

Seminar on b -value

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INTRODUCTION

After the crucial contribution made by C. Richter in 1935, concerning the concept of earthquake magnitude, it has been revealed that earthquakes are not uniformly distributed in time, space and magnitude. Distribution of earthquakes with respect to magnitudes exhibits **scale invariability**, appears to be **self-similar** and obeys a **power law** or **fractal scaling**. This implies the absence of a characteristic event size (theoretical limits on the maximum earthquake size). An empirical formula

$$\log N = a - bM \tag{1}$$

$$N = 10^{a - bM}$$

known in the east as Ishimoto and Iida (1939) relation and in the west as the Gutenberg and Richter (1942) relation defines the distribution of earthquakes with respect to the magnitude. For a certain region and time interval, eq.(1), provides the number of earthquakes, N , with magnitude, M , where a and b are positive, real constants. a describes the seismic activity (log number of events with $M=0$). It is determined by the event rate and for certain region depends upon the volume and time window considered. b , which is typically close to 1, is a tectonic parameter describing the relative abundance of large to smaller shocks. It seems to represent properties of the seismic medium in some respect, like stress and/or material conditions in the focal region. Relation (1) is usually referred to as the Gutenberg-Richter (G-R) **magnitude-frequency relationship** (MFR).

Other geophysical quantities show self-similarity as well. For instance, self-similarity is apparent in frequency of occurrence of fault as a function of length on different scales. Faults look much the same over scales from centimeters to tens or even hundreds of kilometers (about 8 decades). Also the areas of faults follow the power law. Japanese studies reveal that also displacements are self-similar. River drainage systems may serve as another example. Power law can be visualized even outside physics, e.g. in the vascular system of cat brain surface (Matsuo et al., 1990)..

The main objective of the present note is to discuss the essential characteristics of b -values, evaluation techniques and their applicability in earthquake prediction. Two case studies are added to demonstrate the practical approach.

Space and temporal variations of the b -value have earlier been employed in numerous seismicity studies. After the pioneering works of Mogi (1962), Scholz (1968) and Wyss (1973), they have been extensively used (with varying degree of success) by other workers

e.g. to identify volumes of active magma bodies (Wiemer and Benoit, 1996; Wiemer et al., 1998), roots of regional volcanism (Monterroso and Kulhanek, 2003) and to forecast major tectonic earthquakes (Monterroso, 2003; Nuannin et al., 2005). *b*-value studies related to induced seismicity have been carried out during the last, say, 25 years with results published in numerous scientific papers.

A comprehensive summary of achievements can be found e.g. in Gibowicz and Lasocki (2001).

DEVIATIONS FROM LINEARITY AND VARIATIONS

Equation (1) represents a linear relation between $\log N$ and M . Although observational data are generally well described through eq.(1), the linear relation holds only for magnitudes in a certain range $M_1 \leq M \leq M_2$. For small and large ($M \sim 7.3$ and larger) magnitudes, the frequency decreases more rapidly than linearly and consequently a non-linear fit may in some cases be a better approximation of observed data. There are at least two explanations for the deviations from linearity

- **At small magnitudes**, it is the incompleteness of data (catalogs). However, recent studies suggest that the decrease of *b* (below the threshold magnitude) is not just an artifact of catalog incompleteness but that small earthquakes are really not as numerous as a constant *b*-value extrapolated from larger events would predict and so the decline in frequency may to a certain extent be real.
- **At large magnitudes**, it is the saturation of magnitude scales, in other words a problem associated with the way magnitudes are measured. Another reason is the length of available catalogs (they are often too short) with missing rarer large earthquakes. Hence, calculated magnitudes cannot cope with the most energetic earthquakes and catalogs often cover too short a period to yield an accurate count of larger events. In general, recurrence times beyond the time span of data (catalog) should be treated with caution.

To address this problem, we can make use of the seismic moment and/or moment magnitude, which better (when compared with body-wave or surface-wave magnitude) indicate the size of large earthquakes. Using the moment-magnitude definition (Hanks and Kanamori, 1979)

$$M_w = (2\log M_0)/3 - 10.73 \quad (2)$$

where M_0 is moment scalar in dyn-cm, in eq.(1) yields

$$\log N = a - b[(2\log M_0)/3 - 10.73] = \alpha - \beta \log M_0 \quad (3)$$

where $\beta = b/1.5 \sim 2/3$.

This new linear relation, with slope β has a general validity. A closer look at the moment-frequency distribution reveals again a deviation of data at small moments (as expected) but also at large moments which is rather puzzling since moments do not saturate. For moments approximately above 10^{27} dyn-cm ($M \sim 7.3$), the data are more consistent with $\beta \geq 1$ than $\beta = 2/3$. In other words, there are fewer earthquakes than expected for a given (large) moment.

Changes in b or β towards higher magnitudes may result from the inability of fault dimensions, notably W , to keep growing indefinitely with the increasing earthquake size. A significant increase in b is observed around $M=7.3$ or around $M_0=10^{27}$ dyne-cm (Pacheco et al., 1992) is interpreted as expressing the simultaneous saturation of W and the slip, D , on the fault. The latter is another important parameter in this context. When we compare the rate at which earthquakes occur with that predicted by plate rates it has been declared to be too small. The implication that much motion does not take place on the earthquake faults seems irresistible. Perhaps the missing deformation occurs on creeping faults as continuous deformation.

In spite of some opposition from e.g. Kagan, (1999) who advocates that b is essentially constant, others (e.g. Wiemer and Benoit, 1996; Ayele and Kulhanek, 1997; Wiemer et al., 1998; Gerstenberger et al. 2001) supported by numerous observations argue that significant spatial and temporal variations in b exist. It is true that over long time and large areas $b\sim 1$, nevertheless significant variations are observed in limited areas and over short time intervals. Fluctuations with duration of years are known for large earthquakes. Near or local events are often associated with b -variations of duration of days or even hours (Grunthal et al., 1982). Some examples of the observed scatter are

- 1.0 $\leq b \leq$ 1.6 Mogi, global seismicity, $b\sim 1.0$ for $\text{lat.} \geq 40^\circ$, whereas $b\sim 1.6$ for $\text{lat.} \leq 40^\circ$
- 0.3 $\leq b \leq$ 1.8 Hurtig and Stiller (1984), global seismicity
- 0.6 $\leq b \leq$ 1.5 Udias and Mezcua (1997), global seismicity
- 0.8 $\leq b \leq$ 1.2 McNally (1989), global seismicity
- 0.5 $\leq b \leq$ 1.5 McGarr (1984), mining tremors (South Africa) and tectonic earthquakes
- 0.6 $\leq b \leq$ 1.6 Monterroso and Kulhanek (2003), Central America seismicity
- 0.6 $\leq b \leq$ 2.6 Nuannin et al.(2002), mining tremors, Zinkgruvan, Sweden

There are several plausible explanations for observed variations in b -values such as

- **High and low stresses** cause earthquake series with low and high b -values (Scholz, 1968; Wyss, 1973). This observation is employed to study stress levels and structural anomalies in the crust and/or upper mantle (subduction) e.g. in earthquake prediction and in identifying volumes of active magma bodies (Wiemer and Benoit 1996; Wiemer et al., 1998) or roots of regional volcanism (Monterroso and Kulhanek, 2003).
- **Material heterogeneity** (Mogi, 1962). Large heterogeneities correspond to higher b -values.
- Laboratory tests showed (Warren and Latham, 1970) that an increase of **thermal gradients** caused an increase of b from 1.2 to 2.7.
- **Aftershocks** have large b -value, **foreshocks**, on the other hand, show low b (e.g. Suyehiro et al., 1964). For example, $b=0.6$ has been found in the 1975 Haicheng earthquake foreshock series, while the aftershocks give $b=0.9$.

- **Swarms** show often large departure from $b \sim 1$. Sometimes b is as large as 2.5, i.e. in swarms no (fewer) large earthquakes accompany the occurrence of small magnitude events. Swarms are often associated with volcanic activity. In volcanic regions faults are simply not large (to generate large earthquakes) or continuous stresses are substantially heterogeneous. Swarms, by definition, lack a clear main shock and result from processes such as migration of magmatic fluids or caldera development.
- It has been observed that b varies **laterally and with depth**. b -values much smaller than 1 were identified. Low b implies shorter recurrence time. Patches with low b may be interpreted as possible asperities (stress concentrations) reflecting variations in frictional properties along the fault, which may control the recurrence of the next large event.
- **Paleoseismic studies** deviate sometimes from the seismologically determined G-R relation. But the opposite is also observed, i.e. instrumental data show that large earthquakes ($M \geq 7.2$) are less frequent than expected from smaller events (cf saturation effect).
- With **small time-sampling**, b -value is reasonable well estimated from smaller earthquakes, but not for the large ones.
- Current research reveals that thrust-fault event are associated with lower b -values when compared with normal-fault events, i.e. b is likely to be also dependent on the **focal mechanism**.

In this presentation, we accept the explanation of Scholz (1968) and Wyss (1973), i.e. we expect an inverse correlation between the magnitude of observed b and the level of stress accumulated in and around the source volume. It should be also emphasized that the FMD (2) defines the distribution of magnitudes only. Thus, it may serve as a one-parameter approach to forecast “large” events. For practical predictions, it is of course recommended, whenever possible, to supplement the b -value technique with observations of other independent parameters.

CUMULATIVE AND INCREMENTAL DISTRIBUTIONS

G-R relation (1) applies to cumulative number N as well as to incremental numbers n . In other words, N is the cumulative number of earthquakes with magnitude larger than M , while n is the number of events with magnitudes in the range $M \pm \Delta M$ (incremental or interval distribution). A choice of proper ΔM is a crucial task in any incremental b evaluation. Reported magnitudes are not continuous quantities, as assumed in eq.(1), but discrete quantities determined with accuracy of 0.1 magnitude unit in most of global catalogs. Thus, a proper choice of ΔM will be a compromise between the magnitude sampling as close to 0.1 as possible and statistically large numbers of events in each magnitude group.

We prefer to work with the cumulative distribution, it provides better linear fit since numbers are larger and less degraded by statistics of small numbers. It also circumvents the problem of designing a proper ΔM . Both distributions are used to describe earthquake distributions globally and/or in limited regions large (regions, countries) and small (population centers, mines, water reservoirs). Once a and b in relation (1) are determined for a given region and

time window, then one already has the information necessary to assess parameters of seismic hazard (maximum expected magnitude, return period). However, there is a danger of pushing the relation too far. It is not possible to extrapolate, without limits, the frequency of the smaller events to determine the threat of the rare large earthquakes.

PROBLEMS

1. Assume $\log N = a - bM$ has been determined from observations over 1 year and as expected $b \sim 1$. Then a gives the maximum expected magnitude in 1 year interval.

The largest (i.e. 1 earthquake) will have the magnitude

$$\begin{aligned} \log 1 &= a - M \\ 0 &= a - M \\ a &= M \quad \text{gives the max. expected earthquake magnitude in one year.} \end{aligned}$$

If b differs from 1, then

$$\begin{aligned} \log 1 &= a - bM \\ 0 &= a - bM \quad \text{and} \quad M_{max} = a/b \end{aligned}$$

2. Assume the relation $\log N = a - bM$, with $b \sim 1$. From data of one year we obtain

$$\log N = 3.5 - M \quad \text{and for earthquakes of magnitude 3.5 we have}$$

$$\log N = 3.5 - 3.5 = 0 \quad \text{i.e. } N = 1 \quad \text{or in words one event with } M=3.5 \text{ per year.}$$

If we now consider a magnitude 4.5 then

$$\log N = 3.5 - 4.5 = -1 \quad \text{and } N=0.1, \text{ which means one event (} M=4.5 \text{) in 10 years.}$$

3. From a gold mine in South Africa, $\log N = 2.19 - 0.63M$ from 100 days of observations.

We obtain

M	N
2	8.5
2.5	4.1
3	2

If events of $M \geq 2.5$ are taken as minimum size tremors that might be expected to cause damage, then the above table indicates that about 6 (4.1+2) instances of disruption of the mining operation may occur per 100 days.

This and similar procedure can obviously not be extrapolated upward indefinitely to high magnitudes. For example, $a=2.19$ and $b=0.63$ would imply an event of $M=6$ once every 11 years. In fact at this mine events with M significantly in excess of 4 do not seem to occur in

spite of the fact that the mine has been in operation for many times the return period for an event of $M=6$.

PRACTICAL b -VALUE EVALUATION

An essential requirement for a reliable b determination is the availability of high-quality data set. It is advised that several and not only one catalog is employed whenever possible.

DATA

Requirements of an acceptable catalog include

- **Uniform magnitudes** must be available. For global/regional catalogs, we prefer to use the surface-wave magnitude, M_s . If data include also intermediate- and deep-focus events, body-wave magnitude, m_b , has to be used. When processing catalogs with large events ($M \geq 7$), we prefer to work with moment magnitudes, M_w , to circumvent the saturation effects. In local catalogs (mining areas water reservoirs, construction sites) a local magnitude, M_L , will be used. It is important that reported magnitudes are homogeneous throughout the whole catalog.
- **Time span** of the catalog must be at least comparable and possibly larger than the return period of the largest expected event.
- Depending upon the type of the study, it may be desirable to generate **sub-catalogs** corresponding to geographic sub-regions with different specific (e.g. tectonic) characteristics. Each sub-catalog is treated separately.
- To guarantee the **completeness** of data, analysis will comprise only events with magnitudes equal to or larger than the threshold magnitude, M_c . Magnitude M_c can be determined from G-R diagrams by eye or more objectively by taking the derivative of the G-R diagrams.
- To avoid dependent data, the catalog has to be **de-clustered** by deleting all foreshocks and aftershocks. Obviously, this step is usually relevant merely for global/regional catalogs comprising large events with long aftershock/foreshock series.
- It is always desirable that the catalog covers a **large span of magnitudes**. A span of three or more magnitude units is usually required.
- Prior to the analysis, cumulative numbers of events as a function of time are employed to unveil more drastic changes of reporting rates. **Consistent observatory operations** during the period of review are prerequisite.
- In some special studies it is needed to examine the **temporal data stability** or **stationarity** (usually when studying space variations of b). Original data are separated into several sub-sets for different time periods and evaluated separately.

TECHNIQUES

There are at least three techniques, currently utilized in b -value determinations after all event with magnitudes less than M_c have been deleted

- linear fit by eye
- linear least-squares fit
- maximum-likelihood estimation (Utsu, 1965)

$$b = \log_{10} e / (M_{av} - M_{min}) = 0.43 / (M_{av} - M_{min}) \quad (4)$$

where M_{av} is the mean of the observed magnitudes and M_{min} is the minimum or threshold magnitude in the group of events for complete reporting. The standard error of the maximum-likelihood estimation of b is approximately b/\sqrt{N} , for large N (the number of earthquakes with $M \geq M_{min}$). b follows the χ^2 statistical distribution and the statistical significance of the difference in the b -value, for two different earthquake groups, can be tested by the F-distribution (Utsu, 1966). 95% confidence limits are $\pm 1.96b/\sqrt{N}$. Relation (4) is defined for continuous exponential distribution and care must be taken when using discrete magnitude values.

For complete data, $M \geq M_c$, both spatial and temporal variations in b are examined by making use of sliding space- and time-windows, respectively. In both cases we have the option to employ

- constant-size windows
- or
- constant-number of events in the window

The former approach implies a varying number of events for each b calculation and consequently each b is determined with different statistical significance. Selection of a proper window length will depend on data available and may complicate the analysis. The latter technique implies different window size as the window moves over the grid in which b -values are to be determined. This in its turn has a negative impact on the time scale (for temporal variation examinations) which now becomes non-linear. Constant-number of events technique may suffer from a drawback generated by time intervals with low level seismicity (e.g. due to a pause in mining operations). If this is the case, long time windows will be applied resulting in window lengths much larger than grid-elements and an undesired strong smoothing effect will take place. The final window size (usually determined empirically) will be a reasonable compromise between required resolution and smoothing between grid nodes.

Selection of proper ΔM , for incremental distributions, is governed by scarcity of data. Generally speaking, ΔM should be small to approximate well continuous magnitudes in eq. (1), but at the same time each magnitude group should comprise large numbers of data. These are obviously two opposing requirements and a proper compromise must be found.

When studying spatial and/or temporal variations (anomalies) in b -values, results must be stable (robust) and not dependent on personal choice of input parameters. It is advised to carry out tests with different catalogs, catalog time spans, window lengths, threshold magnitudes, magnitude sampling, etc.

CASE STUDIES

- Inverse correlation between induced seismicity and b -value, observed in the Zinkgruvan mine, Sweden (see the attached reprint).
- Spatial and temporal b value anomalies preceding the devastating off coast of NW Sumatra earthquake of December 26, 2004 (see the attached reprint).

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