Automata Theory: The Foundations of Computer Science

Anatoly Zavyalov

University of Toronto

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About me

- I am entering my fourth year as an undergraduate at the University of Toronto, studying math, computer science, and physics.
- My research interests are theoretical computer science (especially automata theory), and discrete math in general. Previously, I have also done research in astronomy.

Photo Credit: Anastasia Zhurikhina

P vs. NP

Who wants to win $\$1,000,000$?

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- We'll talk about a key component of understanding the problem: automata theory.

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    - Computational linguistics

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- **Applications of finite automata**: parsing, number theory
- **Turing machine**: how we abstractly represent computers
- **Turing completeness**: systems that are as powerful as computers

A dangerous gumball machine

A gumball machine charges 25¢ for a gumball, and exact change is needed. The only types of coins you can choose from are 5¢, 10¢, and 25¢. If you put in more than 25¢, the gumball machine explodes.
In what ways can you get a gumball without the gumball machine exploding?

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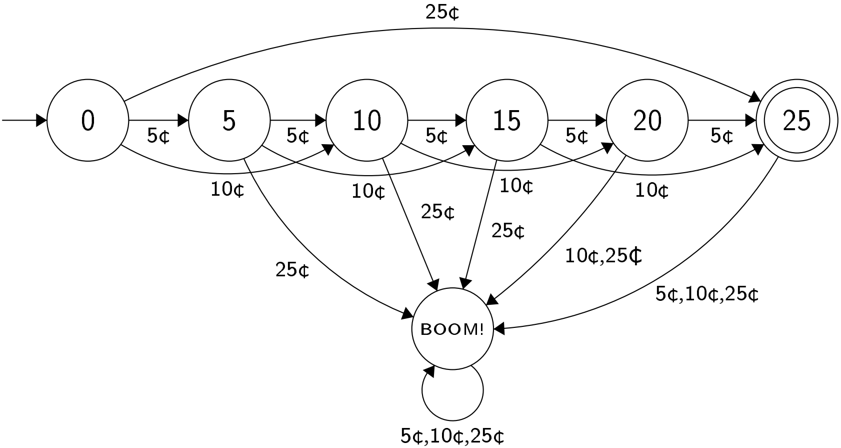
- 25¢
- 5¢ 5¢ 10¢ 5¢
- 10¢ 5¢ 5¢ 5¢

But not:

- 5¢ 5¢
- 10¢ 25¢ (BOOM!)

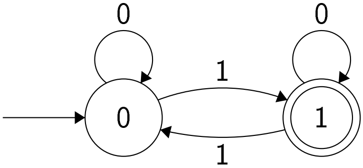
Gum

Here is a finite automaton for the gumball machine:



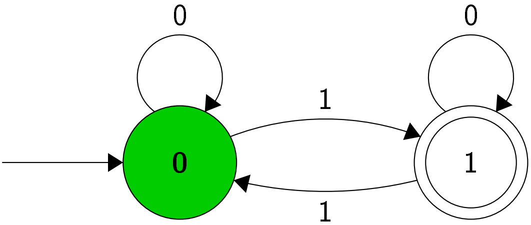
The states tracks how much money has been paid so far. Once the 25 state is reached, the fare is accepted.

Finite Automaton



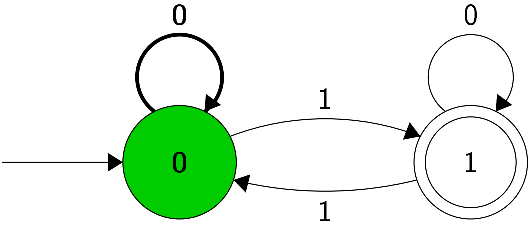
- The automaton starts at the initial state (arrow going in).
- We feed the input into the automaton character by character by following the transitions.
- A string x is accepted by an automaton if it ends on a final (double-circled) state after feeding it through the automaton.

Example



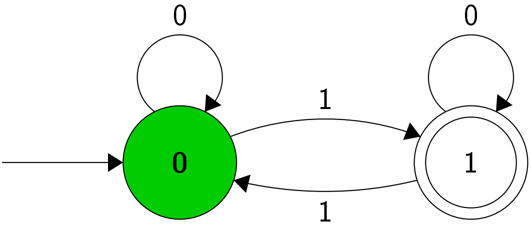
0 1 0 1 1 0

Example



$\mathbf{0}\,1\,0\,1\,1\,0$

Example



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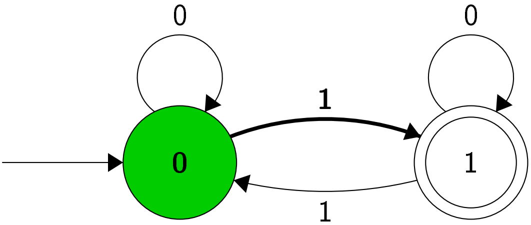

0 **1** 0 1 1 0

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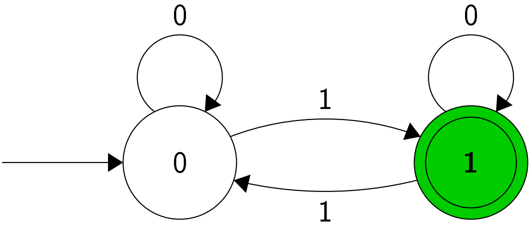

0 1 0 1 1 0

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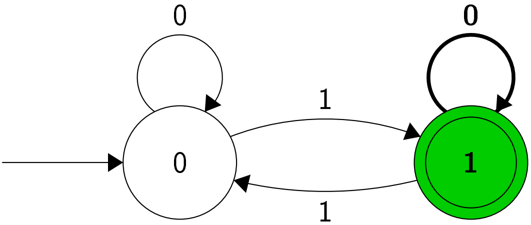
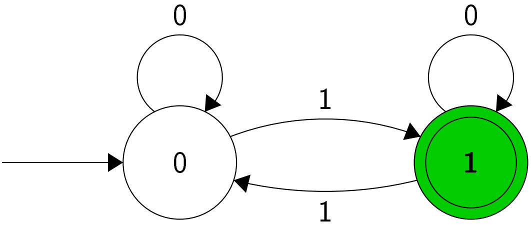


0 1 **0** 1 1 0

Example



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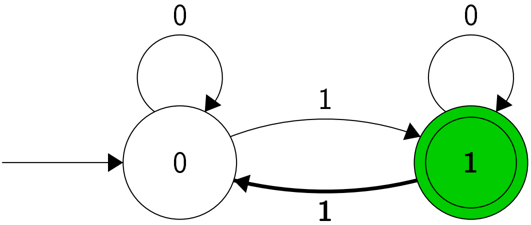

0 1 0 **1** 1 0

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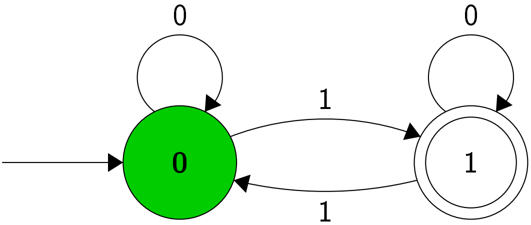

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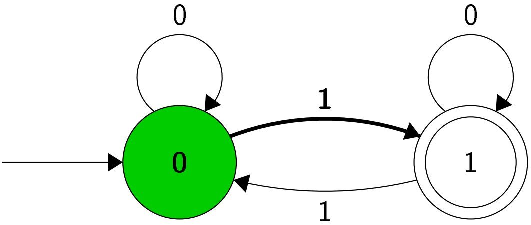

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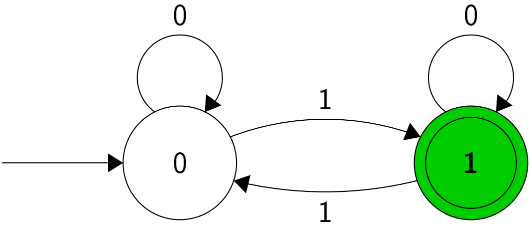

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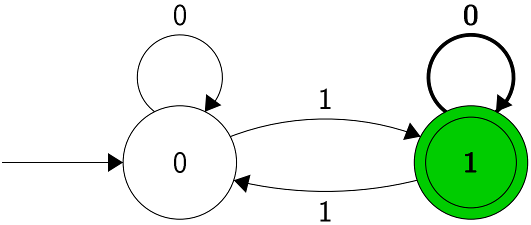
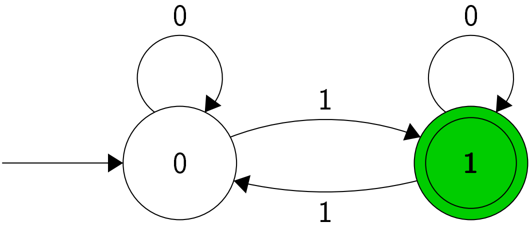


0 1 0 1 1 **0**

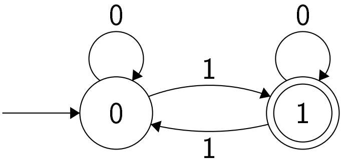
Example



0 1 0 1 1 0

We end in a final (double-circled) state, so **010110** is accepted!

DFA Example



What kinds of strings does this automaton accept?

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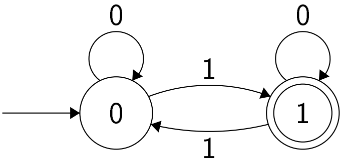

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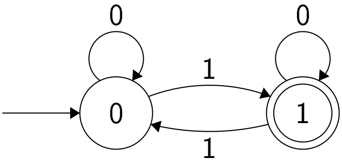

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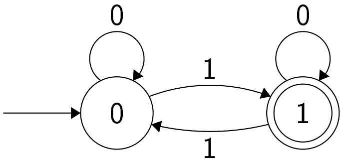

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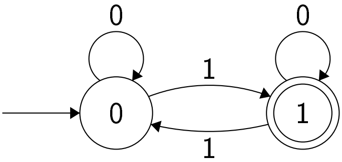

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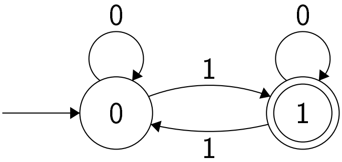

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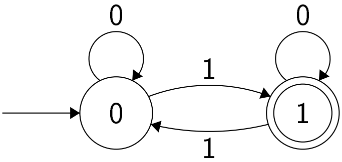

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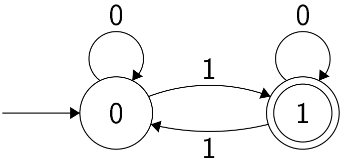

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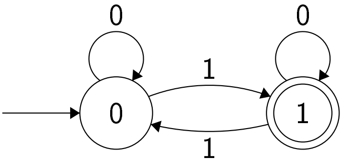

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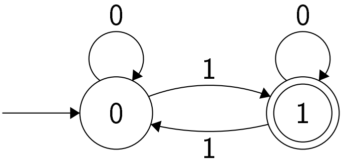

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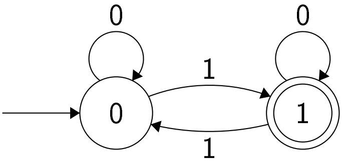

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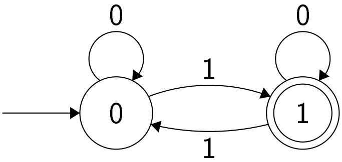

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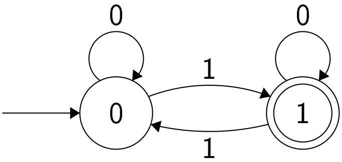

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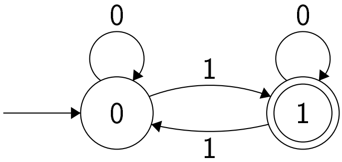

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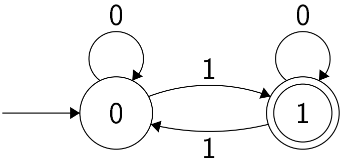

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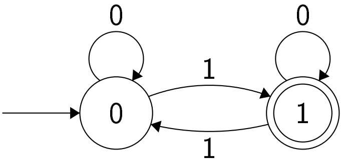

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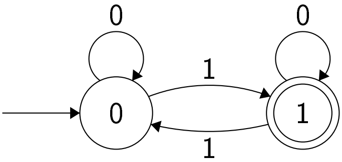

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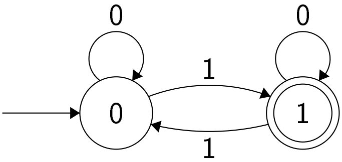
This automaton accepts a binary string x if and only if

DFA Example



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This automaton accepts a binary string x if and only if the number of 1s in x is odd, or equivalently if the sum of the digits of x is odd.

Applications to Parsing

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```
<variable name>=<value>
```

where the variable name does not start with a digit 0-9.

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For example:

- `year=2023`
- `name="Anatoly"`
- `location="SigmaCamp"`

Applications to Parsing

Let's make an automaton that accepts valid Python declarations of integers, i.e. strings of the form

`<variable name>=<digits in 0-9>`

where the variable name does not start with a digit. The allowed characters are a-z, `A-Z`, `0-9`, and =.

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- `age=88`
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But it should not accept:

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- `SIGMA=2023CAMP`

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- `SIGMA=2023CAMP`
- `2023month=8` (variable name can't start with a number)

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Let's do it on the board!

Application: Sum of three squares

We'll say that an integer N is "unfriendly" if it is the sum of three squares of integers:

$$N = x^2 + y^2 + z^2 \quad \text{for some } x, y, z \in \mathbb{Z}$$

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For example, 38 is unfriendly, as

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$28, 15, 7, 240, 92, 348$ are examples of friendly integers.

Application: Sum of three squares

Legendre's three square theorem says that an integer N is a sum of three squares of integers $N = x^2 + y^2 + z^2$ if and only if n is **not** of the form

$$N = 4^a(8b + 7)$$

where a and b are non-negative integers.

Application: Sum of three squares

Legendre's three square theorem says that an integer N is a sum of three squares of integers $N = x^2 + y^2 + z^2$ if and only if n is **not** of the form

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We will make an automaton that decides whether or not N is friendly by reading its binary representation.

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- Lastly, $(4^a(8b+7))_2$ looks like

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So N is friendly if and only if its binary representation is of the form

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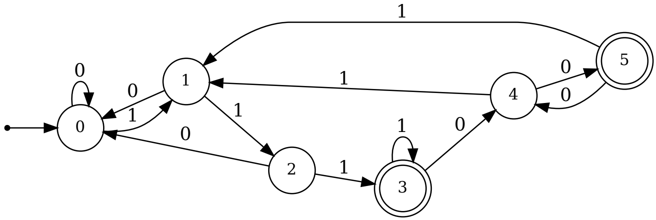

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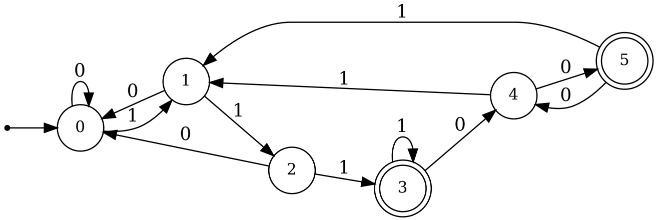

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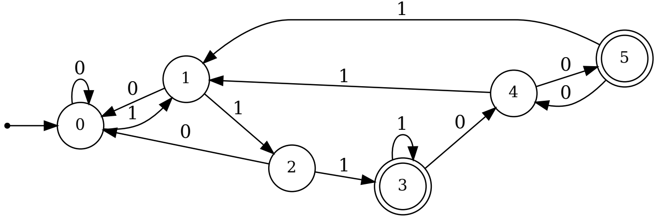
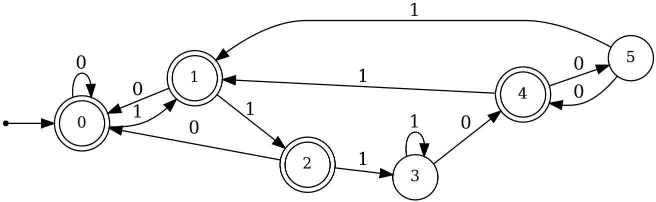


So this automaton accepts $(N)_2$ if and only if N is friendly.
What about an automaton for unfriendly N?

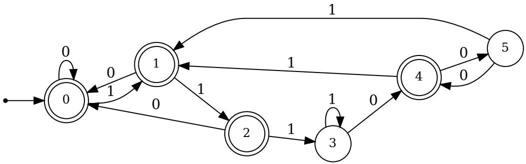
Example: Sum of three squares

To accept all $(N)_2$ if and only if N is unfriendly, just flip the final states:



Then everything that wasn't accepted before is now accepted, and vice versa.

Limitations of finite automata



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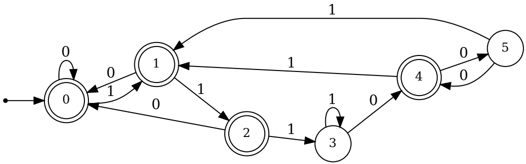

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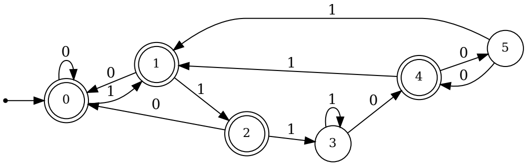
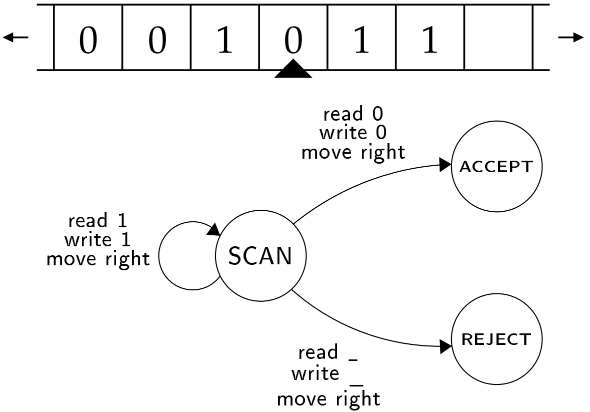


- Finite automata are quite limited, as their number of states is fixed!
- For example, no finite automaton can accept only strings of the form 0…01…1, where the number of 0s and 1s is the same.
  - Intuitively, it's because finite automata can't "count" arbitrarily high.

Turing machine



- To make our automaton more powerful, we're going to give it an infinitely long tape that it can read from and write to, which will act as its memory. The tape consists of infinitely many cells.

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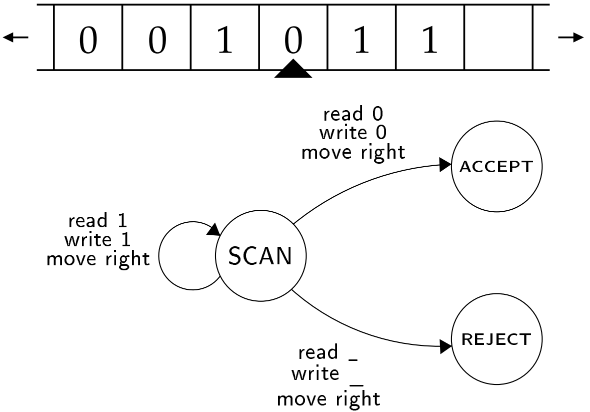
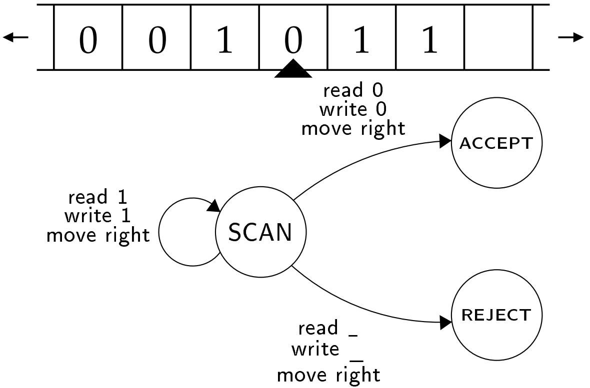


- To make our automaton more powerful, we're going to give it an infinitely long tape that it can read from and write to, which will act as its memory. The tape consists of infinitely many cells.
- This is called a Turing machine.

How Turing machines work



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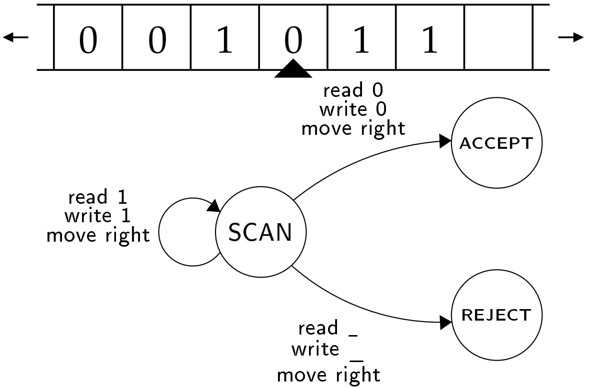

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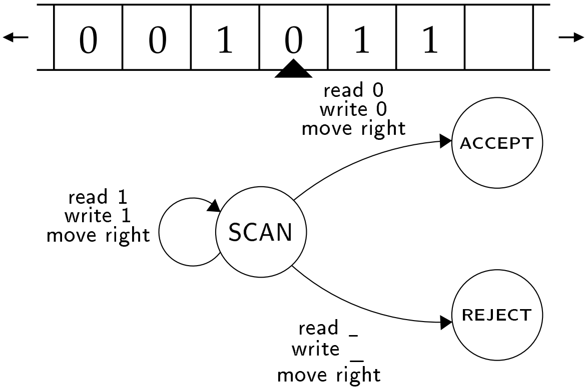

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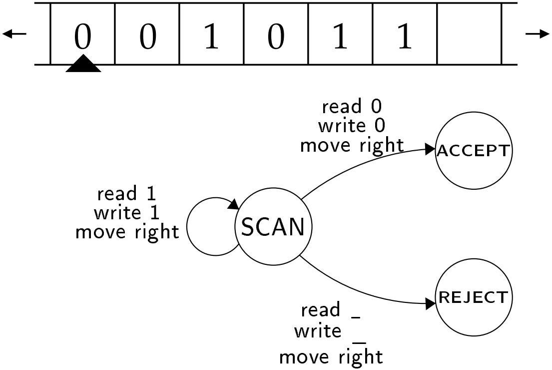
- Turing machines can accept or reject an input.

Turing machine example

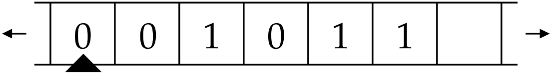


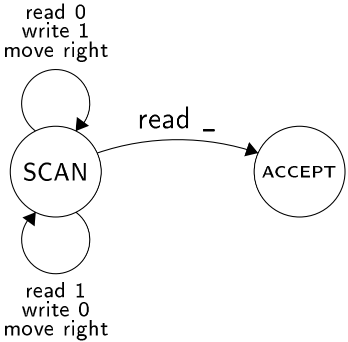
This Turing machine will read an input comprised of 1s and 0s and:

- accept if the input has any 1s, and
- reject if the input has no 1s.

Another example

This Turing machine will read an input comprised of 1s and 0s and flip every 0 to a 1 and vice versa. Once it is done, it will accept. The output is the bitwise complement of the input.





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Church-Turing Thesis

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- So, Turing machines can do everything that computers can do.
- To study how efficient an algorithm is on a computer, we can study how efficient it is on a Turing machine.

Turing completeness

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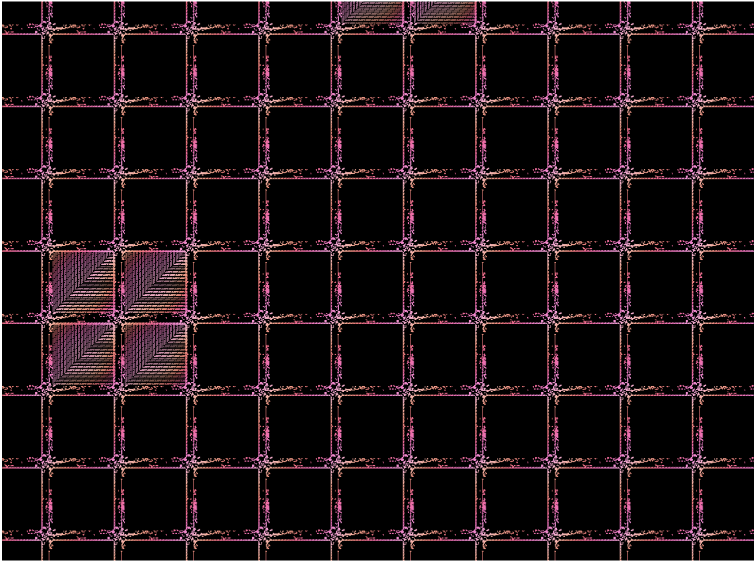
- Anything that's Turing complete can simulate any classical computer.

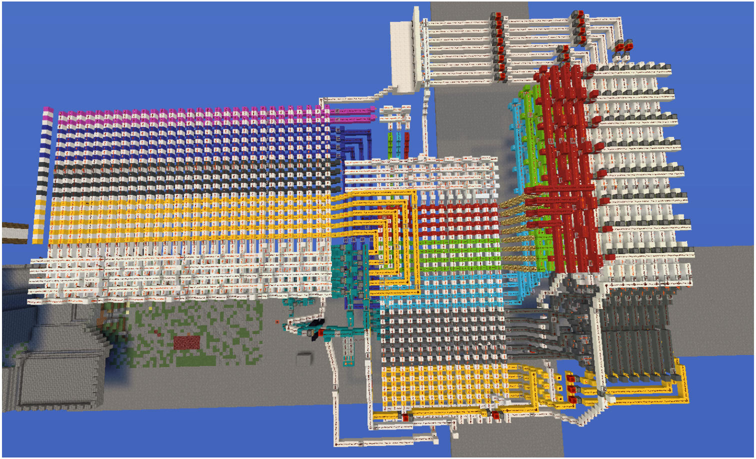
Turing completeness

Conway's Game of Life can simulate itself!



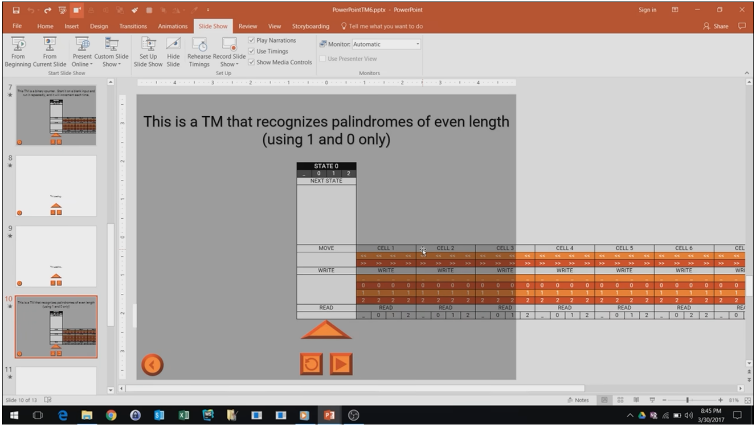
Turing completeness

Computers can be built in Minecraft using redstone!



Turing completeness

Microsoft PowerPoint can simulate Turing machines!



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- By adding an infinite tape to a finite automaton, we get a Turing machine, which are equivalent in power to computers.
- Instead of making a program in Python, write it in PowerPoint!

Thank you!
Any questions?