Subtyping Supports Safe Session Substitution

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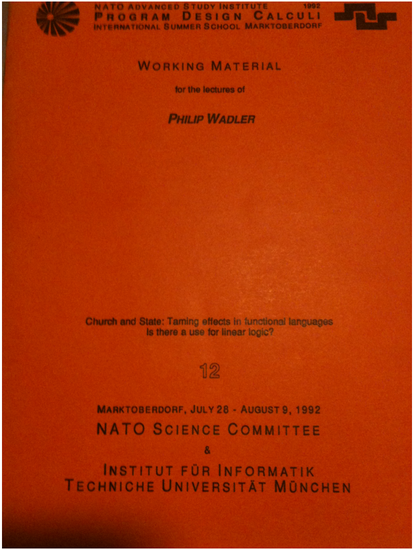
Meeting Phil



NATO ADVANCED STUDY INSTITUTE 1992
PROGRAM DESIGN CALCULI
INTERNATIONAL SUMMER SCHOOL MARKTOBERDORF

WORKING MATERIAL

for the lectures of

PHILIP WADLER

Church and State: Taming effects in functional languages
Is there a use for linear logic?

12

MARKTOBERDORF, JULY 28 - AUGUST 9, 1992

NATO SCIENCE COMMITTEE

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INSTITUT FÜR INFORMATIK
TECHNICHE UNIVERSITÄT MÜNCHEN

Session Types

- Describe a communication protocol as a type, and use type checking to guarantee correctness of communication.

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- The original papers:

  Honda, "Types for Dyadic Interaction", CONCUR 1993.

  Takeuchi, Honda & Kubo, "An Interaction-Based Language and its Typing System", PARLE 1994.

  Honda, Vasconcelos & Kubo, "Language Primitives and Type Discipline for Structured Communication-Based Programming", ESOP 1998.

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- Session types codify the structure of communication and make it available to programming tools.
- EPSRC Programme Grant "From Data Types to Session Types: A Basis for Concurrency and Distribution" (SG, Phil Wadler and Nobuko Yoshida).

The Maths Server: Types / Protocols

- The session type of the server's channel endpoint:

$$S \quad = \quad \&\langle\ \mathsf{add} : ?[\mathsf{int}].?[\mathsf{int}].![\mathsf{int}].\mathsf{end}, \\ \qquad\qquad \mathsf{eq} : ?[\mathsf{int}].?[\mathsf{int}].![\mathsf{bool}].\mathsf{end}\ \rangle$$

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- Duality: $S = \overline{C}$

Upgrading the Maths Server

- newserver adds a new service and extends an existing service:

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- A theory of subtyping needs to allow this interaction to be typechecked.

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- How can both definitions be correct?

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- We have to understand which expressions we are interested in.
- Gay and Hole: safe substitutability of channels.
- Honda et al.: safe substitutability of processes.
- This has become folklore in the session types community.

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- In Gay and Hole's pi-calculus session type system, this is how an old client can safely connect to a new server.

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- Gay (2016): derive the definition of subtyping from the structure of the type safety proof.

Process-Oriented Subtyping (Honda et al.)

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- This approach is natural if processes can be sent on channels (higher-order pi) or when combining pi and lambda.

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- The difference between channel-oriented and process-oriented subtyping is explained by contravariance of the function type constructor.

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