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# The Value Of Runway Time Slots For Airlines

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# THE VALUE OF RUNWAY TIME SLOTS FOR AIRLINES

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In flight scheduling, airlines usually determine optimal timing for their flights to respond to time-dependent demand and the requirement of frequency plans, of available fleets and of aircraft routings. Nevertheless, it is unavoidable that some flights cannot actually operate at their expected time because of the capacity limit of the airport runway. Thus, adjustments have to be made by altering some flights from their optimal times. Scarce runway time slots represent a resource whose value to the airline may be determined from the impact of such re-scheduling on the objective function of the original schedule. In this paper, we first analyze the relationship between rescheduling of flights and airline profit. To assess the impact of flight rescheduling a minimum-cost flow model is constructed. Solving this model gives a new optimal schedule under the condition of rescheduling specific flights at specific time slots. Based on this new optimal schedule the value of specific time slot at specific airport runway is calculated. The method is demonstrated on a sample airline flight network. The model developed can be used for congestion pricing, runway slot auctioning of adjusting airline scheduling programs to accommodate runway capacity constraints.

*Keywords*: flight scheduling, time-dependent demand, value of runway time slot, minimum-cost flow model.

## 1. INTRODUCTION

In optimizing their flight schedules airlines allocate available aircraft to flights, on the basis of their network structure and frequency plan, time-dependent demand and the fleet available, and with the objective of profit or revenue maximization, or cost minimization. Under a given optimal flight schedule for an airline, flights are assigned specific departure and arrival times at specific airports. This originally optimal schedule may have to be modified if the confluence of, say departure times from a particular airport as resulting from the schedules of different airlines will result in a traffic flow rate that exceeds the capacity of that airport. In the extreme case the optimal departure times of a number of flights of different airlines, or even the same airline, may coincide. Therefore, in practical operations, it is unavoidable to remove some flights from their optimal schedule. For a typical flight this may result in: (i) a change of the expected revenue caused by the change of time-dependent demand; and (ii) a change of the related aircraft routing which may cause further perturbations in the system, such as to connecting flights. Depending on the value of these impacts, it can be said that the different runway slots at an airport will have

different values to different flights, or airlines. One way to value runway time slots is to ask how much an airline should be willing to pay for specific ones? The purpose of this paper is to provide a method of assessing the runway time slot value for a specific airline. This is done by linking the value (measured in terms of revenue or profit) of an airline schedule to the specific time slots used at an airport. Perturbations to the schedule caused by a shortage in these slots can be used to assess their impact of the shortage, and consequently the value of the time slots.

In next section, a minimum-cost flow model is used to modify an optimal schedule under time slot shortage. This is followed, in section 3, by the estimation of slot value by comparing the original and the new schedules. The method is demonstrated, in Section 4, on a sample flight network with 10 airports, 20 aircraft and 91 flights. Conclusions are presented in Section 5.

## 2. IMPACT OF FLIGHT REASSIGNMENT

### 2.1 *Relationship between Flight Rescheduling and Airline Profit*

Assuming that the operating cost of a particular flight is independent of the specific departure time, within the short time perturbations in question, we simplify the analysis by measuring time dependent demand in dollars of profit rather than in passengers. The time-dependent profit of a flight is defined as the difference of its revenue and its operating cost. In the following discussion we begin with an original schedule that is optimal under conditions of no time slot constraints. An "Original Schedule" in this paper refers to an optimal flight network and time schedules in the absence of time slot constraints. In this paper, it is used as a base to estimate the impact of changing time slots, and thereby to assess values of time slots. The optimization of this kind of schedule itself is beyond the scope of this paper.

Fig.1 illustrates a schedule table for a case with an aircraft performing two flight departures from two sequenced airports. In this figure, with time on the vertical axis the original schedule is represented by the solid line in the schedule panel in the center of the figure. Demands for flights 1 and 2 are shown to the right and to the left of the schedule panel respectively. If the first flight departure has to be advanced or delayed due to a runway scheduling constraint, then the arrival of the aircraft at the second airport, and consequently the departure of the second flight will also have to be adjusted. In the case of the flight being advanced (shown by the thick dashed lines), the demand for Flight 1 will decrease. But, the aircraft will arrive at Airport 2 earlier and this provides a possibility for Flight 2 to depart Airport 2 earlier when there is higher demand for it. In the case of the flight 1 being delayed (represented by thin dashed lines) it will get higher demand, while Flight 2 will get the lower demand because of the delay of the departure of flight 2. Thus an important question is how to assess the impact, to the airline, of changing a flight's precise schedule, or in other words, how much should an airline be prepared to pay to maintain its use of an originally scheduled runway time slot.
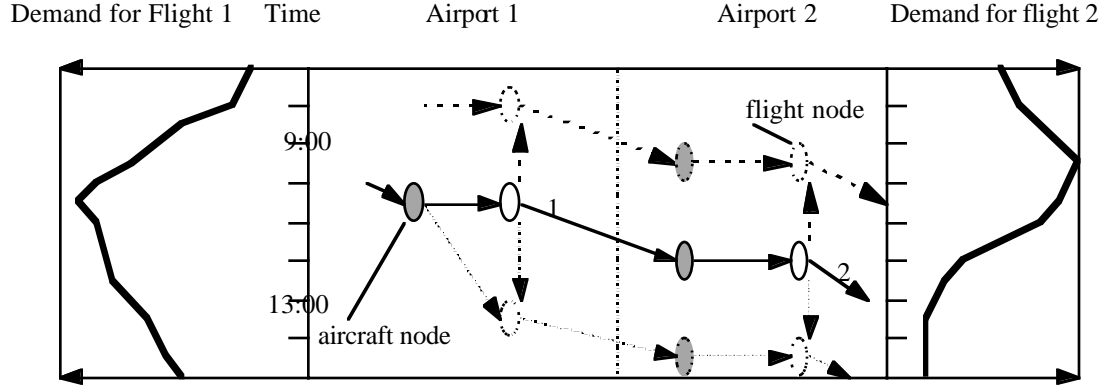
Fig.1. Impact of flight reassignment

Denote by $R_0$ the airline profit under the expected schedule. As mentioned earlier, a flight reassignment may cause a series of effects on the schedule. If we can find the new optimal schedule for the airline based on the flight reassignment and denote by $R_{new}$ the profit of the new schedule, then we can assess the impact $\Delta R$ of the flight reassignment, that is

$$\Delta R = R_0 - R_{new}.$$

### 2.2 *The Impact of a Schedule Change*

To develop a method for finding the new optimal schedule based on the original schedule and specific flight reassignment we modify a scheduling model we developed in [1] and [2] for dealing with schedule perturbation.

The flight sequence taken by an individual aircraft is called a *flight link*. As shown in Fig.1, when a flight is rescheduled some other flights may be affected. We call these flights the *related flights*. For each related flight, its scheduled time may change in the new optimal schedule. In this model we define a set of alternative times for these flights. As shown in Fig.2, flight *f* is considered as a ghost flight group $S_f =: \{f_1, f_2, ..., f_k\}$ in which every flight is set at a specific time. In the new optimal schedule no more than one flight in $S_f$ will be selected, and the scheduled time of flight *f* is determined according to the time of the selected ghost flight. Thus, the new optimal schedule is that group of ghost flights that maximizes the total profit for the airline, subject to the constraints of aircraft routings.

Based on the time-dependent demand we can compute the time-dependent profit of each alternative flight time. The alternative time can be determined by analyzing a profit curve such as the one shown in Fig. 2. Actually, we can set the alternative time at every point of interval $\Delta t$ during the possible time period. Of course, the choice of the size of the interval $\Delta t$ depends on the desired accuracy.
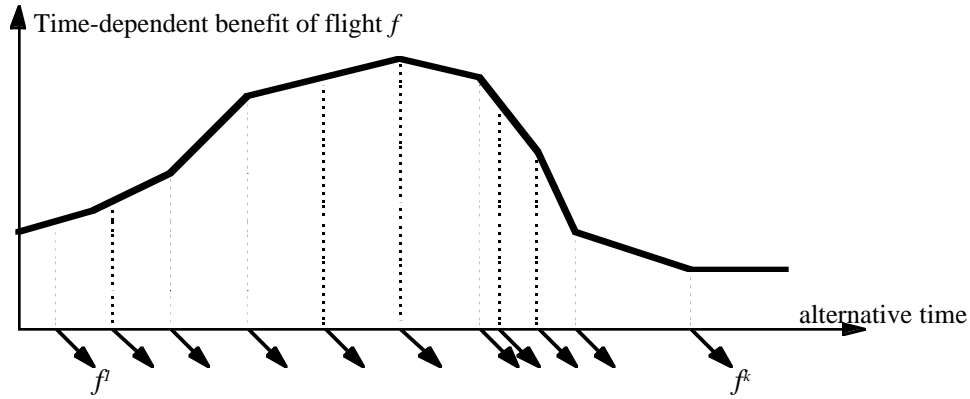
Fig. 2. Alternative time for flight $f$

Define

$A$         index for aircraft nodes.

$F_a$       subset of $F$ consisting of candidate flights considered for aircraft $a$. The scheduled time of every flight in subset $F_a$ must be later than the ready time of aircraft $a$.

$A_f$      subset of $A$ consisting of candidate aircraft considered for flight $f$.

$L(f,a)$    relationship between flight nodes and aircraft nodes in different stations. If $f$ and $a$ in are connected, then $L(f,a)=1$, otherwise $L(f,a)=0$.

$b_f$       profit of flight $f$.

$s_{af}$      cost of reassigning aircraft $a$ to flight $f$, including the cost caused by holding the aircraft or accelerating the related operations to meet the need of short time connection, etc.

Now we can express the model for new optimal schedule. In order to simplify the scheduling diagram the network is divided into two layers, as shown in Figs.3 and 4. Fig.3 illustrates the connecting relationship and Fig.4 illustrates the possibility of aircraft ferrying. In this network, we use two node types: aircraft nodes and flight nodes, and we define the following:

    *flight link*   is a flight sequence taken by the same aircraft;

    *leading flight*   is the first flight of a flight link;

    *leading aircraft node* is the one assigned to a leading flight.

In order to describe the scheduling diagrams used to analyze the reassignments, consider that only time $t^*$ is available for flight $f^*$ at Station 1. An alternative flight group $S_f$ is defined. The features of the network are defined as follows:

(*i*) Every node representing an aircraft (the filled cycle) is connected by arcs with the flight nodes (the unfilled cycles) to which the aircraft is possibly assigned. The cost of an arc is the cost $s_{af}$ of that particular assignment.

(*ii*) Every flight node, say $f_1$, is connected by an arc with only one aircraft node at the station where this flight is going. The cost of this arc is the negative of the flight's profit, i.e. $-b_{f1}$.

(*iii*) Every leading aircraft node (the starting node of a flight link, e.g. node *a* in Fig.3) is supposed to have a supply of 1 which is going to the unique destination (node *D* in Fig.3).

(*iv*) Node *D* represents a destination with demand of *m* (*m* indicates the total number of aircraft available in the scheduling). In Fig.3 Node *D* appears in every station just for avoiding the congestion of the figure. Node *D* is connected with every aircraft node of Fig.3 by arcs with cost of 0.

(*v*) For every aircraft node of Fig.3 there is a group of flight nodes representing the possibility of ferrying the aircraft to other stations (as shown in Fig.4). These ferrying flights are divided into two types. The first type is for Aircraft *a\**, (1) if Aircraft *a\** is a leading aircraft, the ferrying flight's schedule time is set to be the ready time of *a\** and the flying time is set to 0; the costs of the arcs connecting Aircraft *a\** and the ferrying flights and the ferrying flights' profit are set to 0. This is because that the assignment of *a\** to a ferrying flight is equivalent to abandoning Flight *f\** and scheduling *a\** to lead at the station where *a\** is going to be ferried; (2) if *a\** is not a leading aircraft, the ferrying flight's schedule time is set to be the time *t\** (the time available for flight *f\**), In other words, *a\** can only be ferried at the available time *t\**; the profit is the corresponding profit at time *t\**. The second type of ferrying flights are for other aircraft nodes of Fig.3 than *a\**, such as $a_{i2}$ in Fig.4. The schedule time for these ferrying flights is set to be the ready time of $a_{i2}$; the ferrying flight's profit is set to be the negative of the corresponding profit, $-b_a^t$.

It should be pointed out that some aircraft nodes of Fig.4, say $a_{i1}$, do not need to be connected with ferrying flights, because $a_{i1}$ itself follows a ferrying flight so has no need to be ferried once more.
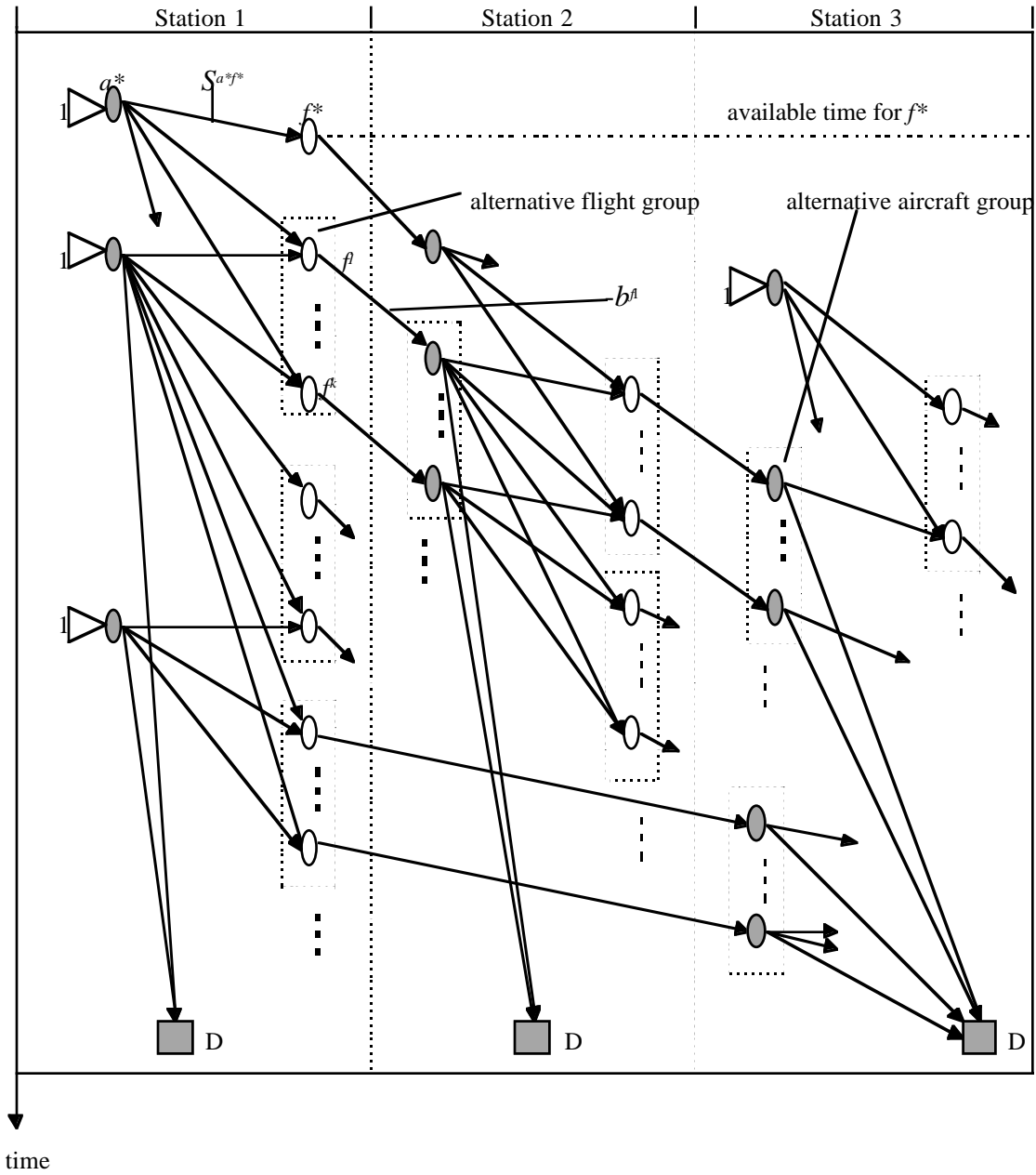
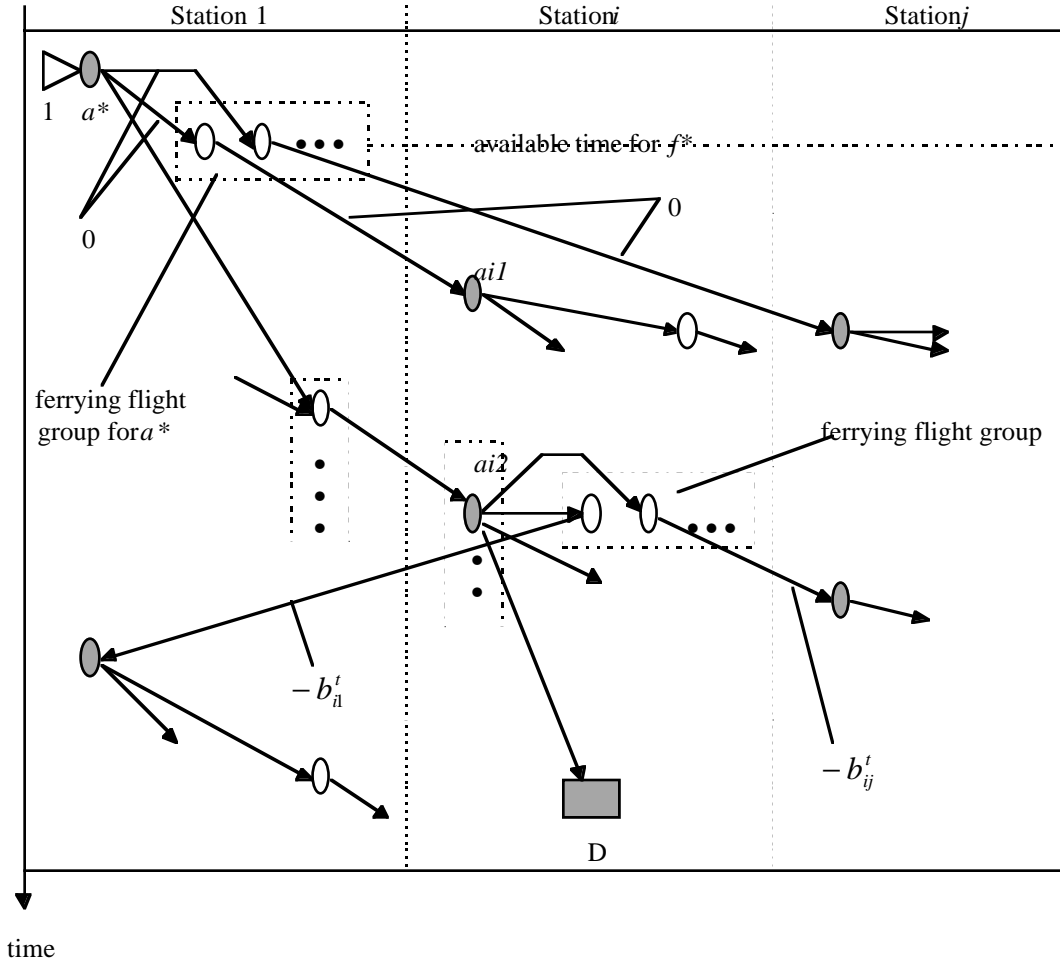Fig.3. The first layer of the network

time

Fig.4. The second layer of the network

If the decision variables are defined as

$$x_{af} = \begin{cases} 1, & \text{if aircraft } a \text{ is assigned to flight } f; \\ 0, & \text{otherwise.} \end{cases}$$

The above minimum-cost flow problem can mathematically formulated as:

$$\min \quad \varphi = \sum_{a,f} s_{af} x_{af} - \sum_{f \in F} b_f \sum_{a \in A_f} x_{af} \tag{1}$$

$$\text{s.t.} \quad \sum_{f \in S_i} \sum_{a \in A_f} x_{af} \le 1 , \quad \forall S_i , \tag{2}$$

$$\sum_{a \in A_f} x_{af} \ge \sum_{f' \in F_{a'}} x_{a'f'} , \quad \forall L(f,a') = 1 , \tag{3}$$

$$x_{af} \in \{0,1\} , \tag{4}$$

where, constraint (2) guarantees that for each flight group no more than one flight is selected, while constraint (3) guarantees the correct connection of aircraft routings. For the completeness of constraint (2) and (3) please see Cao & Kanafani (1997a).

Thus, what we need to do now is to solve the minimum-cost flow problem. The path of every supply represents a flight link of the new schedule. For this kind of problem there are several algorithms available. In this paper we use the method and program of Bertekas (1985).

## 3.  VALUE OF RUNWAY TIME SLOTS

### 3.1  *Value of Expected Slots*

For convenience of expression, we consider the expected time slot for Flight *i* at Airport *A*. As mentioned before we treat the value of a time slot as the profit impact to the airline if this time slot is assigned to the airline. Thus, the value of the expected time slot can be considered as the profit difference of the expected schedule and the optimal schedule of canceling Flight *i*. So, what we need to do is to find the optimal schedule based on cancellation of Flight *i*. In this case, the aircraft previously assigned to Flight *i* will become surplus and may be assigned to other flights or ferried to other airport if necessary. Use the model described in Eqs. 1-4 to find the optimal scheduling without Flight *i* but with a surplus aircraft at Airport *A,* denote the new total profit by $R_c$. Then the value of this expected slot can be assessed as

$$V_e = R_0 - R_c \ .$$

Based on the supposition that the original schedule was optimized $V_e$ should never be negative. On the other hand, an interesting thing is that $V_e$ may be greater than the profit of Flight *i.* This case may occur when the surplus at Airport A has to be ferried to the airport where it was planned to go in the expected schedule, this means a non-profit or low-profit flight. As shown in Fig. 5, after canceling Flight *i,* suppose the new optimal schedule still assigns Aircraft *Ai* to Flight *j* at the same time slot, then, besides losing the profit of Flight *i* the new schedule needs to pay the cost of ferrying Aircraft $A_i$. So,

$V_e$ = profit of Flight *i* +cost of ferrying Aircraft $A_i$ - profit of the ferrying flight.
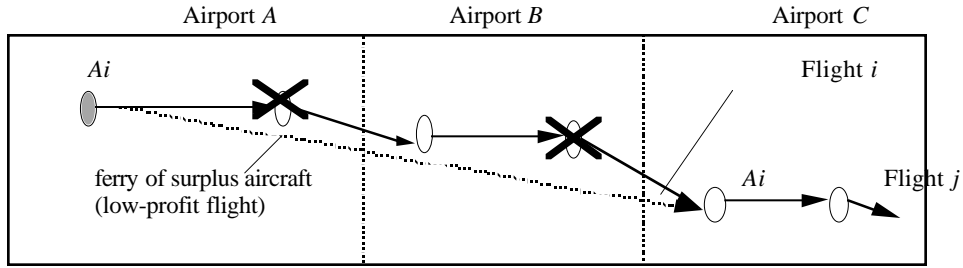
8

Fig.5.  A low-profit flight

### 3.2 *Value of Other Slots*

With no loss of generality we suppose the considered flight is $f$ and the time slot is $k$. Based on the condition that Flight $f$ is assigned at time slot $k$ we can find a new schedule using the model in Section 2. Denoting the total profit of the new schedule by $R_k$, then, the value of slot $k$ for Flight $f$ is

$$V_k = R_k - R_c,$$

Where, as mentioned before, $R_c$ is the maximum profit under canceling flight $f$.

To illustrate the calculation of the value of an expected slot, we use the flow chart in Fig. 6. Consider Flight $f$. Denote by $R_0$ the total profit for the original schedule (disregarding time slot availability). Suppose Flight $f$ is scheduled at time slot $j$ in the original schedule. The relationship between the reschedule and value of time slots can be illustrated as follows:
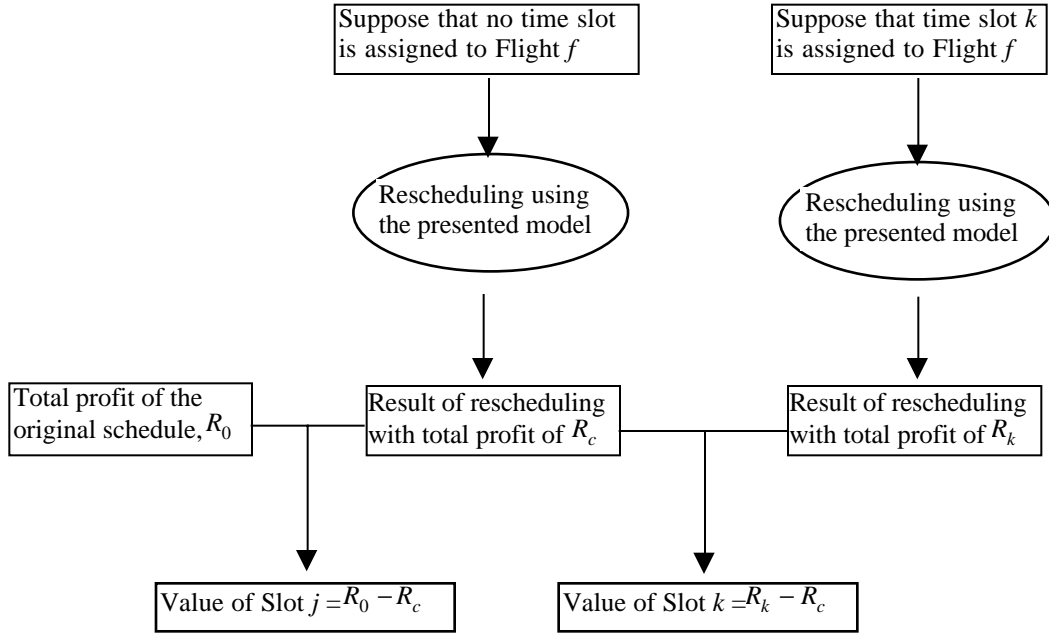
Fig. 6 Valuation of Runway Slots

## 4. SOME RESULTS AND ANALYSIS FOR A SAMPLE FLIGHT NETWORK

### 4.1 *A Sample Flight Network*

An example network consists of 10 airports, 20 aircraft, and 91 flights. For convenience the operations period is set to start at time 0. Appendix A gives the full data of the sample flight network, where Table 1 is the scheduled time, profit and original stations of flights, and Table 2 shows the flight links and their starting stations and starting time in the original schedule. To ensure the original schedule is optimal for the airline we set that:

(*i*) The highest profit for each flight exactly appears at the expected time of the flight. The profit at other time slots is generated sequentially by subtracting a number of 100~450 (generated randomly) from its adjacent time slot's profit.

(*ii*) There is no waiting time for any aircraft at any station, i.e. the ready time of every aircraft is set to be the scheduled time of the flight it will take.

(*iii*) The time span of each flight link is set to be either close the maximum working hours of the corresponding aircraft or limited by the ending time of operation. This means that every aircraft contributes maximally to its airline.

Other parameters include:

(*iv*) The ferrying cost for aircraft $a$ from station $i$ to station $j$ is set to be

$$FC_{ij}^{a} = \begin{cases} 0, & \text{if station } i \text{ is the first station of the flight link taken by aircraft } a; \\ 2000, & \text{otherwise.} \end{cases}$$

(*v*) The holding cost of aircraft at airports for a time slot is set to be 20.
(*vi*) The span of time slots is set to 10 minutes.

The model as developed applies to any given original flight network and schedule. It isalso adaptable to hub-and-spoke flight networks. Using Fig. 7, which represents a hub-spoke system. In this case, the difference might be caused is the time-dependent demands which are used as data in this paper. For example, when we change a time slot at the hub for a flight from H to D, the demands of A→D, B→D and C→D may also change. This just adds some additional work to the data preparation, but requires no change in the model.
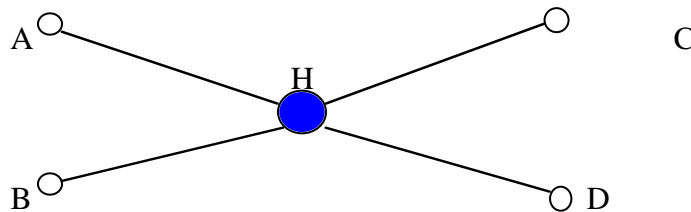


Fig. 7 Hub-and-Spoke Network

4.2 *Computational Results and Analysis*

We select one flight for each airport. Appendix B summarizes the comparison of time-dependent profit and value of various time slots. In this sample we just consider the time slots for taking off, but obviously similar results can be provided, using the same method, for landing time slots. The computational results can be classified as following three groups:

(*i*) Fig. (a), (c) and (g) are the cases for leading flights. For these cases, the value of the delayed slots decreases sharply. This is because the delay of leading flights may cause change of connecting flight link. The changes of time for connecting flights results in the decrement of profit since the highest profit is supposed to be at the expected time slot for every flight. On the other hand, for the advanced slots, there is little difference between the profit and the value. This is because the leading aircraft can be held at its second station and perform the later flight of the same flight link at the expected time slots. The difference of the two curves at advanced part is exactly the holding cost of the aircraft at its second station.

In all these three examples the value of the expected time slot is equal to the related demand (profit). This means that in the new optimal schedule, when the lead flight is canceled, the related aircraft is set at its previously second station to take the rest of the flight links. In other cases, the above tactics may not be optimal and the value of expected time slot is not necessarily equal to the related profit. Fig. (a) looks odd because the expected time slot of Flight 1 is 1 and there is no earlier slot.

*(ii)* Fig. (b), (d), (f), (h), (i), (j) are the cases for flights at the middle of flight links. The peak value may be either smaller [in Fig. (i) ] or greater [in other 5 figures] than the related profit. For example, in Fig.(b), if Flight 12 is abandoned there will be difficulty with the aircraft connection to its down-stream flights (Flights 28 and 18). This means that the new optimal schedule costs more for aircraft connection or assigns the down-stream flights at the time slots with lower profit. So, the expected time slot for Flight 12 at Station 2 has a larger value than the related profit. On the other hand, in Fig. (i), if Flight 80 is abandoned, Aircraft 19 (leading at Station 10) abandons Flight 86 at Station 10, creates a new flight from Station 10 to Station 7 and then takes the connecting Flight 61 at its expected time slot. The profit of the new flight is a little bit greater than that of Flight 86 (but smaller than the total profit of Flights 86 and 80 for sure according to the optimality of the original scheduling). So, the peak value is a little bit smaller than the related profit.

*(iii)* Fig. (e) is the case for flight at the end of flight link. This is a reversion of case *(i)*.

## 5. CONCLUSIONS

We have developed a method for assessing the value of runway time slots to airlines. The method is based on a minimum-cost flow model which gives the optimal schedule under the condition that some flights are required to be re-assigned to specific time slots. This model has a number of possible uses. It can form a basis for creating a bidding environment in which valuable runway time slots are auctioned off to the highest bidding airline. Alternatively it can be used to determine the optimal congestion toll to be charged for using runways during peak periods. The model can also be used to assess the impact of flight delay to an airline daily schedule. The computational results on a sample flight network show some important concepts quantitatively. In real world applications, the model might provide a basis for an auctioning or a bidding strategy as a variation on the current practice of allocating slots to airlines. Knowledge of slot values would improve an airline's bidding strategy, or it could enhance an airports allocation strategy depending on the respective objectives.

A useful extension of this work would be to introduce the value of time slots explicitly into airline scheduling programs. The added complexity of time-variable demand (or profit) will make these program far more complex than they now are, but also more useful.

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