PHYSICS-INFORMED DEEP LEARNING OF RATE-AND-STATE FAULT FRICTION

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We show that neural networks are able to infer sub-surface friction parameters along a strike-slip fault governed by a nonlinear rate-and-state friction law.

Antiplane strike-slip fault

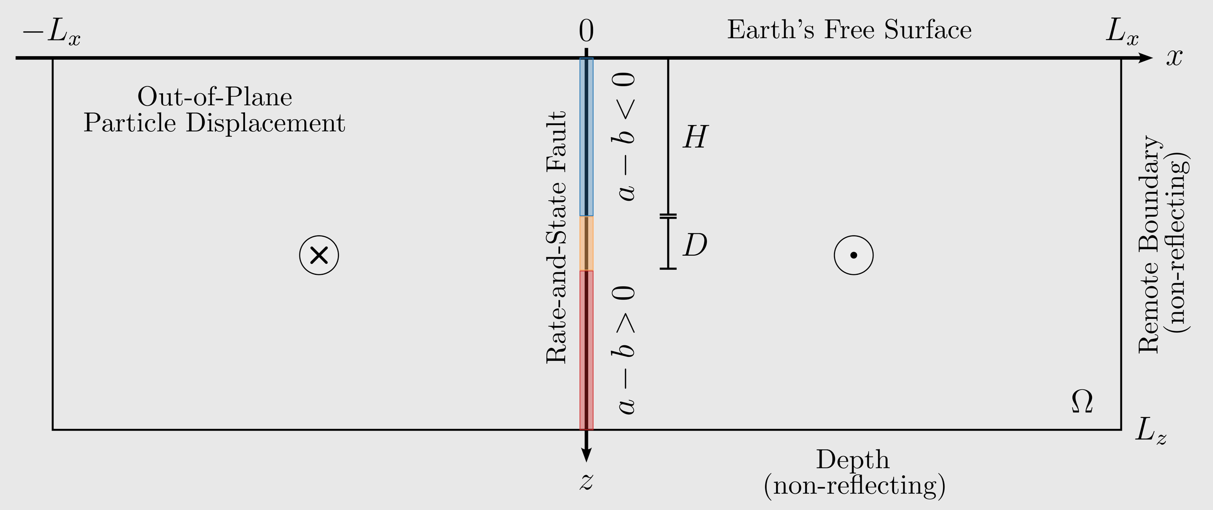

Fig. 1: 2D schematic of a strike-slip fault governed by nonlinear, depth-dependent friction. Out of plane displacements are denoted by circles and boundary conditions are labeled at each relevant surface.

State evolution (aging law)	$G(V,\psi) = \frac{bV_0}{D_c}\exp\left(\frac{f_0-\psi}{b} - \frac{\lvert V\rvert}{V_0}\right)$
Fault strength	$F(V,\psi) = (\bar{\sigma}_n)\, a \ln\left(\frac{V}{V_0}\right) + \psi,$
State ODE	$\psi_t = G(V,\psi)$ for $t \geq 0$; $\psi(0) = \psi_0$
IBVP	$u_{tt} - c^2\Delta u = S,$ on $\widehat{\Omega}$; $\tau = F,$ on $x=0$; $\mu(\nabla\cdot\mathbf{n}) = 0,$ on $z=0$; $\mu(\nabla u\cdot\mathbf{n}) + Zu_t = 0,$ on $\{x=L_x\}\cup\{z=L_z\}$; $u = u_0,$ at $t=0$; $u_t = v_0$ at $t=0$

Depth-dependence of $\alpha = a - b$

$$\alpha(z) = \begin{cases} \alpha_{\min} & 0 < z < H \\ (z-H)*((\alpha_{\max}-\alpha_{\min})/D) + \alpha_{\min} & H \leq z \leq H+D \\ \alpha_{\max} & H+D < z, \end{cases}$$

Parameter	L_x	L_z	H	D	μ	ρ	$\alpha_{\min}$	$\alpha_{\max}$	f_0	$\bar{\sigma}_n$	V_0	D_c
Value	25 km	25 km	12 km	5 km	32 GPa	2.67 kg/m^3	-0.005	0.015	0.6	50 MPa	10^{-6} m/s	2 m

Physics-informed neural networks

Feed-forward deep neural network:
A single hidden layer with weight W and bias b

$$\ell(\mathbf{y};\theta) = \varphi(W\mathbf{y}+b), \text{ where } \theta = (W,b)$$

The recursive definition

$$\ell_0 = \mathbf{y}, \qquad \ell_k = \phi_k(W_k\ell_{k-1}+b_k), \quad \text{for } 0<k<L,$$

defines a feed-forward, deep neural network:

$$\mathcal{N}(\mathbf{y};\theta) = W_L\ell_{L-1} + b_L$$

PINN architecture:
Given a generic initial-boundary-value problem(IBVP)

$$\mathcal{L}[\mathbf{u};\lambda](\mathbf{x}) = \mathbf{k}(\mathbf{x}), \quad \mathbf{x}\in\widehat{\Omega},$$
$$\mathcal{B}[\mathbf{u};\lambda](\mathbf{x}) = \mathbf{g}(\mathbf{x}), \quad \mathbf{x}\in\partial\widehat{\Omega},$$

we define a neural network $\mathcal{N}$ which aspires to be the IBVP solution u. To this end we define the loss components

$$MSE_{\widehat{\Omega}}(\theta) = \frac{1}{N_{\widehat{\Omega}}}\sum_{i=1}^{N_{\widehat{\Omega}}}\left|\mathcal{L}[\mathcal{N};\lambda](\mathbf{x}^i_{\widehat{\Omega}};\theta) - \mathbf{k}^i\right|^2,$$

$$MSE_{\partial}(\theta) = \frac{1}{N_{\partial}}\sum_{i=1}^{N_{\partial}}\left|\mathcal{B}[\mathcal{N};\lambda](\mathbf{x}^i_{\partial};\theta) - \mathbf{g}^i\right|^2,$$

over collocation points $\{\mathbf{x}^i_{\widehat{\Omega}}\}_{i=1}^{N_{\widehat{\Omega}}}$ and $\{\mathbf{x}^i_{\partial}\}_{i=1}^{N_{\partial}}$. Then, $\mathcal{N}(\mathbf{x};\theta^*) = u(\mathbf{x})$ when

$$\theta^* = \arg\min_{\theta} MSE_{\widehat{\Omega}}(\theta) + MSE_{\partial}(\theta).$$

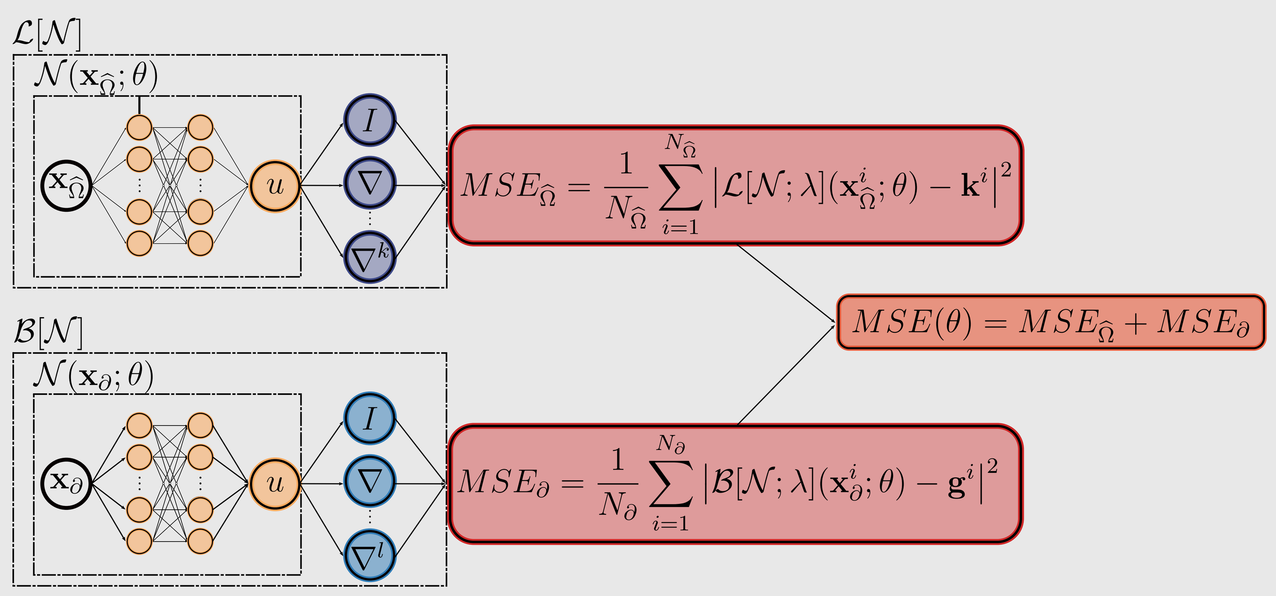

Fig. 2: A schematic of the PINN architecture for solving a generalized boundary value problem. Displacement approximation network $\mathcal{N}$ is trained on interior and boundary subdomains which are governed by operators $\mathcal{L}$ and $\mathcal{B}$, respectively.

Results

PINN is effective in solving the 1D coupled nonlinear fault problem.

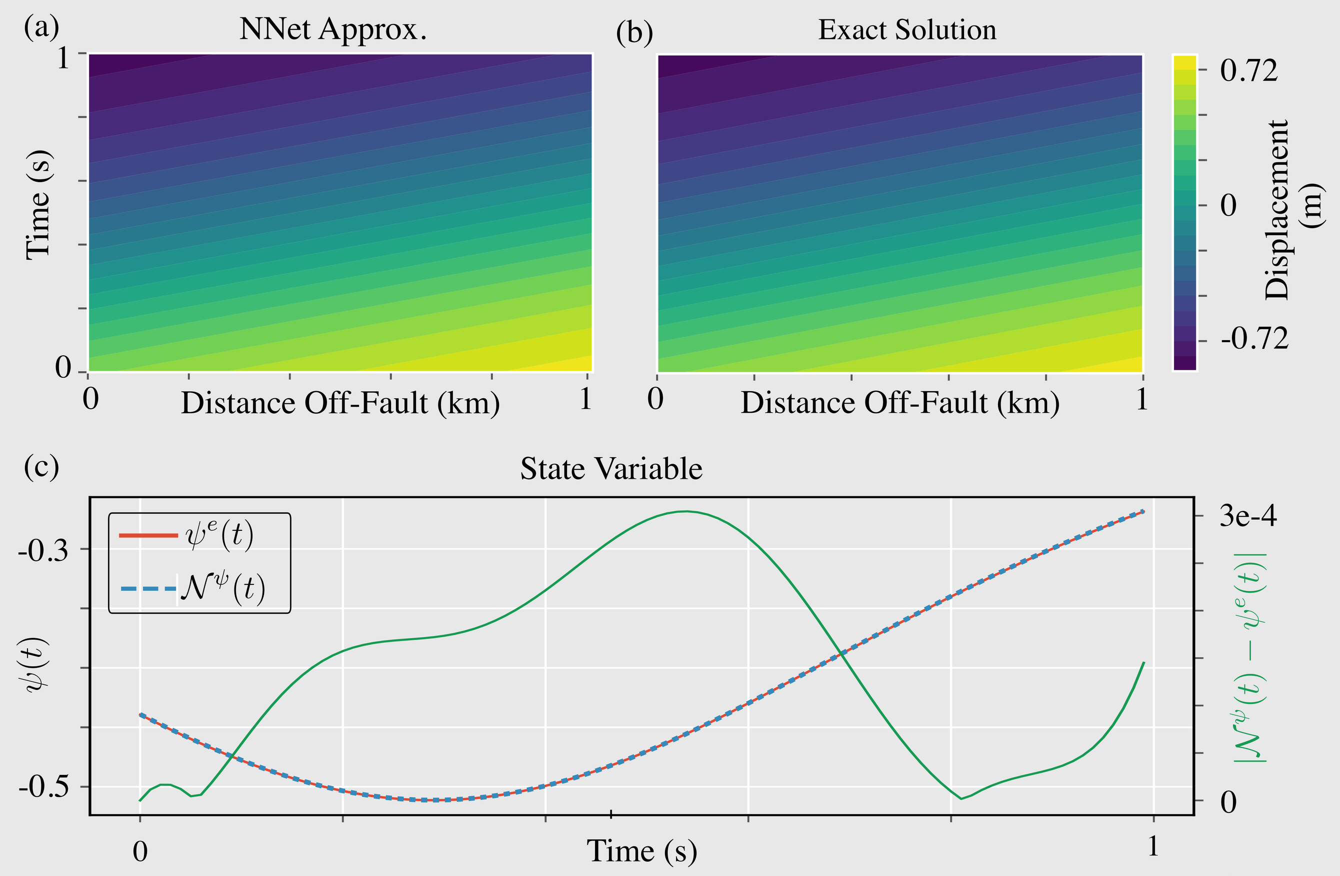

Fig. 3: Comparison of results from 1D illustration showing the (a) displacement network approximation $\mathcal{N}$ with (b) manufactured solution u^e. Additionally, the (c) state approximation network $\mathcal{N}^\psi$ is plotted against the manufactured state ψ^e along with their absolute error $|\mathcal{N}^\psi(t) - \psi^e(t)|$. Absolute displacement error was averaged over 1000 randomly sampled points and measured to be $|\mathcal{N} - u^e|_{\text{avg}} = 1.57e-5$.

PINN solves a 2D antiplane problem and is consistent over various scales of the domain.

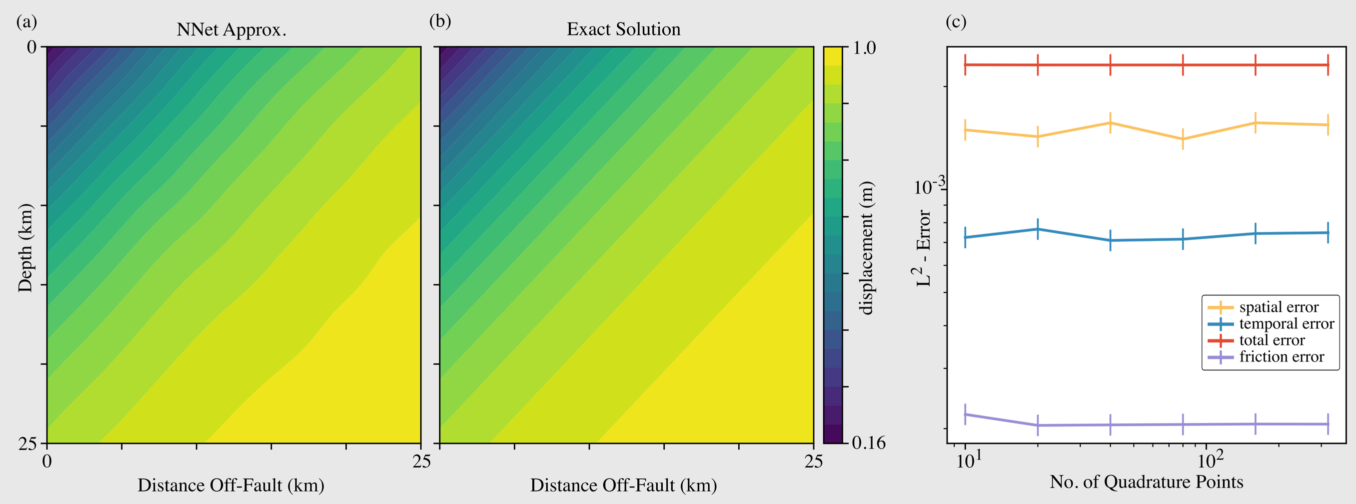

Fig. 4: (a) 2D displacement plot for a PINN trained to solve the inverse problem using hard enforcement of initial conditions compared to (b) the manufactured displacements. (c) L^2-errors for displacement in space, time, and spacetime (along with L^2-errors for the friction parameter) are computed on a uniform grid using Simpson's rule as a quadrature. Errors are then recorded over several mesh refinements.

Loss components decrease over 30 training iterations and the inferred friction parameter is learned in the first few training iterations.

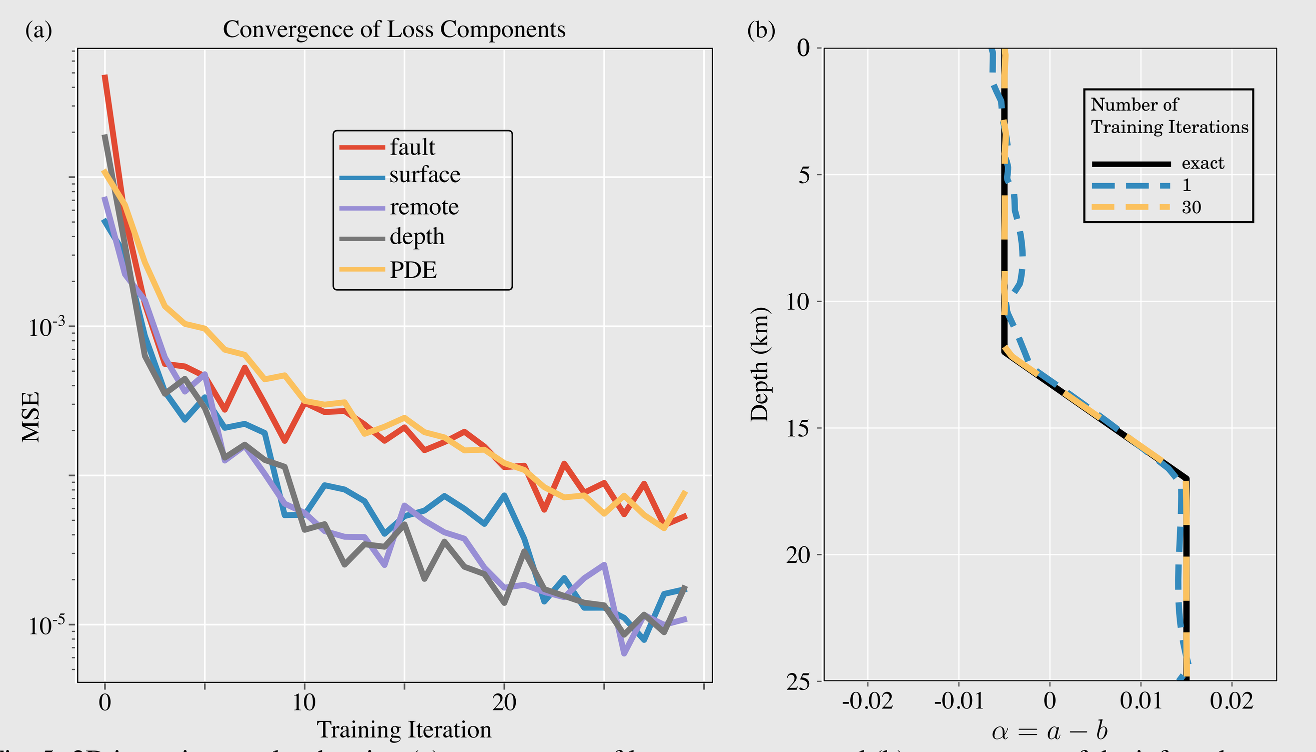

Fig. 5: 2D inversion results showing (a) convergence of loss components and (b) convergence of the inferred parameter approximation.

PINN for solving the antiplane strike-slip fault problem

Hard enforcement of initial conditions:
By defining trainable networks $\mathcal{N}_u, \mathcal{N}_\psi$ we can define trial functions

$$\Phi(\mathbf{x},t) = u_0(\mathbf{x}) + tv_0(\mathbf{x}) + t^2\mathcal{N}_u(\mathbf{x},t),$$
$$\phi(t) = \psi_0 + t\mathcal{N}_\psi(t),$$

satisfying the initial conditions of the above IBVP and state ODE exactly.

IBVP loss components:

$$MSE_\Omega = \frac{1}{N_\Omega}\sum_{i=1}^{N_\Omega}\left|\Phi_{tt} - c^2\Delta\Phi - S\right|^2,$$

$$MSE_f = \frac{1}{N_f}\sum_{i=1}^{N_f}\left| - \mu\Phi_x - F\big((1/2)\Phi,\phi\big)\right|^2,$$

$$MSE_s = \frac{1}{N_s}\sum_{i=1}^{N_s}\left| - \mu\Phi_z\right|^2,$$

$$MSE_{r,d} = \frac{1}{N_{r,d}}\sum_{i=1}^{N_{r,d}}\left|Z\Phi_t + \mu(\nabla\Phi\cdot\mathbf{n})\right|^2,$$

State component loss:

$$MSE_\psi = \frac{1}{N_\psi}\sum_{i=1}^{N_\psi}\left|\phi_t - G\big((1/2)\Phi,\phi\big)\right|$$

Objective function to be minimized:

$$MSE = \sum_{\xi\in\chi} MSI_\xi, \quad \text{where } \chi = \{\Omega, f, s, r, d, \psi\}$$

Summary

- Neural network inference of subsurface friction parameters.
- Multi-network training to solve system of coupled partial differential equations.
- Mesh-free solution retains good accuracy across multiple domain resolutions.
- Hard and soft boundary enforcement are tested for both forward and inverse problems.
- Network rapidly learns fault friction but further training needed for displacements.

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