Walsh Functions for Shock Tracking and their Application to Expansion Tube Simulations

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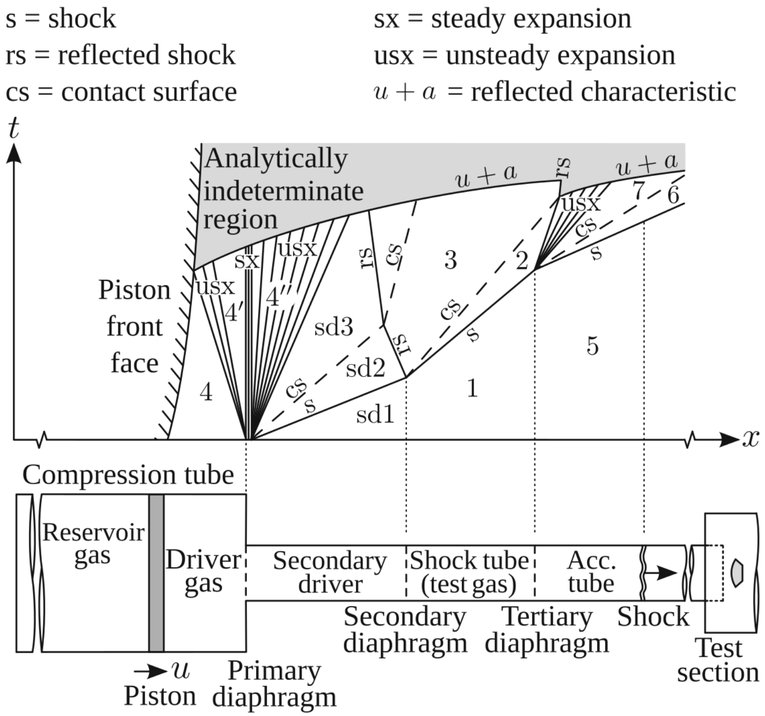
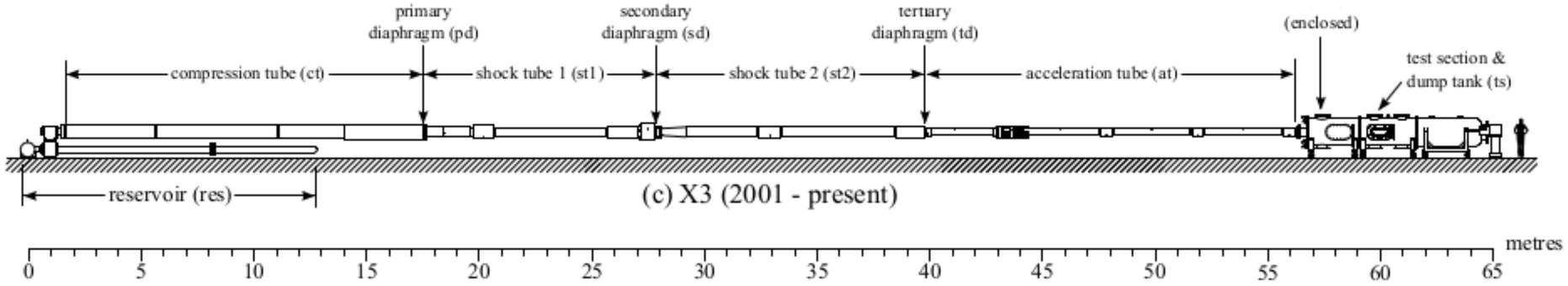
Motivation

Expansion tubes — UQ's X2, X3

- Unsteady processes
  - Free piston driver, unsteady driver gas slug
  - Diaphragm openings
  - Unsteady expansions
- Large boundary layers
  - Shock speed attenuation
- Strong propagating shock





Gildfind, D. E., Jacobs, P. A., Morgan, R. G., Chan, W. Y. K., and Gollan, R. J., "Scramjet test flow reconstruction for a large-scale expansion tube, Part 1: quasi-one-dimensional modelling," Shock Waves, Vol. 28, No. 4, 2018, pp. 877–897. https://doi.org/10.1007/s00193-017-0785-x.

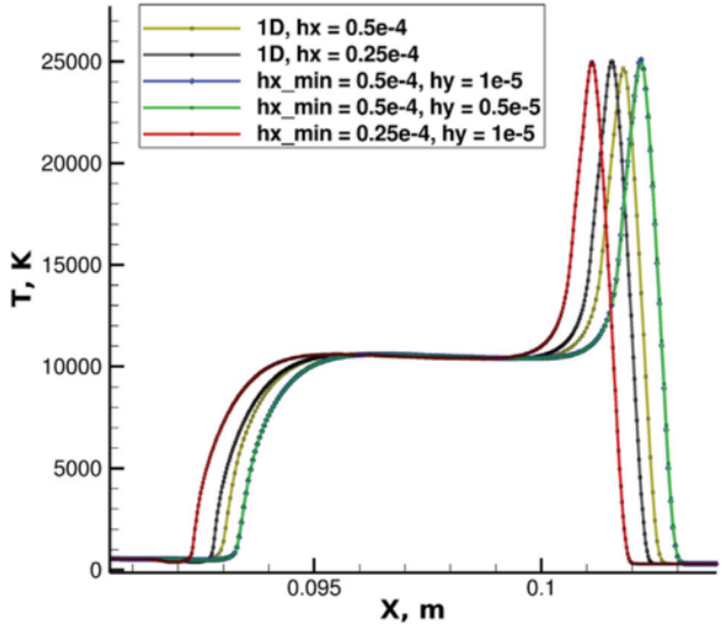
Motivation

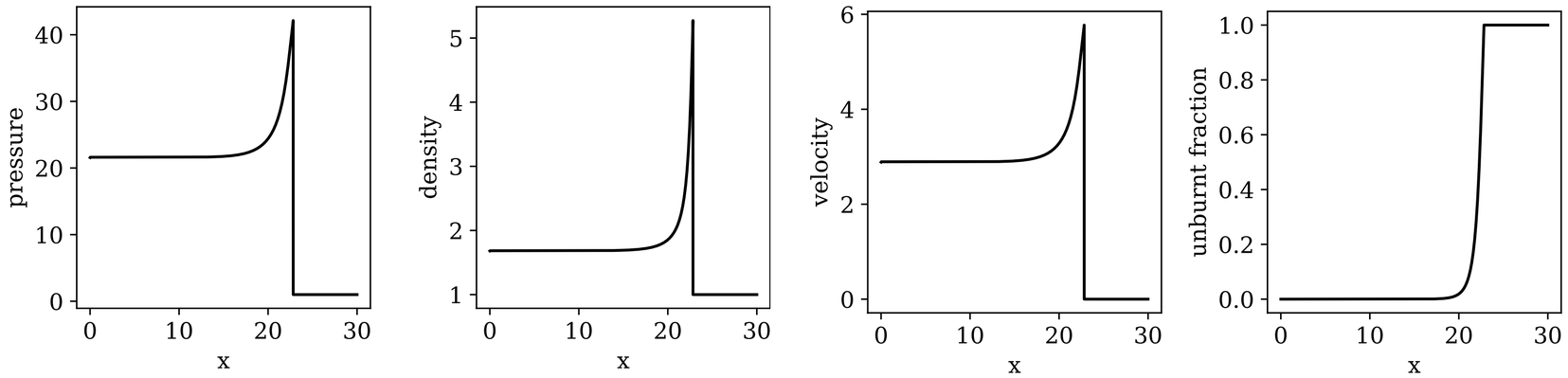
Non-reflected shock tubes — X2 (NRST mode), NASA EAST

- Thermochemical non-equilibrium

Shock — chemistry coupled problem

- CJ detonation wave

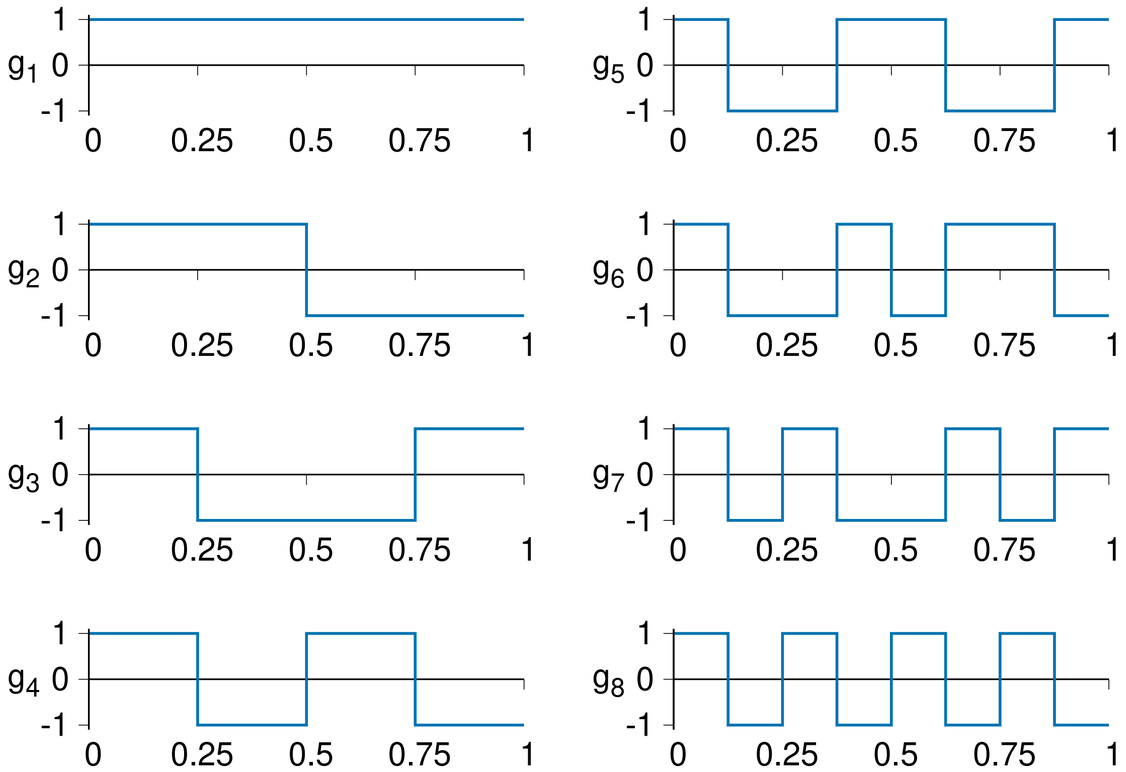




Kotov, D., Yee, H., Panesi, M., Prabhu, D., and Wray, A., “Computational challenges for simulations related to the NASA electric arc shock tube (EAST) experiments,” Journal of Computational Physics, Vol. 269, 2014, p. 215–233. https://doi.org/10.1016/j.jcp.2014.03.021.

Approach

- Need to resolve:
  - Shocks / contacts propagating length of facility
  - Boundary layers
  - Unsteady processes
- Options:
  - Refinement
  - Adaptive mesh refinement
  - Overset meshing
  - Shock tracking

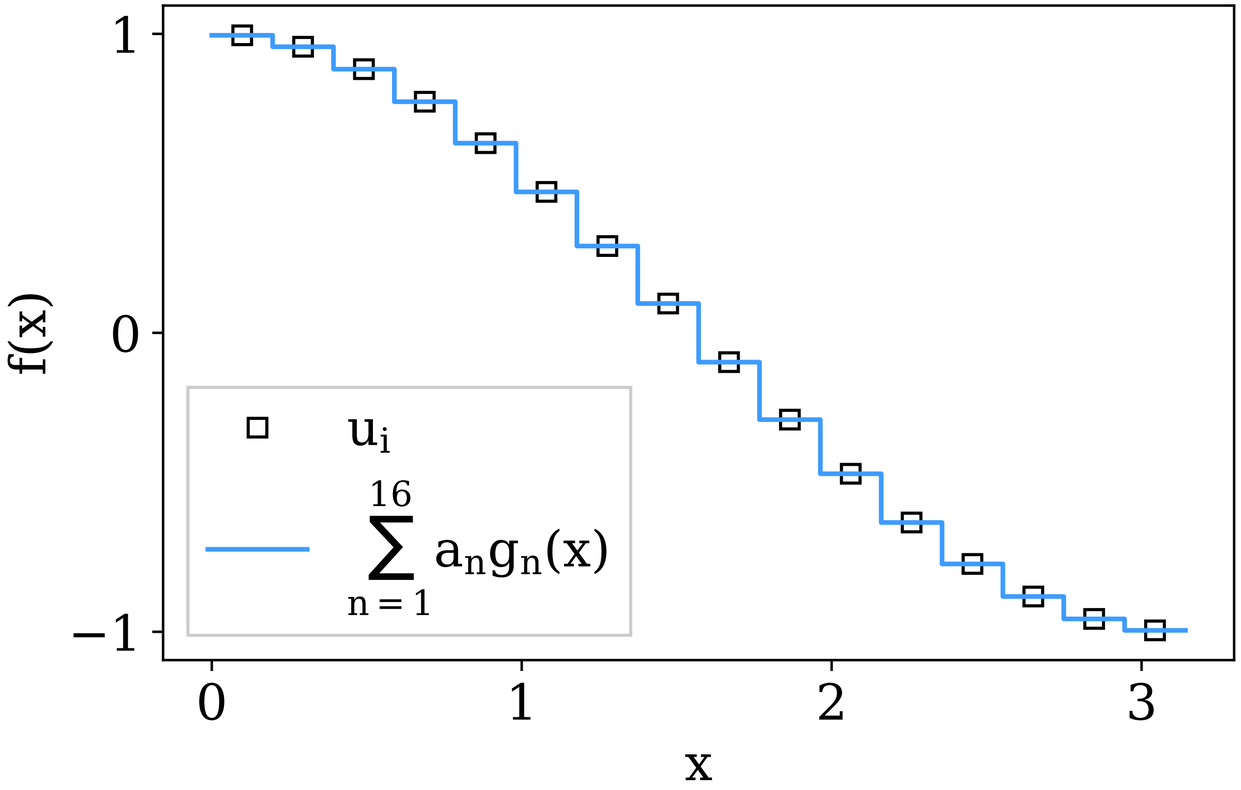


Gnoffo, P. A.,“Solutions of nonlinear differential equations with feature detection using fast Walsh transforms,”Journal of Computational Physics, Vol. 338, 2017, pp. 620–649. https://doi.org/10.1016/j.jcp.2017.03.016.

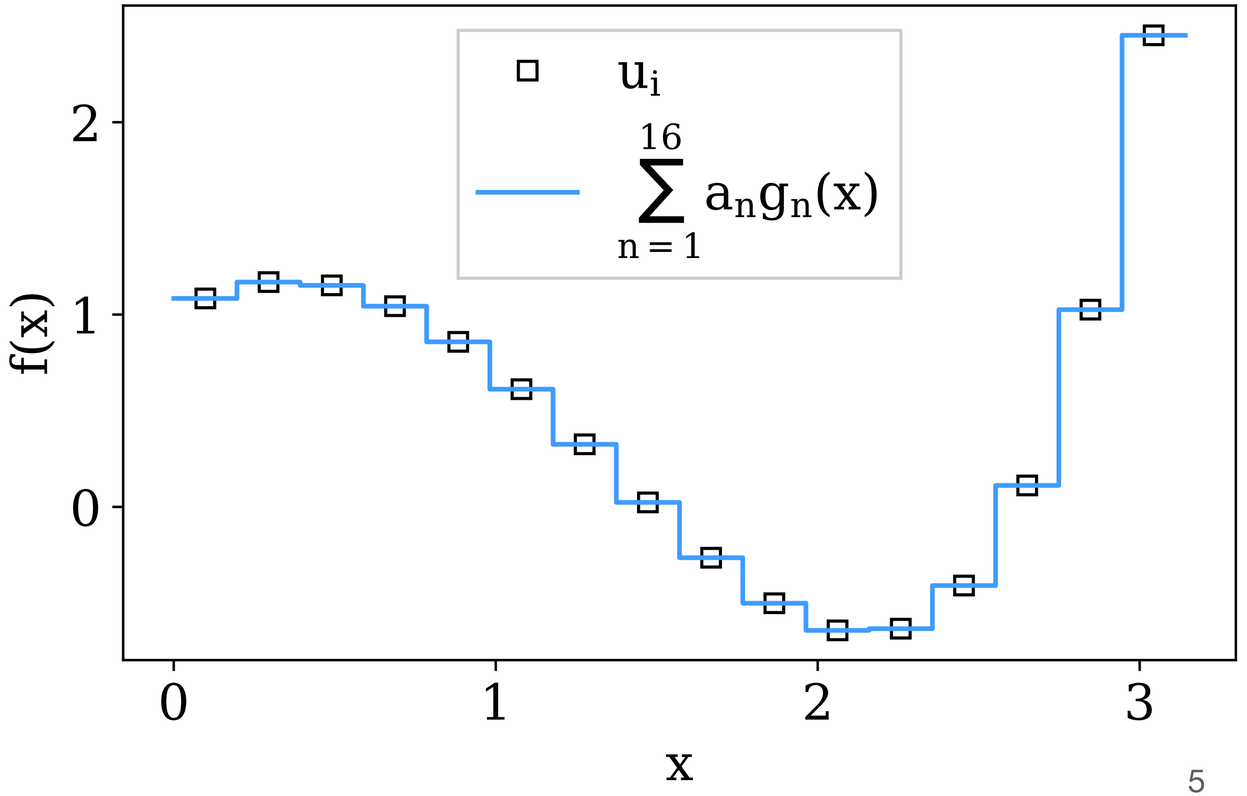
Walsh function interpolation

$$\hat{f}(x) = \sum_{n=1}^{2^p} a_n g_n(x), \qquad a_n = \sum_{i=1}^{N} u_i g_n(x_i) \Delta x$$

$u_i = \cos(x_i)$



$u_i = \exp(x_i) + 2x_i^2$

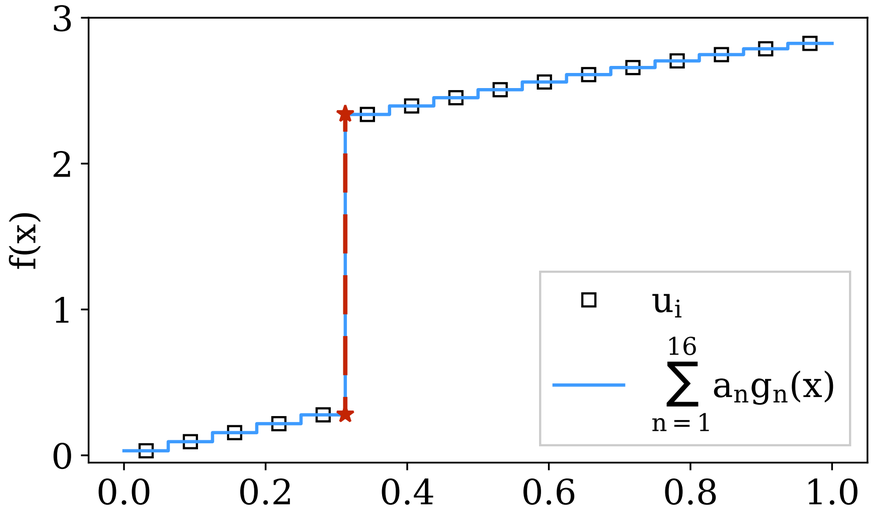


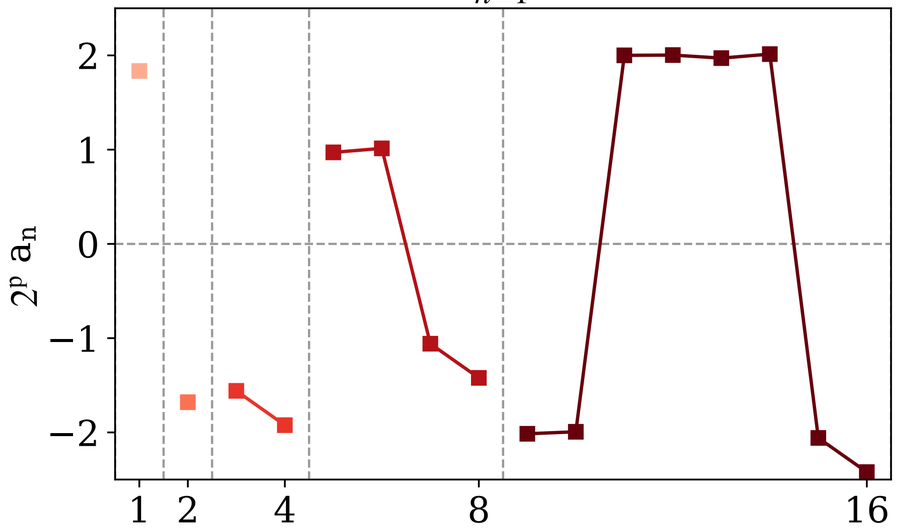
Feature detection with Walsh functions

$$u_i = \sin(x_i) + 2H(x_i - x^*)$$

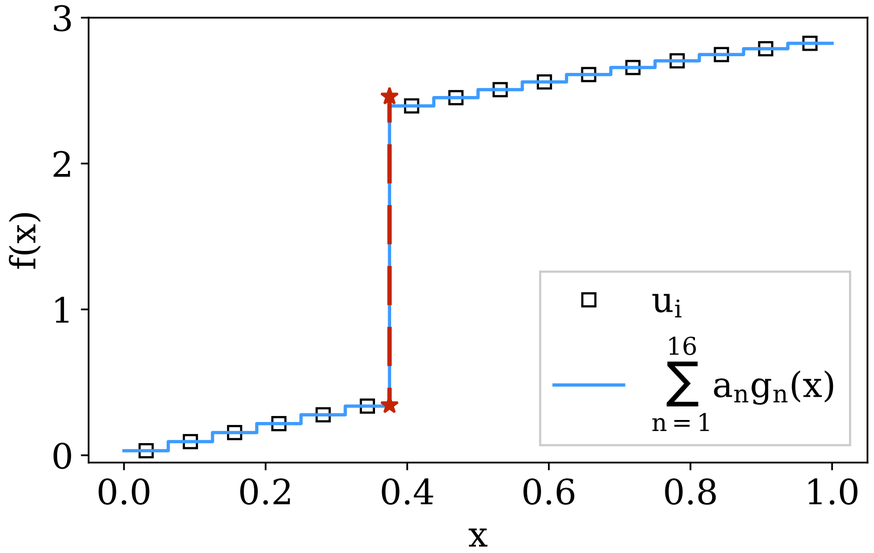
$$\hat{f}(x) = \sum_{n=1}^{2^p} a_n g_n(x),$$

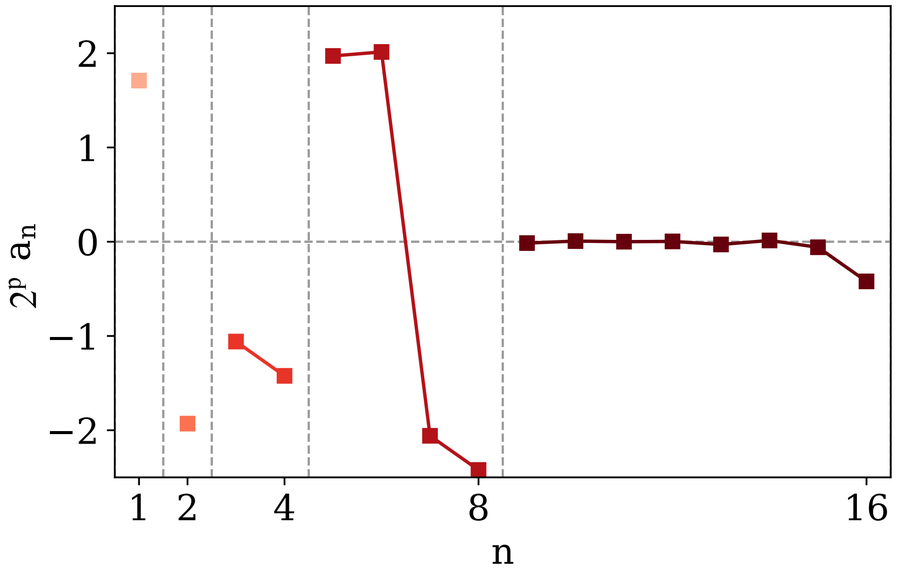
$x^* = 0.3125$





$x^* = 0.375$





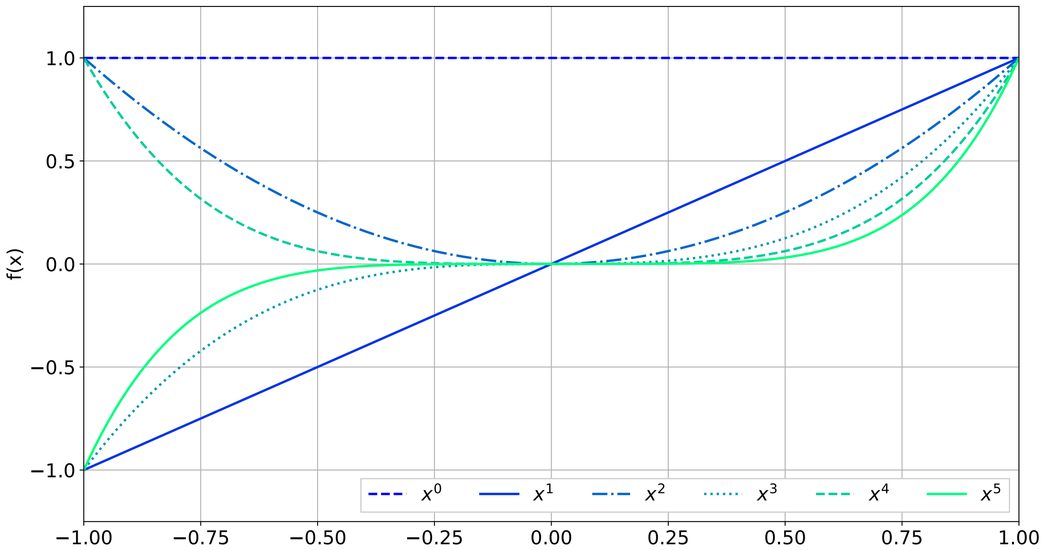
Polynomial curve fit

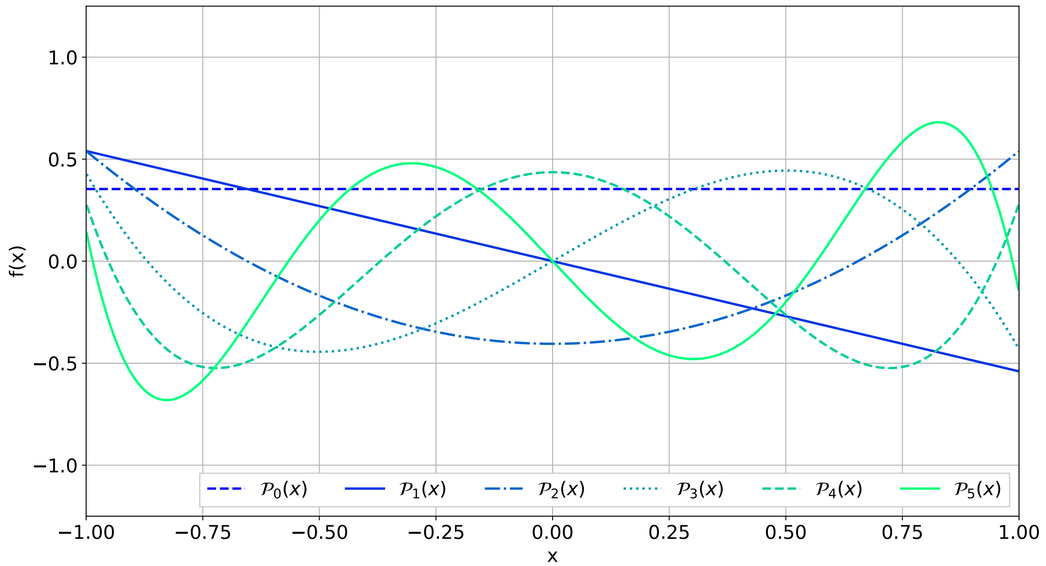
$$\tilde{f}(x) = \sum_{k=0}^{K} c_k \phi_k(x)$$

Least squares fit with smooth basis functions, via Walsh function fit

$$\phi_k(x) \approx \sum_{n=1}^{N} B_{n,k} g_n(x), \qquad B_{n,k} = \sum_{i=1}^{N} \phi_k(x_i) g_n(x_i) \Delta x$$

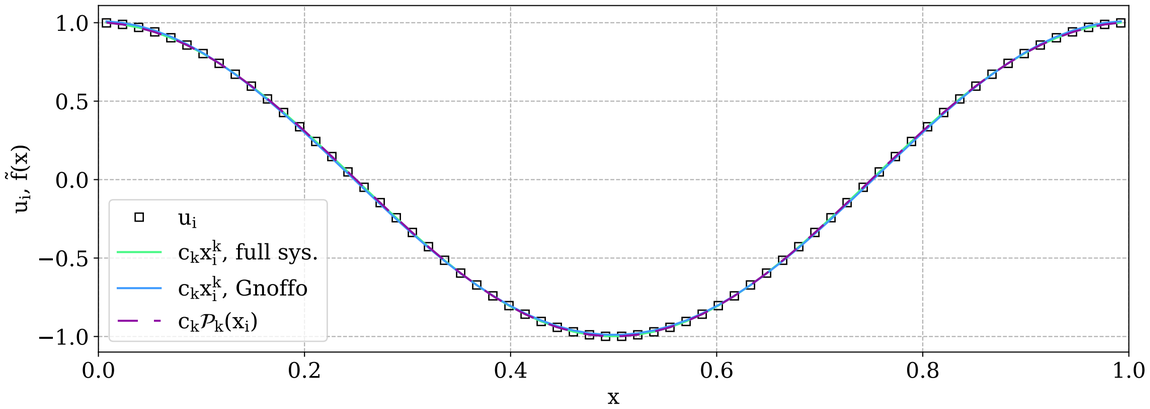
$$c_k = \frac{1}{\Delta x} \sum_{n=1}^{N} a_n B_{n,k}$$

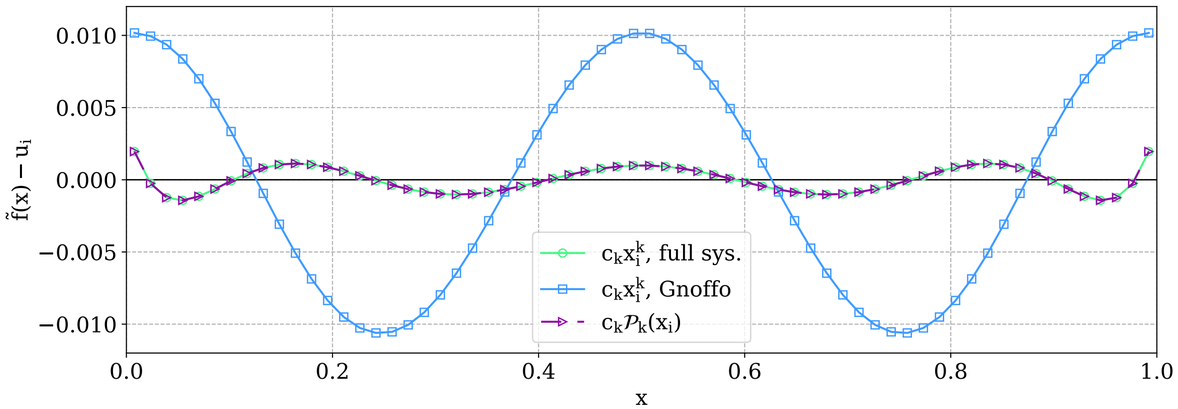




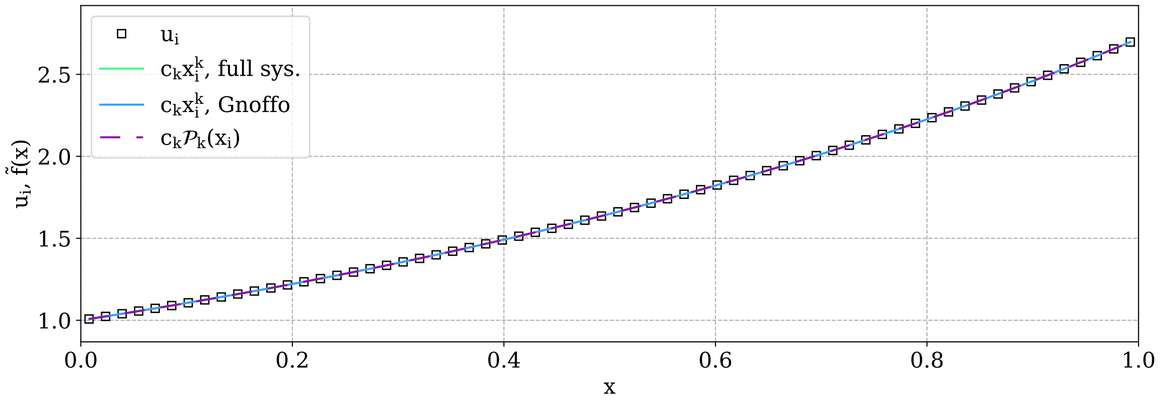
Polynomial curve fit to smooth data

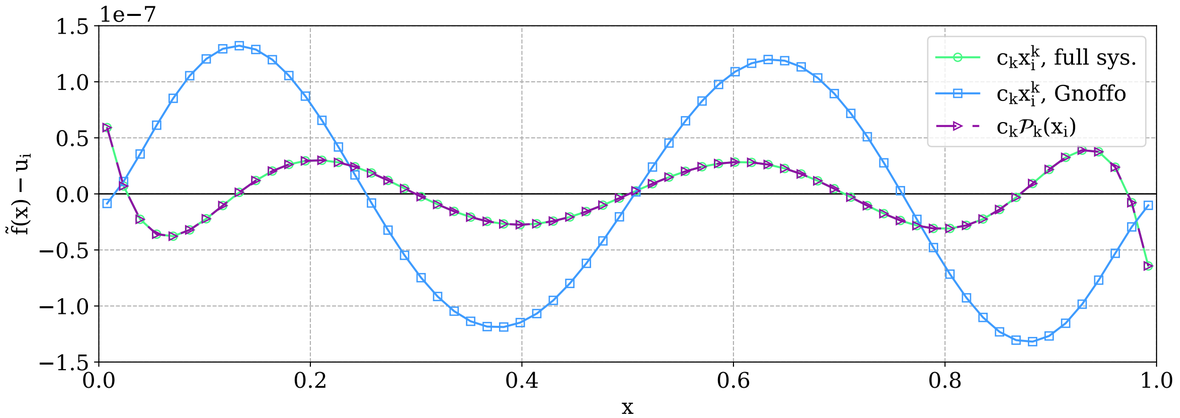
$u_i = \cos(x_i)$



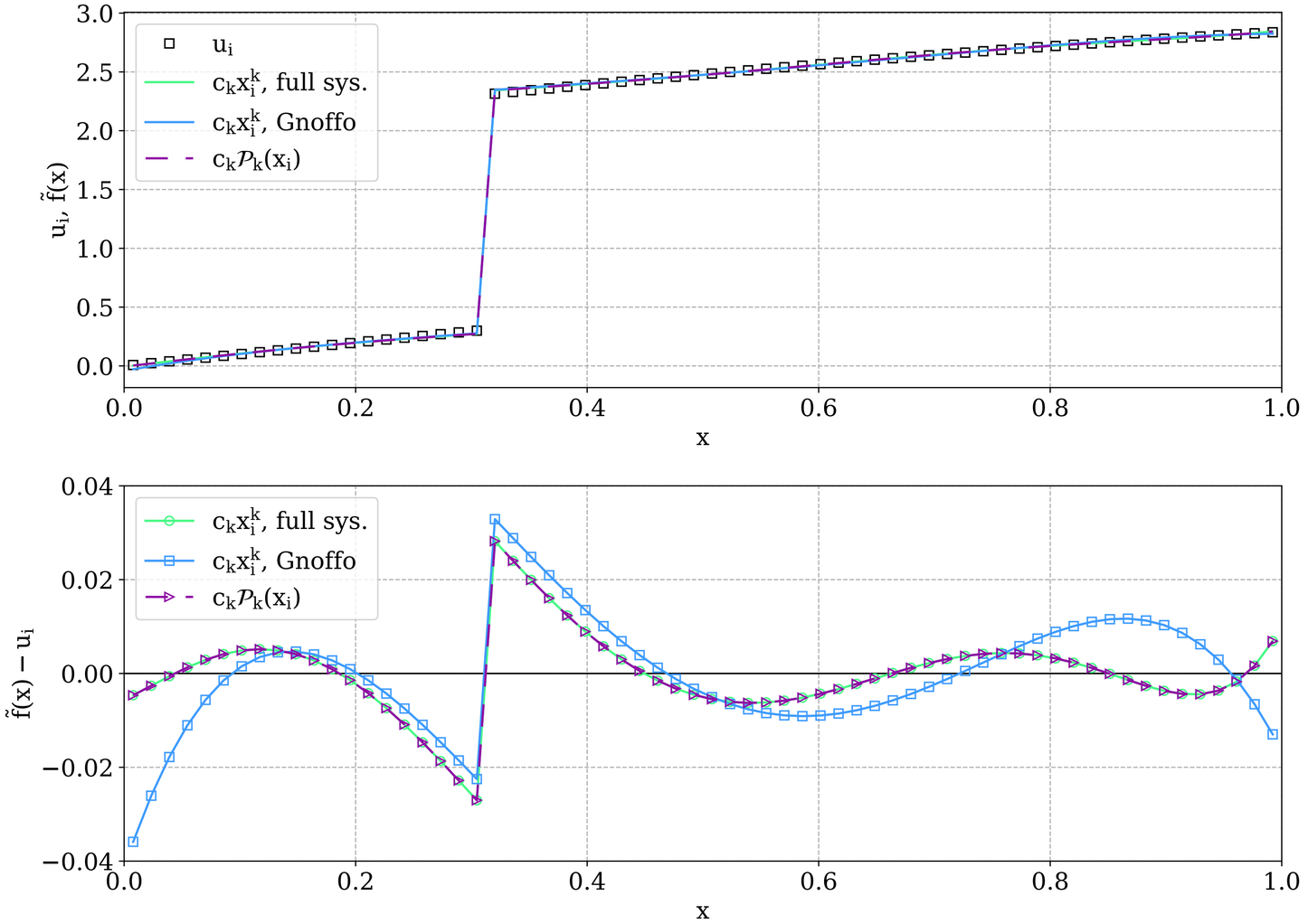


$u_i = \exp(x_i)$

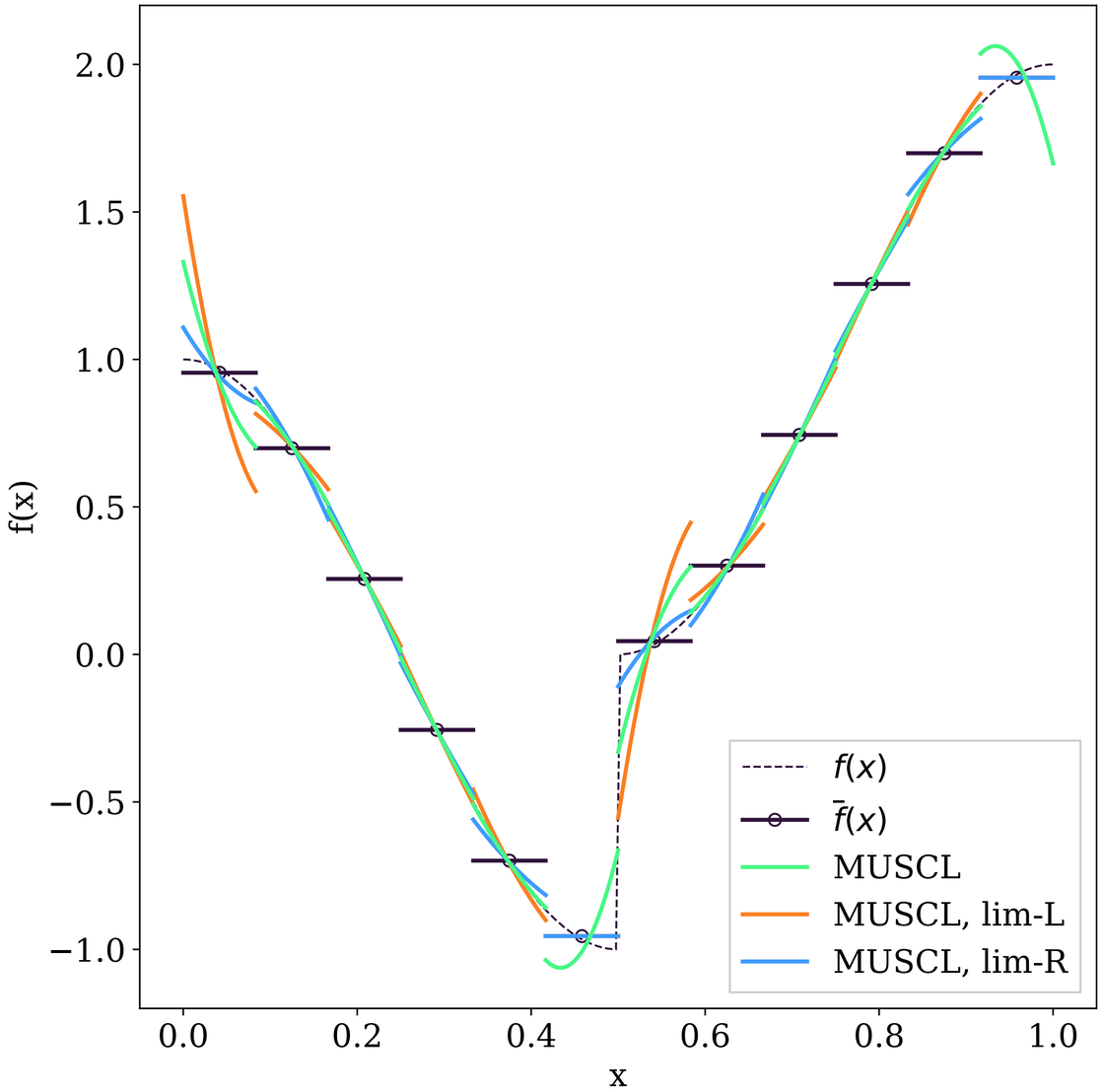


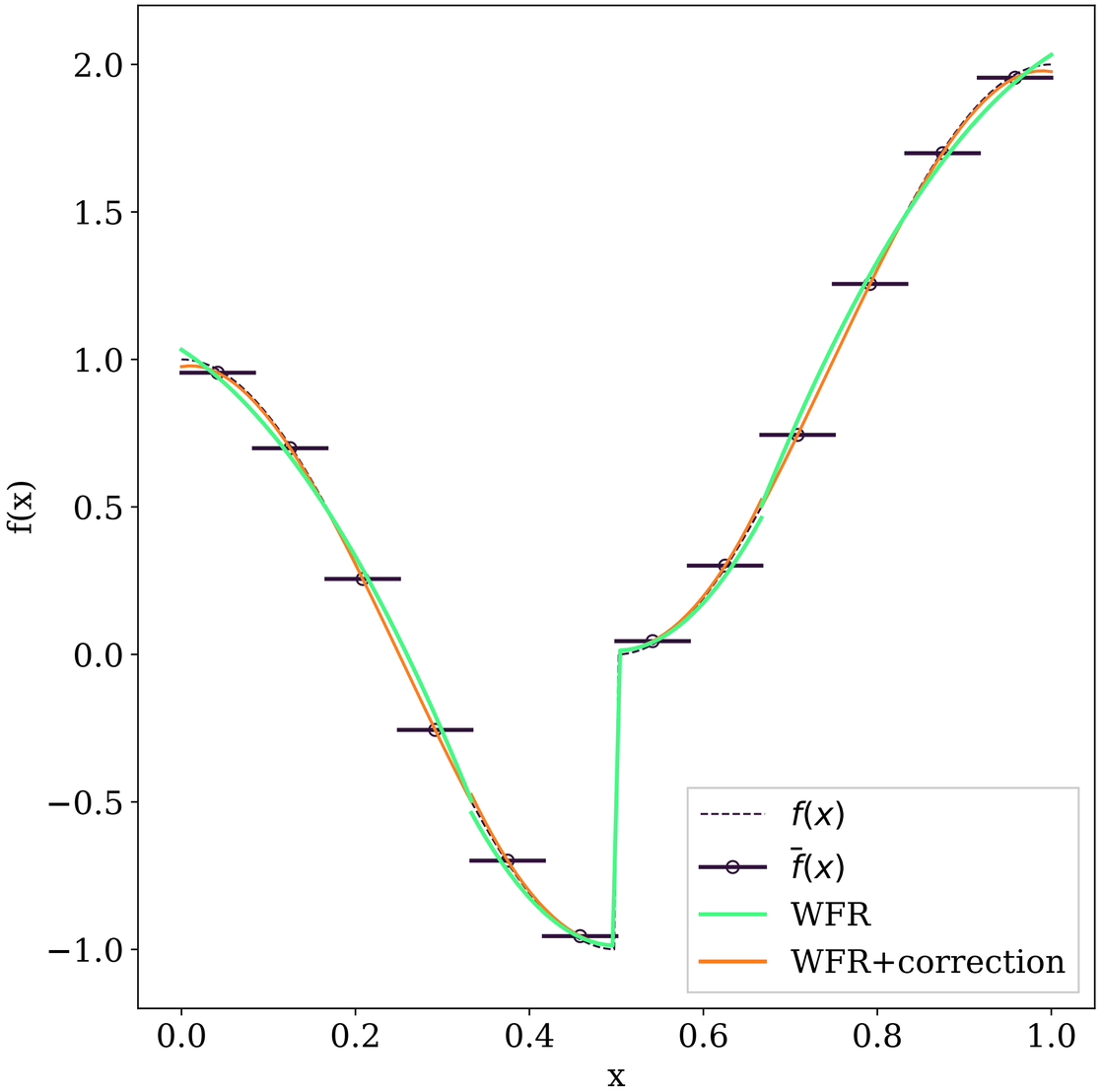


Polynomial curve fit to shocked data



Walsh function reconstruction stencils





Reactive Euler equations

$$\mathbf{u}_t + F(\mathbf{u})_x = S(\mathbf{u})$$

$$\mathbf{u} = \begin{pmatrix} \rho \\ \rho u \\ \rho e_t \\ \rho_2 \end{pmatrix}, \qquad F(\mathbf{u}) = \begin{pmatrix} \rho u \\ \rho u^2 + p \\ \rho u h_t \\ \rho_2 u \end{pmatrix}, \qquad S(\mathbf{u}) = \begin{pmatrix} 0 \\ 0 \\ 0 \\ -K(T)\rho_2 \end{pmatrix}$$

$$p = \rho(\gamma - 1)\left(e_t - \frac{1}{2}u^2 - q_0 f_2\right), \quad T = \frac{p}{\rho R}, \quad h_t = e_t + \frac{p}{\rho}, \quad K(T) = K_0 \exp\left(\frac{-T_{ign}}{T}\right)$$

$$\rho_2 = \rho f_2, \quad f_1 + f_2 = 1$$

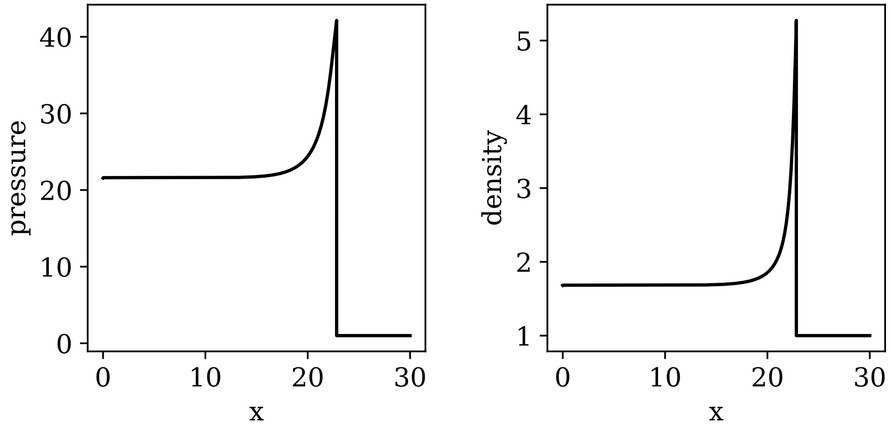
$$\mathbf{u}_t^* + F(\mathbf{u})_x = 0$$

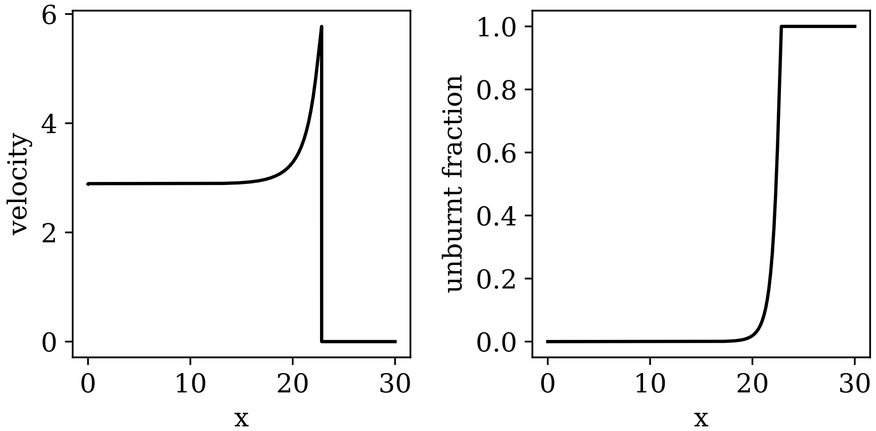
$$\mathbf{u}_t = S(\mathbf{u}^*)$$

CJ detonation wave — ZND solution

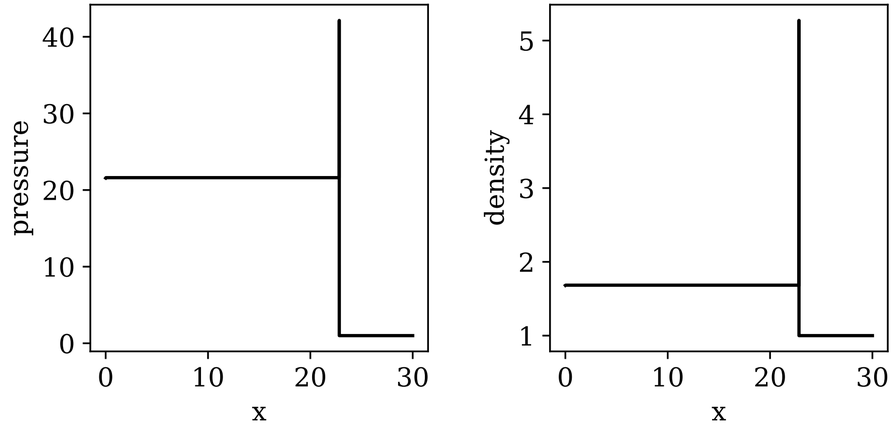
Zel’dovich, von Neumann, Döring

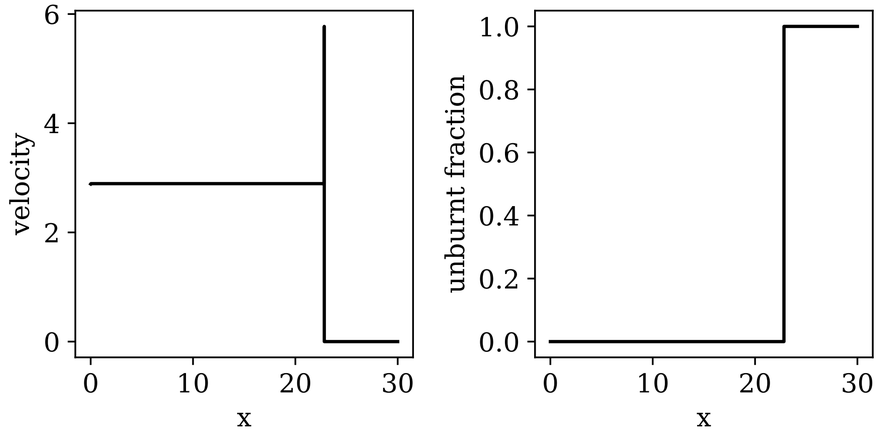
$K_0 = 32$



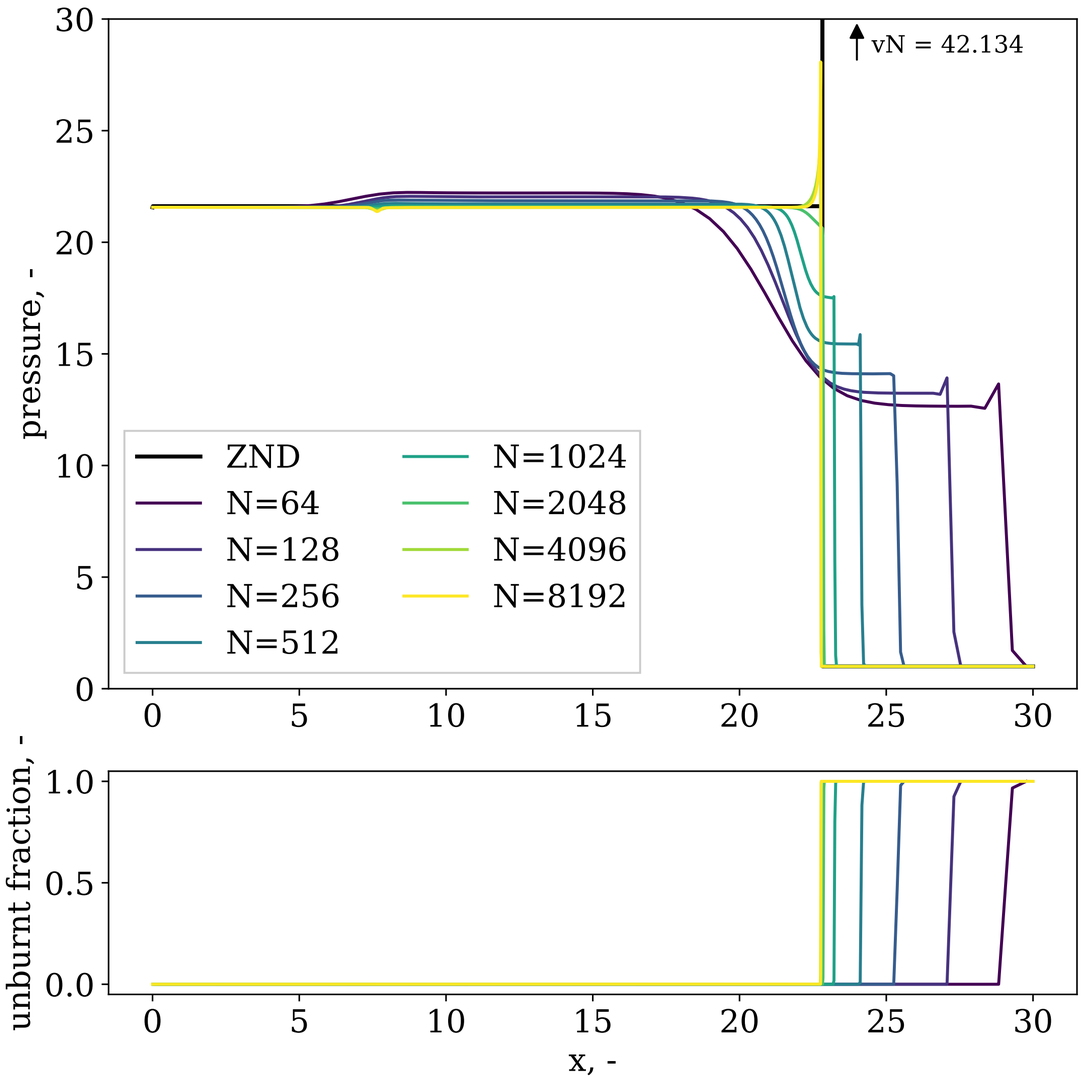


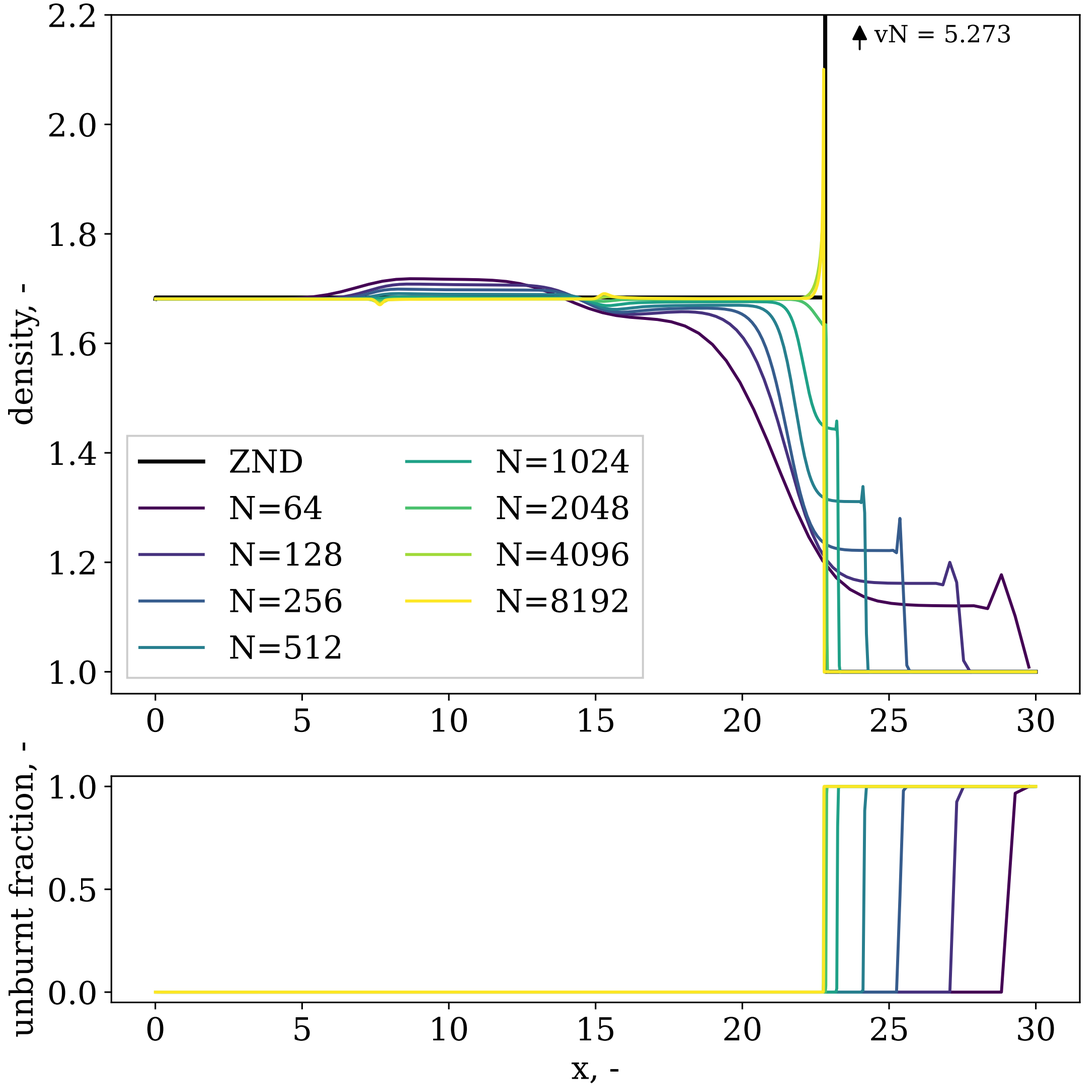
$K_0 = 16418$



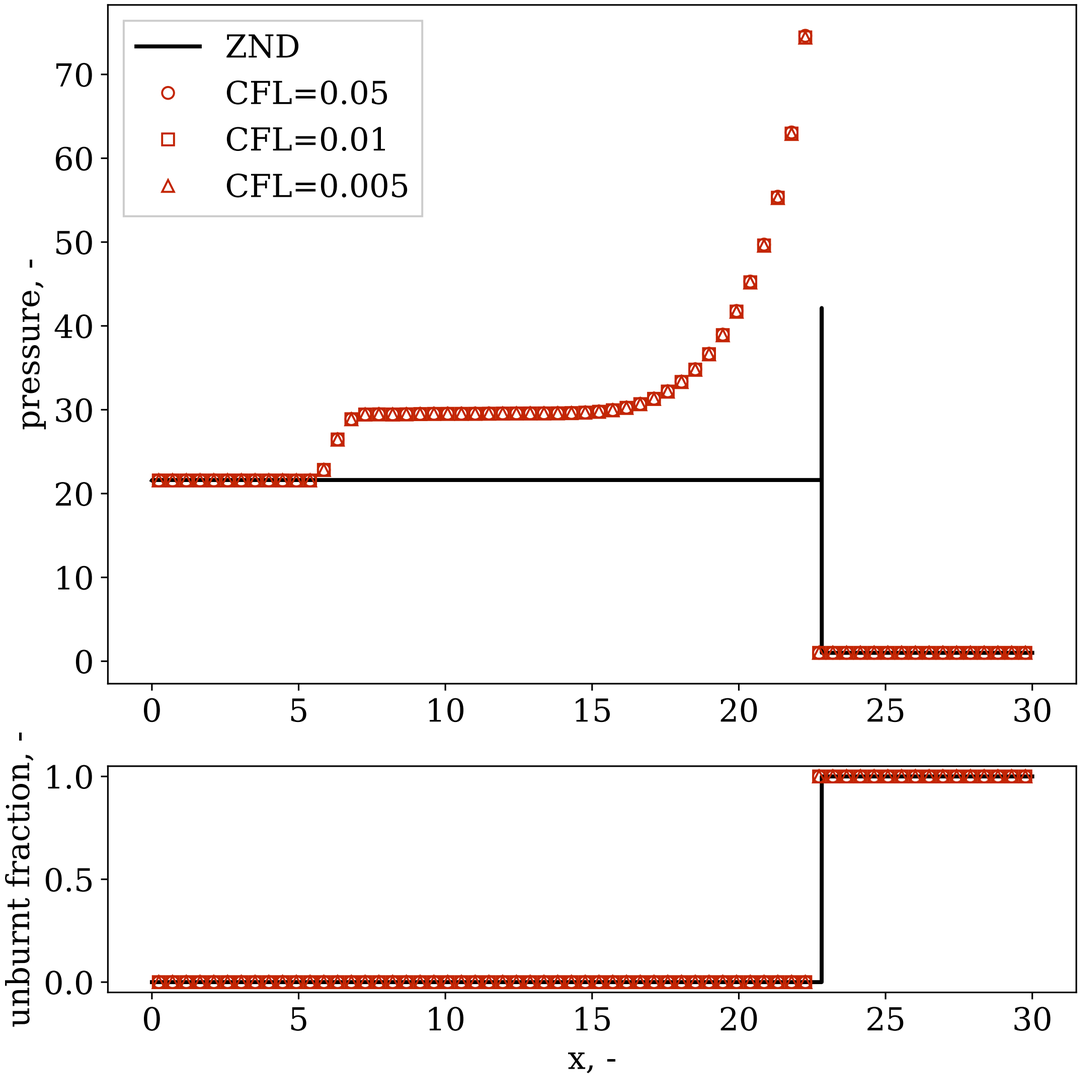


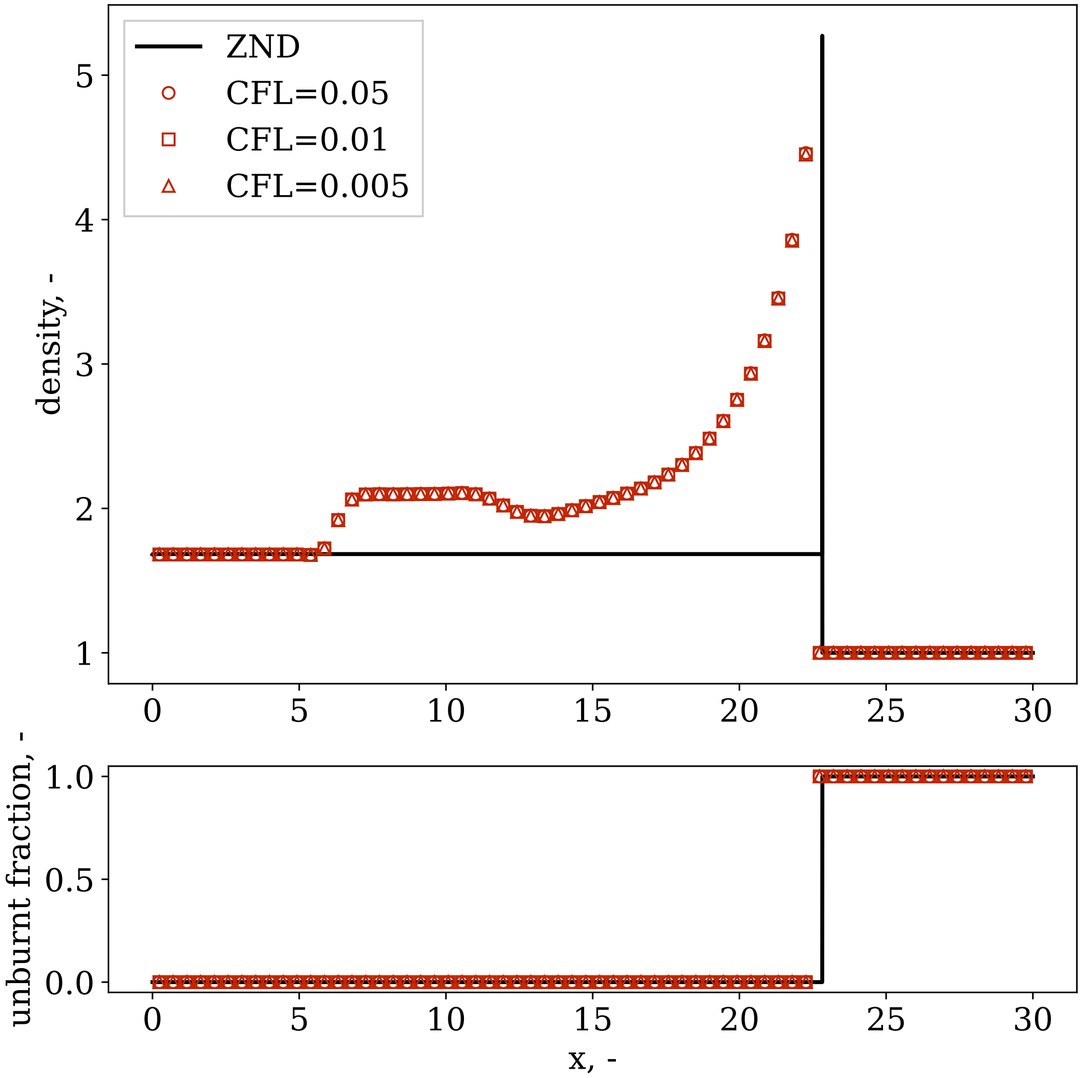
CJ detonation wave — shock capturing



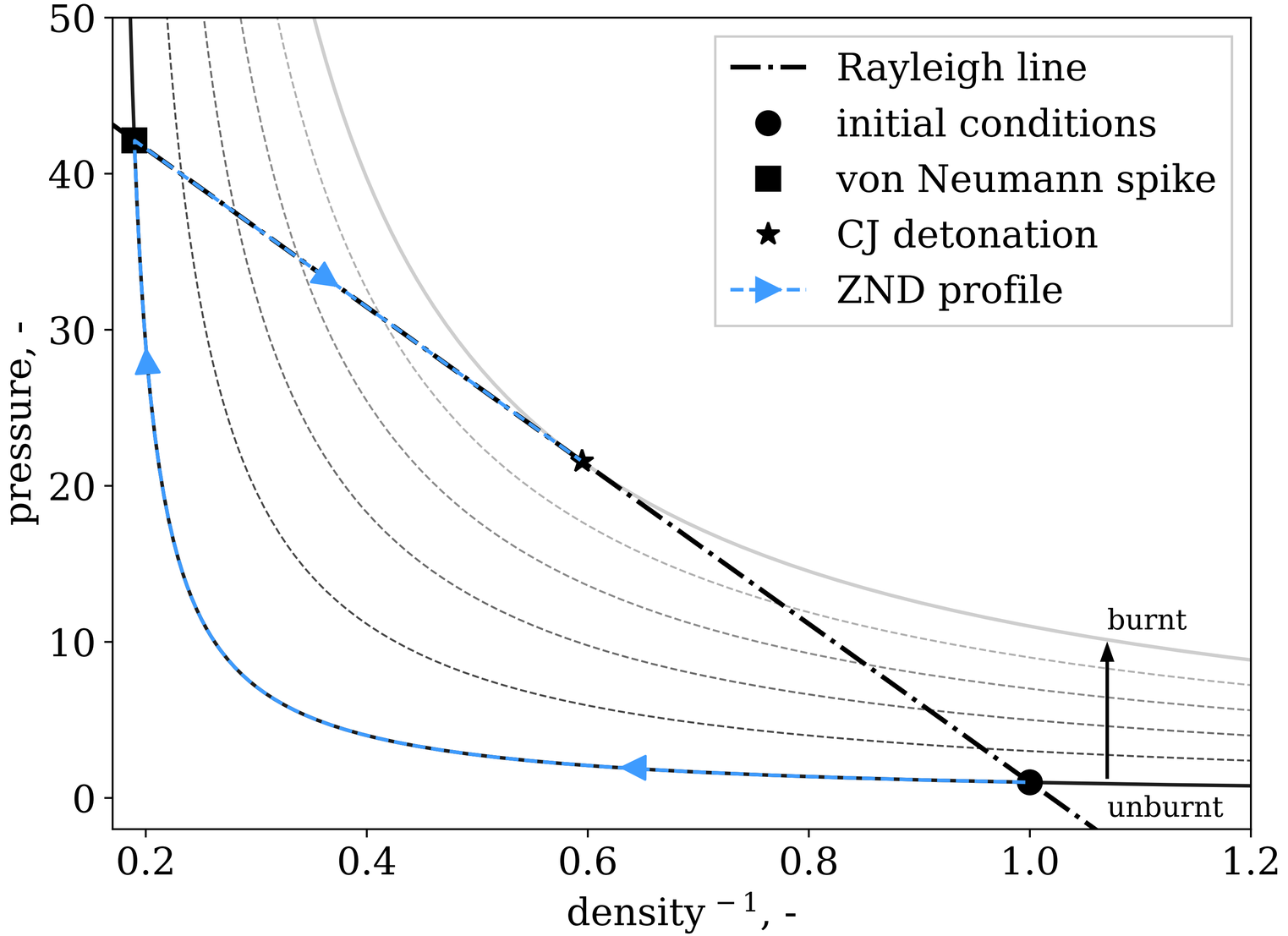


CJ detonation wave — shock tracking

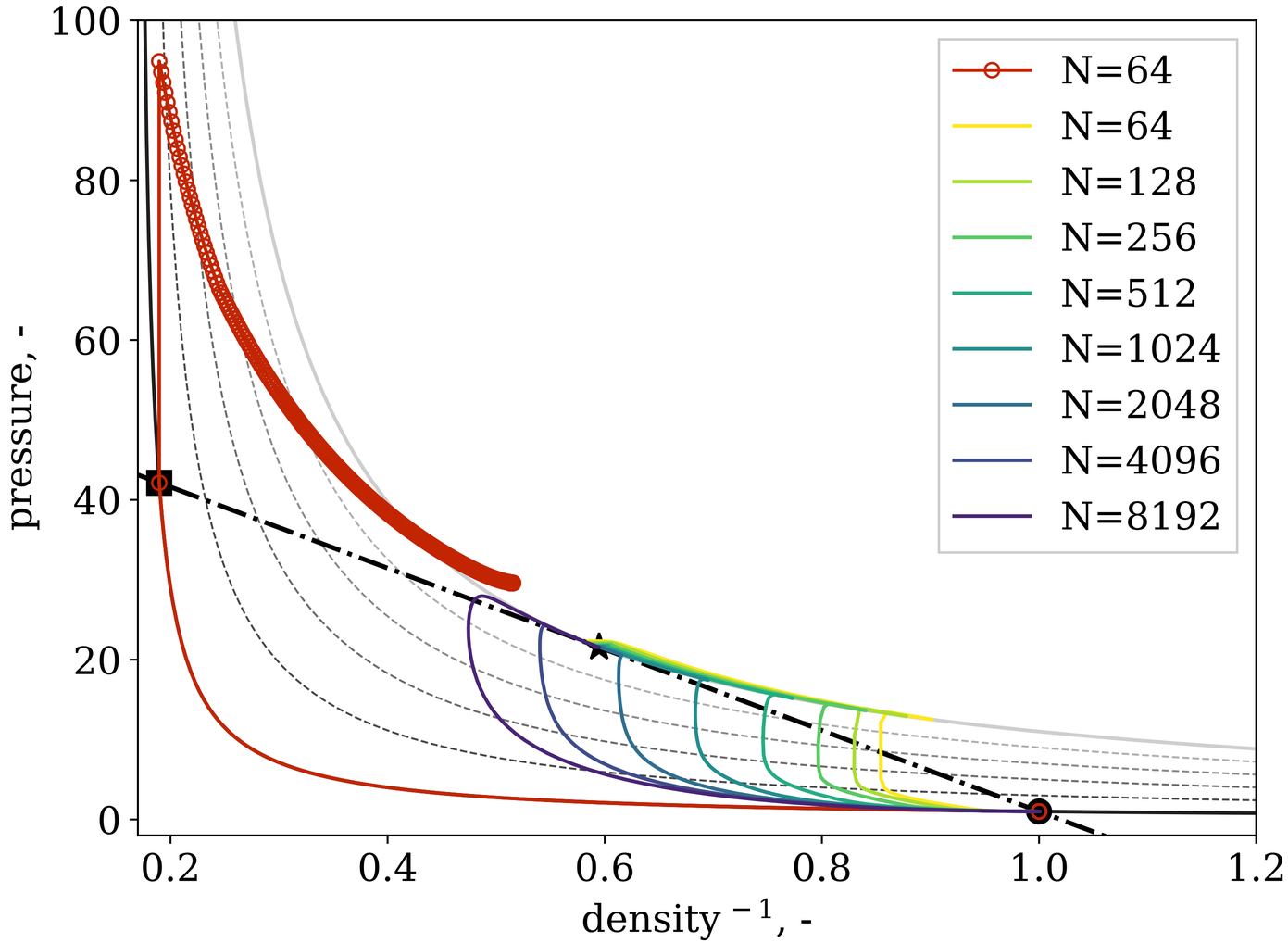


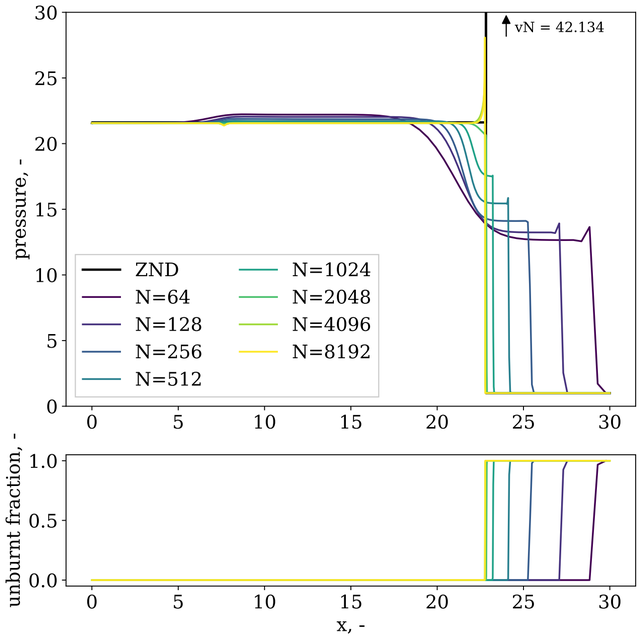


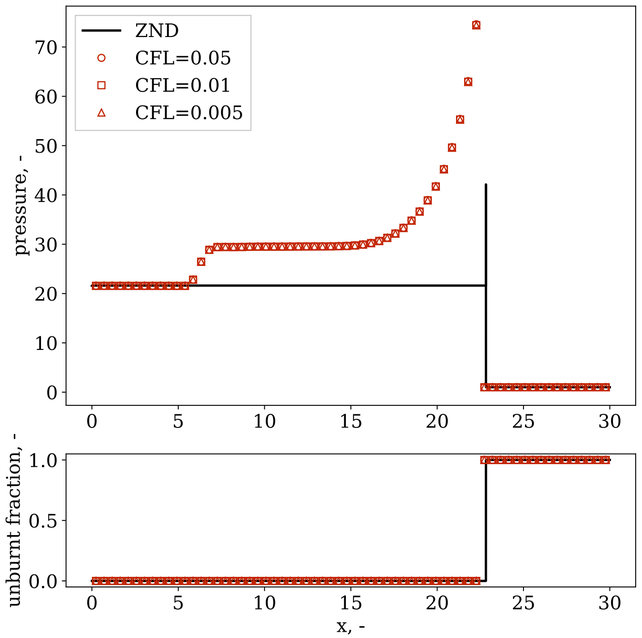
P-v plots


Rayleigh line
initial conditions
von Neumann spike
CJ detonation
ZND profile
burnt
unburnt
pressure, -
density $^{-1}$, -
0
10
20
30
40
50
0.2
0.4
0.6
0.8
1.0
1.2

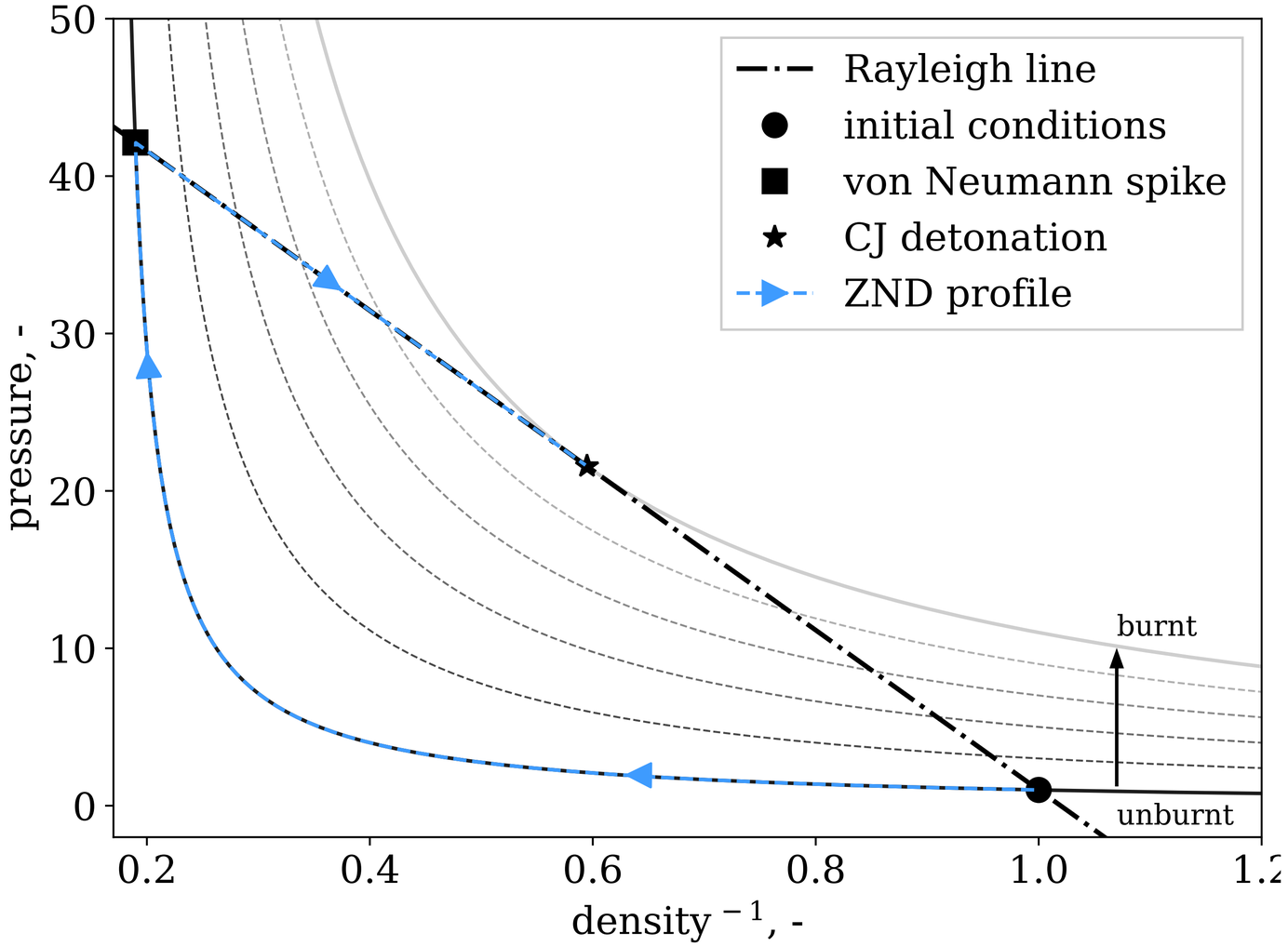
P-v plots

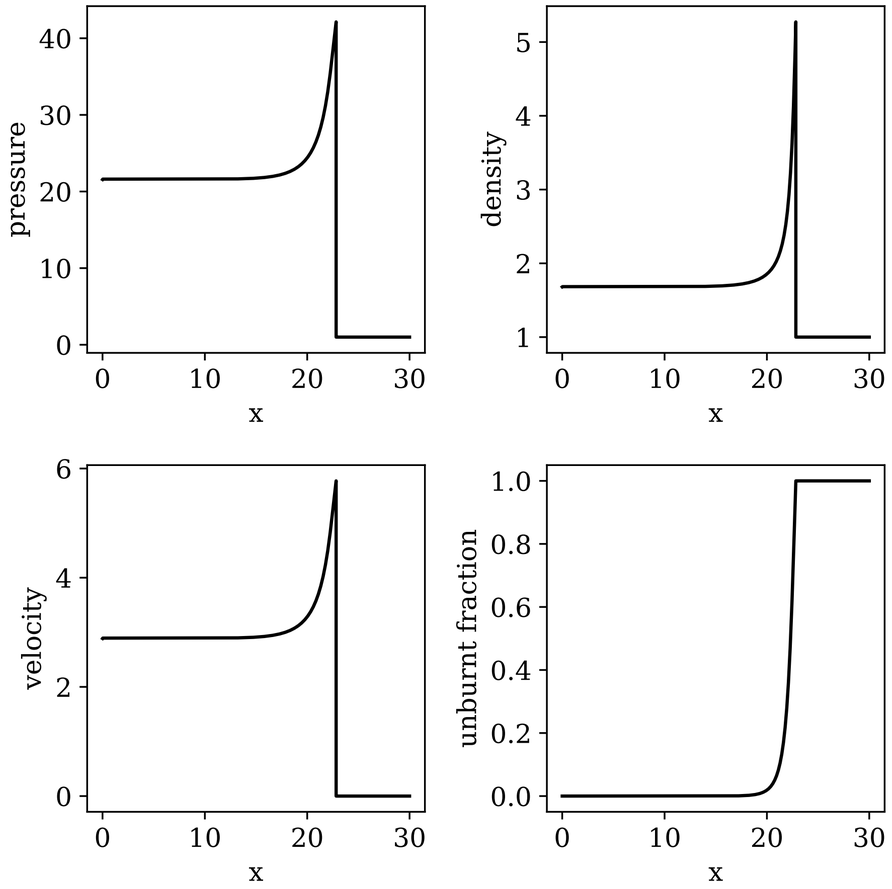

pressure, -
density $^{-1}$, -
N=64
N=64
N=128
N=256
N=512
N=1024
N=2048
N=4096
N=8192


vN = 42.134
pressure, -
ZND
N=64
N=128
N=256
N=512
N=1024
N=2048
N=4096
N=8192
unburnt fraction, -
x, -

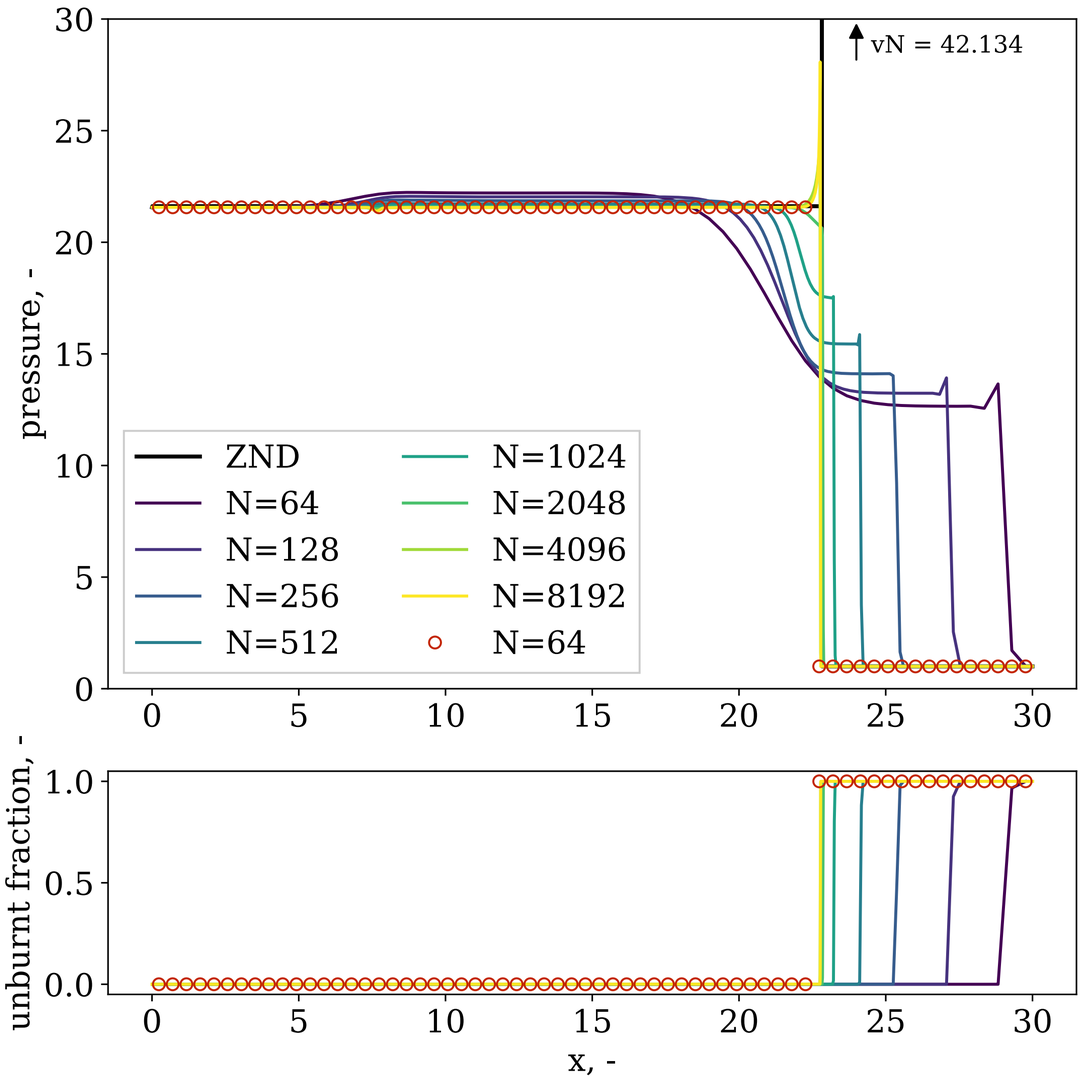

ZND
CFL=0.05
CFL=0.01
CFL=0.005
pressure, -
unburnt fraction, -
x, -

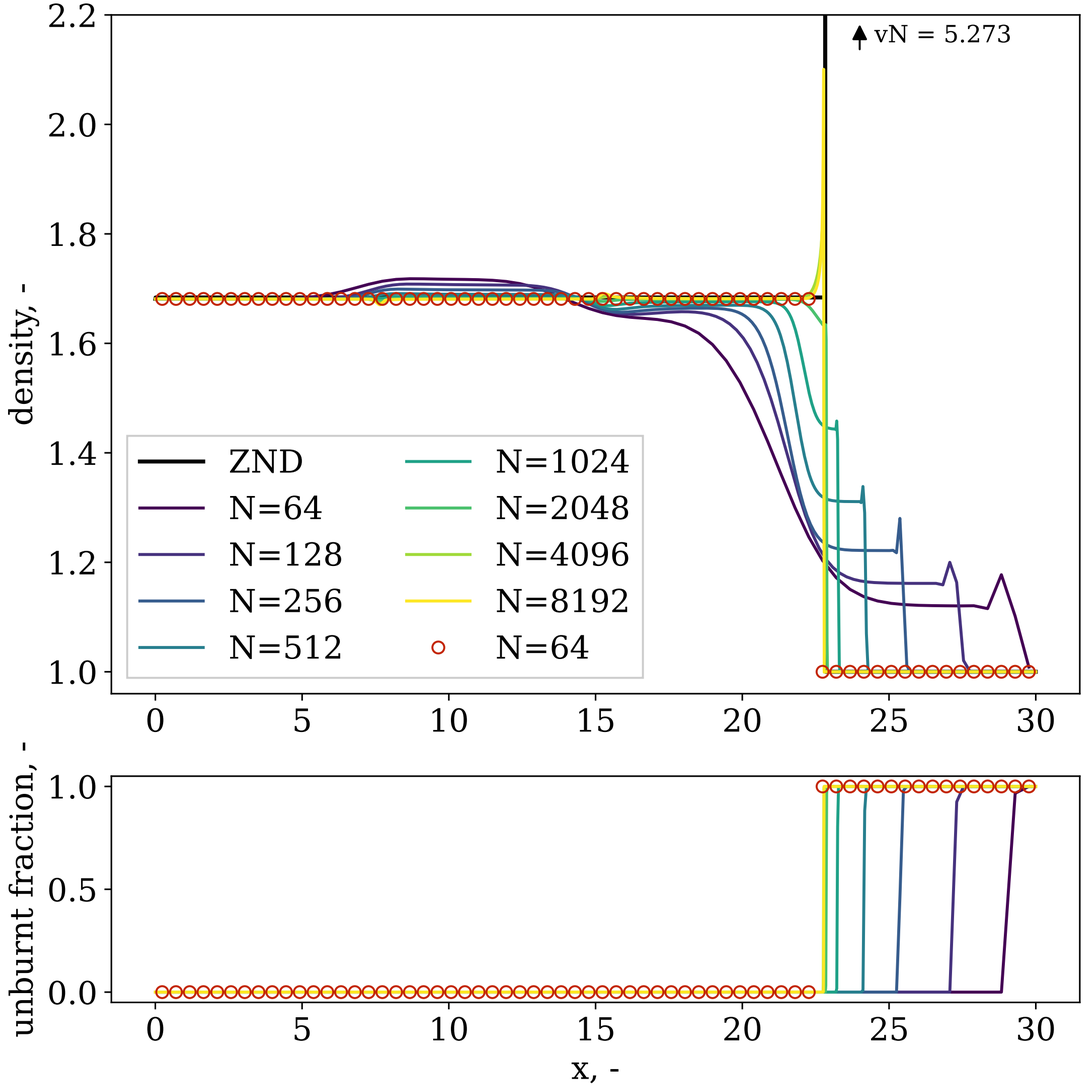
ZND gas model look-up table





CJ detonation wave — shock tracking





Concluding remarks

- Gnoffo's shock-tracking approach modified to reduce computational costs and improve accuracy
- Shock-tracking demonstrated on detonation wave problem
  - Modified gas model required for this shock-tracking approach on coarse domains

- Method has been extended to 2-d with basic shock-tracking