ELEMENTARY PROBLEMS AND SOLUTIONS

Edited by A.P. Hillman
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Send all communications regarding Elementary Problems and Solutions to Professor A. P. Hillman, Department of Mathematics and Statistics, University of New Mexico, Albuquerque, New Mexico. Each problem or solution should be submitted in legible form, preferably typed in double spacing, on a separate sheet or sheets in the format used below. Solutions should be received within three months of the publication date.

B-88 *Proposed by John Wessner, Melbourne, Florida*

Let $L_0, L_2, L_4, L_6, \ldots$ be the Lucas numbers $2, 3, 7, 18, \ldots$. Show that

$$L_{2k} \equiv 2(-1)^k \pmod 5 \quad .$$

B-89 *Proposed by Robert S. Seamons, Yakima Valley College, Yakima, Washington*

Let F_n and L_n be the n-th Fibonacci and n-th Lucas number, respectively. Let $[x]$ be the greatest integer function. Show that $L_{2m} = 1 + [\sqrt{5}\, F_{2m}]$ for all positive integers m.

B-90 *Proposed by Phil Mana, University of New Mexico, Albuquerque, New Mexico*

Let $b_1, b_2, \ldots$ be the sequence $3, 7, 47, \ldots$ with recurrence relation $b_{n+1} = b_n^2 - 2$. Show that the roots of

$$x^2 - 2b_n x + 4 = 0$$

are expressible in the form $c + d\sqrt{5}$, where c and d are integers.

B-91 *Proposed by Douglas Lind, University of Virginia, Charlottesville, Virginia*

If F_n is the n-th Fibonacci number, show that

$$\sum_{j=1}^{\infty} (1/F_j)$$

converges while

$$\sum_{j=3}^{\infty} (1/\ln F_j)$$

diverges.

B-92 *Proposed by J.L. Brown, Jr., The Pennsylvania State University*

Let (x, y) denote the g.c.d. of positive integers x and y. Show that $(F_m, F_n) = (F_m, F_{m+n}) = (F_n, F_{m+n})$ for all positive integers m and n.

B-93 *Proposed by Martin Pettet, Toronto, Ontario, Canada*

Show that if n is a positive prime, $L_n \equiv 1 \pmod{n}$. Is the converse true?

SOLUTIONS

The deadlines for submitting solutions to the Elementary Problems Section have proved to be unrealistic and are being changed as of this issue. We take this opportunity to list some errata and some of the solvers whose solutions were received after copy for this section went into production.

ERRATA

B-33 The solution printed on page 235, Vol. 2, No. 3, was submitted by Charles R. Wall, Texas Christian University, Ft. Worth, Texas. This problem was also solved by John H. Halton, B. Litvack, and the proposer.

B-39 In the note after the solution to B-39 on page 327, Vol. 2, No. 4, the reference to the inequality $a^{n-1} < F_n < a^n$ for $n > 1$ in An Introduction to the Theory of Numbers, by Niven and Zuckerman, should state that their initial conditions on the F_n are $F_1 = 0$ and $F_2 = 1$.

B-57 On the next to last line of the solution on page 160, Vol. 3, No. 2, the second "=" sign should obviously be a ">" sign.

SOLUTIONS RECEIVED AFTER PRODUCTION DATE FOR PROBLEM SECTION

Problem	Solvers
B-17	Ken Siler
B-19	F. D. Parker
B-20	S. L. Basin
B-22	Ken Siler
B-24	Sr. Mary De Sales McNabb, Charles R. Wall
B-25	Douglas Lind, Charles R. Wall
B-26	Gurdial Singh
B-29	Douglas Lind
B-30	Douglas Lind
B-35	Denis Hanson
B-38	Brian Scott
B-44	J. L. Brown, Jr.
B-45	J. L. Brown, Jr.
B-46	Clyde A. Bridger; C. A. Church, Jr.; Kenneth E. Newcomer; Charles R. Wall
B-47	J. L. Brown, Jr.; Charles R. Wall
B-48	J. L. Brown, Jr.; Douglas Lind; Charles R. Wall
B-49	Douglas Lind, Charles R. Wall
B-50	J. L. Brown, Jr.; C. B. A. Peck; Charles R. Wall
B-52	M. N. S. Swamy, Charles R. Wall
B-53	M. N. S. Swamy, Charles R. Wall
B-54	M. N. S. Swamy, Charles R. Wall
B-55	Howard L. Walton
B-70	Dermott A. Breault; James E. Desmond; John E. Homer, Jr.; George Ledin, Jr.; C. B. A. Peck; Dean B. Priest
B-71	Dermott A. Breault; J. L. Brown, Jr.; John E. Homer, Jr; George Ledin, Jr.; C. B. A. Peck; Dean B. Priest
B-72	James E. Desmond; George Ledin, Jr.
B-74	Douglas Lind
B-75	J. L. Brown, Jr.; Douglas Lind.

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Corrected

B-86 *Proposed by Verner E. Hoggatt, Jr., San Jose State College, San Jose, California*

Show that the squares of every third Fibonacci number satisfy

$$y_{n+3} - 17y_{n+2} - 17y_{n+1} + y_n = 0 \quad .$$