Title

Design of optimum discrete finite duration orthogonal Nyquist signals

Journal

IEEE Transactions on Signal Processing, 36(4)

ISSN

1053-587X

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Publication Date

1988-04-01

DOI

10.1109/29.1565

Peer reviewed

hardware this conversion can be implemented by simply toggling the sign bit and considering the binary fraction point to be at the extreme left position. For example, if $x = 1.001(= -0.875)$ and $y = 0.100(= 0.5)$, then $x' = 0.001(= 0.625)$ and $y' = 0.1100(= 0.75)$. Sequences X and Y for x' and y' are generated as before. However, the multiplication is now performed by inverted XOR gates instead of AND gates. This ensures that Z has a 1 whenever X and Y are both 0 or 1. The number of 1's are counted in Z to obtain z'. The counters are initially offset by 1 to account for the missing 0 from the PN sequence. To get the desired product z, we use the inverse mapping $z = 2z' - 1$. In hardware this is achieved by toggling the most significant bit of z', which now becomes the sign bit of z and the answer is in two's complement binary notation.

The above technique can be applied to computation of inner product of vectors used in digital filtering, correlation, Fourier transformation, etc. [7]. An efficient implementation for this basic operation in signal processing is based on merged arithmetic [6]. In this design the boundaries between the multipliers and adders are dissolved. Let the required inner product be $z = x_1 \times y_1 + x_2 \times y_2 = z_1 + z_2$. The input data are provided in n-bit two's complement form. The product sequences for z_1' and z_2' are each 2^{2n} bits long. Using one composite $(2^{2n+1} | 2n + 1)$ parallel counter we obtain the product z'. After inverse mapping we get z'', which is half the desired value z. This automatic scaling by 2 (on account of doubling of the product sequence) is required to ensure that the numbers involved in computation remain in the range 0 to 1.

VI. Conclusion

In summary, we have presented a sparse unary coding technique for binary numbers with built-in fault tolerance and the property that the product can be computed accurately by the use of simple logic gates. We have also included how this multiplication technique can be used to implement digital filters using bit-serial or parallel architecture in VLSI.

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Design of Optimum Discrete Finite Duration Orthogonal Nyquist Signals

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Abstract—**This correspondence presents a design technique for construction of a discrete finite duration signal whose spectral energy is maximized in a given band. The constructed signal is also orthogonal to the block shifted version of itself.**

Manuscript received May 23, 1986; revised November 4, 1987. This work was supported by the Office of Naval Research under Contract N00014-84-K-0598.

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IEEE Log Number 8719030.

I. Introduction

The objective of this correspondence is to present a procedure for designing discrete signals which are of finite duration and whose energy is concentrated in a given frequency band. The generated signal also has no intersymbol interference, i.e., the signal is orthogonal to its block shifted version.

We limit our discussion to low-pass signals as any bandpass signal can be represented by a modulated low-pass signal. The problem of obtaining a finite duration continuous signal whose energy is concentrated in a given base band was investigated by Ville and Bouzitat [1] and generalized by Slepian and Pollack [2] and then by Landau and Pollack [3]. It has been known that the optimum signal of one baud time (interval time between successive pulses or a sample period), whose energy is concentrated in a given base band, is the truncated first prolate spheroidal wave function. However, for signals of more than one baud time duration, it is necessary to remove intersymbol interference. There are two common constraints which are used to remove intersymbol interference. The first constraint is that the desired signal has zero values at all sampling instances but one. This is often referred to as the zero crossing property. Such signals are called "Nyquist signals." A raised cosine signal falls in this category. This constraint leads to a linear design technique and was recently utilized by Panayirci and Tugbay [5]. The other constraint is that the desired time limited signal is orthogonal to its shifted versions, i.e., its autocorrelation function is a Nyquist signal. We call them orthogonal Nyquist signals. This orthogonality property is superior to the zero crossing in that the former is less sensitive to additive noise of zero mean on the signal. This is one of the reasons that matched or correlation filters are often used to process received signals. The orthogonality constraint leads to a nonlinear problem. Halpern [6] has utilized the variational principle to analyze this problem, and has also utilized the prolate spheroidal functions to expand the unknown signal.

An alternate approach to the design of a finite time signal with its energy concentrated in a given band has been taken by MacCall [7], Gerst and Diamond [8], and Dines and Hazony [9], where they produced a time limited signal by forming an entire function in the transformed domain. Detailed discussion of this method is given in Papoulis [10]. The k-pulse concept due to Kennaugh [11] is along these lines.

In this correspondence, we present a design technique for discrete finite duration optimal signal with the orthogonality constraints. We deal with discrete (sampled) signals rather than the continuous ones which have been dealt with by all the previous authors. In our formulation, we minimize a constrained nonlinear functional by the method of steepest descent. We feel that this iterative method is computationally faster (especially for a few baud time) than expanding the unknown signal in terms of prolate spheroidal functions and the solving for their unknown coefficients.

II. Formulation of the Problem

Let us assume a discrete signal sequence $f(m)$ is defined for $m = 0, 1, 2, \cdots, N_m - 1$ and is identically zero outside these N_m values. If the signal sequence extends over N_c baud times and consists of N_s samples per baud time, then we have $N_m = N_c N_s$. The DFT of the signal is defined by

$$F(l) = \frac{1}{\sqrt{N_l}} \sum_{m=0}^{N_m - 1} f(m) \exp\left[\frac{-j2\pi lm}{N_l}\right], \quad l = 0, 1, \cdots, N_l - 1 \tag{1}$$

and its inverse transform by

TABLE I
COMPARISON OF TWO METHODS FOR DESIGNING DISCRETE ORTHOGONAL NYQUIST SIGNALS (BAND MEANS THE ACTUAL BANDWIDTH DEVIDED BY BAUD RATE)

Band (in baud rate)		0.6	0.8	1.0	1.4	1.8
In-Band Energy J (in percentage)	[6] (Halpern)	not available	0.924	0.998	0.9994	0.99949
	New Method	0.96595	0.9983	0.9996	0.99999	>0.99999

$$f(m) = \frac{1}{\sqrt{N_l}} \sum_{l=0}^{N_l - 1} F(l) \exp\left[\frac{+j2\pi l m}{N_l}\right], \quad m = 0, 1, \cdots, N_m - 1 \tag{2}$$

where N_l is the number of samples for the DFT sequence $F(l)$. In the frequency domain if we assume N_r samples per baud rate (inverse of baud time) then $N_l = N_r N_s$. Note that $N_r \geq N_c$, and increasing N_r increases resolution in frequency domain.

Now our objective is to maximize the energy J within the set $\Phi = \{-N_b \leq l \leq N_b\}$ which corresponds to a frequency band of N_b/N_r baud rate, or equivalently, minimize the energy out of the above set, under the orthogonality constraint

$$\sum_{m=0}^{N_m - 1} f(m) f(m + kN_s) - \delta(k) = 0, \quad k = 0, 1, \cdots, N_c - 1 \tag{3}$$

where $\delta(k)$ is the delta function. So that the cost function can be defined as

$$\begin{aligned} J_c &= W \sum_{l \notin \Phi} |F(l)|^2 + \sum_{k=0}^{N_c - 1} d_k e_k \\ &= W \Bigg[\sum_{n=0}^{N_m - 1} f^2(n) - 1/N_l \sum_{m=0}^{N_m - 1} \sum_{n=0}^{N_m - 1} \\ &\quad \cdot f(m) f(n) \frac{\sin\left\{\frac{2\pi(n-m)}{N_l}\left(N_b + \frac{1}{2}\right)\right\}}{\sin\left\{\pi \frac{(n-m)}{N_l}\right\}} \Bigg] \\ &\quad + \sum_{k=0}^{N_c - 1} d_k \left[\sum_{m=0}^{N_m - 1} f(m) f(m + kN_s) - \delta(k)\right]^2 \end{aligned} \tag{4}$$

where W is the weight on the out-of-band energy, e_k's are the errors associated with the constraint equations (3), and d_k's are the corresponding weights.

The minimization process can be outlined as follows while details are available in [12].

Step 1: Choose an initial guess of $f(m)$ and initial values of W and d_k's.

Step 2: Compute the gradient vector of J_c with respect to $f(m)$ and find an optimum step length to update the signal sequence $f(m)$.

Step 3: If the norm of the previous gradient vector is not small enough, go to step 2. Otherwise, see whether the orthogonality errors $|e_k|$ ($k = 1, \cdots, N_c - 1$) are small enough. If the errors are small enough, stop the process. If the errors are still considered to be large, increase each d_k ($k = 1, \cdots, N_c - 1$) by a factor and then go to step 2.

Note that throughout the process, d_0 may be a fixed nonzero value since the absolute value of the total energy is not important. However, the weight W can be increased during the process if the in-band energy is unexpectedly low; because the cost function may have more than one local minima and increasing W can make one jump out of an undesired local minimum.

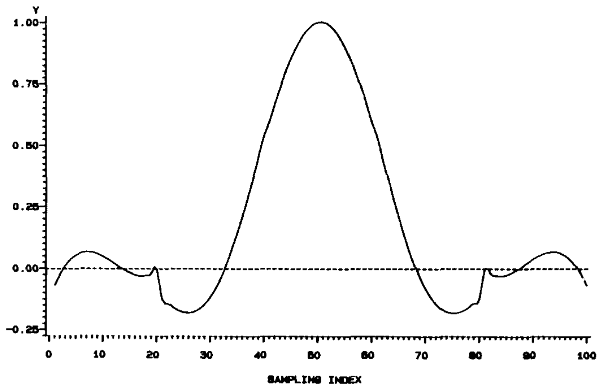

Fig. 1. Orthogonal Nyquist signal. $N_s = 20$; $N_c = 5$; band = 0.6 and the in-band energy $J = 0.98681$ percent (with $N_r = 100$).

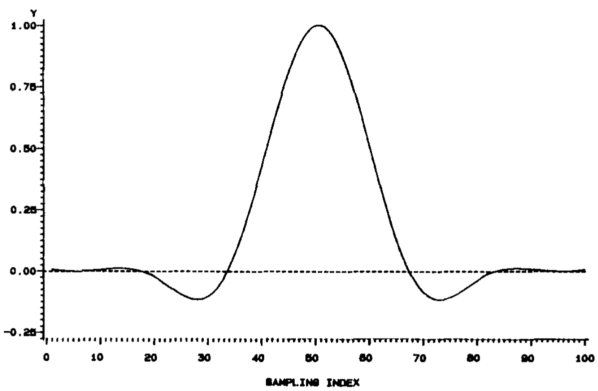

Fig. 2. Orthogonal Nyquist signal. $N_s = 20$; $N_c = 5$; band = 0.8 and the in-band energy $J = 0.99955$ percent (with $N_r = 100$).

III. NUMERICAL RESULTS

We summarize our numerical experiments by presenting a table and two figures. Table I is for three baud time signals with the resulting normalized orthogonality errors all less than 0.00001. As we see, the in-band energies are improved over those obtained by the previous approach [6].

Fig. 1 shows that severe discontinuity in the optimized waveform spanning five baud times (which did not occur in three baud time signal case) appears if the bandwidth is chosen to be very small. The optimization process was repeated with several initial guesses but all ended up with the same discontinuous solution.[1] However, when we increased the band, the discontinuity disappeared. Fig. 2 shows the result where band is 0.8 baud rate as opposed to 0.6 of Fig. 1.

[1]By discontinuous signal we mean the discrete sequence where large jumps exist. This term should not be confused with the continuous or discontinuous analog signals.

One would obtain a continuous signal by expanding the signal in terms of sampled prolate spheroidal or Legendre functions. However, if the band is greater than 0.8, for example, then the "continuous" signal can be obtained in a more efficient way by the present approach which exploits more degrees of freedom than the prolate spheroidal expansion approach.

IV. Conclusion

A nonlinear optimization technique is presented for the design of optimum discrete finite duration orthogonal Nyquist signals. This approach is believed to be more efficient than expanding the signal in terms of finite duration functions (like prolate spheroidal, Legendre, and so on), particularly when the signals last a few baud times.

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Performance Contours of Autoregressive Estimates

SHIPING LI AND BRADLEY W. DICKINSON

***Abstract*—Given an autoregressive (AR) model, to what region of parameter space will parameter estimates belong if the prediction error variance is not minimized? Given a set of distorted LMS estimates, in what region will the true parameters lie? We investigate these questions. Several examples involving second-order AR models are given.**

I. Introduction

The least mean-square (LMS) criterion has been widely used in system identification and parameter estimation for numerous applications. There is a vast body of literature on this subject. In this correspondence, we explore relations between parameter estimates and the prediction error variance functional for autoregressive models. Given a pth-order AR process, we obtain an explicit formula which relates the true parameters and their estimates (or distorted versions) to the variance of the prediction error process. For a second-order AR model, we determine contours within which the parameter estimates must lie, corresponding to a set of prediction error variance values. Conversely, for a given set of parameter estimates, we obtain contours within which the true parameters would lie, corresponding to a set of prediction error variance values. These results provide some interesting insights about AR modeling problems.

Suppose we have an AR(p) process $x(n)$ described by

$$x(n) + \sum_{i=1}^{p} a_i x(n-i) = w(n) \tag{1}$$

where $w(n)$ is a white Gaussian process with zero mean and variance σ_w^2, and the parameters $\{a_i,\ 1 \le i \le p\}$ are such that the roots $\{z_i,\ 1 \le i \le p\}$ of

$$A(z) = 1 + \sum_{i=1}^{p} a_i z^{-i} = 0 \tag{2}$$

are located inside the unit circle. The LMS estimation of an AR model can be schematically described as in Fig. 1, where $\hat{A}(z) = 1 + \Sigma_{i=1}^{p} \hat{a}_i z^{-i}$ and $\{\hat{a}_i,\ 1 \le i \le p\}$ are estimated by minimizing the prediction error variance $E\{e^2(n)\}$. It can be shown that

$$\sigma_e^2 = E\{e^2(n)\} \ge E\{w^2(n)\} = \sigma_w^2 \tag{3}$$

where equality holds if and only if $\hat{a}_i = a_i,\ 1 \le i \le p$. That is, the prediction error process $e(n)$ will have minimum power if and only if the estimates $\{\hat{a}_i\}$ are exactly the true parameters $\{a_i\}$.

In practice, the estimates $\{\hat{a}_i\}$ must be obtained from a finite sample of observations of $\{x(n)\}$ and will never be exact; consequently, $\sigma_e^2 > \sigma_w^2$. The Cramer–Rao lower bound for mean-square parameter estimation error provides one means of characterizing relationships between parameter estimates and "true" parameter values. In our analysis, we determine such relationships that arise from structural considerations, without regard to a criterion for estimator quality. These considerations may be of importance not only in situations involving parameter estimation, but also in cases where numerical accuracy of solving for "theoretical" LMS parameter values is a concern.

Two questions of interest will be considered. Given an AR(p) process, if we set the prediction error variance $\sigma_e^2 = c\sigma_w^2$, $c > 1$, in what region will the estimates $\{\hat{a}_i,\ 1 \le i \le p\}$ be? Alternately, given a set of estimates $\{\hat{a}_i,\ 1 \le i \le p\}$, in what region will the true parameters be and with what distortion in terms of prediction error variance which we try to minimize?

II. Results

From Fig. 1, we see that the prediction error variance can be expressed as

$$\sigma_e^2 = E\{e^2(n)\} = \frac{\sigma_w^2}{2\pi j} \oint \frac{\hat{A}(z)}{A(z)} \frac{\hat{A}(z^{-1})}{A(z^{-1})} z^{-1}\, dz \tag{4}$$

where the integration is around the unit circle. Using the Residue Theorem and writing the integration result in terms of the corresponding roots, we have

$$\frac{\sigma_e^2}{\sigma_w^2} = \prod_{i=1}^{p} \frac{\hat{z}_i}{z_i} + \sum_{k=1}^{p} \frac{(z_k - \hat{z}_k)}{z_k} \cdot \prod_{\substack{i=1\\ i \ne k}}^{p} \frac{(z_k - \hat{z}_i)}{(z_k - z_i)} \prod_{j=1}^{p} \frac{(1 - z_k \hat{z}_j)}{(1 - z_k z_j)} \tag{5}$$

Manuscript received July 28, 1986; revised April 24, 1987. This work was supported by the National Science Foundation under Grant ECS84-05460 and by the U.S. Air Force under Grant AFOSR-84-0381.

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IEEE Log Number 8719025.

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