AdS/CFT: from Gauge Theories to Conformal Higher spins

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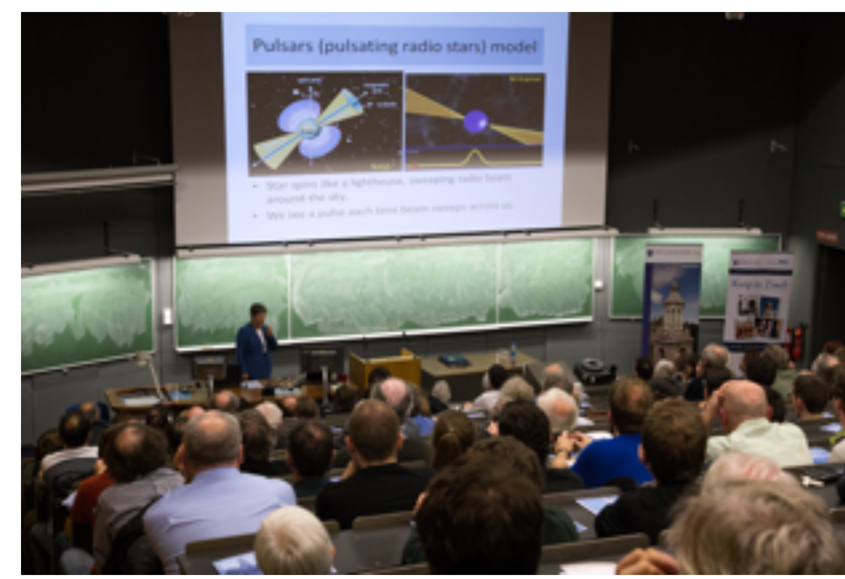
  - **QFT, Quantum Gravity, String theory, Integrable systems:** R. Britto, S. Frolov, J. Manschot, T. McLoughlin, A. Parnachev, S. Shatashvili, D. Volin.
  - **Lattice QCD:** M. Krstic Marinkovic, M. Peardon, S. Ryan, S. Sint.



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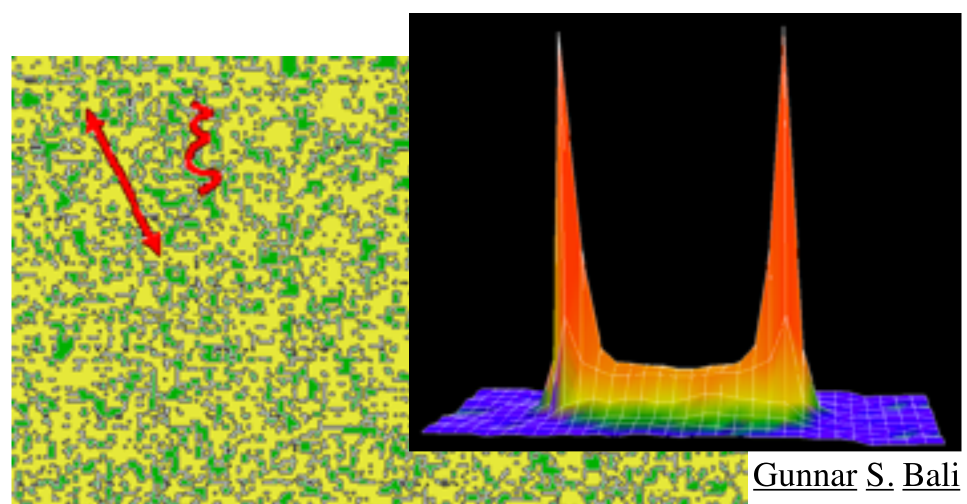
Introduction

- Many problems in physics involve a large number of interacting degrees of freedom: any macroscopic gas, liquid or crystal will have 10^{23}.
- Of course we can often characterise the system by much smaller number of parameters as the physics are unchanged by reducing the system down to a minimum size the correlation length ξ.
- When ξ is small there are a variety of techniques but when the number of d.o.f. inside a region of the scale of ξ is large understanding the physics can be more challenging e.g. relativistic QFT and critical phenomena.



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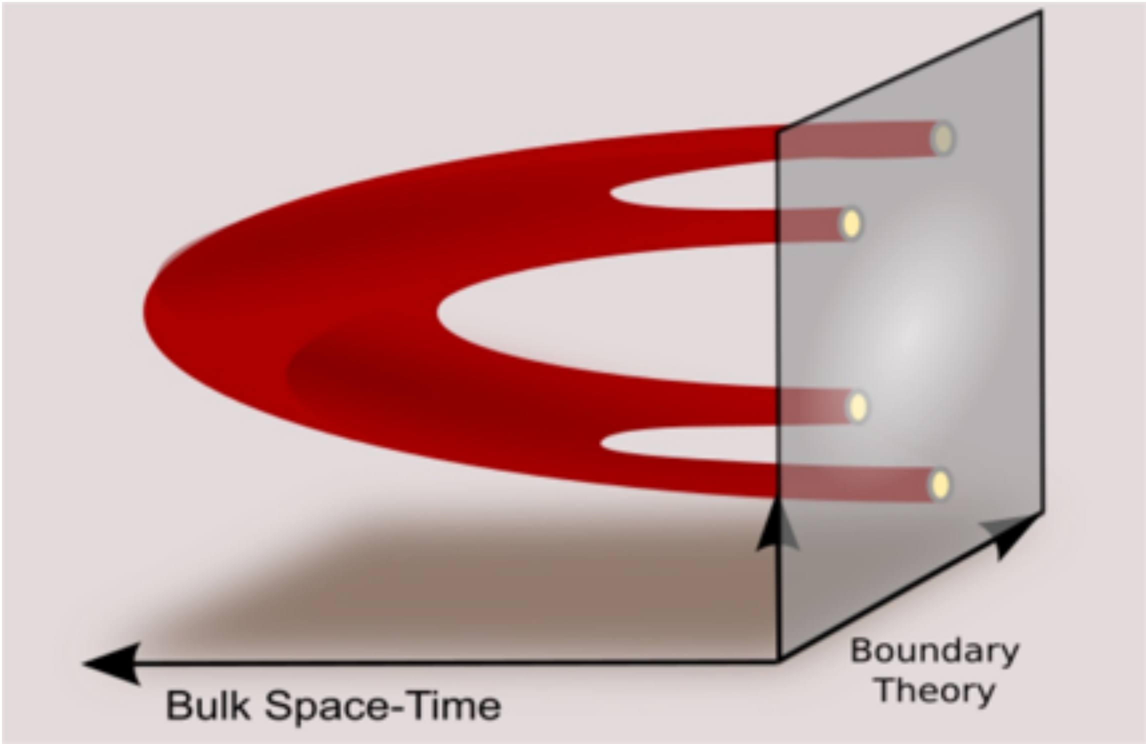
- Such systems are often qualitatively different than few-body problems demonstrating features such as universality and cooperative or emergent behaviour.

Introduction

One approach to certain examples of such systems is via the AdS/CFT correspondence. Conjectured duality between theories of gravity on anti-de Sitter space-times and conformal field theories in one dimension less.

Canonical example is N = 4 super Yang-Mills theory and type IIB string theory

[Maldacena]



Supersymmetric Theories

Supersymmetric generalisations are of both phenomenological and mathematical interest.

- Maximally supersymmetric theory N = 4 SYM:

$$\text{Gauge Fields,} \quad A_\mu \quad \text{Scalars,} \quad \Phi^{IJ} \quad \& \quad \text{Fermions,} \quad \Psi^I$$

  $I = 1,\ldots4$, and all fields are N_c x N_c matrices (we will mostly focus on large N_c limit).

- Operators are composites of fundamental fields e.g. traces of scalars

$$\mathcal{O}(x) = \mathrm{Tr}(\Phi^{I_1}\Phi^{I_2}\ldots\Phi^{I_n})$$

- Unlike QCD this is a conformal field theory, i.e. invariant under dilatations, $D : x^\mu \to ax^\mu$, and inversions which combine with supersymmetry into the supergroup of symmetries PSU(2,2|4).

CFT Correlation Functions

For a conformal field theory, symmetry is sufficient to fix the space-time dependence of the one-, two- and three-point correlation functions.

Consider scalar primary operators, i.e. composites of fields with

$$[D, \mathcal{O}_a(0)] = -i\Delta_a \mathcal{O}_a(0) \qquad\qquad [K, \mathcal{O}_a(0)] = 0$$

so that
$$\langle \mathcal{O}_a(x_1)\mathcal{O}_b(x_2)\rangle = \frac{\delta_{a,b}}{|x_1 - x_2|^{2\Delta_a}}$$

$$\langle \mathcal{O}_a(x_1)\mathcal{O}_b(x_2)\mathcal{O}_c(x_3)\rangle = \frac{c_{abc}}{|x_{12}|^{\Delta_a+\Delta_b-\Delta_c}|x_{23}|^{\Delta_b+\Delta_c-\Delta_a}|x_{31}|^{\Delta_c+\Delta_a-\Delta_b}}$$

The non-trivial dependence on the coupling comes via the scaling dimensions and the structure constants.

AdS/CFT

The duality to string theory in anti-de Sitter space gives a novel method for calculating CFT correlation functions: [Maldacena/GKP/Witten]

$$\langle \mathcal{O}_{a_1}(x_1)\ldots\mathcal{O}_{a_n}(x_n)\rangle = \int DX\ \mathrm{e}^{-\sqrt{\lambda}S[X]}V_{a_1}(x_1;X)\ldots V_{a_n}(x_n;X)$$

with

$$S[X] = \frac{1}{2\pi}\int_\Sigma d^2z\sqrt{h}h^{\alpha\beta}G_{\mu\nu}(X)\partial_\alpha X^\mu\partial_\beta X^\nu$$

- $h^{\alpha\beta}$: 2D metric, for which different gauge fixings are useful.
- Embedding coordinates $X:\Sigma\to\mathcal{M}$. For N = 4 SYM on $\mathbb{R}^4$, $\mathcal{M}=\mathbb{H}_5\times S^5$, where, $\partial\mathbb{H}_5\equiv\mathbb{R}^4$.
- $V_a(x_a;X)$: vertex operators encode positions and charges of CFT fields.

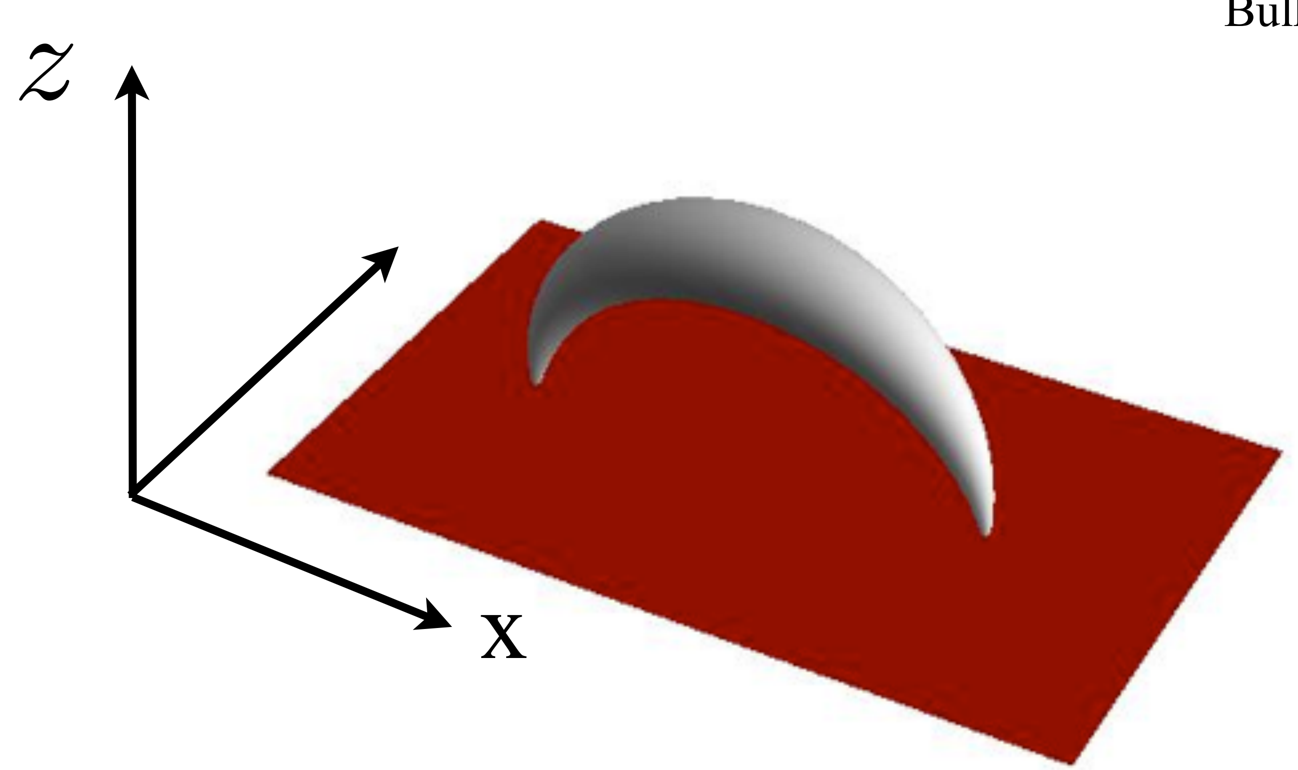
Semiclassical Strings

For $\sqrt{\lambda} >> 1$ the path-integral problem reduces to finding minimal surfaces with specific boundary conditions at $\partial\mathcal{M}$.

Simplest case: two string vertex operators located at the boundary at positions x_1 and x_2 source a surface which hangs into the bulk

$$ds^2_{\text{Bulk}} = \frac{d\vec{x}^2 + dz^2}{z^2} + d\Omega_5^2$$

with $\vec{x} \in \mathbb{R}^4$

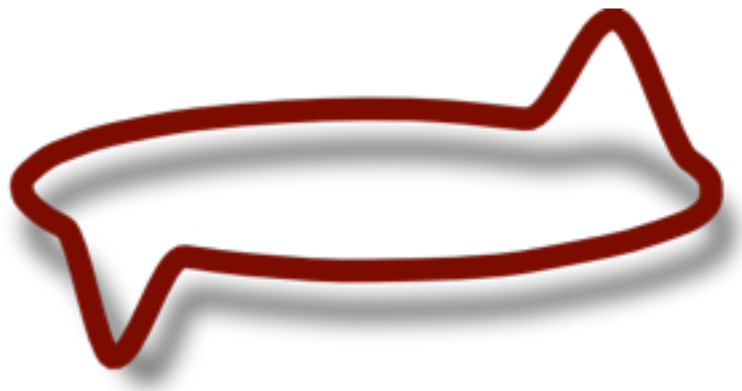


The problem of finding the strong coupling expressions for CFT correlation functions reduces to finding solutions of integrable two-dimensional classical field theories and then reconstructing the surfaces.

While the semiclassical picture is extremely useful, to actually find all order results a different perspective is required...

Lightcone Methods

By choosing lightcone gauge for the worldsheet theory we find a massive, integrable 2D theory of 8 bosons + 8 fermions describing quantum string fluctuations.



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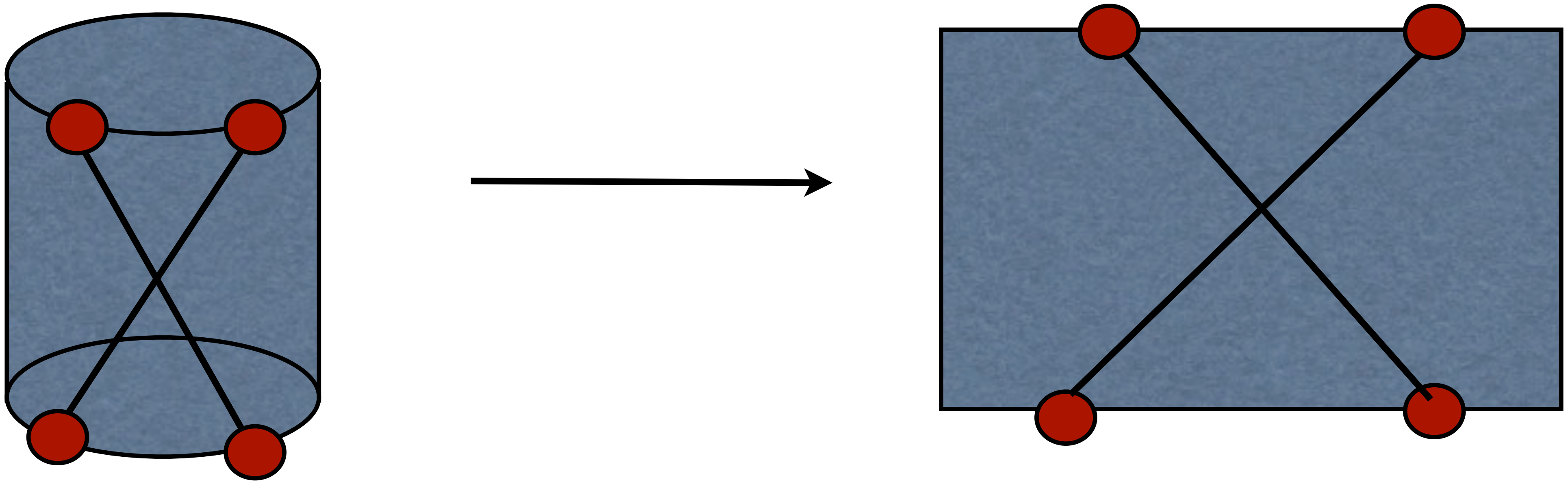

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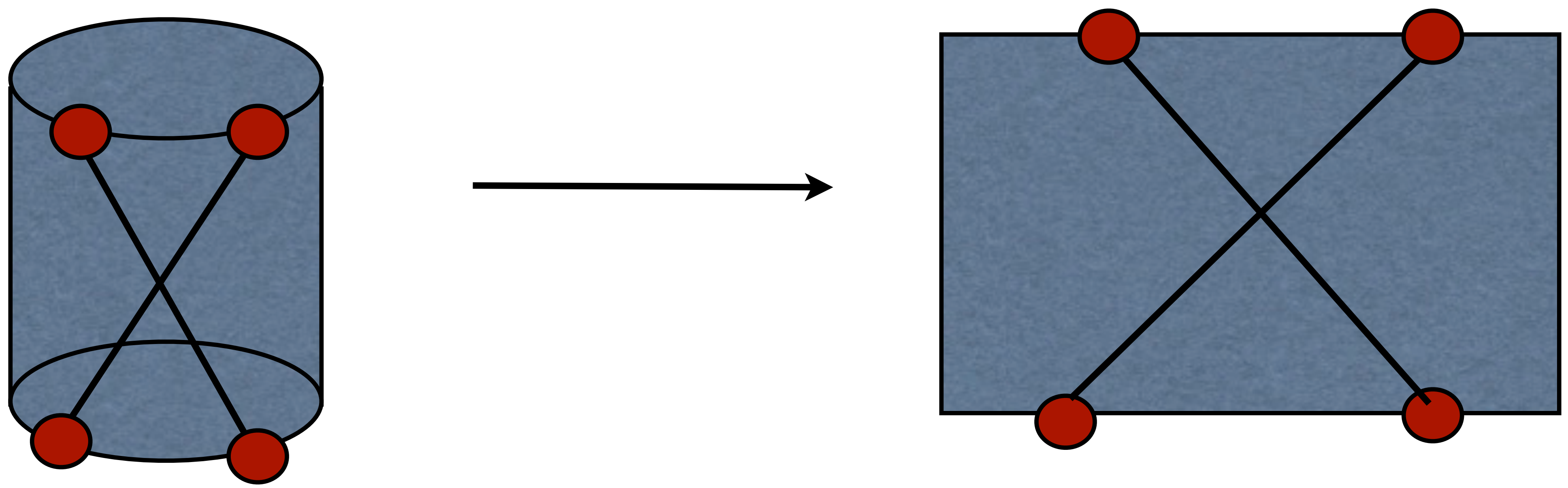


The spectrum of energies is equivalent to the planar spectrum of operator dimensions (determines two-point functions).

<u>Solve for the spectrum of an integrable 2D QFT</u>

$\Longrightarrow$ Find the worldsheet S-matrix and apply B(ethe) A(nsatz).

AdS/CFT S-matrix

The exact (all loop order) S-matrix describing the string/CFT integrable model has been conjectured (BES) and passed many stringent tests

- 4-loop gauge theory calculation (weak coupling) [Beisert, TMcL, Roiban]
- Two-loop string calculation (strong coupling) [Klose, Minahan, TMcL, Zarembo] [Klose, TMcL, Zarembo, Roiban]
- Many more e.g. S-matrix with (TBA/Lüscher) correctly reproduces Konishi anomalous dimension to five-loop order. [Bajnok, Hegedus, Janik, Lukowski], [Arutyunov, Frolov and Suzuki], [Eden et al].

To find finite volume spectrum one can use **TBA/Y-system** [Gromov, Kazakov, (Kozak), Vieira; Bombardelli, Fioravanti, Tateo; Arutyunov, Frolov] or

More recently the **Quantum Spectral Curve** [Gromov, Kazakov, Leurent, Volin] gives extremely high order perturbative results (e.g. 7-loop twist-2), efficient numerical calculations and strong-coupling results.

Recent work has extended these results to consider three-point functions, scattering amplitudes, Wilson loops, defect theories, ...

Deformations

One particular subject that has been of recent interest has been that of deformations. The study of integrable deformations of the superstring sigma model is important as it may give new solvable examples of AdS/CFT duality.

Example: η-deformed theory [Delduc, Magro, Vicedo]

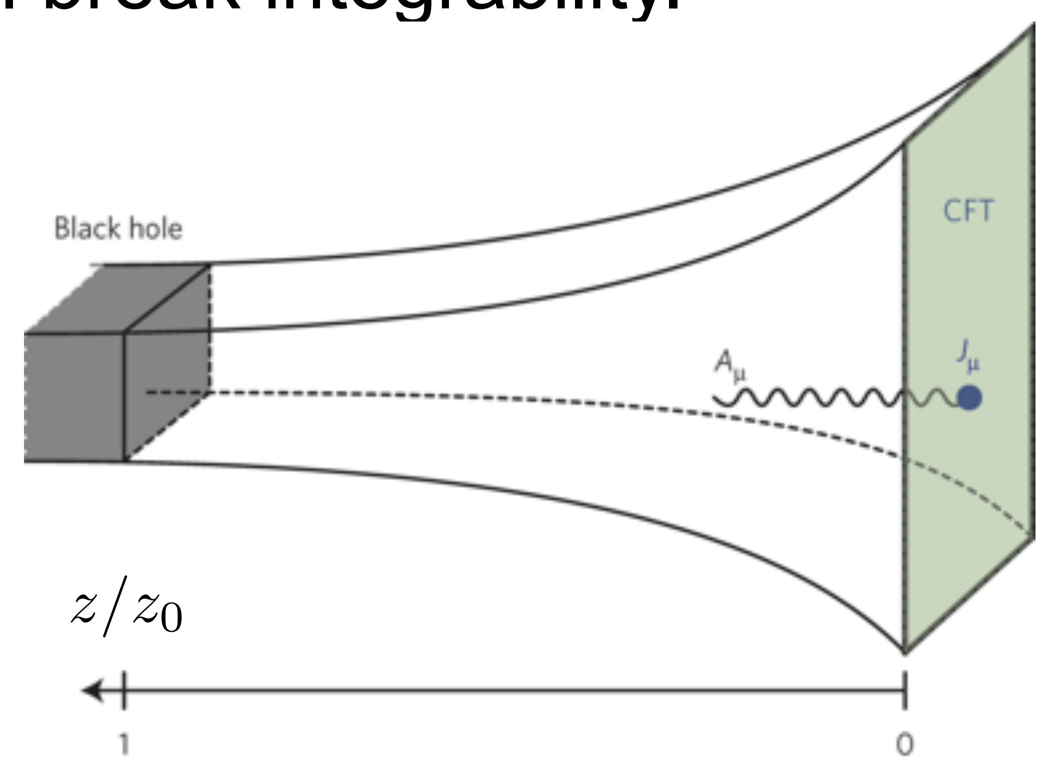
$$\mathcal{L} = \frac{\sqrt{\lambda}(1+\eta^2)}{2\pi}(\sqrt{h}h^{\alpha\beta} - \epsilon^{\alpha\beta})\mathrm{str}\left[\tilde{d}(A_\alpha)\frac{1}{1-\eta R_g \circ d}(A_\beta)\right]$$

However, many interesting deformations will break integrability.

Example: Finite Temperature AdS/CFT

$$ds^2 = R^2\left(\frac{-f(z)dt^2 + d\vec{x}^2 + \frac{dz^2}{f(z)}}{z^2}\right)$$

where $f(z) = 1 - \frac{(\pi R^2 T_H)^4}{z^4}$



Witczak-Krempa, Sørensen & Sachdev

Integrable Form Factors

In the theory of integrable 2D models one of the most interesting problems is the study of matrix elements of local operators

$$f^{\mathcal{O}}(\theta_1, \theta_2, \ldots, \theta_n) = \langle 0|\mathcal{O}(\tau, \sigma)|\theta_1, \theta_2, \ldots, \theta_n\rangle$$

For an integrable theory there exist axioms, e.g. due to Smirnov and generalised to world-sheet theory [Klose, McL], which can be used to find exact results. Currently not known but can in principle be extracted from Hexagon approach [Basso, Komatsu, Vieira; Jiang].

Is there a useful form factor perturbation theory? For example given a non-integrable perturbation of an integrable Hamiltonian

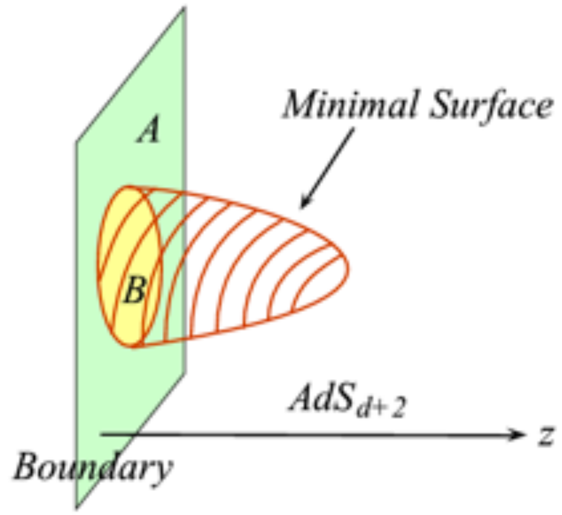
$$H = H_{\text{int}} + g \int d\sigma \ \mathcal{O}(\sigma)$$

the correction to the S-matrix is:

$$\delta S(\theta) \propto -ig \frac{f^{\mathcal{O}}(\theta_1 + i\pi, \theta_2 + i\pi, \theta_1, \theta_2)}{m^2 \sinh \theta}$$

AdS/CFT Applications

There are many more extensions of AdS/CFT to studying more interesting physical systems and quantities:



- Entanglement Entropy [Ryu and Takayanagi]

- Measuring complexity of thermal QFT states

[Brown, Roberts, Susskind, Swingle, Zhao; Abad, Kulaxizi, Parnachev].

$$\frac{d\mathcal{C}}{dt} \leq 2M$$

- Finite density and Fermi Liquids

[Karch, Son and A. O. Starinets; Kulaxizi and Parnachev].

- Holographic Quantum Hall Effect in terms of intersecting D-branes [Keski-Vakkuri and Kraus; Kristjansen and Semenoff; Parnachev and Mezzalira].

many more....

Higher Spin Theories

Most of these applications are studied in the context of string theory where the dual theory is supersymmetric and involves adjoint matter. However there are examples of holography involving only non-supersymmetric matter in vector representation:

A second important example starts with the free 3-d vector model consisting of N massless scalar fields ψ^I. The dual bulk theory is Vasiliev higher spin theory on four dimensional anti-de Sitter space [Sezgin & Sundell '02, Klebanov & Polyakov '02]. Depending on the boundary conditions on bulk fields the dual theory is either free theory or interacting.

O(N) vector model

- For the free theory there are an infinite number of conserved currents

$$\bar{J}^{\mu_1 \dots \mu_s} \sim \psi^I \partial_{(\mu_1} \dots \partial_{\mu_s)} \psi^I$$

- As the theory is conformal can project onto a traceless currents and minimally couple the theory with the couplings being the boundary values of the bulk fields. The correspondence states that the effective action $J^{\mu_1 \dots \mu_s}$ of the boundary theory

$$W[\phi] = N \log \det(-\partial^2 + \sum_s \phi_{\mu_1 \dots \mu_s} J^{\mu_1 \dots \mu_s}) \ .$$

is equal to the bulk partition function of Vasiliev Higher Spin theor specified boundary values.

The local UV divergent part of this action can be taken as defining a consistent interacting theory of conformal higher-spin fields [Tseyltin '02, Segal '02, Bekaert et al '10].

Thank you!

Twistor Theory

Starting point is the twistor equation

$$\nabla_{A'}^{(A}\omega^{B)} = 0$$

- A, $A'= 0,1$ are two-component spinor indices.
- $\nabla_{AA'} = \nabla_a$ is the covariant derivative where we make use of the bi-spinor notation for vectors:

$$x^{AA'} = \frac{1}{\sqrt{2}} \begin{pmatrix} x^0 + x^3 & x^1 + ix^2 \\ x^1 - ix^2 & x^0 - x^3 \end{pmatrix}$$

Flat Twistor Space

In Minkowski space the twistor equation has non-trivial solutions:

$$\omega^A = \overset{\circ}{\omega}{}^A + i x^{AA'} \overset{\circ}{\pi}{}_{A'}$$

$$\pi_{A'} = \overset{\circ}{\pi}{}_{A'}$$

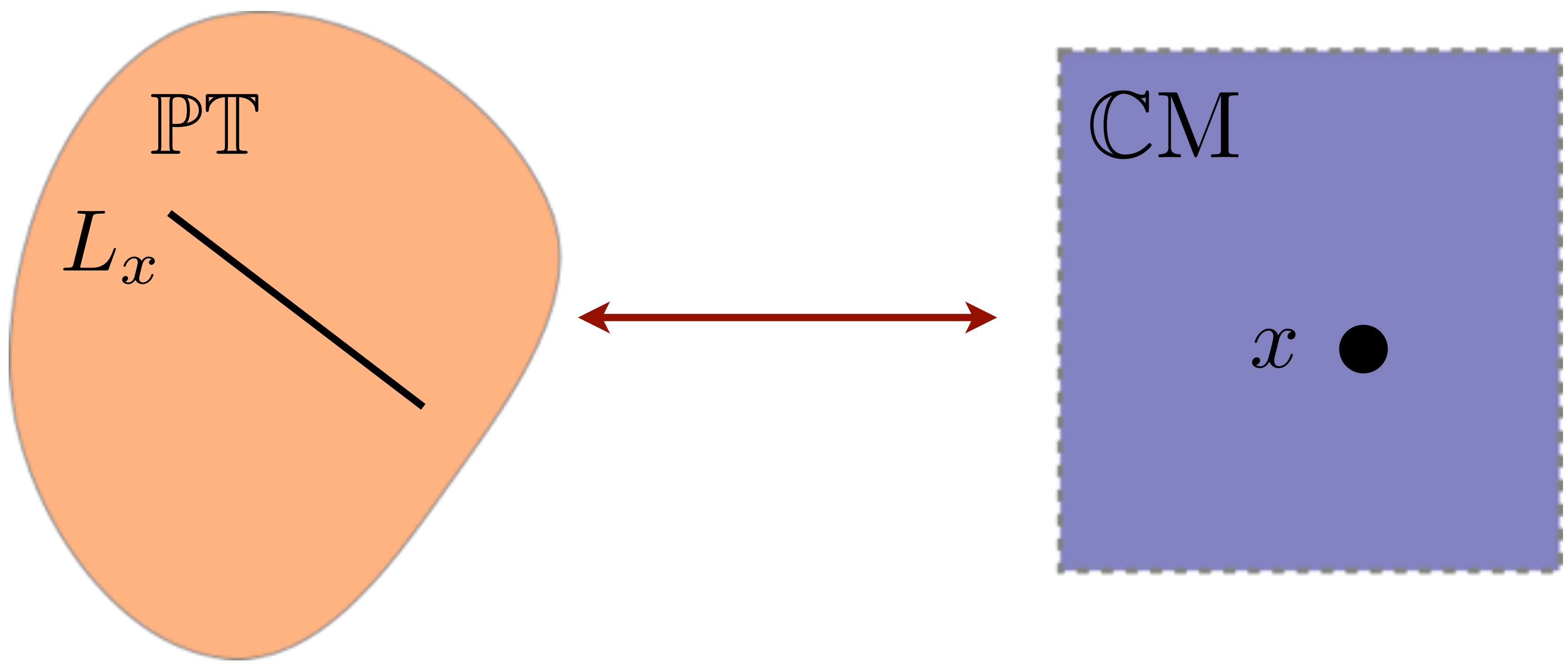
- $\overset{\circ}{\omega}{}^A$, $\overset{\circ}{\pi}{}_{A'}$ are constant spinor fields defining a four complex-dimensional vector space of solutions called twistor space, $\mathbb{T}_\alpha \simeq \mathbb{C}^4$.
- We denote elements of this solution space W_α and we can represent them in a non-conformally invariant fashion by the pair of fields

$$W_\alpha = (\pi_{A'}, \omega^A)$$

- We can similarly define dual twistors as solutions of $\nabla_A^{(A'}\mu^{B')} = 0$

$$Z^\alpha = (\mu^{A'}, \lambda_A)$$

Conversely we identify points in $\mathbb{C}M$ with complex projective lines in $\mathbb{PT}$ via the incidence relation



Describe a point by a bi-spinor $P^{\alpha\beta} = Z_1^{[\alpha} Z_2^{\beta]}$ in $\mathbb{PT}$. The vertex of the light-cone at infinity corresponds to a special skew-tensor denoted $I^{\alpha\beta}$.