, neither the target nor the competitor would be bolstered relative to the other. The psychophysical mechanism would then guarantee choosing the decoy.

Comparison grouping provides a unified framework toward understanding the three phenomena. In particular, it explains why difference between the core alternatives exists in the similarity effect but disappears in the compromise effect, as comparison grouping can be modified by changing the similarity between the decoy and the core alternatives.

[1] This is just one way of specifying the inverse relationship between frequency ratio and similarity, which should be viewed as qualitative rather than quantitative. Note that in simulations of the other two effects, frequency ratios are not determined by the same function and just roughly reflect the inverse relationship.

Conclusion

We propose a stochastic comparison-grouping theory cast in a connectionist model to explain three important violations of rational choice. In addition, this model lends us understanding of the decision processes involved in these tasks.

A comparison is made between this model and previous accounts of the same findings. It extends Guo & Holyoak's (2002a, 2002b) model by incorporating insights from experimental data (Russo & Rosen, 1975; Satomura et al., 1997). In addition, it better accounts for individual differences in choice by introducing randomness in intial beliefs to the model. In comparison with Roe et al.'s (2001) model, both models use similarity relationship, but in different manners. Their model uses variable lateral inhibition that increases with inter-alternative similarity, whereas the current model proposes a similarity-based grouping mechanism. In addition, both models suggest momentarily shifted attention, again in different manners. In their model attention shifts from attribute to attribute, whereas in the present model attention shifts across different types of pairwise comparisons. The assumptions of this model seem more consistent with the aforementioned experimental data. Future studies are in order to further test the relative merits of the two models.

One apparent problem of the present model is that the modeling seems to depend on manually specified parameter values rather than psychological principles. Our justification is that these parameters specify linear transformations that do not alter the essence of the modeling assumptions. In addition, the same set of parameter values applies to all three phenomena.

Finally, this model is consistent with theoretical frameworks that relate cognition to perceptual processes (c. f., Medin, Goldstone, & Markman, 1995, Goldstone & Barsalou, 1998), and its proposed perceptual mechanisms might help us understand decision behavior at large.

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