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ACCURATE NUMERICAL METHODS FOR STUDYING THE NONLINEAR WAVE-DYNAMICS OF TENSEGRITY METAMATERIALS

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Abstract. *This paper presents presents an effective numerical approach to the nonlinear dynamics of columns of tensegrity prisms subject to impulsive compressive loading. The equations of motions of the analyzed structures are formulated in vector form, by modeling the cables as deformable members and the bars as rigid bodies. The given numerical results investigate the wave dynamics of tensegrity columns, with focus on the propagation of compression solitary waves with variable size and amplitude throughout the system, as a function of the applied impact velocity and the state of prestress of the structure. The engineering potential of the examined structures as tunable acoustic actuators is discussed.*

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1 INTRODUCTION

Recent studies have shown that tensegrity lattices exhibit a tunable geometrically nonlinear response, which switches from stiffening to softening by playing with a number of mechanical, geometrical, and prestress variables [1, 2, 3, 4]. Tensegrity lattices consists of networks of prestressable truss structures, obtained by connecting compressive members (bars or struts) through pre-stretched tensile elements (cables or strings). Special attention is receiving the formulation of analytical and numerical procedures for the optimal design of such structures, due to both their easy control (geometry, size, topology and prestress control) [5, 6], and the fact that such structures provide minimum mass systems under different loading conditions [5, 7, 8, 9, 10, 11, 12].

The importance of protecting materials and buildings against impacts with external objects is well known (cf., e.g., [13, 14]). Equally, there is growing interest in research into noninvasive tools to target defects in materials, and for monitoring structural health in materials and structures [16, 17, 18, 19]. Highly efficient and unconventional mechanisms for protecting materials and focusing mechanical waves through the use of rarefaction and compression solitary waves have recently been discovered by [4, 15]. It is worth noting that arrays of tensegrity lattices with elastically hardening response can be employed to fabricate tunable focus acoustic lenses that support extremely compact solitary waves [4, 15].

Three-dimensional finite element (FE) models of lattice structures usually make use of tetrahedral elements with a large number of degrees of freedom [20]. Such models are hardly applicable to dynamic simulations, even for lattices constituted by a small number of cells. A key goal of the present work is to develop efficient and accurate models of tensegrity lattices that make use of 3D assemblies of one-dimensional models for bars and strings. By describing the bars as rigid members and the cables as elastically deformable elements, we develop the dynamics of an arbitrary tensegrity network in vector form. Sucha formulation proves to be useful in order to coupling the proposed model with standard FE models that may interact with tensegrity networks. The time-integration of the equations of motion is conducted through a Runge-Kutta algorithm that accounts for a rigidity constraint of the bars [21].

IWe apply the proposed numerical model to investigate the nonlinear wave dynamics of tensegrity columns under impact loading, by establishing comparisons with the alternative model proposed in Ref. [22]. We show that our 3D modeling of tensegrity columns allows us to detect different strain wave profiles, as a function of the applied prestress, and a rigidity parameter describing the kinematics of the terminal bases. Such tunable response can be profitably used to build tensegrity actuators, which can subjected to different levels of prestress, so as to generate solitary waves with different phases that coalesce at a focal point in an adjacent host medium [16, 17].

2 VECTOR FORM OF THE DYNAMICS OF TENSEGRITY NETWORKS

Let us consider a tensegrity network made up of n_n nodes (or joints), n_b bars and n_s cables. The joints are frictionless hinges, and each member carries only axial forces. The bars (i.e., the compressed members) are assumed to behave as straight rigid bodies (rods) with uniform mass density, constant cross-section, and negligible rotational inertia about the longitudinal axis. The cables are instead modeled as straight elastic springs that can carry only tensile forces.

The generic node i, with $i \in [1, ..., n_n]$, is located by the vector $\mathbf{n}_i \in \mathbb{R}^3$ in the three-dimensional Euclidean space, and is loaded with an external force vector $\mathbf{w}_i \in \mathbb{R}^3$. By suitably collecting the vectors $\mathbf{n}_i$ and $\mathbf{w}_i$, we introduce the following nodal and force matrices:

$$\mathbf{N} = \begin{bmatrix}\mathbf{n}_1 & \mathbf{n}_2 & ... & \mathbf{n}_i & ... & \mathbf{n}_{n_n}\end{bmatrix} \in \mathbb{R}^{3\times n_n} \tag{1}$$

$$\mathbf{W} = \begin{bmatrix}\mathbf{w}_1 & \mathbf{w}_2 & ... & \mathbf{w}_i & ... & \mathbf{w}_{n_n}\end{bmatrix} \in \mathbb{R}^{3\times n_n} \tag{2}$$

The k-th bar (or cable) k of the network, with $k \in [1, ..., n_b]$ (or $k \in [1, ..., n_s]$), is located by the vector $\mathbf{b}_k \in \mathbb{R}^3$ (or $\mathbf{s}_k \in \mathbb{R}^3$). For example, if the k-th bar connects nodes i and j, then $\mathbf{b}_k = \mathbf{n}_j - \mathbf{n}_i$. By stacking up the bar and string vectors, we obtain the following matrices describing the geometry of all bars and cables:

$$\mathbf{B} = \begin{bmatrix}\mathbf{b}_1 & \mathbf{b}_2 & ... & \mathbf{b}_k & ... & \mathbf{b}_{n_b}\end{bmatrix} \in \mathbb{R}^{3\times n_b}, \tag{3}$$

$$\mathbf{S} = \begin{bmatrix}\mathbf{s}_1 & \mathbf{s}_2 & ... & \mathbf{s}_k & ... & \mathbf{s}_{n_s}\end{bmatrix} \in \mathbb{R}^{3\times n_s} \tag{4}$$

The center of mass of the k-th bar between nodes i and j is located by the vector $\mathbf{r}_k = (\mathbf{n}_i + \mathbf{n}_j)/2$. Collecting all the $\mathbf{r}_k$ vectors, we get the matrix:

$$\mathbf{R} = \begin{bmatrix}\mathbf{r}_1 & \mathbf{r}_2 & ... & \mathbf{r}_k & ... & \mathbf{r}_{n_b}\end{bmatrix} \in \mathbb{R}^{3\times n_b} \tag{5}$$

It is useful to rewrite the above matrices as follows:

$$\mathbf{B} = \mathbf{N}\mathbf{C}_{\mathbf{B}}^{\mathbf{T}}, \quad \mathbf{S} = \mathbf{N}\mathbf{C}_{\mathbf{S}}^{\mathbf{T}}, \quad \mathbf{R} = \mathbf{N}\mathbf{C}_{\mathbf{R}}^{\mathbf{T}} \tag{6}$$

where $\mathbf{C_B} \in \mathbb{R}^{n_b\times n_n}$ and $\mathbf{C_S} \in \mathbb{R}^{n_s\times n_n}$ are *connectivity* matrices of bars and cables, respectively. The general i^{th} row of $\mathbf{C_B}$ (or $\mathbf{C_S}$) corresponds to the i^{th} bar (or cable), and the element $\mathbf{C_B}_{ij}$ (or $\mathbf{C_S}_{ij}$) is equal to: -1 if vector $\mathbf{b}_i$ (or $\mathbf{s}_i$) is directed away from node j^{th}, 1 if vector $\mathbf{b}_i$ (or $\mathbf{s}_i$) is directed toward node j^{th}, and 0 if vector $\mathbf{b}_i$ (or $\mathbf{s}_i$) does not touch node j. Similarly, the i^{th} row of $\mathbf{C_R} \in \mathbb{R}^{n_b\times n_n}$ corresponds to the bar $\mathbf{b}_i$, and the element $\mathbf{C_R}_{ij}$ is equal to: 1 if vector $\mathbf{b}_i$ is touching node j, or 0 if vector $\mathbf{b}_i$ does not touch node j. Following Ref. [5], we say that a tensegrity network is of *class n*, if the maximum number of bars concurring in each node is equal to n.

Let us consider now the generic cable (say the k-th one) with Young modulus of the material E_{sk}, cross-section area A_{sk}, rest length L_k, and stretched length s_k (i.e. $s_k = \|\mathbf{s}_k\|$, and $s_k \geq L_k$). We define the stiffness k_{sk} and the prestrain p_k through the following equations:

$$k_{sk} = \frac{E_{sk}A_{sk}}{L_k}, \tag{7}$$

$$p_k = \frac{s_k - L_k}{L_k} \tag{8}$$

The force density carried by the current cable is given by the following (unilateral) constituve equation (*elastic, no-compression response*):

$$\gamma_k = \max\left[k_{sk}\left(1 - \frac{L_k}{s_k}\right), 0\right], \qquad if : s_k \geq L_k, \tag{9}$$

$$\gamma_k = 0, \qquad if : s_k < L_k \tag{10}$$

As we shall see in the sequel, it is convenient to collect all the quantities γ_k into the diagonal matrix: $\hat{\boldsymbol{\gamma}} = diag\begin{pmatrix}\gamma_1 & \gamma_2 & \dots & \gamma_{n_s}\end{pmatrix} \in \mathbb{R}^{n_s \times n_s}$.
On adopting the matrix from of the tensegrity dynamics presented in Ref. [24], we write the equations of motion of a class 1 tensegrity network as follows:

$$\ddot{\mathbf{N}}\mathbf{M} + \mathbf{N}\mathbf{K} = \mathbf{W} \tag{11}$$

where:

$$\mathbf{M} = \mathbf{C_B^T}\hat{\boldsymbol{m}}\mathbf{C_B}\frac{1}{12} + \mathbf{C_R^T}\hat{\boldsymbol{m}}\mathbf{C_R} \in \mathbb{R}^{n_n \times n_n} \tag{12}$$

$$\mathbf{K} = \mathbf{C_S^T}\hat{\boldsymbol{\gamma}}\mathbf{C_S} - \mathbf{C_B^T}\hat{\boldsymbol{\lambda}}\mathbf{C_B} \in \mathbb{R}^{n_n \times n_n} \tag{13}$$

and:

$$\hat{\boldsymbol{m}} = diag\left(m_1, m_2, \dots, m_{n_b}\right) \in \mathbb{R}^{n_b \times n_b} \tag{14}$$

$$-\hat{\boldsymbol{\lambda}} = \left\lfloor \dot{\mathbf{B}}^{\mathbf{T}}\dot{\mathbf{B}} \right\rfloor \hat{\boldsymbol{m}}\hat{\ell}^{-2}\frac{1}{12} + \left\lfloor \mathbf{B^T}\left(\mathbf{W} - \mathbf{S}\hat{\boldsymbol{\gamma}}\mathbf{C_S}\right)\mathbf{C_B^T} \right\rfloor \hat{\ell}^{-2}\frac{1}{2} \in \mathbb{R}^{n_b \times n_b} \tag{15}$$

$$\hat{\ell}^{-2} = diag\left(\|\mathbf{b}_1\|^{-2}, \|\mathbf{b}_2\|^{-2}, \dots, \|\mathbf{b}_{n_b}\|^{-2}\right) \in \mathbb{R}^{n_b \times n_b} \tag{16}$$

The generalization of the above equations to the case of a class k system is straightforward, by making recourse to the *Lagrange multipliers* technique illustrated in [25].

The vector form of the equations of motions (11) is as follows:

$$\mathbf{M}_n\ddot{\mathbf{n}} + \mathbf{K}_n\mathbf{n} = \mathbf{w} \in \mathbb{R}^{3n_n} \tag{17}$$

where: $\mathbf{n} = vec\left(\mathbf{N}\right)$, $\mathbf{M}_n = \mathbf{M} \otimes \mathbf{I}_3$, and $\mathbf{K}_n = \mathbf{K} \otimes \mathbf{I}_3$. On applying the vectorizing operator to (11), we obtain:

$$vec\left(\ddot{\mathbf{N}}\mathbf{M}\right) + vec\left(\mathbf{N}\mathbf{K}\right) = vec\left(\mathbf{W}\right) \in \mathbb{R}^{3n_n} \tag{18}$$

which implies, after some calculations:

$$\left(\mathbf{M}^T \otimes \mathbf{I}_3\right)\ddot{\mathbf{n}} + \left(\mathbf{K}^T \otimes \mathbf{I}_3\right)\mathbf{n} = \mathbf{w} \in \mathbb{R}^{3n_n} \tag{19}$$

Since $\mathbf{M}$ and $\mathbf{K}$ are symmetric, upon defining $\mathbf{M}_n = \mathbf{M} \otimes \mathbf{I}_3$ and $\mathbf{K}_n = \mathbf{K} \otimes \mathbf{I}_3$, we can finally reduce Eqn. (19) to the vector form (17). We employ the Runge-Kutta integration algorithm described in Ref. [21] to perform the time-integration of Eqn. (17). Such an algorithm prevents numerical violations of the rigidity constraint of the bars, ensuring that the bar vectors $\mathbf{b}_k$ remain constant at each time step.

3 WAVE DYNAMICS OF TENSEGRITY COLUMNS

Let us examine the wave dynamics of tensegrity columns which may feature either flexible (Fig. 1(a-b)) or rigid bases (Fig. 1(c-d)) in each prism, and are equipped with $n_p = 50$ right-handed prisms and cables featuring Young modulus $E_s = 5.48x10^6\ N/m^2$, and cross-section radius $r_s = 0.14mm$. The lattice constant a of the anlyzed columns is set equal to 5 mm, while the reference height of each prism is equal to $h_0 = 6.2mm$, giving a total height of the column of $0.31m$. The bases of the prisms forming the coulmn are endowed with lumped masses and the total mass of each unit is equal to $0.0249kg$. We characterize the state of prestrain/prestress of the column through the cross-string prestrain p. We refer to the quantity $\epsilon = (h-h_0)/h$ as the *axial strain* of the prism (positive when the prism is stretched from the reference configuration), h denoting the current (deformed) height of the generic prism. The wave dynamics of the analyzed systems is studied through the numerical model given in Sect. 2, by applying different initial velocities v_0 to the nodes of the free end (right-end), which are directed along the axis of the column, so as to generate a compressive impulsive loading (impact velocites).

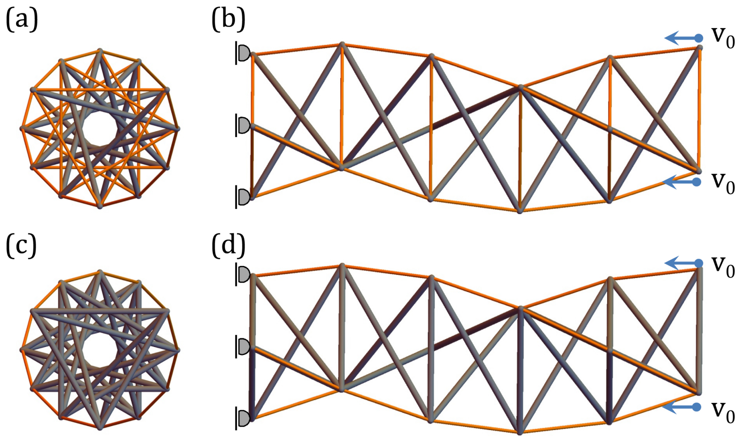

Figure 1: Top views and front views of tensegrity columns of right handed prisms with flexible bases (a-b: FB columns), and rigid bases (c-d: RB colums).

We compare the dynamics of columns composed of prisms with flexible bases (hereafter referred to as FB-columns) with that of columns composed of prisms equipped with rigid bases (RB-columns). The analyzed columns have the same geometric and mass data of those analized in [22]. We consider tensegrity columns subject to a state of prestress in the reference configuration, which is obtained by applying a prestrain $p = 0.002$ to the cross cables. The compression wave dynamics of prestressed FB columns is illustrated in Fig. 2, for impact velocities ranging between $v_0 = 0.1m/s$, and $v_0 = 0.15m/s$. We apply larger impact velocities to the prestressed column, as compared to the column under zero prestress, to account for the prestress-induced increse in the acoustic impedence of the system. We note the propagation of leading compression pulses with oscillatory tails, which span approximatively 4 prisms at $t = 0.5s$, and exhibit speed varying from $0.248m/s$ for $v_0 = 0.1m/s$ to $0.372m/s$ for $v_0 = 0.15m/s$ (Fig. 2). The leading strain pulses illustrated in Fig. 2 have amplitudes of: $\epsilon = 5.4 \times 10^{-2}$ for $v_0 = 0.1m/s$; $\epsilon = 6.97 \times 10^{-2}$ for $v_0 = 0.125m/s$; and $\epsilon = 8.5 \times 10^{-2}$ for $v_0 = 0.15m/s$.

The response of prestressed RB columns to impact velocities ranging between $v_0 = 0.1m/s$, and $v_0 = 0.15m/s$ is illustrated in Fig. 3. One observes the propagation of compact solitary pulses spanning about 3 units and featuring the following amplitudes: $\epsilon = 8.93 \times 10^{-2}$ for $v_0 = 0.1m/s$; $\epsilon = 0.106$ for $v_0 = 0.125 \div 0.15m/s$. The mean speeds of the compression pulses are: $0.186m/s$ for $v_0 = 0.1m/s$, and $0.248m/s$ for $v_0 = 0.125 \div 0.15m/s$.

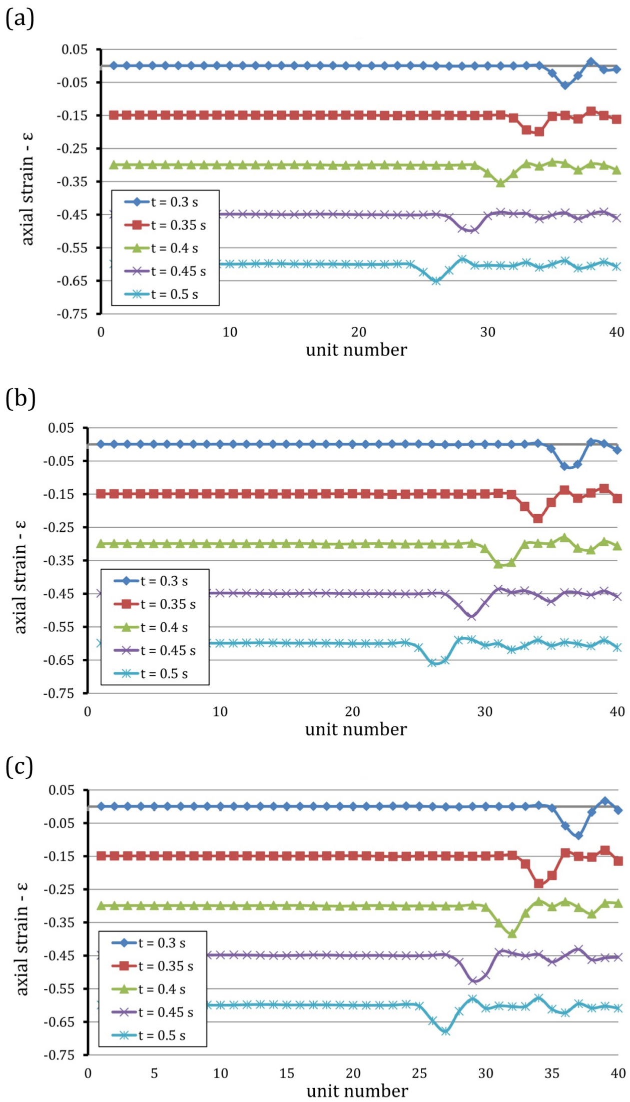

Figure 2: Axial strain wave profiles in a FB chain of 50 prisms under cross string prestrain $p = 0.002$, for various initial velocities: (a) $v_0 = 0.1 m/s$, (b) $v_0 = 0.125 m/s$, (c) $v_0 = 0.15 m/s$.

4 CONCLUSIONS

We have presented a three-dimensional numerical model for the dynamics of arbitrary tensegrity networks that accounts for a rigidity constraint of the compressed members, elastic response of cable elements, and vector form of the equations of motions. Such a model can be easily coupled with standard FE models of bodies and structures interacting with tensegrity networks, and proves to be useful for studying the highly nonlinear dynamics of tensegrity metamaterials. It has been applied to investigate the wave dynamics of tensegrity columns traversed by propagating compressive strains waves under impulsive impact loading.

The numerical results presented in Sect. 3 allow us to conclude that the more rigid is the response of the bases of the units, the more compact is the nature of the compressive solitary pulses that traverse tensegrity columns, under initial compressive disturbances. We are led to conclude that it is possible to exploit the use of highly nonlinear dynamic response in tensegrity units to create novel metamaterials that will enable unconventional wavefocusing methodologies. Arrays of tensegrity columns may indeed be employed to fabricate tunable focus acoustic

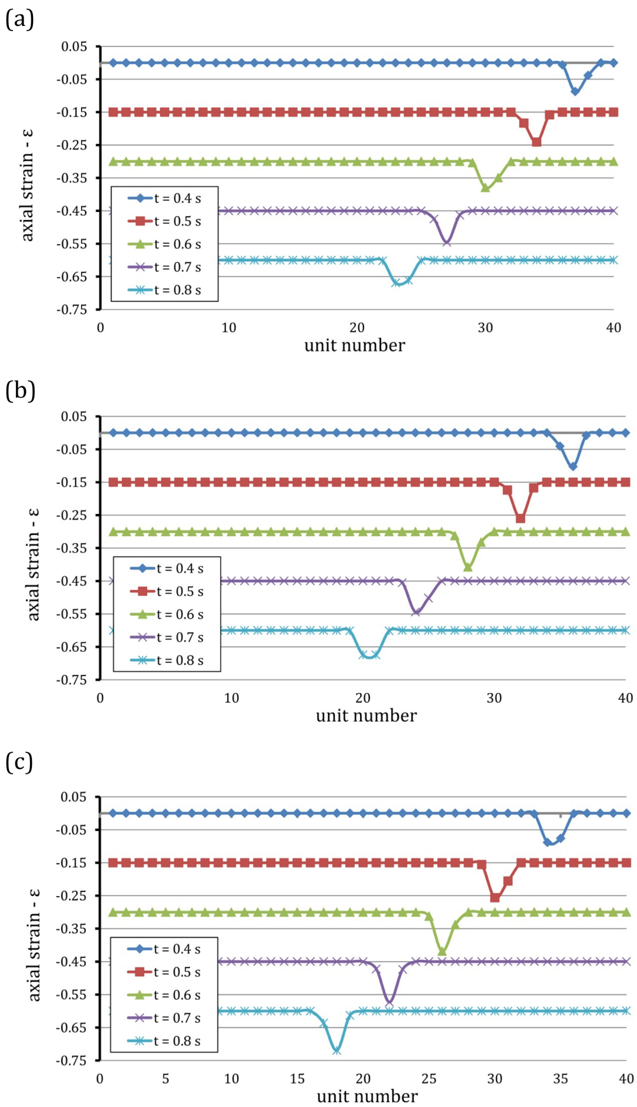

Figure 3: Axial strain wave profiles in a RB chain of 50 prisms under cross string prestrain $p = 0.002$, for various initial velocities: (a) $v_0 = 0.1 m/s$, (b) $v_0 = 0.125 m/s$, (c) $v_0 = 0.15 m/s$.

lenses supporting extremely compact solitary waves. Such lattices can be subjected to different levels of prestress, so as to generate compact solitary waves with different phases within the lens, which will coalesce at a focal point [16, 17] in an adjacent host medium (i.e., a material defect to be targeted). The 3D modeling presented in this work offers a very useful tool to simulate the mechanical response of such spatial arrays of tensegrity columns, and can also be used to deal with their design by computation. As compared to acoustic lenses based on arrays of granular metamaterials [16, 17], tensegrity acoustic lenses will profit from the adjustable width of compression solitary waves in such metamaterials (cf. Sect. 3). While compression solitary waves in uniform granular chains have a constant width, which is independent of the amplitude [29], the width of similar waves in tensegrity metamaterials changes with amplitude and speed, and the solitary wave tends to concentrate on a single lattice spacing in the high energy regime [15, 4].

We address specific studies about engineering applications of tensegrity networks to future work. A key goal of such a research will regard the design of 3D innovative devices for monitoring structural health and damage detection in materials and structures. Combined tensegrity actuators and sensors will be tested to detect the mechanical properties and/or the presence of damage in materials and structures through closed-loop identification procedures [26, 18, 19]. A second goal will regard the design, manufacture and testing of effective impact mitigation systems based on tensegrity metamaterials with softening-type response. Such nonlinear metamaterials will be able to transform compressive disturbances into solitary rarefaction waves with progressively vanishing oscillatory tail, and/or rarefaction shock-like waves [4]. Finally, we address to future studies the employment of tensegrity concepts in a variety of mechanical problems involving innvovative materials and structures [61]-[65].

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