Viscoelastic earthquake sequence simulations in complex geometries

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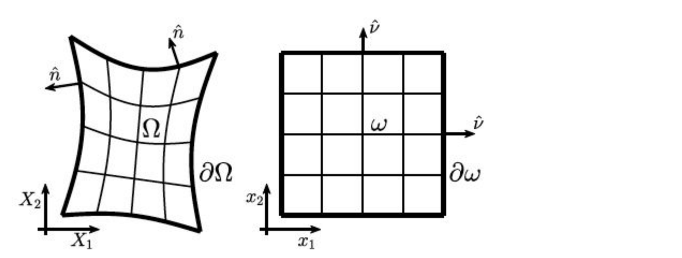
Abstract

Earthquake sequence simulations are used to study earthquake source processes, relate fault slip to observable ground motions and crustal deformation, and set initial stresses for dynamic rupture simulations. Most previous earthquake sequence simulations use boundary element or boundary integral equation formulations of elasticity that are limited to planar faults and/or homogeneous elastic solids. Here we utilize our recently developed finite difference discretization of the elastic operator on curvilinear multiblock meshes (Almquist and Dunham, 2020b) in 2D plane strain fully dynamic earthquake sequence simulations with rate-and-state fault friction. The method switches between the fully dynamic problem with explicit time steps in the coseismic phase and the quasi-static problem with adaptive time steps in the interseismic phase, as in Duru et al. (2019). We account for spatially variable material properties, power-law viscoelasticity, and complex geometries. Target applications of our method include earthquake sequences on rough faults, dipping faults, and branching faults. Viscoelasticity relaxes stress concentrations during the interseismic period, thereby allowing us to load complex fault systems by displacing the remote boundaries instead of utilizing the backslip approximation. In addition, by coupling to the acoustic wave equation for velocity potential in an overlying ocean layer, and accounting for gravity following Lotto and Dunham (2105), we will be able to simulate earthquake and tsunami histories for offshore faults.

Multiblock Finite Differences for Elastodynamics and Viscoelasticity in Complex Geometries

We use summation-by-parts (SBP) finite differences on multiblock meshes. Complex domain is divided into four-sided blocks, with coordinate mapping unique to each block used to map to unit square; finite differences applied on cartesian mesh in unit square. Boundary and interface conditions enforced weakly using penalty method. Overall method is high order accurate and energy stable.



$$\rho \ddot{u}_J = \partial_I C_{IJKL} \partial_K u_L + f_J \qquad J\rho \ddot{u}_j = \partial_i F_{Ii} J C_{IjK\ell} F_{Kk} \partial_k u_\ell + J f_j.$$

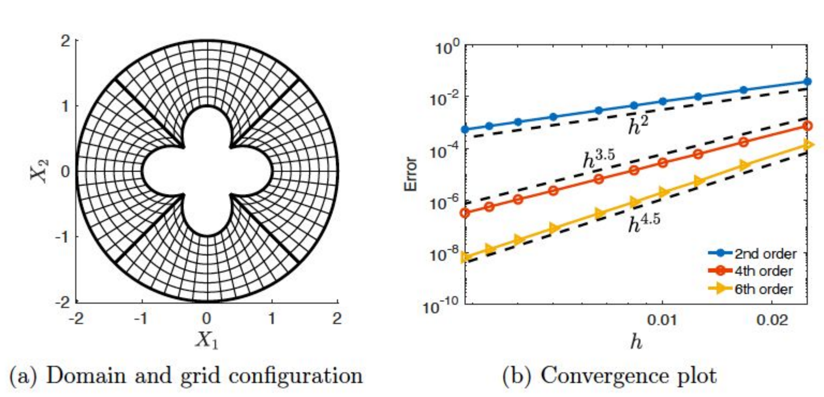
$$F_{Ii} = \partial x_i / \partial X_I \qquad J = \det[(F^{-1})_{iI}]$$

Figure 1: Coordinate transform method for handling complex geometries. (From Almquist and Dunham, 2020b)

Why a new code and numerical method? Our group has utilized a dynamic rupture code FDMAP (Dunham et al., 2011; Kozdon et al., 2013) that solves velocity-stress (first-order hyperbolic) formulation of elastic wave equation in complex geometries. That method *cannot* be used for earthquake sequence simulations; instead, to solve both dynamic and quasi-static problem, we must solve displacement (second-order) formulation of elasticity. Erickson and Day (2016) did this for quasi-dynamic sequence simulations; here we extend their method to dynamics, with heterogeneous properties, complex geometries, and with an improved treatment of boundary and interface conditions.

Quasi-static means neglecting inertia, $\cancel{\rho \ddot{u}_J} = \partial_I C_{IJKL} \partial_K u_L + f_J$

creating a problem with time-dependence only from evolving fault slip



(a) Domain and grid configuration (b) Convergence plot

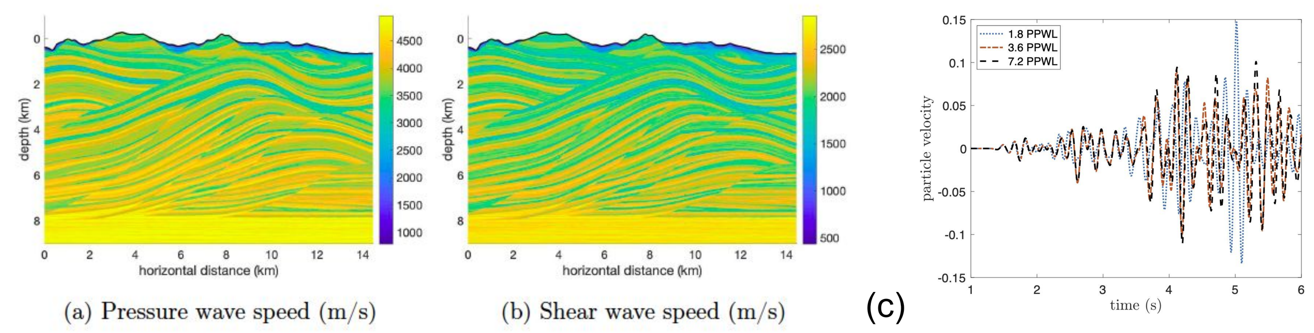
Figure 2: Convergence test for anisotropic elastic wave equation on multiblock mesh. Order of accuracy reported for interior difference operators; boundary operators are less accurate, leading to global convergence rates a bit less than interior accuracy. (From Almquist and Dunham, 2020b)

Viscoelasticity. We have extended our plane strain quasi-static solver from linear elasticity to nonlinear viscoelasticity, extending previous work on the antiplane shear problem by Allison and Dunham (2018) and Duru et al. (2019). Viscous strains appear as additional dependent variables that are updated according to a power-law viscous flow law. Convergence tests (not shown here) verify the accuracy of the implementation.

Wave Propagation Examples

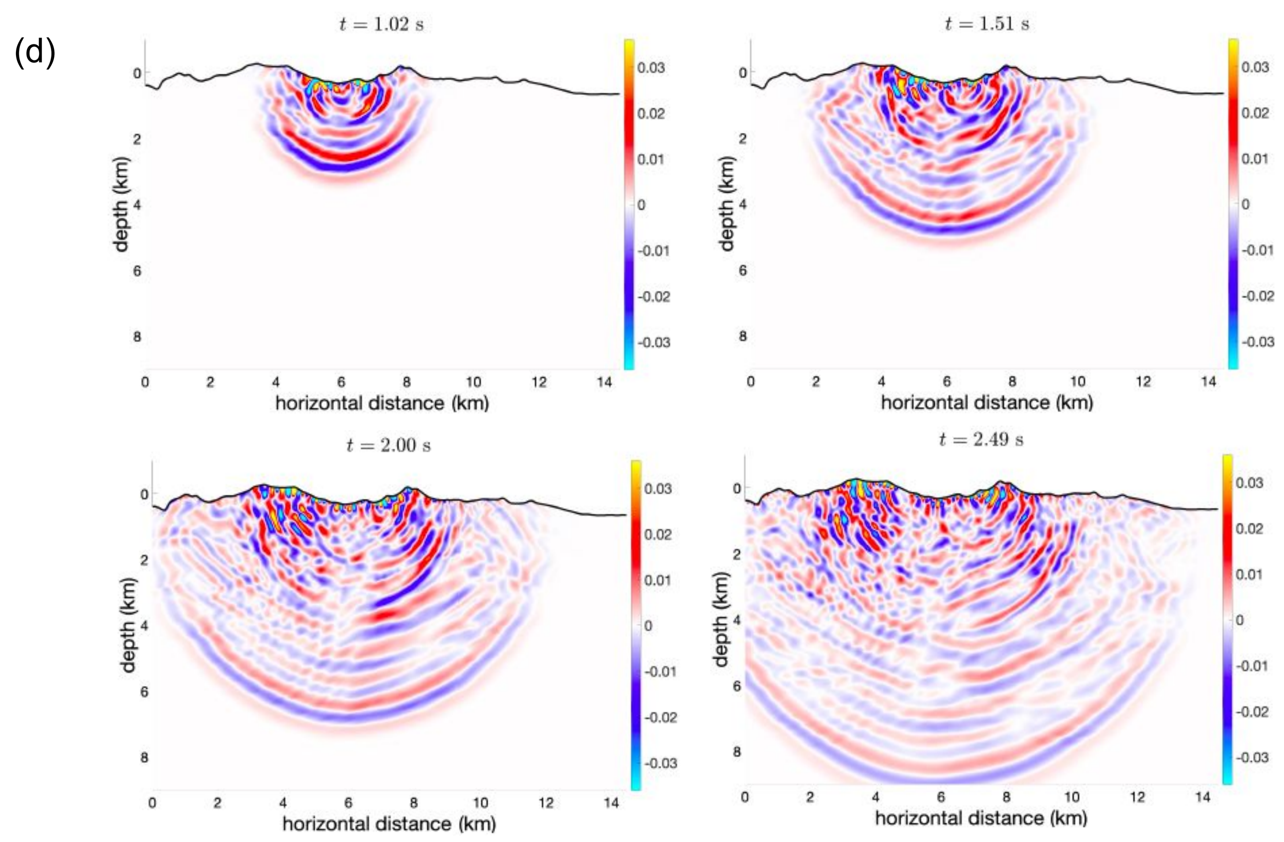
Here we demonstrate code capabilities for simulation of elastic and acoustic waves in complex geometries.

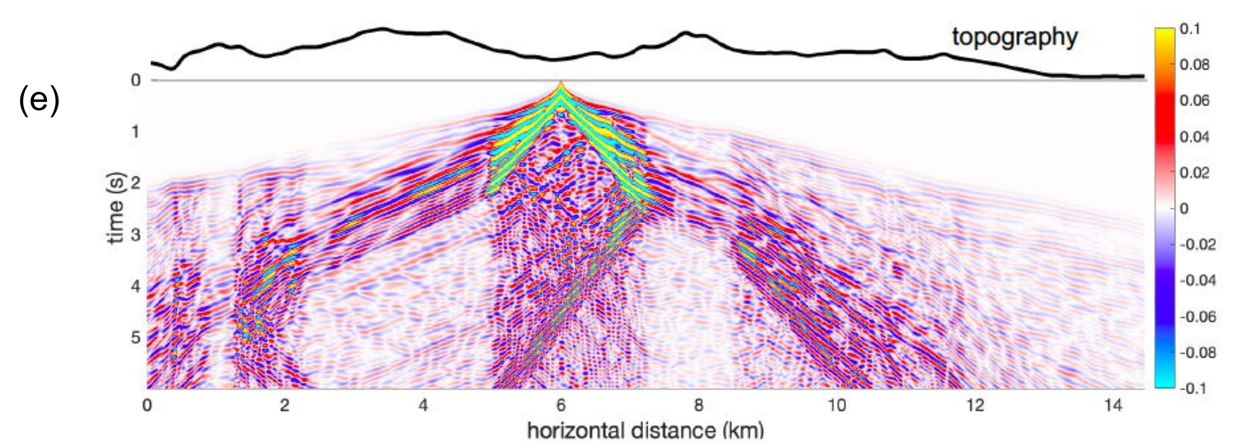
Example 1: *Elastic wave propagation in a complex structural model with topography.*



(a) Pressure wave speed (m/s) (b) Shear wave speed (m/s) (c)

Figure 3: (a) and (b) SEG SEAM Foothills model. (c) Solution on surface at x=10 km convergences with mesh refinement (PPWL=points per minimum wavelength). (d) Vertical particle velocity (nondimensional) from a Ricker wavelet point source on the free surface. (e) Record section of wavefield on surface. 6th order finite difference operators and 4th order explicit Runge-Kutta time stepping. Supergrid absorbing layers on exterior boundaries. (From Almquist and Dunham, 2020b)





Example 2: *Elastic-acoustic wave propagation in a complex structural model. By slightly modifying the free surface boundary condition (Lotto and Dunham, 2015) we can also simulate tsunami generation and propagation in this same simulation.*

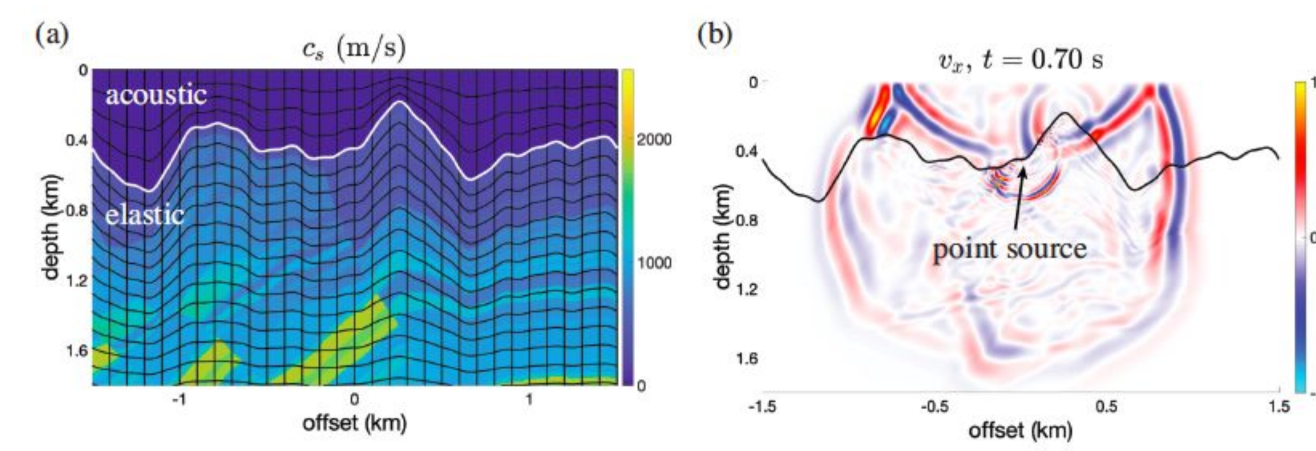


Figure 4: (a) Structural model of ocean on heterogeneous solid with variable bathymetry. Ocean and solid have separate meshes with collocated grid points along seafloor interface. Ocean obeys acoustic wave equation for velocity potential; solid obeys elastic wave equation for displacements. (b) Horizontal particle velocity; note discontinuity across seafloor. (From Almquist and Dunham, in preparation)

Earthquake Sequence Simulation Capabilities

A work in progress... We have completed implementation and testing of both quasi-dynamic and fully dynamic earthquake sequence simulation capabilities. This was done by combining

- viscoelastic solver (except during fully dynamic ruptures)
- elastodynamic solver with friction (during fully dynamic ruptures)
- rate-and-state friction with adaptive time-stepping, using adaptive Runge-Kutta method with embedded error estimate and error control on slip and state variable (Erickson and Dunham, 2014; Allison and Dunham, 2018).

Rigorous verification and convergence tests (not shown) were done using the Method of Manufactured Solutions (MMS).

We are currently working on setting up relevant application problems, like the upcoming 2D plane strain dipping fault SEAS benchmark.

Open-Source Code

We are committed to providing our code to the community.

- acoustic code (Almquist and Dunham, 2020a): https://laplace-curvilinear.sourceforge.io/
- elastic code (Almquist and Dunham, 2020b): https://sourceforge.net/projects/elastic-curvilinear/
- viscoelastic and earthquake sequence code: forthcoming...

Future Directions

We will verify our code on SEAS benchmark problems and use it to study

- earthquake sequences on rough faults, branching faults, and other geometrical complexities (effects of residual stress concentrations);
- dipping faults, especially subduction zones, accounting for transition to distributed viscous flow at depth (as in Allison and Dunham, 2018);
- fault zone fluid transport and pore pressure evolution (as in Zhu et al., 2020) to capture fluid-driven slow slip events and fault valving.

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